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A FUZZY INFERENCE SYSTEM APPLIED TO VALUE OF INFORMATION ASSESSMENT FOR OIL AND GAS INDUSTRY

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Abstract: *Value of information is a widely accepted methodology for evaluating the need to acquire new data in the oil and gas industry. In the conventional approach to estimating the value of information, the outcomes of a project assessment relate to the decision reached following Boolean logic. However, human thinking logic is more complex and include the ability to process uncertainty. In addition, in value of information assessment, it is often desirable to make decisions based on multiple economic criteria, which, independently evaluated, may suggest opposite decisions. Artificial intelligence has been used successfully in several areas of knowledge, increasing and enhancing analytical capabilities. This paper aims to enrich the value of information methodology by integrating fuzzy logic into the decision-making process; this integration makes it possible to develop a human thinking assessment and coherently combine several economic criteria. To the authors' knowledge, this is the first use of a fuzzy inference system in the domain of value of information. The methodology is successfully applied to a case study of an oil and gas subsurface assessment where the results of the standard and fuzzy methodologies are compared, leading to a more robust and complete evaluation.*

Key words: *Value of information, fuzzy logic, fuzzy inference system, oil and gas industry, uncertainty.*

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1. Introduction

1.1. Review of Value of information

Value of information (VoI) is a prescriptive methodology embedded in the discipline of decision analysis that has the aim of assessing the value assocted with gathering information. To that end, the methodology maximizes an objective function, which defines the value of a project.

Grayson (1960), Raiffa and Schlaifer (1961) and Newendorp (1967) were the pioneers in the field of decision making for data acquisition in the oil and gas industry. Subsequently, more research and applications, such as those of Warren (1983), Lohrenz (1988), Demirmen (1996), Newendorp and Schuyler (2000) and Koninx (2000), among others, expanded the scope of the subject, adding more robustness to the methodology.

Recently, more applications have emerged—like those of Clemen (1996), Coopersmith and Cunningham (2002), Suslick and Schiozer (2004)—which enrich the process of assessing the VoI decision problem from a methodological perspective. Several papers, such as Walls (2005) and Vilela et al (2017), have discussed the use of utility theory in VoI assessment in the oil and gas industry; similarly, Santos and Schiozer (2017) discussed the impact of the risk attitude of the decision makers in VoI assessments; Kullawan et al (2017) developed a discretized- -programming approach, based on value of information, to optimize stochastic-dynamic the geosteering operations; Steineder et al (2018) discussed the maximization of the VoI on a horizontal polymer pilot project; all these researchers used one or more crisp decision criteria to make decisions.

In the oil and gas industry, the scope of a project varies from the complex exploitation of hydrocarbon fields to theoretical reservoir studies or laboratory tests; project's economic benefits are calculated based on the estimated figures of hydrocarbons' production and price, operating cost, taxes, royalties, and investments. All these figures carry uncertainties because it is not possible to predict their future fluctuations accurately—in particular, future hydrocarbon production is uncertain due to a combination of:

(a) the uncertainties associated with the reservoir parameters (permeability, thickness, top reservoir, well producibility, aquifer support, etc.);

(b) the uncertainties associated with the methods used to estimate future production based on the reservoir parameters (dynamic reservoir models, decline curve analysis, etc.)

On occasion, additional data can be acquired to change the uncertainty in the reservoir parameters; however, acquiring data involves a cost that could be greater than the benefits of the data. Changes in the reservoir parameters' uncertainties translate into changes in the value of the project. In general, acquiring additional data makes sense in cases in which the outcome from the data acquisition can change the decisions being made.

For a project with uncertain outcomes, the VoI is the difference between the expected value (EV) of the project with and without the newly acquired data (Clemen, 1996):

$$VOI = EV_{with\ information} - EV_{without\ information} \tag{1}$$

where both values, $EV_{with\ information}$ and $EV_{without\ information}$, assess the outcome of the project in these circumstances and refer to a future situation.

In the 'without-information' case, for κ possible scenarios (which include endorsing the project with the current knowledge and uncertainties and the alternative of relinquishing it), the EV of the project corresponding to the j^{th} scenario is defined as:

$$V\left(a_j\right)=\sum_{i=1}^{n} u_{ji}\, p\left(s_i\right) \tag{2}$$

where u_{ji} is the value of the state of nature s_i for the scenario a_j and $p(s_i)$ is the prior probability of the state of nature s_i. The most often used decision criterion is to select the alternative that maximizes the EV:

$$EV\left(a^*\right)=\max_j EV\left(a_j\right) \tag{3}$$

Equation (3) is the $EV_{without\ information}$ term shown in Equation (1).

Similarly, in the '*with-information'* alternative, for κ possible scenarios and for each possible data outcome, x_k, the EV for the j^{th} alternative is:

$$EV\left(a_j|x_k\right)=\sum_{i=1}^{n} u_{ji}\, p(s_i \mid x_k) \tag{4}$$

where u_{ji} is the value of the state of nature s_i for the scenario a_j; $p(s_i|x_k)$ is the posterior probability of the state s_i given the outcome x_k; and the term $EV(a_j|x_k)$ is the expected project value for the j^{th} alternative given the outcome x_k.

Similarly, as in the '*without-information'* case, the optimum alternative in the '*with-information'* case for a given data outcome x_k (EV conditioned on the outcome x_k) is the one that maximizes the EV:

$$EV\left(a^*|x_k\right)=\max_j EV(a_j \mid x_k) \tag{5}$$

The unconditional maximum EV (which is the EV of the project considering the data acquisition outcomes) is the sum of the conditional EV weighted with the corresponding marginal probabilities:

$$EV\left(a^*\right)=\sum_{k=1}^{n} EV(a^* \mid x_k)\, p(x_k) \tag{6}$$

The VoI is the difference between the estimates of EV in Equation (6) and Equation (3).

So far, the discussion has focused on the classical methodology to assess the VoI in a decision problem in which the output values (hydrocarbon production, total benefits, etc.) are uncertain due to uncertainties in the input variables; these uncertainties have been included using probabilistic measures. In the next section, we will include the imprecision in the input variables by making use of fuzzy logic.

1.2. Review Fuzzy Logic

Fuzzy logic, pioneered by Zadeh (1965), is one of the most prolific areas of artificial intelligence, which has enriched the analysis of challenging and complex problems.

Bellman and Zadeh (1970) introduced an important distinction between randomness and fuzziness: while randomness relates the uncertainty concerning membership or non-membership of an object or event to a non-fuzzy set (a crisp set), fuzziness deals with classes in which there may be degrees of membership (between the full- and the no-membership relationship).

These distinct sources of uncertainties are managed during different phases of the VoI assessment:

Phase 1) Assessing: assesses, using one or more decision criteria, whether the new data add value to the project or not.

Phase 2) Categorization: relates the values obtained during the assessing phase to the options available for the decision problem.

During the Phase 1, the uncertain nature of the input variables (e.g. reservoir parameters) and outcome values (e.g. financial benefits or economic parameter values) is captured using probabilistic analysis based on EV calculations.

In the standard VoI approach, Phase 2 is implemented using crisp criteria to make decisions that do not correspond with human fuzzy thinking for making decisions. Following Bellman and Zadeh (1970), the uncertainty related to fuzziness is a major source of uncertainty in many decision processes.

In classical set theory, the events (values) belong (or not) to sets in a crisp manner that is represented by the "characteristic function", defined by Equation (7), which is a mapping from the input variables to the Boolean set $\{0,1\}$:

$$\mu_M = \begin{cases} 1, & x_k \in M \\ 0, & otherwise \end{cases} \tag{7}$$

Thus, an event (e.g. X) belongs totally or not at all (1 or 0) to a set; these kinds of relationships follow Boolean logic.

As a practical example, in subsurface reservoirs, the characteristic function allows establishing a Boolean relationship (1 or 0, i.e. totally belongs or totally excluded) between quantitative input variables (e.g. reservoir depth of 5000 ft) and descriptive terms (e.g. deep reservoir). Fuzzy logic extends the mapping between events and sets using the membership function (MF) to include all the values between 0 and 1, [0,1]; mathematically, the MF is a mapping from a given universe of discourse "X" to the continuous unit intervals that are the membership values. Equation (8) shows the mathematical expression for the MF:

$$\mu_{\tilde{M}}(x) = \{ y \ / y \in [0,1] \} \tag{8}$$

which shows that the values of the MF belong to the interval [0,1]. The membership values measure the degree of belonging of each event to a given set, representing the "degree of membership" of the mentioned event to that set. In this logic, an event (e.g. reservoir depth of 5000 ft) belongs partially (with a value between 0 and 1) to a set (e.g. deep reservoir).

In the standard VoI, the results of the assessment are a set of crisp values that measure the project benefits or losses of the different alternatives under evaluation. Comparing those values with a set of threshold values, a decision is made regarding the project fate; however, a decision made in this manner is limited because it does not follow the human thinking for decision making which works with fuzzy categories like "the project is viable to endorse", "the project is unviable to endorse" or "the project needs reframing".

1.3. Review Fuzzy Inference Systems

In practice, fuzzy logic is implemented through a process called a "fuzzy inference system" (FIS). A FIS is a non-linear procedure that derives its output based on fuzzy reasoning and a set of IF-THEN rules. The FIS performs approximate reasoning like the human brain, albeit in a much more straightforward manner.

The FIS is one of the most prolific applications of fuzzy logic. It has been used recently in very different areas and within various problem domains, such as: the assessment of water quality in rivers (Ocampo, 2008), the improvement of image expansion quality (Sakalli et al, 1999), the differential diagnosis of non-toxic thyropathy (Guo and Ling, 2008), the development of a fuzzy logic controller for a traffic junction (Pappis and Mamdani, 1997), maintenance scheduling of Smart Grid systems (Malakhov et al, 2012), the design of fire monitoring sensors in coal mines using fuzzy logic (Muduli et al, 2017), the estimation of the impact of tax legislation reforms on the tax potential (Musayev et al, 2016), pipeline risk assessment (Jamshidi et al., 2013), depression diagnosis (Chattopadhyay, 2014), river discharge prediction assessments (Jayawardena et al., 2014), geological strength index calculation and slope stability assessments (Sonmez et al, 2004), the regulation of industrial reactors (Ghasem, 2006) and the use of a fuzzy logic approach for file management and organization (Gupta, 2011). In the domain of the oil and gas industry several applications of FIS have been reported such as the streamline based fuzzy logic workflow to redistribute water injection by accounting for operational constraints and number of supported producers in a pattern (Bukhamseen et al, 2017), the identification of horizontal well placement (Popa, 2013), for estimating strength of rock using FIS (Sari, 2016), for predicting the rate of penetration in shale formations (Ahmed et al, 2019). Fuzzy logic has been used in combination with others Artificial Intelligence techniques such as Adaptative Neuro-Fuzzy Inference system (ANFIS) on practical applications, e.g. for predicting the inflow performance of vertical wells producing two-phase flow (Basfar et al, 2018) or to predict geomechanical failure parameters (Alloush et al, 2017); FIS has also been used in conjunction with Analytical Hierarchical processes to evaluate the water injection performance in heterogeneous reservoirs (Oluwajuwon and Olugbenga, 2018).

From the point of view of applications, there are two kinds of FIS (Guillaume, 2001):

1) Fuzzy expert systems or fuzzy controllers: fuzzy rules built on expert knowledge. This kind of FIS uses the ability of fuzzy logic to model natural language.

2) Automatic learning from data: neural networks have become the most popular tool using a numerical performance index, typically based on the mean square error. These kinds of development are distinguished by their accuracy, and their main drawback is their "black-box" approach.

For the current application, we will focus on the first kind of FIS.

From a methodological perspective, the FIS can be understood as a general procedure that transforms a set of input variables into a set of outputs, following the workflow shown in Figure 1.

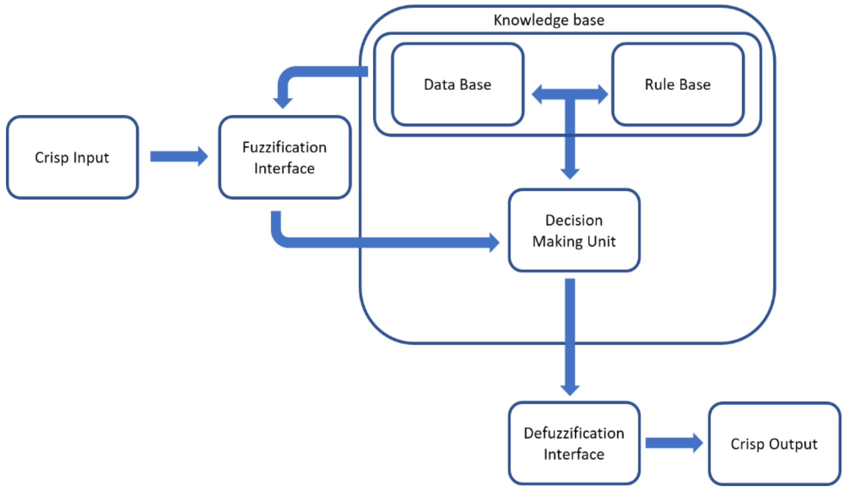

Figure 1. Fuzzy inference system

As shown in Figure 1, FIS as a procedure entailing five blocks in which the inputs and outputs are in crisp form.

For a Mandani FIS, shown in Figure 1, the outcome is a crisp number, independently of the number of crisp parameters used the asses the value of the project (e.g. NPV, DPI, IRR, etc.); this is FIS aggregation process; in general, higher FIS values means higher value of the project and vice versa.

1.4. Objective of this research work

The objective of this work is to investigate whether considering the fuzzy nature of human thinking can impact the decision's assessment in VoI problems, especially in oil and gas projects; to reach this objective we integrate Fuzzy Inference System into the VoI assessment.

2. Application

2.1. Reservoir Description

An exploration campaign conducted in Algeria discovered a medium-sized oil field located at 5200 ft. TVD SS. Four wells were drilled—the discovery well and three appraisal wells. The range of original oil in place (minimum and maximum figures) has been assessed; the fluid characteristics are known based on samples taken from the appraisal wells. The operator's technical team has estimated, based on the available information, technical experience, and analogue fields, that the main source of subsurface uncertainty is the well productivity. The four wells drilled were tested for six hours; however, considerable uncertainty remains regarding well productivity due to reasons described in Table 1.

Table 1. Causes of well productivity uncertainty

Reason for uncertainty	Comment
Quality and reliability of the well test	Possible calibration issues on well testing equipment
Duration of the tests	Well test period too short, no enough to reach stabilized flow

Based on the information gathered during the exploration phase and from similar fields in the same basin, a material balance model is built to represent the forecast oil rate for the high-, medium- and low-development scenarios, as shown in Figure 2.

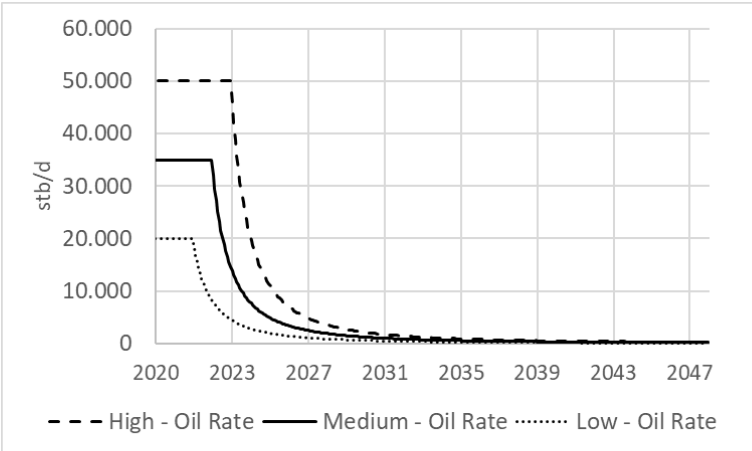

Figure 2. High, medium and low cases of the oil production rate

The difference between the profiles is the well model used in each case. The full development of the field includes twelve vertical wells, four of which have already been drilled—at present these are "suspended", to be used in the development phase of the project if a decision is taken to move the project forwards; otherwise, those wells should be abandoned entirely.

The rig will be available in four months, and each well can be drilled in two months; the duration of the campaign to drill and complete the remaining eight wells included in the development plan is sixteen months. The first period of oil production was planned to have a fixed plateau rate followed by a period of oil rate decline (Figure 2).

2.2. Decision Problem

At this stage, the operator company must decide whether acquiring additional information would increase the project's value.

Alternative A: without-information. The decision on project development is made based on the current information using the expected value (EV) of the net present value (NPV) and the discounted profit to investment ratio (DPI), which is discussed further in Section 3.3. Prior probabilities are assigned according to the technical team members' judgment on the subjective probabilities of realizing the different states of nature; the economic parameters are estimated based on the assumptions and assessments included in the high-, medium- and low-production scenarios. If this option is chosen, the first oil can be reached in two years' time, including facilities and wells.

Alternative B: with-information. Additional information is acquired regarding the uncertain parameters of the reservoir and, subsequently, based on the outcomes of the data obtained, a decision is made on the future development of the project. The operator's technical team has estimated, based on the reservoir and fluid properties, that, to obtain meaningful well test results, the minimum well test duration per well should be four months. It was decided that two of the appraisal wells could be used to perform an extended well test (EWT) in each one. Following these assumptions, there will be a delay of one year (four months rig move + eight months EWT) compared with the without-information alternative.

After the test results have been gathered, the technical team expects to have a more certain criterion to assign well deliverability, although uncertainty will still be present because the data are not perfect. The cost associated with the well test in these two wells is US$20 million. It should be considered that, if the project is relinquished now, the US$90 million already spent on exploration and appraisal will be lost; additionally, the abandonment cost for the 4 drilled wells, US$4 million, and the facilities' abandonment cost, US$10 million, should be added to the economic evaluation. If new data are acquired and afterwards the decision is made to abandon the project, the cost of the data acquisition must be added to the previously mentioned costs.

The outcome of the assessment of the alternatives without-information and with-information will result in one of the following options:

1) the project is viable to endorse: it will proceed to the development phase, which necessitates a large investment;

2) the project is not viable to endorse : it will be relinquished, carrying the losses associated with the exploration costs;

3) the project needs additional analysis before deciding: it will be reframed.

2.3. Economic Parameters for Decision Making

Two economic parameters are used to make the decision: the net present value (NPV) and the discounted profit to investment ratio (DPI). The NPV is the yearly net cash flow discounted to the weighted average cost of capital (WACC—the average rate of return with which a company expects to compensate all its different investors, in which the weights are the fraction of each financing source in the company's target capital structure), which in this case is 10.5%; the DPI is the result of dividing the discounted net cash flow by the discounted sum of the investment using the WACC. The values of NPV and DPI are shown in Section 2.4.2.

2.4. Classical VoI Assessment

As discussed in Section 1.1, the VoI is described by Equation (1); in this section, the classical approach to the VoI is discussed.

2.4.1. Decision rules

Based on the operator's portfolio of projects, the criterion for making decisions on projects with a financial investment higher than US$500 million is: A) a project with NPV lower than US$100 million is unviable to endorse, which means that it is relinquished, B) a project with NPV higher than US$500 million is viable to endorse and, C) a project with NPV between US$100 million and US$500 million is reframed to find alternative development options.

Regarding the DPI: A) a project with DPI higher than 0.5 is viable to endorse, B) a project with DPI lower than 0.0 is unviable to endorse and, C) a project with DPI between 0.0 and 0.5 should be reframed.

2.4.2. VoI assessment for the without-information and with-information alternatives

For the without-information alternative, Table 2 shows the prior probabilities, the calculated NPV, and DPI of each state of nature and the EV of the without-information alternative.

Table 2. Prior probabilities, NPV, DPI and expected values for the *without-information* alternative

State of nature	Prior probabilities (%)	NPV (US$ million)	DPI (fract.)
S1=High	25	2,139	2.27
S2=Medium	40	414	0.42
S3=Low	35	-631	-0.61
EVNPV-A1 (US$ million)	479		
EVNPV-A2 (US$ million)	-102		
EVNPV (US$ million)	479		
EVDPI-A1 (fraction)	0.52		
EVDPI-A2 (fraction)	-1.00		
EVDPI (fraction)	0.52		

For the with-information alternative, the technical team members estimated the reliability probabilities for the well test. It is acknowledged that, in a developed field, wells perform differently depending on their location and well test results are representative of a specific location; additionally, the duration of the test, although designed to capture the well performance, might not be long enough to assess the long-range well operation. Table 3 shows the reliability probabilities of the well test estimated by the technical team members.

Table 3. Reliability probability of the well test

Reliability probability	X1=High productivity	X2=Medium productivity	X3=Low productivity
$p(x_k \| s_1)$	0.9	0.1	0.0
$p(x_k \| s_2)$	0.1	0.8	0.1
$p(x_k \| s_3)$	0.0	0.1	0.9

Reliability probabilities are used together with prior probabilities to obtain posterior probabilities, which are combined with the project values to generate the expected value of the net present value (EVNPV) and the EV of the discounted profit to investment ratio (EVDPI). The results of these assessments are shown in Table 4.

Table 4. Posterior probabilities, residual probabilities and expected values for the *with-information* alternative

	X1=High productivity	X2=Medium productivity	X3=Low productivity
$p(s_1\|x_k)$	0.85	0.07	0.00
$p(s_2\|x_k)$	0.15	0.84	0.11
$p(s_3\|x_k)$	0.00	0.09	0.89
$p(x_k)$	0.27	0.38	0.36
$EVNPV(A_1\|x_k)$	1,667	357	-497
$EVNPV(A_1\|x_k)$	-114	-114	-114
$EVNPV(A_1\|x_k)$	1.667	357	-114
$\underline{EVNPV(US\$\,million)}$	537		
$EVDPI(A_1\|x_k)$	1.76	0.37	-0.48
$EVDPI(A_1\|x_k)$	-1.00	-1.00	-1.00
$EVDPI(A_1\|x_k)$	1.76	0.37	-0.48
$\underline{EVDPI(\,fraction)}$	0.44		

2.4.3. Results of the VoI using the classical approach

Decision based on NPV: Based on the results obtained in Section 2.4.2, and using the decision rules (Section 2.4.1), it can be concluded that, the without information project should be reframed and the with information project should be endorsed; in this manner, acquiring data increase the project's value; Summarizing, according to the NVP figures, the preferred alternative is to perform the well test (the with-information alternative) and, depending on the data outcomes, decide whether the project should be endorsed or not.

Decision based on DPI: Using DPI as the decision criterion, the without-information alternative suggests endorsing the project, while the with-information alternative suggests reframing the project; summarizing, the alternative that maximizes the DPI is the without-information one, which means that it is not recommended to perform the well test.

At this stage, using two financial criteria, we have two contrasting recommendations about the future of the project.

Making a decision using these crisp criteria does not include the sophisticated elements used by human thinking in which, the criteria may overlap. In addition, from the independent NPV and DPI results, it is not clear which is the optimum alternative unless some form of weighted valuation is made by combining the two economic parameters into one.

2.5. FIS VoI Assessment

Up to this stage, the criterion to decide the future of the project has been based on linguistic variables like "not endorse", "endorse", "viable", "unviable", "high", "medium" and "low". Indeed, a crisp relationship is established between the NPV and DPI and the linguistic variables: if the NPV is less than US$ ***X*** million, the project is "unviable to endorse", if the NPV is higher than US$ ***Y*** million, the project is "viable to endorse" and if the NVP is higher than ***X*** but lower than ***Y***, the project should be reframed; similar relationships apply to the DPI criterion.

However, it is worth recognizing that these criteria are fuzzy and not always aligned. The fuzziness occurs because, if the project NPV is US\$ $\boldsymbol{X} - \boldsymbol{\varepsilon}$ million, where $\boldsymbol{\varepsilon}$ is a given amount, the crisp logic decision criterion catalogues the project as "unviable to endorse", although $\boldsymbol{\varepsilon}$ could be "small" compared with $\boldsymbol{X}$. In the same manner, if the project value is US\$ $\boldsymbol{Y} + \boldsymbol{\varepsilon}$, in a crisp decision, the project is catalogued as "viable to endorse", although $\boldsymbol{\varepsilon}$ could be "small" compared with $\boldsymbol{\varepsilon}$.

The no alignment between the criteria happens because very often the two indices, NPV and DPI, can produce a contradictory assessment of the same problem; for example, it could be the case that, using the NPV, the project is "viable to endorse" but, using the DPI, the project is "unviable to endorse" or vice versa, as has been witnessed in this case study.

These cases suggest that fuzzy logic can be used advantageously to make VoI decisions by providing a more versatile tool to assess these decision problems; fuzzy logic is implemented through the FIS.

2.5.1. FIS building and application

The FIS used in this work was developed using MATLAB. The input parameters in the FIS are the NPV and DPI; for each input parameter, six Membership Functions are built, representing the linguistic variables high NVP or NVP viable to endorse, low NVP or NVP unviable to endorse, mid NVP or NVP for reframing, high DPI or DPI viable to endorse, low DPI or DPI unviable to endorse and mid DPI or DPI for reframing; the corresponding MFs are: NVP HIGH, NVP MID, NVP LOW, DPI HIGH, DPI MID and DPI LOW.

In MATLAB, a set of predefined MFs—triangular-shaped functions—are selected. These MFs are chosen because they capture the technical team members' interpretation of the degree to which the NPV and DPI figures belong to the three categories into which the range of potential values are divided. Equation (9) shows the mathematical form of the triangular-shaped MF:

$$f(x;a,b,c) = \begin{cases} 0 & x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & c \le x \end{cases} \tag{9}$$

The comparison between the output of the FIS for the with-information and that for the without-information alternative indicates which alternative has more value (the better decision).

A Mamdani FIS with the centroid defuzzification method was used in this assessment. Figure 3 shows the design of the FIS using MATLAB.

In Section 1.3 we show the Figure 2 which describe the FIS process; that figure is shown below but now numbering the steps, in order, we are following in this work

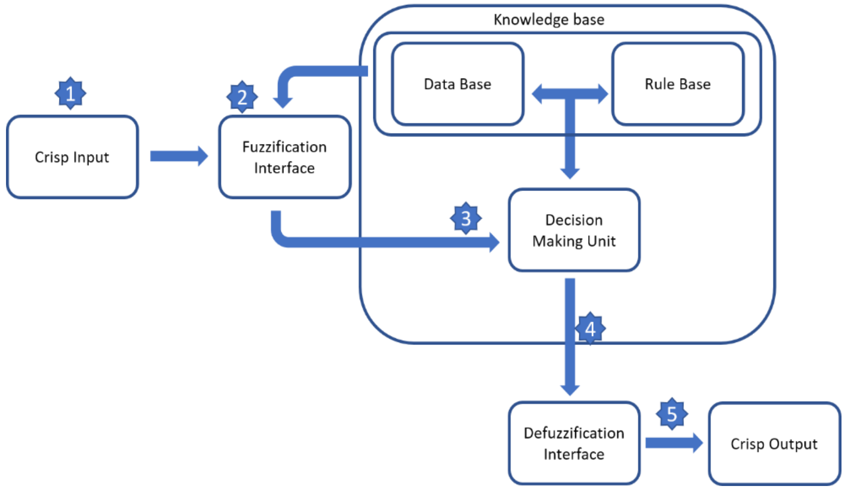

Figure 3. FIS implementation

Step1: the crisp data is generated, in this case, the project value, NPV and DPI

Step 2: data is fuzzified using the membership function located in the data base; those MF describe the degree of belonging of different input values which is defined according with the analyst belief. In this work, the MF used for NPV and DPI are shown in Figure 4 and Figure 5.

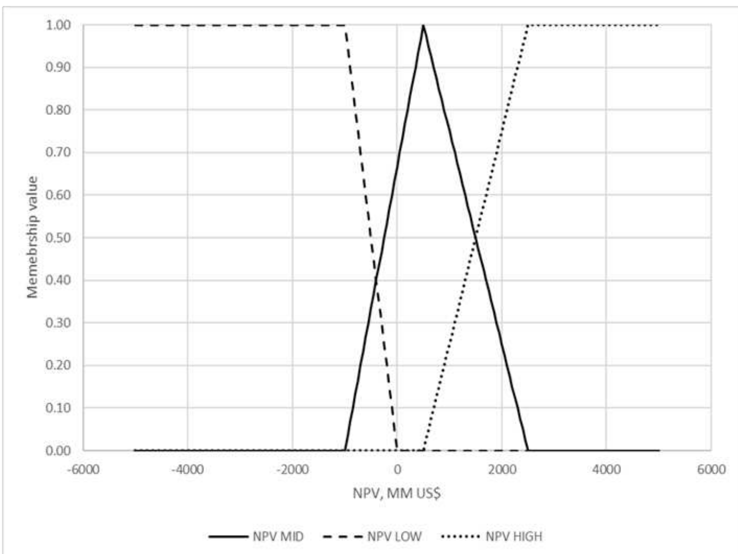

Figure 4. Membership Function for NPV (input)

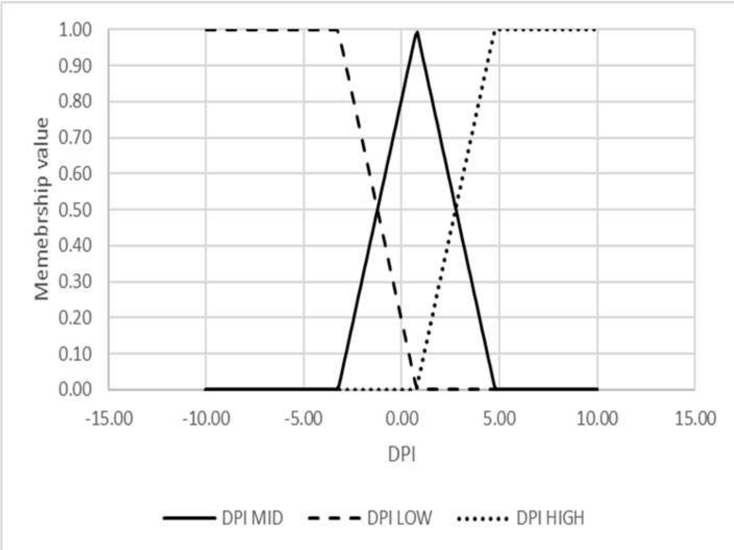

Figure 5. Membership Function for DPI (input)

Step 3: once input variables are fuzzified, the decision rules, which are part of the knowledge base, are applied to the membership functions in the Decision-making unit; in the Mandani type inference, the decision rules are a mapping from the input membership function to the output membership functions, which are also part of the knowledge base; the rules aggregation process generate the fuzzy outcome.

The output membership functions used in this work, describing the different decision options are show in Figure 6.

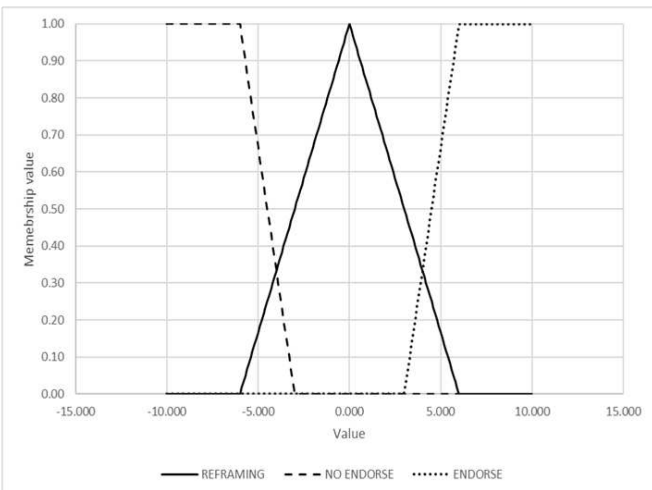

Figure 6. Membership Function for the decision (output)

The decision rules indicate the manner in which the two fuzzy financial parameters combine to result in a fuzzy decision. In this work we define the rules shown in Table 5 to include the cases of interest,

Table 5. Fuzzy rules

RULES	IF	THEN
Rule 1	(NPV is NPV_HIGH) & (DPI is DPI_HIGH)	ENDORSE
Rule 2	(NPV is NPV_HIGH) & (DPI is DPI_MID)	ENDORSE
Rule 3	(NPV is NPV_HIGH) & (DPI is DPI_LOW)	REFRAMING
Rule 4	(NPV is NPV_MID) & (DPI is DPI_LOW)	REFRAMING
Rule 5	(NPV is NPV_LOW) & (DPI is DPI_LOW)	NO ENDORSE
Rule 6	(NPV is NPV_LOW) & (DPI is DPI_HIGH)	REFRAMING
Rule 7	(NPV is NPV_MID) & (DPI is DPI_MID)	ENDORSE

Step 4: fuzzy output gets into the defuzzification interface to generate crisp output.

Step 5: the value of the project is the crisp out; different crisp outputs are compared and, the one with the higher value is the optimum decision.

The MFs of the NVP are chosen in accordance with past decisions taken by the decision maker, as discussed in Section 1.2. The rationale for the selection of these MFs is that, for very high or very low NPV values, the NPV belongs to only one set, the NPV_HIGH or the NPV_LOW, with a membership value of 1; for the intermediate NPV value, the NPV belongs partially to the three fuzzy sets. This fuzzy representation of the criteria for categorizing the project is based on past decisions made by the field operator company. The selection of the MFs needs to be updated once more decisions have been taken.

The MFs for the DPI are chosen following the same procedure used for the NPV

The authors define a set of seven rules that determine the logic of this decision; these rules (IF-THEN rules) are made by pairs of NPV and DPI figures and a consequential sentence (THEN). The rules do not pretend to be exhaustive but must be coherent. All the rules were built using the AND connector; although, in general, they can be defined equally well with OR.

2.5.2. FIS applied to the without-information and with-information alternatives; VOI assessment

Referred to Section 1.3, the outcome of FIS (a crisp number) is the value of the project resulting from aggregating the project's values in terms of NPV and DPI; in addition, the FIS assessment includes the imprecision in the terms used to decide whether a project worth or not to endorse (Section 2.2).

For evaluating the project using the FIS developed in Section 2.5.1, the crisp values for NPV and DPI estimated in Section 2.4.2 Table 2 (US$479 million and 0.52), are input in the FIS; the outcome of the assessment made by the FIS indicates that the value associated with the without-information alternative is **7.2**.

Similarly, in the with-information alternative, the NPV and DPI figures (US$537 million and 0.44), contained in Table 3, are input in the FIS; as a result, the FIS assessment for the with-information alternative is **6.97**.

Due to the fact that, the value of the FIS for the without-information alternative is higher than the value of the FIS for the with-information case, the best alternative for the decision problem discussed is to endorse the project now and move it forward to the development phase without acquiring additional data.

This result is explained by the fact that, although the data acquisition reduces the uncertainty regarding well deliverability, the cost of this data acquisition, in terms of the additional investment and oil production delay, is higher than the increased project value due to uncertainty reduction.

3. Conclusions

In this study, an FIS has been successfully implemented with the aim of assessing the VoI of an oil and gas project. In the discussed case study, the use of the FIS was able to introduce the fuzzy thinking of the decision maker into a subsurface VoI assessment while removing ambiguity coming from the use of more than one economic parameter for decision making.

The proposed methodology for VoI assessment using FIS has improved the conventional approach because:

1) instead of using a Boolean relationship between project valuation and project decision, the FIS uses a fuzzy human thinking approach to make decisions;

2) the FIS uses a coherent method to integrate more than one criterion into the assessment, while, in the conventional VoI approach, when more than one criterion is used, they can reach contradictory outcomes which conduct to inconclusive assessment.

In addition to the aspects discussed above, the FIS provides a tool for “self-learning” in which the quality of the VoI assessments can be improved through continuous updating of the decision-making unit, knowledge base, and fuzzification and defuzzification interfaces with actual decisions, progressively generating a more robust FIS and making the system act closer and closer to the way in which humans make decisions. The FIS brings the VoI methodology closer to the decision maker’s reasoning.

These are important advantages of the fuzzy compared with the classical VoI assessment.

The fuzzy approach for VoI assessment requires a longer and more complex analysis of the data to be acquired and their outcomes. However, this additional effort worth due to the impact it has in the decision.

As a summary, the use of the FIS makes it possible to have a system that can integrate the linguistic variables that are part of human language, reasoning, and understanding, but not necessarily part of the Boolean logic used in the standard VoI, into the prescriptive VoI assessment.

VoI assessment using the FIS brings the decision-making process one step forward with respect to the classical VoI approach. To have tools and methods that replicate the human reasoning process for assessing VoI increases the confidence of the decision maker in those procedures, thereby increasing their use and making the tools more reliable.

Decisions are made by human, and because human thinking is approximated more accurately by imprecise logic than by crisp logic, this research work successfully develop a methodology that integrate the human logic in the VoI assessment, in special to problems in the oil and gas industry; the integration of the imprecise thinking and terminology in the VoI is made through the use of FIS.

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EVALUATION OF SOLID WASTE TREATMENT METHODS IN LIBYA BY USING THE ANALYTIC HIERARCHY PROCESS

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Abstract: *Evaluation and selection of the appropriate method for solid waste treatment (SWT) in Libya are a complex problem and require an extensive evaluation process. This is because it is very difficult to develop a selection criterion that can precisely describe the preference of one method over another. Waste management is the collection, transport, treatment, recycling or disposal and monitoring waste materials. In this paper, four treatment systems for waste management in Libya are evaluated using the analytic hierarchy process (AHP) in respect to four main criteria and twenty-two sub-criteria. The treatment systems for waste management are anaerobic digestion, landfilling, incineration and compost. The selected criteria used in the evaluation of four treatment systems are environmental impacts, socio-cultural aspects, technical aspects and economic aspects. According to the evaluation, anaerobic digestion ranks the highest in classification in Libya. Compost ranks higher than landfilling and incineration. Furthermore, it should be noted that the rank of waste treatment systems can be changed according to the future technological developments.*

Key words: *Waste management, multi-criteria evaluation, AHP, Libya.*

1. Introduction

During the earliest periods, solid wastes were conveniently and unobtrusively disposed in large open land spaces, as the density of the population was low. As the population and economic growth increases, the solid household waste also increases. However, the population and economic growth not only lead to an increase in volume

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of solid household waste but also to great changes in its specification and contents (Abduelmajid et al., 2015). The concept of eliminating waste completely is highly unrealistic. However, the best approach is to handle solid waste in such a manner that does not damage the environment, while utilizing methods supported by the denizens of the community who are directly impacted by the solid waste management (SWM) program in an area (Khan & Faisal, 2008). Therefore, waste management is a priority issue regarding protection of the environment and conservation of natural resources.

Libya is a north African country located along the southern coast of the Mediterranean basin. Its total land area is about 1.8 million km2, most of which (95%) is a desert, whereas the rest is either rangeland (4%), or agricultural land (0.5%), and less than 0.5% is a scattered forested area. Due to rapid expansion of industry, urbanization and increasing population, particularly in large cities which are located on the coast, has increased the amount of solid waste generated in Libya significantly (Badi et al., 2016). In Libya issues related to sound municipal solid waste (MSW) management including waste reduction and disposal have not been addressed adequately and the collection and the separation treatment of solid waste are still neglected.

In this paper, criteria for the assessment of the municipal waste management technologies are analyzed and evaluated. The technology assessment indices calculated with these methods were applied as criterions for multi-criteria analysis, which evaluates individual variants of municipal waste management systems. Indices evaluating the performance of the system can be determined with due regard to the technical, environmental, economic, social and other objectives, bearing in mind specific features of the area involved.

The aim of this study is to evaluate different waste management methods and their applicability in Libya based on Multi-Criteria Decision Analysis (MCDA). The common methods used in this study are those recommended in the waste management laws and regulations, such as composting, anaerobic digestion, incineration with thermal energy recovery (electricity and heat), and landfill without any form of energy recovery.

2. Literature review

This literature review studies and investigates various waste management methods and a multi-criteria decision analysis including waste reduction and disposal that is applied to the SWM.

Javaheri et al. (2006) presented study includes multi criteria evaluation method under the name of weighted linear combination by using geographical information technology to evaluate the suitability of the vicinity of Giroft city in Kerman province of Iran for landfill. The major criteria used in the study were geomorphologic, hydrologic, humanistic and land use. The results of the study were afford strategy to the decision makers of Giroft city by a variety of options (Javaheri et al., 2006).

Manaf et al. (2009) evaluated the generation, characteristics and management of solid waste in Malaysia. It was concluded that the efficiency of solid waste management in Malaysia will be increased towards achieving Vision 2020 as a developed country (Manaf et al., 2009). A case study was conducted by Sawalem et al. (2009) to evaluate hospital waste management in Libya. The study found that several factors such as the type of healthcare establishment, level of instrumentation and location affect waste generation rates. The results showed that the highest generation rates at Tripoli medical center are attributed to a larger number of patients due to being in the capital of Libya (Sawalem et al., 2009).

Gebril et al. (2010) presented an overview of the current SWM practices in Benghazi, Libya. The objective of the study was to investigate the current practices and challenges that faced MSW management in Benghazi. It was found that several issues affected in the SWM such as lack of suitable facilities and inadequate management and technical skills, improper bin collection and route planning (Abdelsalam & Gebril, 2010). Generowicz et al. (2011) combined the best available techniques, technology quality method and multi-criteria analysis in order to develop indices for evaluating municipal waste management systems. The results showed that incineration of waste is much more beneficial than disposal (Generowicz et al., 2011).

Tabasi and Marthandan (2013) presented an overview on the existing researches in the area of clinical waste management. The objective of the study was to investigate different findings regarding associated factors on quantity of waste generation to find, integrate and enhance accessibility to hospital key factors in waste generation forecasting. The results showed that the number of patients, number of beds, bed occupancy rate and type of hospitals were the most important factors in waste generation (Tabasi & Marthandan, 2013). Norkhadijah et al. (2013) investigated the challenges which can be faced to find a suitable place for future landfill in Malaysia. Based on the fact that limited space is available for landfill development, the conclusion of the study was that, landfill cannot be the ultimate option for much longer (Norkhadijah et al., 2013).

A study was conducted by Gebril (2013) to determine the causes of solid waste pollution in Benghazi City, in Libya and its surrounding areas. The results showed that solid waste pollution in the city and its surrounding areas is the outcome of poor planning and environmental management, population growth, lack of hardware and equipment for the collection and transport of waste from the city to the landfill site (Ali, 2013). Hamad et al. (2014) presented an overview on solid waste that can be used as a source of bioenergy in Libya including industrial solid waste and health care wastes as biomass sources. The aim of the study was to investigate whether or not solid waste can be used as a source of bioenergy in Libya. The results showed that organic matter represents 59% of waste, followed by paper–cardboard 12%, plastic 8%, miscellaneous 8%, metals7%, glass4%, and wood 2%. The technology of incineration is recognized as a renewable source of energy and is playing an increasingly important role in MSW management in Libya (Hamad et al., 2014).

Najjar et al. (2015) conducted a study to estimate the percentage of total plastic and PVC in particular, in solid household waste in the city of Tripoli, Libya. The results concluded that the weight percentages of plastic waste and PVC were about 10.52% and 1.36%, respectively. The percentage of PVC from plastic waste was only 12.94% (Abduelmajid et al., 2015). Babalola (2015) presented a multi-criteria decision analysis to evaluate different waste management options and their applicability in Japan. The results showed that anaerobic digestion should be chosen as the best food and biodegradable waste treatment option concerning resource recovery (Babalola, 2015).

A study was carried out by Moftah et al. (2016) to evaluate the generation, composition and density of household solid waste in Tripoli city, Libya. It was concluded that the total generation quantity, daily generation rate, total volume and density were in Tripoli city agreed with those for African and Arabic countries. The study showed that Tripoli suffers from insufficient MSW management and lack of sanitary landfills (Moftah et al., 2016). Jovanović (2016) presented a method for selection an optimal waste management system in the city of Kragujevac, Serbia through an integrated application of the life cycle assessment method and MCDM methods. Six different waste management strategies for the territory of the city were formulated and eight parameters were selected (Jovanović et al., 2016).

Omran et al. (2017) conducted a study in the City of Al-Bayda, Libya dealing with solid waste management. The aim of the study was to investigate the major problems facing the city in dealing with SWM in terms of generation, collection, handling, transportation, recycling and disposal of MSW. The conclusion was, there were major factors impacting the decision-making and operational processes of MSW that include lack of resources and services that significantly affect the disposal of waste and inadequate number of waste collection containers. This makes the distance to these containers for many households excessive and thus leading to an increasing likelihood of dumping solid waste in open areas and roadsides (Omran et al., 2017).

By reviewing the previous studies specifically, the studies that dealt with SWM in Libya, it can be noted that, vast majority of them focused on the classification of solid waste management rather than the selection of the technology treatment. To fill this research gap, this paper examines the selection of the appropriate method for the solid waste treatment.

3. Variants of municipal waste management technology

Several types of recycling, energy recovery or waste neutralization technologies are used in a system of waste management. Each of them shows different technical and environmental characteristics.

3.1. Composting

Composting is a biological process in which the organic matter current in waste is converted into enriched inorganic nutrients. The manure obtained has high nitrogen, phosphorus, and potassium content. Composting is often described as nature's way of recycling is a key ingredient in organic farming. At the simplest level, the process of composting only requires making a heap of wetted organic material (leaves, food waste) and waiting for the materials to breakdown into humus like substance by undergoing biological decomposition after a period of weeks or months (Ladan, 2014). The quality of compost depends upon the waste being composted. There are a number of biological or compost related technologies. These are open windrow, vermi-composting, enclosed composting and fermentation (Thompson-Smeddle, 2011).

3.2. Anaerobic digestion

Anaerobic digestion (AD) is a naturally occurring biological process that uses microorganisms to break down organic material in the absence of oxygen. In other words, AD is a process that makes any organic waste can be biologically transformed into another form, in the absence of oxygen. The production of biogas and other energy-rich organic compounds is mainly produced from the degradation of organic waste by microbial organisms (Arshad et al., 2011). A series of metabolic reactions such as hydrolysis, acid genesis, acetogenesis and methanogenesis are involved in the process of anaerobic decomposition (Charles et a., 2009). Anaerobic digestion can be applicable for a wide range of material including municipal, agricultural and industrial wastes and plant residues (Chen et a., 2008).

3.3. Incineration

Incineration, or thermal oxidation is the process of oxidizing combustible materials by raising the temperature of the material above its auto-ignition point. The process is done in the presence of oxygen, and maintaining it at a high temperature for sufficient time to complete combustion to carbon dioxide and water (EPA-CICA, 2003). Any non-combustible materials (e.g. metals, glass, stones) remain as a solid, known as incinerator bottom ash that always contains a small amount of residual carbon (Defr, 2007). The efficiency of the combustion process is affected by the factors such as time, temperature, turbulence (for mixing) and the availability of oxygen. These factors provide the basic design parameters for volatile organic compounds oxidation systems (ICAC, 1999).

3.4. Landfilling

Landfilling is the ultimate disposal process for the SWM. The process is simply dumping the waste in trenches or cells with leveling and compacting by trash compactors to reduce the size and the thickness of the layers, and finally the waste is covered by soil (Aljaradin, 2014). The quantity of MSW for land disposal can be considerably reduced by setting up waste processing facilities and recycling the waste materials as much as possible.

4. Multi-criteria decision making

Multi-Attribute Decision Making (MADM) is the most well-known branch of decision making. MADM models deal with decision making problems under the presence of a number of decision criteria. This class of models is very often called multi-criteria decision making (or MCDM). According to many authors MCDM is divided into Multi-Objective Decision Making (or MODM) and Multi-Attribute Decision Making (or MADM) (Shu et al., 1998; Karami, 2011).

MODM is a mathematical programming problem with multiple objective functions. Whereas, the developing of MADM models is based on several alternatives according to some criteria are ranked and selected. Ranking and selecting will be made among decision alternatives described by some criteria through decision-maker knowledge and experience (Karami, 2011; Devi et al., 2009; Chatterjee et al., 2018; Pamučar et al., 2018a). MCDM is approach for finding the optimal alternative from all the feasible alternatives according to some criteria or attributes (Stević et al., 2017; Pamučar et al., 2018b).

5. Analytic Hierarchy Process

Analytical hierarchy process is a common MCDM method. It is developed by Saaty to provide a flexible and easily understood way of analysing complex problems (Saaty, 1979; Saaty, 1990). According to (Chai et al., 2013) it has been found that AHP method was used more than any other MCDM methods (Chai et al., 2013). It breaks a complex problem into hierarchy or levels, and then making comparisons between possible pairs in a matrix to give a weight for each factor and also a consistency ratio. The AHP utilises a tree structure in order to simplify complex decision-making problems resulting in simplified sub problems, which can easily be examined. The AHP method can be distinguished in four main steps:

- Creation of a tree structure, which comprises of one goal, the criteria, and alternative solutions.
- Evaluation of each alternative solution in relation to each criterion.
- Calculation of the weighting factor of the criteria with subjective evaluation using pairwise comparisons.
- Synthesis of the results of stages 2 and 3 so as to calculate the overall evaluation of each alternative regarding the degree of achievement of each goal. Figure 1 presents the tree structure for the four SWT systems.

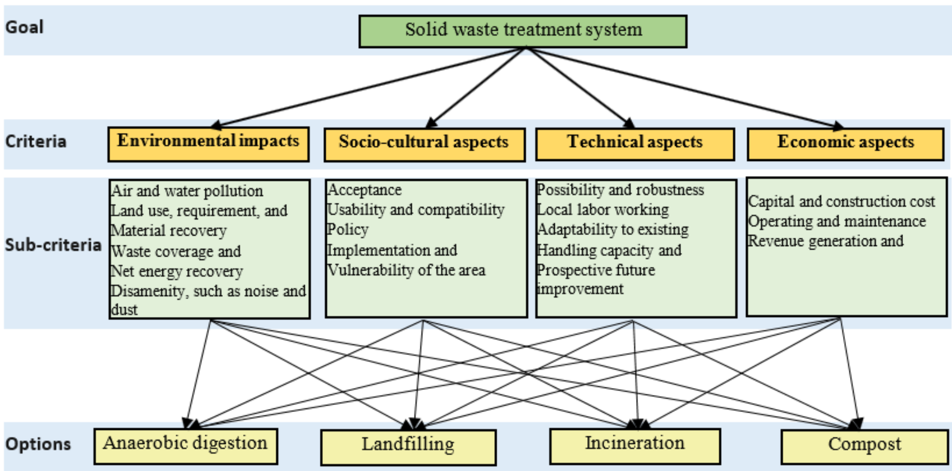

Figure 1. Tree structure for the four SWT systems

In the AHP method, pairwise comparisons permit the decision maker to concentrate only on one element at a time. Specifically, to explore how strongly important is one criterion related to another with regards to the goal?" The comparisons are the input into a matrix. If the matrix is sufficiently consistent, priorities can then be calculated with formula (1).

$$AW = \lambda_{\max} w \tag{1}$$

where A is the comparison matrix, λmax is the principal eigenvalue and W is the priority vector. The AHP model gives feedback to the decision maker on the consistency of the entered judgments through the measurement of consistency ratio (CR) by using formulas (2) and (3).

$$CR = \frac{CI}{RI} \tag{2}$$

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{3}$$

where CI is the consistency index, n is the dimension of the comparison matrix, λmax is the principal eigenvalue and RI is the ratio index. The ratio index or Random Consistency index (RI) is given in Table 1. If the consistency ratio is less than 0.1 (<10%) the matrix is regarded as consistent, otherwise the matrix is inconsistent and it is suggested to modify the comparisons in order to reduce the inconsistency (Saaty, 1980). If all sub-priorities are available, they are aggregated with a weighted sum in order to obtain the overall priorities of the alternatives so as a final judgment can be made based on the ranking (Saaty, 1980; Saaty & Vargas, 2001).

Table 1. Random Consistency Index (RI)

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

6. Analysis of the results

The quantity of MSW generated in Libya is estimated at 3.2 million tons/year (Sawalem et al., 2009; Ali, 2013). The treatment of solid waste is done by throwing in open dumps designated by the relevant authorities and in many cases random dumps that are not controlled by the state. Lack of suitable facilities, inadequate management and technical skills, improper bin collection and shortages in solid waste plants are among the important issues resulting in poor collection and transportation of municipal solid wastes in Libya (Abdelsalam & Gebril, 2010). However, in Libya few MSW plants were established in several cities as shown in Figure 2. These plants are suffering from many obstacles, because all of them are outdated and need to be updated or replaced. For example, the MSW plant in Misurata, which was opened in 1982 with a capacity of 120 tons per day, currently the capacity is only 60 tons.

Figure 2. Distribution of solid waste plants in Libya

For the evaluation of the four treatment systems, with the use of the AHP, 12 cases were carried out. These cases were the base case, equally distributed criteria case, four cases of single-criterion analysis and six cases of multi-criteria analysis. In this paper, qualitative criteria are identified based on questionnaire forms that have been filled in by environmental experts and academic staff university members. Table 2 shows the pairwise comparison matrix of the general and organizational structure of the technology's sub-criteria. In order to facilitate the solution process for the AHP problem, Expert Choice software were used to compute the model.

Table 2. Criteria, sub-criteria and their weights

Criteria	Weight	CR	Sub-criteria	Weight
Environmental	0.581	0.07	Air and water pollution	0.286
			Land use, requirement, and contamination	0.046
			Material recovery	0.062
			Waste coverage and elimination	0.061
			Net energy recovery	0.077
			Disamenity, such as noise and dust	0.049
Socio-Cultural	0.204	0.07	Acceptance	0.119
			Usability and compatibility	0.027
			Policy	0.020
			Implementation and adoptability	0.023
			Vulnerability of the area	0.016
Technical	0.128	0.08	Possibility and robustness	0.054
			Local labor working experience	0.019
			Adaptability to existing systems	0.012
			Handling capacity and continuous process	0.031
			Prospective future improvement	0.012
Economical	0.086	0.08	Capital and construction cost	0.038
			Operating and maintenance cost	0.007
			Revenue generation and marketability	0.009
			Financial planning	0.011
			Employment and job creation	0.016
			Waste volume and composition	0.005

6.1. Base case

In the base case, the criteria weights have been calculated using pairwise comparison according to the AHP method. The following weighting factors are used: Environmental impacts 58%, socio-cultural aspects 20%, technical aspects 13% and economic aspects 9%. The weighting factors were given to each criterion. These percentages indicated that the environmental impact of each alternative option is the primary concern of this case, while socio-cultural aspects follow. Figure 3 presents the rating of alternative options for SWT system. As can be seen from Figure 2, the anaerobic digestion is the best option when a greater emphasis is given to environmental impact. Furthermore, compost and incineration are ranked second and third, respectively. On the contrary, landfill is ranked last.

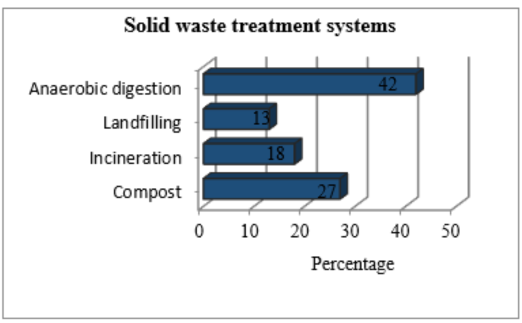

Figure 3. Overall evaluation of SWT system

The next step of the decision process of the AHP method is the sensitivity analysis, where the input data of criteria weighting are slightly modified in order to observe their impact on the results. If the ranking of treatment systems does not change significantly, the results are said to be robust. Bearing in mind that the opinions of experts may vary, a sensitivity study was carried out. The following cases were examined:

6.2. Equally distributed criteria (case 1)

In case1, the following weighting factors are used: Environmental impacts 25%, socio-cultural aspects 25%, technical aspects 25% and economic aspects 25%. Figure 4 presents the rating of alternative options for SWT system for this case. Again, the anaerobic digestion is the best option while landfill is ranked last.

6.3. Single-criterion analysis (cases 2–6)

In the single-criterion analysis (cases 2–6), the evaluation has been carried out with full emphasis to one criterion while the other four criteria are ignored.

6.3.1. Case 2

In case 2, the following weighting factors are used: Environmental impacts 100%, socio-cultural aspects 0%, technical aspects 0% and economic aspects 0%. As can be seen from figure (5), the best option is the anaerobic digestion, landfill ranks last, given the fact that it has a high impact on the environment.

6.3.2. Case 3

In case 3, the following weighting factors are used: Environmental impacts 0%, socio-cultural aspects 100%, technical aspects 0% and economic aspects 0%. Figure 6 gives the overall ranking of SWT system when emphasis is given to socio-cultural aspects. The anaerobic digestion has the highest ranking while incineration receives the last position.

6.3.3. Case 4

In case 4, the following weighting factors are used: Environmental impacts 0%, socio-cultural aspects 0%, technical aspects 100% and economic aspects 0%. As can be seen from Figure 7, the compost has the highest-ranking while incineration receives the last position. This result was expected since the incineration system requires some technical consideration.

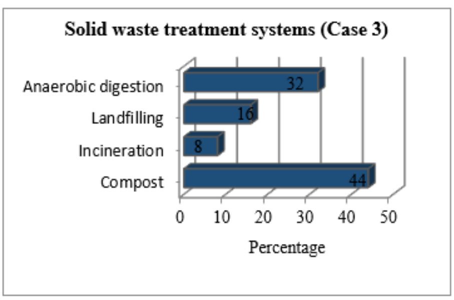

Figure 6. Overall evaluation of SWT system for case 3

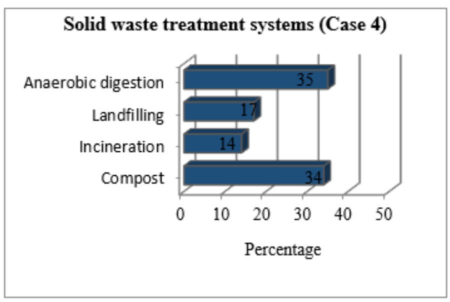

Figure 7. Overall evaluation of SWT system for case 4

6.3.4. Case 5

In case 5, the weighting factors used are: Environmental impacts 0%, socio-cultural aspects 0%, technical aspects 0% and economic aspects 100%. As shown in Figure 8, again the anaerobic digestion has the highest-ranking while the landfilling system receives the last position.

6.4. Multi-criteria analysis (cases 6–11)

According to multi-criteria analysis (cases 6–14), the evaluation of the four-selected SWT system has been carried out by giving greater emphasis (a larger weighting factor) to one criterion without ignoring the rest as was carried out in the single-criterion analysis (cases 6–9). In the last two cases, greater emphasis is given to two criteria at the same time (cases 10–11).

6.4.1. Case 6

In case 6, the following weighting factors are used: Environmental impacts70%, socio-cultural aspects 10%, technical aspects 10% and economic aspects 10%. As can be seen from Figure 9, the best SWT system is the anaerobic digestion while landfilling system receives the last position.

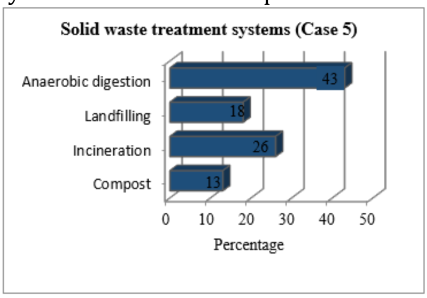

Figure 8. Overall evaluation of SWT system for case 5

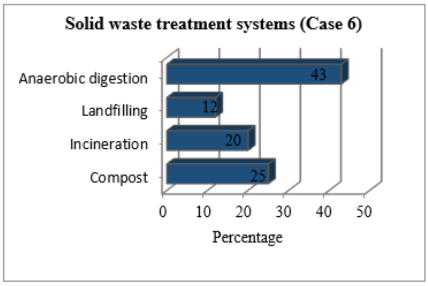

Figure 9. Overall evaluation of SWT system for case 6

6.4.2. Case 7

In case 7, the following weighting factors are used: Environmental impacts 10%, socio-cultural aspects 70%, technical aspects 10% and economic aspects 10%. Figure 10 presents the rating of alternative options for SWT system for case 7. According to this figure, the best waste treatment system is anaerobic digestion and next is compost while incineration receives the last position.

6.4.3. Case 8

In case 8, the following weighting factors are used: Environmental impacts 10%, socio-cultural aspects 10%, technical aspects 70% and economic aspects 10%. According to Figure 11, the best waste treatment system in case 8 is anaerobic digestion, and next is compost while incineration receives the last position. These outcomes are very similar to the results obtained in case 7.

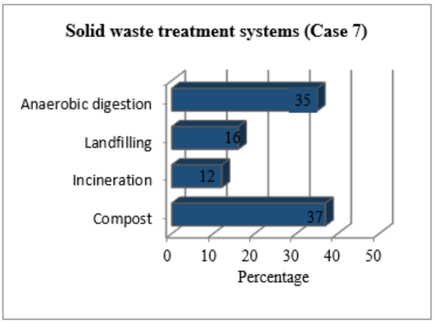

Figure 10. Overall evaluation of SWT system for case 7

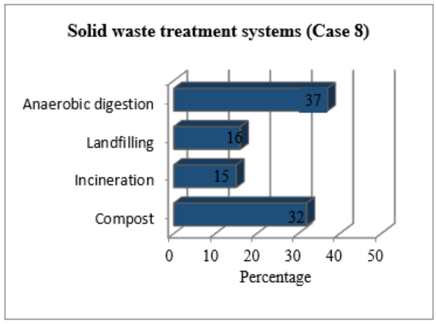

Figure 11. Overall evaluation of SWT system for case 8.

6.4.4. Case 9

In case 9, the following weighting factors are used: Environmental impacts 10%, socio-cultural aspects 10%, technical aspects 10% and economic aspects70%. As can be seen from Figure 12, the best SWT system is the anaerobic digestion while incineration system receives the last position.

6.4.5. Case 10

In case 10, the following weighting factors are used: Environmental impacts 35%, socio-cultural aspects 35%, technical aspects 15% and economic aspects 15%. As can be seen from Figure 13, the best SWT system is the anaerobic digestion while incineration system receives the last position.

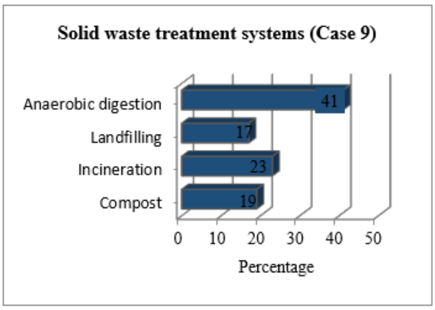

Figure 12. Overall evaluation of SWT SWT system for case 9

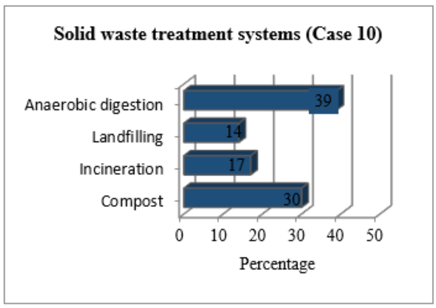

Figure 13. Overall evaluation of system for case 10

6.4.6. Case 11

In case 11, the following weighting factors are used: Environmental impacts 35%, socio-cultural aspects 15%, technical aspects15% and economic aspects 35%. As can be seen from Figure 14, the best SWT system is the anaerobic digestion while landfilling system receives the last position.

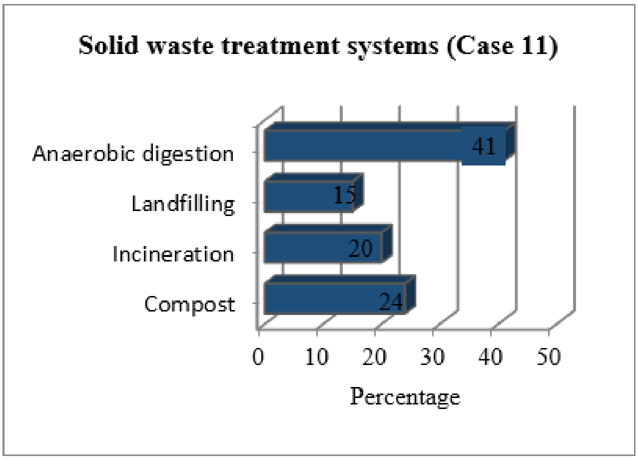

Figure 14. Overall evaluation of SWT system for case 11

Table 3 presents the criteria weights for the 12 scenarios conducted to the case study which is described above, and Table 4 summarises an overall evaluation and ranking of the four SWT systems under examination. The evaluation of the SWT systems was carried out using the AHP for 12 cases. These consisted of the base case, the equally distributed criteria, four cases of single-criterion analysis and six cases of multi-criteria analysis. Each treatment system presents a solution for the solid waste management system with a certain degree of trade-off between benefit and its consequences related to environmental, social, technical and economic issues. Sensitivity analysis is conducted to evaluate the robustness of the selected treatment options. A "What if Analysis" Figure 15 was performed to see if there were any changes among the selected treatment options. The results display no changes in the ranked results, as the anaerobic alternative remained the most suitable option for the treatment of the SWM. As can be seen from Figure 15, the majority of cases, the anaerobic digestion is considered to be better than the other systems (Landfilling, Incineration, and Compost) and is higher in ranking. On the contrary, the landfilling and incineration systems rank last in most of the cases. More specifically, in most of the cases (10 out of 12), the first in ranking SWT system is considered to be the anaerobic digestion and the worst (7 out of 12) is Landfilling. There is a need for improvement in the design of this treatment system, site location, size and management of the disposal sites. Existing practices must be improved immediately as they create environmental problems. It should be noted that the rank of SWT systems can be changed according to the future system development.

Table 3. Overall criteria weights for each scenario.

Criteria	Criteria Weights for each case (%)											
	Base Case	Case #01	Case #02	Case #03	Case #04	Case #05	Case #06	Case #07	Case #08	Case #09	Case #10	Case #11
Environmental impacts	58%	25%	100%	00%	00%	00%	70%	10%	10%	10%	35%	35%
Socio-cultural aspects	20%	25%	00%	100%	00%	00%	10%	70%	10%	10%	35%	15%
Technical aspects	13%	25%	00%	00%	100%	00%	10%	10%	70%	10%	15%	15%
Economic aspects	09%	25%	00%	00%	00%	100%	10%	10%	10%	70%	15%	35%

Table 4. Overall evaluation and ranking SWT system for each case

Solid waste treatment system	Base Case		Case #01		Case #02		Case #03		Case #04		Case #05	
	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank
Anaerobic digestion	42%	1	39.7%	1	46.2%	1	39.1%	1	34.7%	2	38.4%	1
Landfilling	13%	4	15.7%	4	9.9%	4	16.5%	3	17.0%	3	19.3%	4
Incineration	18%	3	16.2%	3	21.7%	3	8.8%	4	13.5%	4	20.9%	3
Compost	27%	2	28.5%	2	22.2%	2	35.7%	2	34.8%	1	20.4%	2

Solid waste treatment system	Case #06		Case #07		Case #08		Case #09		Case #010		Case #011	
	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank	Score (%)	Rank
Anaerobic digestion	43.6%	1	37.7%	1	35.2%	1	37.2%	1	38.9%	1	38.8%	1
Landfilling	12.2%	4	17.2%	3	17.5%	3	18.9%	3	15.9%	3	16.5%	4
Incineration	19.5%	3	10.6%	4	13.4%	4	17.9%	4	14.5%	4	17.0%	3
Compost	24.7%	2	34.5%	2	33.9%	2	26.0%	2	30.7%	2	27.7%	2

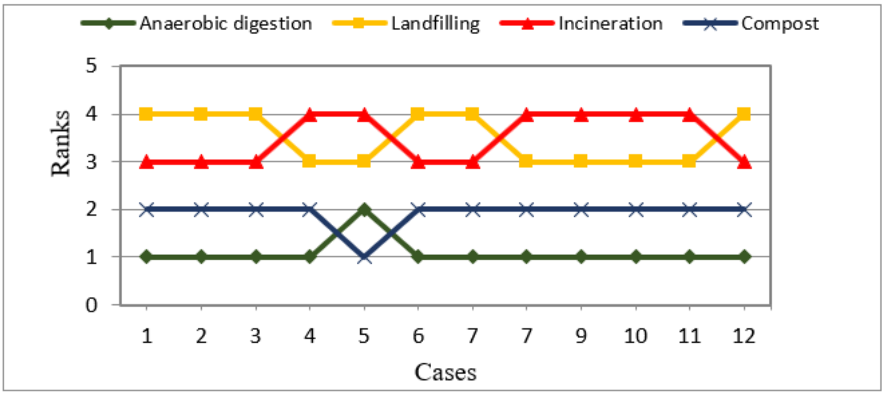

Figure 15. Sensitivity analysis for the 12 cases of the SWM

We feel the proposed method plays an important role in ranking of waste treatment systems, especially when it is in a situation where dynamic, and complex real-world problems. One of the most important advantages of the proposed approach is that it is based on a pair-wise comparison. Moreover, the method computes the inconsistency index, which is used to determine whether a respondent answered similar items in a consistent manner.

7. Conclusion

Undoubtedly, the waste treatment system in Libya is very poor, for instance, more than 97% of the waste is dumped in uncontrolled open areas. As a result, Libyan authorities need to take urgent steps in order to address the current situation. In this study, the multi-criteria decision-making approach is identified as a useful means for an integrated evaluation of the appropriate treatment options for the SWM. The methodology presented here can be used as a well-organized, strategic decision supporting tool for decision makers, politicians, and planners. It is essential to have consistent goals and objective information about the evaluation process of anaerobic digestion suitability for solid waste treatment based on environmental, sociocultural, technical, and economic aspects. Clearly, the anaerobic digestion and composting treatment systems are the two most preferred alternatives. Furthermore, a large part of the used fertilizer in the agriculture is imported from abroad, and most of the local fertilizer industries are not competitive in the today market. Also, the results show that the incineration alternative is in the last order, due to the inability to compete with current power generation methods in the country (e, g. Power generation using fuel oil and natural gas). Furthermore, Libya is also considered as rich country with renewable energy resources such as solar and wind energy. However, waste incineration is not a competitive alternative renewable energy. The performance of the treatment options based on the criteria mentioned earlier is a robust one similar to the synthesis results.

As anaerobic digestion is based on a naturally occurring biological process which produces biogas through anaerobic digestion, this can lead to reduce the main environmental problems of increasing organic waste production and increasing carbon dioxide in the atmosphere. Moreover, investments in this waste management facility can be considered to offer another source of revenue generation for waste management practitioners. Consequently, they facilitate and at the same time lighten the burden of waste management incurred by the municipal government. Anaerobic digestion technology has tremendous application in the future for sustainability of both environment and agriculture, with the production of energy as an extra benefit.

As the municipal governments do not have adequate options to dispose their waste, it is suggested that the proposed project must be implemented within a period of two to three years. It is needed that local authority's search for potential investors, with technical advice and support from international organizations in this aspect, to achieve this feasible project.

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A FUZZY GOAL PROGRAMMING METHOD TO SOLVE CONGESTION MANAGEMENT PROBLEM USING GENETIC ALGORITHM

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Abstract: *The objective of this work is to present a priority-based fuzzy goal programming (FGP) method for solving the congestion management (CM) problem in electric power transmission lines by employing genetic algorithm (GA). To formulate the model for this problem, membership functions which are associated with the fuzzy model goals are converted into membership goals by assigning highest membership value (unity) as goal level and adding under- and over-deviational variables to each of them. In solution process, a GA computational scheme is addressed within the framework of FGP model to achieve aspired goal levels of goals according to their priorities in imprecise environment. The standard IEEE 30-Bus 6-Generator test system is taken as a case example to show the effectiveness of the approach. A comparison of model solution is also compared with solution of another approach studied previously.*

Keywords: *Congestion Management; Fuzzy Goal Programming; Genetic Algorithm; Membership Function; Overload Alleviation; Particle Swarm Optimization*

1. Introduction

Congestion in thermal power supply system in Bhattacharya et al. (2001) refers to overloading situation in transmission lines when thermal bounds and line capacities of the power supply system are violated in Chung et al. (2015). Congestion actually occurs when power flow in a transmission line is higher than the flow allowed by operating reliability limits in Bachtiar Nappu & Arief (2016). As such, congestion in power system would have to be rectified as and when needed to ensure system security. Further, a lack of paying proper attention to congestion of the system may lead to widespread blackouts which give birth to negative impact to social and economic perspectives. Therefore, congestion management in Emami & Sadri (2012)

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appears as one the key issues to maintain security and reliability of transmission network.

The mathematical programming method for estimating voltage dropping and line loading for out of service of each network element was first introduced in Abiad & Stagg (1963) in 1963. Then, different classical optimization methods based on load flow were studied for CM in Mamandur & Berg (1978), and Medicherla et al. (1979) in the past century. The decomposition of spot prices to reveal congestion cost component in a pool model was presented in Finney et al. (1997). The DC-optimal power flow (DC-OPF) based approach to compute congestion cost was also propounded by Singh et al. (1998). The real-time operational environment based CM was studied in Fang & David (1999) and Wang & Song (2000) in last two decades. An optimal dispatch with the consideration of dynamic security constraints for CM was discussed in Singh & David (2000). Rau (2000) presented the AC-OPF driven approach to CM along with congestion cost allocation. Then, an effective model to location of unified power flow controller (UPFC) for CM was deeply studied in Verma et al. (2001). The use of *Thyristor-Controlled Series Compensation* (*TCSC)* to reduce congestion cost is also presented in Lee (2002). To manage congestion, a minimum load curtailment problem was proposed in Rodrigues & DaSilva (2003). An OPF model with multiplicity of objectives and a set of voltage security constraints was also discussed in Milano et al. (2003) with regard to avoiding congestion through the use of location marginal price. The use of rescheduling of generation and load with voltage security constraints for CM was also discussed in Yamin & Shahidepour (2003). An efficient CM approach using real and reactive power rescheduling via optimal allocation of reactive power resources was proposed in Kumar et al. (2004). A simple cost effective model for generation rescheduling and load shedding was also studied in Talukdar et al. (2005) in the past.

The heuristic methods in Hazra & Sinha (2007); Dutta & Singh (2008); Balaraman & Kamaraj (2010) for global optimizations have been made successfully to solve CM problems in the recent past. Hazra & Sinha (2009) put forth an efficient approach based on fuzzy estimation for identifying collapse sequences to reach the optimal solution of a CM problem. Fuzzily described adaptive bacterial foraging algorithm and gravitational search method have also been studied in Venkaiah & Vinod Kumar (2011) and Vijaya Kumar et al. (2013) previously.

To overcome the various drawbacks associated with the previous approaches concerning CM in thermal power supply system, a *priority-based* FGP method for multiobjective decision making (MODM) is addressed in this paper to model CM problem and a GA computational scheme is adapted to reach decision in imprecise premises. In model formulation, fuzzy representations of different objectives are considered for minimization of overload alleviation and operation cost subject to various constraints associated with the problem. The experimental test on standard IEEE 6-Generator 30-bus system is made to expound the effective use of the method. The solution is also compared with solution achieved by using Particle Swarm Optimization (PSO) technique in Hazra & Sinha (2007) is performed to present superiority of the proposed method.

Now, FGP model formulation of a MODM problem is discussed in the section 2.

2. FGP problem formulation

In fuzzy environment, objectives are generally described fuzzily, whereas structural resource constraints may be fuzzy or crisp and that depends on how the model parameters are involved there in the decision situation.

In line with the work of Dubois (1987), the generic form of a Fuzzy Programming (FP) problem can be exhibited as follows.

Find X for:

$$F_k(X)\begin{Bmatrix}\gtrsim\\ \lesssim\end{Bmatrix}g_k;$$

$$\textit{Subject to}:$$

$$X \in S = \begin{Bmatrix}\le\\ =\\ \ge\end{Bmatrix}b,\ \left\{X \in R^n; b^T \in R^m\right\} \tag{1}$$

$$X^L \le X \le X^U;$$

$$X \ge 0$$

where X is a vector of decision variables, g_k be the imprecise goal level of kth objective $F_k(X)$, k = 1,2,...., K, $\gtrsim$ and $\lesssim$ indicate fuzziness of $\ge$ and $\le$ restrictions, respectively, and where A is a real matrix and b is a constant vector and T means transposition, X^L and X^U denote the vectors of lower- and upper-limits, respectively, of the vector X, and where L and U indicate lower and upper, respectively. Also, it is assumed that the feasible region $S(\neq \varphi)$ is bounded.

Now, characterization of fuzzy goals is by associated membership functions concerned with measuring degree of achievement of each of them in a decision making horizon.

2.1. Characterization of membership function

Let $t_{\ell k}$ and t_{uk} be lower- and upper-tolerance ranges, respectively, regarding achievement of aspired level g_k of kth fuzzy goal.

Then, membership function, say $\mu_k(X)$, associated with $F_k(X)$ can be characterized as follows.

For $\gtrsim$ type of constraint, $\mu_k(X)$ appear as in Zimmermann (1987):

$$\mu_k(X) = \begin{cases} 1 & when\ \ F_k(X)\ ^3\ g_k, \\ \dfrac{F_k(X)-(g_k - t_{\ell k})}{t_{\ell k}} & if\ \ \ g_k - t_{\ell k} \le F_k(X) < g_k, \\ 0 & when\ \ F_k(X) < g_k - t_{\ell k}, \end{cases} \tag{2}$$

where $(g_k - t_{\ell k})$ denotes the lower-tolerance limit to achieve the stated fuzzy goal.

Further, for $\lesssim$ type of constraint, $\mu_k(X)$ can be presented as:

$$\mu_k(X) = \begin{cases} 1 & \text{if } F_k(X) \le g_k, \\ \dfrac{(g_k + t_{uk}) - F_k(X)}{t_{uk}} & \text{if } g_k < F_k(X) \le g_k + t_{uk}, \\ 0 & \text{if } F_k(X) > g_k + t_{uk}, \end{cases} \tag{3}$$

where ($g_k + t_{uk}$) denotes the upper-tolerance limit to achieve the stated fuzzy goal.

The membership functions in (2) and (3) can be graphically depicted as in Figure 1 and Figure 2, respectively.

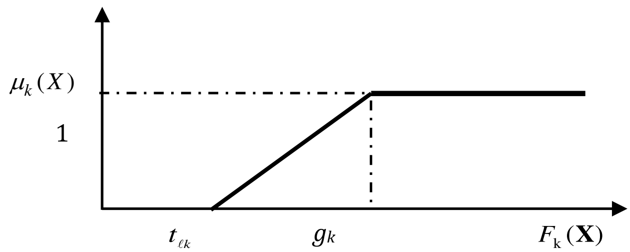

Figure 1. Represented Graph of the membership function in (2)

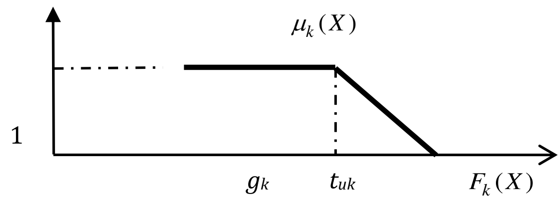

Figure 2. Graph of the membership function in (3)

The formulation of an FGP model under a *pre-emptive* priority structure by defining membership goals is described in section 2.2.

2.2. FGP model

Since in a MODM context, various conflicting goals are dealt for achieving the aspired levels, *priority-based* FGP is adopted by Pal & Chakraborti (2013) for formulating the model of the problem. In *priority-based* FGP, priorities are assigned to goals according to importance of achieving goal levels, where a set of goals which seems equally important for their goal achievements are included at a same priority level and numerical weights are introduced there according to relative weights of importance to achieve goal levels.

The generic form of a *priority-based* FGP model can be presented as follows.

Find so $X(x_1, x_2, ..., x_n)$ as to:

Minimize $Z = [P_1(d^-), P_2(d^-), ..., P_r(d^-), ..., P_R(d^-)]$

and satisfy (4)

$$\frac{F_k(X) - (g_k - t_{\ell k})}{t_{\ell k}} + d_k^- - d_k^+ = 1$$

$$\frac{(g_k + t_{uk}) - F_k(X)}{t_{uk}} + d_k^- - d_k^+ = 1$$

subject to the given constraints set as described in (1).

Here $d_k^-, d_k^+ \geq 0$, $d_k^- d_k^+ = 0$, $k = 1, 2, ..., K$, are under and over-deviational variables introduced to *k*th goal, and where Z represents the vector of R priority achievement function. $P_r(d^-)$ is a linear function of vector of weighted under-deviational variables, and $P_r(d^-)$ is of the form:

$$P_r(d^-) = \sum_{k \in K} w_{rk}^- d_{rk}^-; \quad k = \{1, 2, ..., K\} \tag{5}$$

where d_{rk}^- is renamed for d_k^- to represent it at *r*th priority level, w_{rk}^- (> 0) is the numerical weight associated with d_{rk}^- and it is the weight of importance of achieving *k*th goal level relative to others which are grouped together at *r*th priority level and where w_{rk}^- values are determined in Pal et al. (2003) as:

$$w_{rk}^- = \begin{cases} \frac{1}{(t_{lk})_r} \\ \frac{1}{(t_{uk})_r} \end{cases} \tag{6}$$

for $\mu_k(X)$ in (2) and (3), respectively, where $(t_{\ell k})_r$ and $(t_{uk})_r$ are used to present $t_{\ell k}$ and t_{uk}, respectively, at *r*th priority level. Also, the relationship among the priorities is $P_1 >>> P_2 >>> ... >>> P_r >>> ... >>> P_R$, where ">>>" implies "much greater than".

In the formulated model, the notion of using *pre-emptive priorities* is that the goals which are at *r*th priority level P_r are preferred most to achieve the corresponding aspired levels before taking the achievement problem of goals included at next lower priority level P_{r+1}.

Now, to design the model of a CM problem, it is worth noting that objectives and some system constraints are with nonlinear characteristics. To avoid computational complexity in Awerbach et al. (1976) with nonlinearity in model goals and constraints as well as to overcome the burden of hand calculations for linearization of them using approximation technique in Pal et al. (2009), GA as a goal satisfier in Deb (2002) for multiobjective decision analysis is considered for searching solution of the problem. The GA computational scheme is presented in the section 3.

3. GA Computational scheme for CM problem

The three probabilistically defined operators in Goldberg (1989): *selection*, *crossover* and *mutation* are used to generate new population (i.e., new solution candidates) in the GA scheme to search solution. The real-value coded chromosomes are considered to perform operations with GA in random fashion. To evaluate a function, say $Eval\ (E)_v$, the fitness score of a chromosome, say *v,* according to maximization or minimization of an objective function defined by decision maker (DM) in the decision making context. In the proposed MODM model, since $Eval\ (E)_v$ is a single-objective linear program, roulette-wheel selection, arithmetic crossover and uniform mutation are adapted to search decision of the problem.

The algorithmic steps of GA computational process are described in the following section 3.1.

3.1. GA algorithm

Step 1. Representation and initialization.

Let E denote the double vector representation of chromosome in a population as $E = (x_1, x_2, ..., x_n)$. The population size is defined by pop_size, and pop_size chromosomes are randomly initialized in the domain of searching solution. .

Step 2. Fitness function.

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as:

$$Eval(E_r)_v = (Z_r)_v = \left(\sum_{k=1}^{K} (w_{rk}^{-} d_{rk}^{-}) \right)_v, v = 1, 2, 3, ..., pop\ size \quad (7)$$

where $(Z_r)_v$ is achievement function (*Z*) in (4) for measuring the fitness value of *v*th chromosome, when attainments of goals included at *r*th priority level P_r is considered.

The best value of a chromosome is determined as

$$E^* = \min\left\{ Eval(E)_v \mid v = 1, 2, ..., pop\ size \right\} \quad (8)$$

in course of searching minimum value of achievement function.

Step 3. Selection Stage.

The simple roulette-wheel scheme is employed for selection of two parents for mating purpose in solution search process.

Step 4. Crossover Stage.

The probability of crossover is defined by parameter p_c. The single-point crossover in Goldberg (1989) is applied here with a view to obtaining offspring that always satisfy linear constraints set.A chromosome is selected as a parent, if for a random number $r \in [0,1], r < p_c$ is satisfied.

For example, if two parents $E_1, E_2 \in S$ are selected, then the arithmetic crossover is defined as: $E_1^1 = \alpha_1 E_1 + \alpha_2 E_2$, $E_2^1 = \alpha_2 E_1 + \alpha_1 E_2$, for generating two offspring E_1^1 and E_2^1, where $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$, $E_1^1, E_2^1 \in S$.

Step 5. Mutation.

A parameter p_m is defined as the probability of mutation. The mutation operation is made uniformly, where for a random number $r \in [0,1]$, a chromosome is selected for mutation provided that $r < p_m$.

Step 6. Termination.

The solution search process terminates when best decision for a chromosome is received at a certain generation number in decision making premises.

The pseudo code of the GA is as follows:

Initialize population of chromosomes $E\ (x)$

Evaluate the initialized population by computing its fitness measure

While not termination criteria do

$x := x+1$

Select $E\ (x+1)$ *from* $E(x)$

Crossover $E(x+1)$

Mutate $E(x+1)$

Evaluate $E(x+1)$

End While

Now, formulation of FGP model of CM problem is discussed in the section 4.

4. CM problem Formulation

The various objectives that are inherently associated with a CM problem are defined as follows.

4.1 Defining the objective functions

(a) *"Overload alleviation" function*.

In decision premises, the alleviation of overload on a transmission line is essentially needed to ensure security and stability of system, and thereby taking preventing measure against happening of system outage. Here, transmission line overload can be alleviated by line switching, generation rescheduling and load shedding.

The alleviation of overload in the system takes the form:

$$F_1 = \sum_{i=1}^{NL} (S_i - S_i^{\max})^2 \tag{9}$$

where, F_1 represents cumulative overload, NL is number of overloaded lines, and where S_i *and* S_i^{max} be the MVA flow and MVA capacity of line *i* in power supply system, respectively. Also, square form of objective is made to avoid masking effect.

(b) *Operational cost function*.

In this context, the total incurring cost for thermal power plant operation and which is associated with CM problem can be expressed as sum of the fuel cost and cost of load shedding. The total operational cost function is expressed as:

$$F_2 = \sum_{i=1}^{NG} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) + \sum_{k=1}^{PL} (a_k^{'} + b_k^{'} L_{shd,k} + c_k^{'} L_{shd,k}^2) \tag{10}$$

where F_2 denotes total operating cost, NG be the number of participating generators, PL is used to represent number of associated loads, P_{Gi} is generation of power from *i*th generator, $L_{shd,k}$ is amount of load shedding at bus *k*, and where a_i, b_i, c_i are cost coefficients of objective associated with generation of power from generator G_i, and $a_k^{'}, b_k^{'}, c_k^{'}$ are cost coefficients of objective associated with load shedding at bus *k*.

(c) *Power-loss function*.

A certain function called real power-loss function which is inherent to a power transmission line and directly affect the ability to transfer power. The mathematical expression of real power-loss function, F_3 (MW) can be defined as in Talukdar et al. (2005):

$$F_3 = \sum_{i,j=1}^{TL} g_l [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_i - \delta_j)] \tag{11}$$

where TL represents total transmission lines, g_l be the conductance of lth line, V_i and V_j are voltage magnitudes, δ_i and δ_j are voltage phase angles at the end buses i and j of lth line, respectively, of the system, where 'cos' designates ***cosine*** function.

4.2. Definitions of system constraints

The constraints on the power generation system r are as follows:

a) *Power balance constraints*.

The power balance constraints appear as:

$$\begin{aligned} P_{Gi} - P_{Di} - V_i \sum_{j=1}^{H} V_j [g_{ij} \cos(\delta_i - \delta_j) + b_{ij} \sin(\delta_i - \delta_j)] = 0 \\ Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{H} V_j [g_{ij} \sin(\delta_i - \delta_j) + b_{ij} \cos(\delta_i - \delta_j)] = 0 \end{aligned} \tag{12}$$

where H be the number of buses, P_{Gi} and Q_{Gi} are real- and reactive-power of the generator connected to ith bus, respectively, and where P_{Di} and Q_{Di} be real- and reactive-power of the load connected to ith bus, respectively, g_{ij} and b_{ij} indicate transfer conductance and susceptance between bus i and bus j, respectively, δ_i and δ_j are bus voltage angles of buses i and j, respectively.

b) *Determining the Generation capacity & voltage constraint*.

Similar to conventional power generation and dispatch system, constraints on power generation and voltage appear as:

$$\begin{aligned} & P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max}, \\ & Q_{G_i}^{\min} \le Q_{G_i} \le Q_{G_i}^{\max}, \\ & V_i^{\min} \le V_i \le V_i^{\max};\ i = 1, 2, \ldots, N. \end{aligned} \tag{13}$$

Now, to show the effective use of the proposed approach, an example is considered in the section 5.

5. Case example

The IEEE 30-bus 6-generator test system Talukdar et al. (2005) is addressed to present the effectiveness of the method. The diagram of the system depicted in Figure 3 below.

The diagram shows that the system is with 6 generators, 41 lines and 30 buses. The total demand on 21 load buses is 283.4 MW.

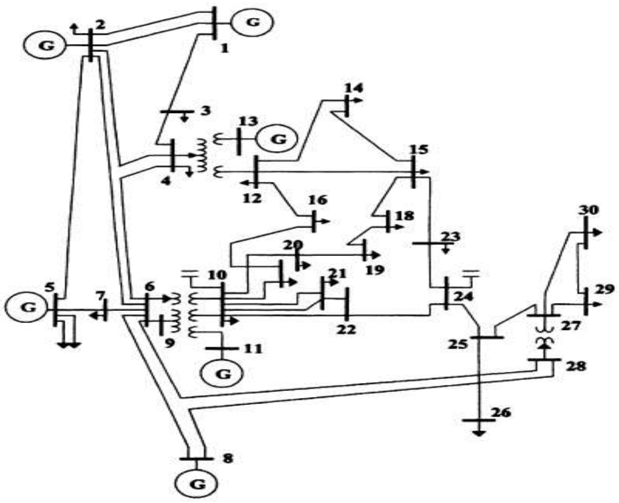

Figure 3. Diagram of IEEE 30-bus test system

The model data were collected from the studies (Talukdar et al., 2005; Hazra & Sinha, 2007) made previously. The cost-coefficients of power generation and that of load shedding are presented in the Table 1 and Table 2, respectively.

Table 1. Power generation cost –coefficient data

Generator Type (T_i)	Maximum gen capacity (MW)	A	b	c
T_1	< 25	0.0	2025.00	1.500
T_2	50	0.0	1875.00	1.425
T_3	100	0.0	1800.00	1.350
T_4	200	0.0	1650.00	1.250
T_5	250	0.0	1575.00	1.500
T_6	300	0.0	1575.00	1.250
T_7	350	0.0	1500.00	1.350
T_8	400	0.0	1500.00	1.250
T_9	500	0.0	1200.00	1.500
T_{10}	> 500	0.0	1200.00	1.000

Table 2. Load shedding cost-coefficient data

Load in a bus (MW)	$a_k^{'}$	$b_k^{'}$	$c_k^{'}$
<=10	0.0	1200	1.00
<=20	0.0	1200	1.50
<=30	0.0	1500	1.25
<=40	0.0	1500	1.35
<=50	0.0	1575	1.25
<=60	0.0	1575	1.5
<=75	0.0	1650	1.25
<=100	0.0	1800	1.35
<=125	0.0	1875	1.425
>125	0.0	2025	1.5

The data associated with transmission lines and loads at buses are presented in the Table 3 and Table 4, respectively.

Table 3. Transmission-line data

Line No.	From Bus No.	To Bus No.	Line Impedance		Line No.	From Bus No.	To Bus No.	Line Impedance	
			R(p.u.)	X(p.u.)				R(p.u.)	X(p.u.)
1	1	2	0.0192	0.0575	22	15	18	0.1070	0.2185
2	1	3	0.0452	0.1852	23	18	19	0.0639	0.1292
3	2	4	0.0570	0.1737	24	19	20	0.0340	0.0680
4	3	4	0.0132	0.0379	25	10	20	0.0936	0.2090
5	2	5	0.0472	0.1983	26	10	17	0.0324	0.0845
6	2	6	0.0581	0.1763	27	10	21	0.0348	0.0749
7	4	6	0.0119	0.0414	28	10	22	0.0727	0.1499
8	5	7	0.0460	0.1160	29	21	22	0.0116	0.0236
9	6	7	0.0267	0.0820	30	15	23	0.1000	0.2020
10	6	8	0.0120	0.0420	31	22	24	0.1150	0.1790
11	6	9	0.0000	0.2080	32	23	24	0.1320	0.2700
12	6	10	0.0000	0.5560	33	24	25	0.1885	0.3292
13	9	11	0.0000	0.2080	34	25	26	0.2544	0.3800
14	9	10	0.0000	0.1100	35	25	27	0.1093	0.2087
15	4	12	0.0000	0.2560	36	28	27	0.000	0.3960
16	12	13	0.0000	0.1400	37	27	29	0.2198	0.4153
17	12	14	0.1231	0.2559	38	27	30	0.3202	0.6027
18	12	15	0.0662	0.1304	39	29	30	0.2399	0.4533
19	12	16	0.0945	0.1987	40	8	28	0.6360	0.2000
20	14	15	0.2210	0.1997	41	6	28	0.0169	0.0599
21	16	17	0.0824	0.1932					

Table 4. Bus-load data

Bus No.	Load		Bus No.	Load	
	P(p.u.)	Q(p.u.)		P(p.u.)	Q(p.u.)
1	0.000	0.000	16	0.035	0.018
2	0.217	0.127	17	0.090	0.058
3	0.024	0.012	18	0.032	0.009
4	0.076	0.016	19	0.095	0.034
5	0.942	0.190	20	0.022	0.007
6	0.000	0.000	21	0.175	0.112
7	0.228	0.109	22	0.000	0.000
8	0.300	0.300	23	0.032	0.016
9	0.000	0.000	24	0.087	0.016
10	0.058	0.020	25	0.000	0.000
11	0.000	0.000	26	0.035	0.023
12	0.112	0.075	27	0.000	0.000
13	0.000	0.000	28	0.000	0.000
14	0.062	0.016	29	0.024	0.009
15	0.082	0.025	30	0.106	0.019

Table 5 exhibits various simulation runs which were carried out in the test system.

Table 5. Simulation runs

Run	Different simulation Cases
1	Overload simulation with reduction of capacity of line 1-2 from 130 MW to 50 MW.
2	Overload simulation with reduction of capacity of line 1-3 and 2-4 from 130 MW to 50 MW and 65 MW to 15 MW.
3	Overload simulation for outage of unit 3 at bus 5 and with reduction of capacity of line 2-5 from 130 MW to 50 MW.

In this case, the Optimization Toolbox under MATLAB (MATLAB R2010a) has been employed to conduct the experiments by employing GA at different stages for program evaluation. The computational environment is Intel Pentium IV with 2.66 GHz. Clock-pulse and 3 GB RAM. In the solution search process, initial population= 50; Roulette-Wheel selection; Single-point crossover with probability= 0.8; Mutation probability= 0.07 and Maximum generation number= 100 are taken into account for exploration and exploitation of search space in the domain of interest.

Then, following the procedure and fitting the data presented in Tables 1 - Table 5, the membership goals can be obtained by addressing the second goal expression in (4).

The executable FGP models for individual three simulation runs under a priority structure considered for the system are presented as follows.

Run-1: Simulation of system under overload by reducing capacity of line 1-2 from 130 MW to 50 MW

The model appears as

Find $(S_{1\text{-}2}, P_{G_i})\ \{i = 1,2,5,8,11,13\}$ so as to:

$$Minimize\ \ Z = \left[P_1\left(\frac{1}{2.50}\right)d_1^-,\ P_2\left\{\left(\frac{1}{27942}\right)d_2^- + \left(\frac{1}{2}\right)d_3^-\right\}\right]$$

and satisfy

$$\mu_{F_1} : [\{3.0 - (S_{1-2} - 50)^2\} / (3.0 - 0.5)] + d_1^- - d_1^+ = 1,$$

$$\mu_{F_2} : [\{547942 - (1650P_1 + 1.25P_1^2 + 1875P_2 + 1.425P_2^2 + 1875P_5 + 1.425P_5^2 + 2025P_8 + 1.5P_8^2 + 2025P_{11} + 1.5P_{11}^2 + 2025P_{13} + 1.5P_{13}^2)\} / (537942 - 530000)\}] + d_2^- - d_2^+ = 1,$$

$$\mu_{F_3} : [(5.50 - F_3) / (5.50 - 3.5)] + d_3^- - d_3^+ = 1,$$

subject to

$$P_{G1} - V_1[V_2\{5.2246\cos(\delta_1 - \delta_2) - 15.6467\sin(\delta_1 - \delta_2)\} + V_3\{1.2437\cos(\delta_1 - \delta_3) - 5.0960\sin(\delta_1 - \delta_3)] = 0 \quad (14)$$

$$P_{G2} - 0.217 - V_2[V_1\{5.2246\cos(\delta_2 - \delta_1) - 15.6467\sin(\delta_2 - \delta_1)\} + V_4\{1.7055\cos(\delta_2 - \delta_4) - 5.1974\sin(\delta_2 - \delta_4) + V_5\{1.1360\cos(\delta_2 - \delta_5) - 4.7725\sin(\delta_2 - \delta_5)\} + V_6\{1.6861\cos(\delta_2 - \delta_6) - 5.1165\sin(\delta_2 - \delta_6)\}] = 0 \quad (15)$$

$$P_{G5} - 0.942 - V_5[V_2\{1.1360\cos(\delta_5 - \delta_2) - 4.7725\sin(\delta_5 - \delta_2)\} + V_7\{2.9540\cos(\delta_5 - \delta_7) - 7.4493\sin(\delta_5 - \delta_7)] = 0 \quad (16)$$

$$P_{G8} - 0.3 - V_8[V_6\{6.2893\cos(\delta_8 - \delta_6) - 22.0126\sin(\delta_8 - \delta_6)\} + V_{28}\{1.4308\cos(\delta_8 - \delta_{28}) - 0.4499\sin(\delta_8 - \delta_{28})] = 0 \quad (17)$$

$$P_{G13} - V_{13}[V_{12}\{-7.1429\sin(\delta_{13} - \delta_{12})\}] = 0 \quad (18)$$

$$P_{G11} - V_{11}[V_9\{-4.8077\sin(\delta_{11} - \delta_9)\}] = 0 \quad (19)$$

$$Q_{G1} - V_1[V_2\{5.2246\sin(\delta_1 - \delta_2) - 15.6467\cos(\delta_1 - \delta_2)\} + V_3\{1.2437\sin(\delta_1 - \delta_3) - 5.0960\cos(\delta_1 - \delta_3)] = 0 \quad (20)$$

$$Q_{G2} - 0.217 - V_2[V_1\{5.2246\sin(\delta_2 - \delta_1) - 15.6467\cos(\delta_2 - \delta_1)\} + V_4\{1.7055\sin(\delta_2 - \delta_4) - 5.1974\cos(\delta_2 - \delta_4) + V_5\{1.1360\sin(\delta_2 - \delta_5) - 4.7725\cos(\delta_2 - \delta_5)\} + V_6\{1.6861\sin(\delta_2 - \delta_6) - 5.1165\cos(\delta_2 - \delta_6)\}] = 0 \quad (21)$$

$$Q_{G5} - 0.942 - V_5[V_2\{1.1360\sin(\delta_5 - \delta_2) - 4.7725\cos(\delta_5 - \delta_2)\} + V_7\{2.9540\sin(\delta_5 - \delta_7) - 7.4493\cos(\delta_5 - \delta_7)] = 0 \quad (22)$$

$$Q_{G8} - 0.3 - V_8[V_6\{6.2893\sin(\delta_8 - \delta_6) - 22.0126\cos(\delta_8 - \delta_6)\} + V_{28}\{1.4308\sin(\delta_8 - \delta_{28}) - 0.4499\cos(\delta_8 - \delta_{28})] = 0 \quad (23)$$

$$Q_{G11} - V_{11}[V_9\{-4.8077\cos(\delta_{11} - \delta_9)\}] = 0 \quad (24)$$

$$Q_{G13} - V_{13}[V_{12}\{-7.1429\cos(\delta_{13} - \delta_{12})\}] = 0 \quad (25)$$

(*Equality constraints*)

$$50 \le P_{G_1} \le 200,\ 20 \le P_{G_2} \le 80,\ 15 \le P_{G_5} \le 50,\ 10 \le P_{G_8} \le 35,\ 10 \le P_{G_{11}} \le 30,\ 12 \le P_{G_{13}} \le 40 \quad (26)$$

$$-20 \le Q_{G_2} \le 100 \text{ , } -15 \le Q_{G_5} \le 80 \text{ , } -15 \le Q_{G_8} \le 60,$$
$$-10 \le Q_{G_{11}} \le 50 \text{ , } -15 \le Q_{G_{13}} \le 60, \qquad (27)$$
$$0.95 \le V_{G_i} \le 1.1 \text{ , } i = 1,2,5,8,11,13$$

(*Generator constraints*)

$$0.85 \le V_{L_i} \le 1.05 \text{ , } i = 2,3,4,5,7,8,10,12,13,14,15,16,17,18,19,20,21,23,24,26,29,30$$

(*Load-bus voltage constraint*)

Run-2: Simulation of system under overload by reducing capacity of line 1-3 and 2-4 from 130 MW to 50 MW and 65 MW to 15 MW

In this case, the executable model is found as:

Find (S_i, P_{Gi}) so as to:

$$Minimize \;\; Z = \left[P_1\left(\frac{1}{15}\right)d_1^-,\, P_2\left\{\left(\frac{1}{21942}\right)d_2^- + \left(\frac{1}{2}\right)d_3^-\right\}\right]$$

and satisfy

$$\mu_{F_1} :[(25-\{(S_{1-3}-50)^2+(S_{2-4}-15)^2\})/(25-10)]+d_1^- - d_1^+ = 1,$$
$$\mu_{F_2} :[\{(551942-(1650P_1+1.25P_1^2+1875P_2+1.425P_2^2+1875P_5+1.425P_5^2+2025P_8+1.5P_8^2$$
$$+2025P_{11}+1.5P_{11}^2+2025P_{13}+1.5P_{13}^2\}/(551942-530000)]+d_2^- - d_2^+ = 1,$$
$$\mu_{F_3} :[(6.50-F_3)/(6.50-4.00)]+d_3^- - d_3^+ = 1,$$

subject to the constraints in (14)-(27).

Run-3: Simulation of system under overload with outage of unit 3 at bus 5 and by reducing capacity of line 2-5 from 130 MW to 50 MW

The executable model is obtained as follows.

Find (S_i, P_{Gi}) so as to:

$$Minimize \;\; Z = \left[P_1\left(\frac{1}{3}\right)d_1^-,\, P_2\left\{\left(\frac{1}{21942}\right)d_2^- + \left(\frac{1}{1.50}\right)d_3^-\right\}\right]$$

and satisfy

$$\mu_{F_1} : [\{5-(S_{2-5}-50)^2\}/(5.00-2.00)]+d_1^- - d_1^+ = 1,$$
$$\mu_{F_2} : \{(551942-(1650P_1+1.25P_1^2+1875P_2+1.425P_2^2+2025P_8+1.5P_8^2+2025P_{11}$$
$$+1.5P_{11}^2+2025P_{13}+1.5P_{13}^2)/(551942-530000)\}+d_2^- - d_2^+ = 1,$$
$$\mu_{F_3} : [(10.00-F_3)/(10.00-8.50)]+d_3^- - d_3^+ = 1,$$

subject to the problem constraints in (14) - (27).

The goal achievement function (Z) defined for the three runs actually describes the evaluation function in GA search process for solving the problem.

The evaluation function for determining the fitness of a chromosome is given as:

$$Eval\,(E_r)_v = (Z_r)_v = \left(\sum_{k=1}^{3}(w_{rk}^- d_{rk}^-)\right)_v, v = 1,2,3,\ldots,50; r = 1,2$$

The best value of objective (Z^*) for the fittest chromosome is determined as:

$$E^* = \min\{Eval(E)_v \mid v = 1, 2, ..., 50\}.$$

The solutions obtained from the three runs of the test system are presented in the Table 6.

Table 6. Solution achievements under different runs

Run	Overload-condition			Solution			
	Line/ Unit	MVA Flow	MVA Capacity	MVA Flow	Power-loss (MW)	Png (ng=1,2,5,8,11,13)	Cost (Rs/ hr)
1	1-2	61.25	50	49.55	3.78	(82.27, 58.59, 50.00, 35.00, 30.00, 31.32)	536633.75
2	1-3 and 2-4	35.44 20.30	50 15	35.50 14.95	4.50	(112.44, 20.46, 50.00, 35.00, 30.00, 40.00)	538219.96
3	2-5 and Out of Unit 3	40.89	50	48.12	8.76	(122.22, 31.24, 0.00, 35.00, 30.00, 40.00) (31.2 MW Load-Shaded)	550064.38

It is clear from the results that the decision is a satisfactory one from the view point of proper management of MVA flow with incurring of minimum operational cost of the power plant in imprecise environment.

To show the effective use of the approach, a performance comparison is made in the section 6.

6. Performance comparison

The PSO technique in Hazra & Sinha (2007) is considered for a solution comparison. The resulting decision is presented in the Table 7.

Table 7. Results of three simulation cases under PSO technique

Case	Overloaded Condition		Solution	
	Line/Unit	MVA Capacity	MVA Flow	Cost (Rs/ hr)
1	Line 1-2	50	49.16	541171
2	Line 1-3 and Line 2-4	50 15	12.31 14.99	542465
3	Line 2-5 and Unit 3 Out	50	49.88	565979

The MVA flow and total incurring cost of the CM problem under the proposed model and PSO technique are diagrammatically presented in Figure 4 and Figure 5, respectively.

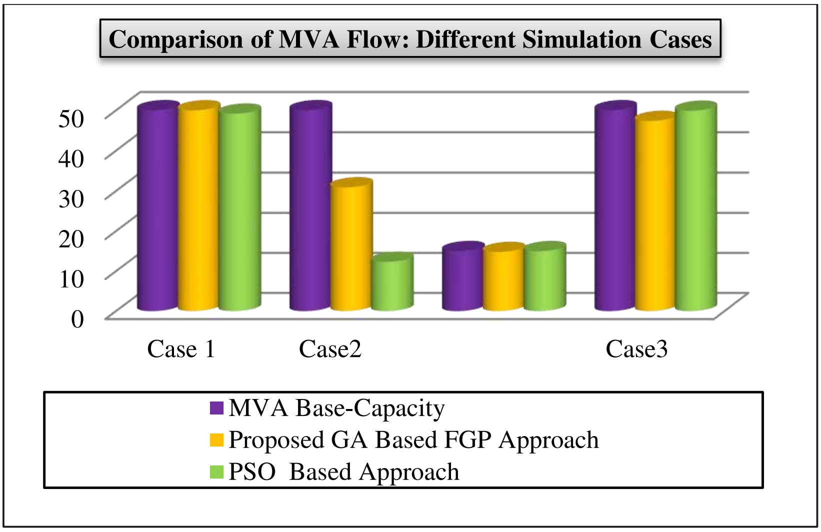

Figure 4. Graphical representation of MVA flow comparison

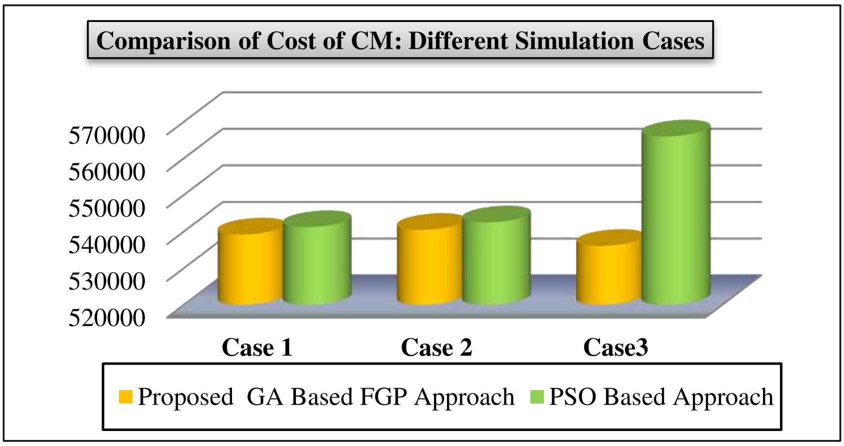

Figure 5. Graphical representation of cost comparison

The result comparisons show that the proposed approach is superior over the PSO to arrive at appropriate decision in imprecise environment.

7. Conclusion

The main merit of the method presented here is that the fuzzy characteristics regarding attainment of objectives values are preserved there in all possible instances of executing the model of the CM problem. Again, computational complexity arising out of the nonlinearity in the goals and constraints associated with the model can easily be avoided here with the use of GA based solution search approach for solving problems in imprecise environment. The proposed method is also advantageous in the sense that here a multi-objective optimization problem can be converted into a goal oriented single objective optimization problem for achieving a compromise solution in the decision making horizon. Further, the proposed approach is flexible enough to accommodate different other restrictions as and when needed for CM in electric power transmission system. However, the use of interval data in Pal (2018), instead of considering fuzziness of model parameters, towards promoting CM performances and thereby improving quality of solution is an interesting alley of research for optimization of a power supply problem.

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AN APPROACH TO RANK PICTURE FUZZY NUMBERS FOR DECISION MAKING PROBLEMS

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Abstract: *Comparison of picture fuzzy numbers (PFNs) are performed using score and accuracy values. But when both of the score and accuracy values are equal, those PFNs are said to be identical. This article presents a novel method to compare the PFNs even when the score and accuracy values of those PFNs are equal. The proposed ranking method is based on positive ideal solution, positive and negative goal differences, and score and accuracy degrees of the picture fuzzy numbers. A new score function is proposed to calculate the actual score value which depends on the positive and negative goal differences and the neutral degree. Finally, a real-life example has been used to illustrate the efficiency of the proposed method.*

Key words*: Picture fuzzy set, picture fuzzy number, positive ideal solution, positive goal difference, negative goal difference.*

1. Introduction

Picture fuzzy set (PFS) (Cong, 2014) is an extension of fuzzy set (FS) (Zadeh, 1965) and intuitionistic fuzzy set (IFS) (Li, 2008, Das et al., 2018, Das et al., 2017), and it can easily manage the uncertain nature of human thoughts by introducing the neutral and refusal membership degrees. In PFS, the authors have divided the hesitation margin of IFS into two parts which are neutral membership degree and refusal membership degree. When both of the neutral and refusal membership degrees are zero, i.e., hesitation margin becomes zero, then the PFS returns to IFS. Sometimes, FS and IFS find it difficult to express the situations when human thoughts involve more options like 'yes', 'abstain', 'no' and 'refusal'. PFS is preferable to handle

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this type of situations using the positive, neutral, negative, and refusal membership degrees. The general election of a country is a good example of such situation where the voters can give their opinions like ‘vote for’, ‘abstain’, ‘vote against’ and the ‘refusal’ of the election (Cuong and Kreinovich, 2013, 2014). Suppose one candidate and 1000 voters are participating in an election process. Among them, 400 voters vote for the candidate, 100 voters are not interested in casting their vote, i.e., they remain to abstain from the voting process, 300 voters are giving their vote against the candidate, and 200 voters refuse to cast their vote for the candidate, i.e., they vote for NOTA. This kind of situation may happen in reality and it is outside the scope of IFS and FS, since FS and IFS don’t support neutrality. As another example, suppose an expert takes the opinion of a person regarding some object. Now the person may say that 0.4 is the possibility that the object is good, 0.3 is the possibility that the object is not good, 0.2 is the possibility that the object is good and as well as not good, and 0.1 is the possibility that (s)he does not know about the object. This issue is also not handled by the FSs or IFSs. Due to having the capability of accepting more opinions, PFS has become an important tool to deal with imprecise and ambiguous information and been applied in many real-life problems by some researchers (Zhang and Xu, 2012, Cuong and Kreinovich, 2013, Cuong, 2013).

Son (2016) investigated the application of PFS in clustering algorithms to exploit the hidden knowledge from a mass of data sets by proposing a hierarchical picture clustering (HPC) method. Motivated by the application of PFS in decision making, Garg (2017) proposed a series of aggregation operators in the context of PFS and presented a decision-making approach using the proposed aggregation operators. Wang et al. (2017) proposed picture fuzzy set based geometric aggregation operator and compared two picture fuzzy numbers (PFNs) using score and accuracy functions. Guiwu (2016) used PFS in decision-making problem and proposed cross entropy of PFSs. Using the idea of cross entropy, the author introduced a new ranking method in PFS environment. Singh (2015) defined correlation coefficient of PFS and applied it to clustering analysis problem. Many authors have contributed to rank the corresponding numbers in the framework of fuzzy sets (Atanassov and Georgiev, 1993, Das et al., 2015, 2017, 2018), intuitionistic fuzzy sets (Atanassov, 1989,Bhatiaand Kumar, 2013, Kumar and Kaur, 2012) and intuitionistic multi fuzzy sets (Li, 2005, Li, 2008, Liu, 2007, Si and Das, 2017). Most of these ranking methods are based on the comparative analysis of a pair of sets, measurement of the distance between the sets, and measurement of the distance of a set from a central point. In the comparative and distance measurement methods, the membership, non-membership and neutral degrees are considered to have similar importance. Another important concern is that the neutral membership degree is considered just like a positive or negative membership degree. No focus is given even when the neutral degree increases or decreases. But in our real life, some situations are totally different and the membership, non-membership and neutral degrees play different roles and have various types of functionality in the decision making process or ranking among them.

In this article, we propose a new method to calculate the score to rank the PFNs using positive ideal solution, negative ideal solution and average neutral value of the alternatives. Neutral degree of PFS has an active role in our proposal. We consider the average value of the neutral degree as a pivot point concerning the all other neutral degrees. Then, we provide a practical example to analyze the proposed method for ranking to take the decision.

Remaining of the article organized is as follows. Some relevant ideas of picture fuzzy sets are recalled in Section 2. We propose the new ranking method in Section 3 followed by a real-life example in Section 4. A comparative study is given in Section 5. Finally, we conclude in Section 6.

2. Preliminaries

This section briefly presents the relevant ideas of picture fuzzy set and some of its operations.

2.1 Picture fuzzy set

A picture fuzzy set (PFS) *A* on the universe X is an object in the form of

$$A=\{(x,\mu_A(x),\eta_A(x),\nu_A(x))|x\in X\} \quad (1)$$

where $\mu_A(x)\in[0,1]$ be the degree of positive membership of *x* in *A*, similarly $\eta_A(x)\in[0,1]$ and $\nu_A(x)\in[0,1]$ are respectively called the degrees of neutral and negative membership of *x* in *A*. These three parameters $(\mu_A(x),\eta_A(x)\ \textit{and}\ \nu_A(x))$ of the picture fuzzy set *A* satisfy the following condition $\forall x\in X, 0\le\mu_A(x)+\eta_A(x)+\nu_A(x)\le 1$.

Then, the degree of refusal membership $\rho_A(x)$ of *x* in *A* can be estimated accordingly,

$$\forall x\in X, \rho_A(x)=1-(\mu_A(x)+\eta_A(x)+\nu_A(x)) \quad (2)$$

The neutral membership $(\eta_A(x))$ of *x* in *A* can be consideredasdegree of positive membership as well as degree of negative membership whereas refusal membership $(\rho_A(x))$ can be explained as not to take care of the system. When, $\forall x\in X, \eta_A(x)=0$, then the PFS reduces into IFS.

For a fixed $x\in A$, $(\mu_A(x),\eta_A(x),\nu_A(x),\rho_A(x))$ is called picture fuzzy number (PFN),where $\mu_A(x)\in[0,1]$, $\eta_A(x)\in[0,1]$, $\nu_A(x)\in[0,1]$, $\rho_A(x)\in[0,1]$ and

$$\mu_A(x)+\eta_A(x)+\nu_A(x)+\rho_A(x)=1 \quad (3)$$

Simply, PFN is representedas $(\mu_A(x),\eta_A(x),\nu_A(x))$.

2.2 Operations on PFS

For two PFSs $A=(\mu_a,\eta_a,\nu_a)$ and $B=(\mu_b,\eta_b,\nu_b)$, Cong (2014) defined some operations as given below.

$$A\cup B=\{(x,\max(\mu_A(x),\mu_B(x)),\min(\eta_A(x),\eta_B(x)),\min(\nu_A(x),\nu_B(x)))|x\in X\} \quad (4)$$

$$A\cap B=\{(x,\min(\mu_A(x),\mu_B(x)),\min(\eta_A(x),\eta_B(x)),\max(\nu_A(x),\nu_B(x)))|x\in X\} \quad (5)$$

$$\overline{A}=\{(x,\nu_A(x),\eta_A(x),\mu_A(x))|x\in X\} \quad (6)$$

Cuong and Kreinovich (2013) and Cuong (2013) defined some properties on PFSs as given below.

$$A \subseteq B \text{ If } \left(\forall x \in X, \mu_A(x) \le \mu_B(x), \eta_A(x) \le \eta_B(x), \nu_A(x) \ge \nu_B(x)\right) \quad (7)$$

$$A = B \text{ If } \left(A \subseteq B \text{ and } B \subseteq A\right) \quad (8)$$

If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$ (9)

$$\overline{\overline{A}} = A \quad (10)$$

2.3 Distance between picture fuzzy sets

Distances between the two PFSs are defined in (Cuong and Son, 2015, Son, 2016). The distance between two PFSs $A = (\mu_a, \eta_a, \nu_a)$ and $B = (\mu_b, \eta_b, \nu_b)$ in $X = \{x_1, x_2, ..., x_n\}$ is calculated as follows.

Normalized Hamming distance

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(\left|\mu_a(x_i) - \mu_b(x_i)\right| + \left|\eta_a(x_i) - \eta_b(x_i)\right| + \left|\nu_a(x_i) - \nu_b(x_i)\right|\right) \quad (11)$$

Normalized Euclidean distance

$$d_E(A,B) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\left(\mu_a(x_i) - \mu_b(x_i)\right)^2 + \left(\eta_a(x_i) - \eta_b(x_i)\right)^2 + \left(\nu_a(x_i) - \nu_b(x_i)\right)^2\right)} \quad (12)$$

Example 1: Let A ={(0.7,0.2,0.1),(0.8,0.1,0.1),(0.7,0.1,0.2)} and B={(0.6,0.2,0.2),(0.8,0.2,0.0), (0.9,0.0,0.1)} are two picture fuzzy sets of dimensions 3. Then

$$d_H(A,B) = \frac{1}{3}\begin{pmatrix} \left(|0.7-0.6| + |0.2-0.2| + |0.1-0.2|\right) \\ +\left(|0.8-0.8| + |0.1-0.2| + |0.1-0.0|\right) \\ +\left(|0.7-0.9| + |0.1-0.0| + |0.2-0.1|\right) \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} (0.1+0.0+0.1) \\ +(0.0+0.1+0.1) \\ +(0.2+0.1+0.1) \end{pmatrix} = \frac{1}{3}(0.2+0.2+0.4) = \frac{0.8}{3} = 0.26$$

Wang et al. (2017) defined some special operations of picture fuzzy set. They proposed the following operations on PFNs $A = (\mu_a, \eta_a, \nu_a)$ and $B = (\mu_b, \eta_b, \nu_b)$.

$$A.B = (\mu_a + \eta_a)(\mu_b + \eta_b) - \eta_a\eta_b,\ \eta_a\eta_b,\ 1-(1-\nu_a)(1-\nu_b) \quad (13)$$

$$A^{\lambda} = (\mu_a + \eta_a)^{\lambda} - \eta_a^{\lambda},\ \eta_a^{\lambda},\ 1-(1-\nu_a)^{\lambda},\ \lambda > 0 \quad (14)$$

Example 2: Let A =(0.7,0.2,0.1) and B=(0.6,0.2,0.2) are two picture fuzzy sets and λ=5.

A.B=(0.7+0.2)*(0.6+0.2)-0.2*0.2, 0.2*0.2, 1-(1-0.1)*(1-0.1)=(0.68,0.04,0.19).

$A^{\lambda}=A^5=(0.7+0.2)^5-(0.2)^5, (0.2)^5, 1-(1-0.1)^5=$ (0.16807-0.00032), 0.00032, 1-0.59

=(0.16,0.00032,0.41)

2.4 Comparison of Picture fuzzy sets

Wang et al. (2017) used the score function and accuracy function to compare the PFSs. Let $C=(\mu_c, \eta_c, \nu_c, \rho_c)$ be a picture fuzzy number, then a score function *S(C)* is being defined as $S(C)=\mu_c-\nu_c$ and the accuracy function *H(C)* is given by $H(C)=\mu_c+\nu_c+\eta_c$ where $S(C)\in[-1,1]$ and $H(C)\in[0,1]$. Then, for two picture fuzzy numbers *C* and *D*

I. If $S(C)>S(D)$, then *C* is higher than *D*, denoted by *C>D*;

II. If $S(C)=S(D)$, then

   a. $H(C)=H(D)$, implies that *C* is equivalent to *D*, denoted by *C=D*;

   b. $H(C)>H(D)$, implies that *C* is higher than *D*, denoted by *C>D*.

Example 3: Let C = (0.7,0.2,0.1) and D=(0.6,0.2,0.2) are two picture fuzzy sets. Now, *S(C)*=0.7-0.1=0.6, *S(D)*=0.6-0.2=0.4, *H(C)*=0.7+0.2+0.1=0.9, *H(D)*=0.6+0.2+0.2=1. Since *S(C)>S(D)*, therefore *C>D*.

3. Proposed method for ranking PFNs

It is known that the ranking of the fuzzy sets depends on the membership value of the elements. Fuzzy numbers with higher membership value are ranked first. In an intuitionistic fuzzy set (IFS), the rank of Intuitionistic fuzzy numbers (IFNs) depend on membership values as well as non-membership values. The IFS which has the highest membership value and smaller non-membership value will have the first rank (Zhang and Xu, 2012). Below, some situations are given, which appears during the ranking of IFNs.

Let $A=(\mu_a, \nu_a)$ and $B=(\mu_b, \nu_b)$ be two IFNs, then

I. If $\mu_a \succ \mu_b$ and $\nu_a \prec \nu_b$ then $A \succ B$

II. If $\mu_a \prec \mu_b$ and $\nu_a \succ \nu_b$ then $A \prec B$

III. If $\mu_a = \mu_b$ and $\nu_a = \nu_b$ then $A = B$

Here, situation 1 and 2 clearly defines the rank between the IFSs *A* and *B*, but unable to provide the rank as given in situation 3, when both the membership and non-membership values are equal. Motivated by the ranking procedure in IFS, Wang et al. (2017) proposed the comparison technique between two PFSs with the help of score and accuracy function. But their proposed method cannot discriminate the PFNs when the score and accuracy values are same. We observed that the neutral membership grade could be considered to contribute in positive membership grade as well as negative membership grade. This motivated us to propose a new ranking method for the PFNs even when the score and accuracy values are equal. The proposed approach is given below in a stepwise manner.

Let $f_i=(\mu_i,\eta_i,\nu_i), i=1,2,\ldots,n$ be the set of PFNs, where μ_i, η_i *and* ν_i respectively denote the positive, neutral and negative membership degree.

Step 1: The positive ideal solution (PIS) $f^{+}=\left(\mu^{+},\eta^{+},\nu^{+}\right)$ of the PFNs $f_i=\left(\mu_i,\eta_i,\nu_i\right)$, $\left(i=1,2.......n\right)$ is determined, where

$$f^{+}=\left(\mu^{+},\eta^{+},\nu^{+}\right)=\left(\max_{i}\mu_i,\min_{i}\eta_i,\min_{i}\nu_i\right) \tag{15}$$

Step 2: The positive goal difference (PGD) μ_i^{*} and negative goal difference (NGD) ν_i^{*} of each of the PFNs $f_i=\left(\mu_i,\eta_i,\nu_i\right)$, $\left(i=1,2,\ldots,n\right)$ are computed by $\mu_i^{*}=\mu^{+}-\mu_i$ and $\nu_i^{*}=\nu_i-\nu^{+}$.

Step 3: Absolute score of each PFN is calculated as

$$p_i=\left(1-\mu_i^{*}\right)-\nu_i^{*} \tag{16}$$

The absolute score of a PFN is computed using the membership and non-membership grade only. It completely ignores the neutral membership grade. However the neutral membership grade has an important contribution in finding the score which is narrated in the following steps.

Step 4: Next the average neutral degree $\bar{\eta}$ is computed, where $\bar{\eta}=\left(\sum_{i=1}^{n}\eta_i\right)/\eta..$

Step 5: Estimate the actual score S_i (given below) of the PFNs $f_i=\left(\mu_i,\eta_i,\nu_i\right)$, $\left(i=1,2,\ldots,n\right)$ using the average neutral degree. When actual scores of the two PFNs f_i and f_j are the same, then go to *Step 6.*

$$S_i=\frac{p_i}{1-\left(\bar{\eta}-\eta_i\right)} \tag{17}$$

Here the actual score will be always a finite value because the difference between average neutral degree and individual neutral degree of an PFN is never equal to 1, i.e., $\left(\bar{\eta}-\eta_i\right)\neq 1$.

Step 6: I) If $\mu_i>\mu_j$ *and* $\eta_i\geq\eta_j$ then $S_i>S_j$.

II) If $\mu_i\geq\mu_j$ *and* $\eta_i<\eta_j$ then

III) If $\nu_i\leq\nu_j$ then $S_i>S_j$ otherwise $S_i<S_j$.

Remark 3.1. Absolute score *pi =1* if $\mu^{+}=\mu_i$ and $\nu^{+}=\nu_i$, i.e., when membership degree highest and non-membership degree is lowest, then the absolute score will be at most.

Remark 3.2. The actual score basically depends on the neutral degree. If the neutral degree of all PFNs are same then actual score equal to absolute score. Similarly, the actual score increases if the neutral degree decreases alternatively actual score decrease when the neutral degree increases.

4. Practical Example

In this section, we present a practical example to demonstrate the evaluation of the students and their ranking concerned with the multiple-choice questions (MCQs) based examination system with picture fuzzy information to illustrate the proposed method. Suppose n be the number of students who are appearing in a competitive examination where the question paper is composed of multiple-choice questions. During the evaluation process, normally this kind of exams assign some positive marks opting for the correct choice and negative marks for opting the wrong choice. This exam system does not consider the non-attempted questions in the evaluation process. In the exam, some students may attempt all the questions while some other students may not attempt all the questions. Now, among the attempted questions, two cases may arise. In the first case, all answers may be correct and in the second case, some answers may be correct while the rest are wrong. Let's consider, μ_i , ν_i and η_i be the percentage of correct answers, wrong answers, and not attempted questions respectively for the i^{th} candidate. In the examination system, we consider that there are no wrong questions or out of syllabus questions. So, $\mu_i + \eta_i + \nu_i = 1$. We assume that there is no refusal membership value of the students in this examination system. The result of the i^{th} student can be presented by a PFN $f_i = (\mu_i, \eta_i, \nu_i)$ where *i=1,2,..,n*. Table 1 shows the results of seven students (T_1, T_2, T_3, T_4, T_5, T_6, T_7) for a particular MCQ based exam using PFNs.

Table 1. Students' results using PFNs

Students	T1	T2	T3	T4	T5	T6	T7
Result	(0.64, 0.22, 0.14)	(0.74, 0.15, 0.11)	(0.72, 0.19, 0.09)	(0.82,0.1, 0.08)	(0.82,0.1 4,0.04)	(0.9,0.05, 0.05)	(0.68,0.1, 0.22)

To find out the ranking of the students, we illustrate the proposed approach as given below.

Step1: Calculate the PIS $f^+ = (0.9, 0.22, 0.03)$ is calculated using Eq. (15) and Table 1.

Step 2: The positive and negative goal differences of individual students are given in Table 2.

Table 2. PGD and NGD of students

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
PGD	0.26	0.16	0.18	0.08	0.08	0.0	0.22
NGD	0.11	0.08	0.06	0.05	0.00	0.02	0.19

Step 3: Table 3 shows the absolute score of each student using Eq. (16).

Table 3. Absolute score of each student

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Absolute Score	0.63	0.76	0.76	0.87	0.92	0.98	0.59

Step 4: Table 4 shows an actual score of each student using Eq. (17).

Table 4. Actual score of each student

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Actual Score	0.68	0.77	0.80	0.84	0.92	0.98	0.59

Now the ranking list of students is found as $T_6 > T_5 > T_4 > T_3 > T_2 > T_1 > T_7$.

5. Comparative Study

To present the comparative study, we have compared the proposed method with the right marks (RM) method (Lesage et al., 2013), multiple mark question (MMQ) method (Tarasowaand Auer, 2013) and score and accuracy function based method (Wang et al., 2017) with the same example stated above in Section 4. As presented in (Lesage et al., 2013), the degree of correctness (μ_i) considers the score of the students which is presented in Table 5. According to the RM method, the more be the score, the higher be the rank. This method does not consider the negative marking for the wrong answers. So the students are privileged to guess the answers to the unknown questions and attempt those questions. At this moment the students utilize the ambiguity of this technique. This technique does not measure the actual knowledge of the students. In Table 5, both the students T_4 and T_5 score 0.82 along with 0.08 and 0.04 incorrect answers respectively. This method does not give any penalty for incorrect answers, and there is no proper method to handle when scores are the same.

Table 5. Score of students according to the right marks method

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Actual Score	0.64	0.74	0.72	0.82	0.82	0.9	0.68

To overcome the drawbacks of right mark method, the authors introduced negative marking (NM) method (Lesage et al., 2013) which measures the score of students as the difference between the degree of correctness (μ_i) and degree of incorrectness (ν_i), which is presented in Table 6. This technique incorporates some penalty for the wrong answers. So, the students try to attempt their known questions to maximize their scores. But the categories of students, who attempt as many questions as possible without adequate knowledge can get the advantage of this technique. This method attempt to minimize the guessing tendency of the students with a penalty of the wrong answers but to guess advantage remains. In this negative marking method, there's no provision for handling the situation when the score of two or more students are the same. In the following table (Table 6), one can find that the score of two students T_2 and T_3 are similar. As a result, we are unable to get the proper rank of the students using the methods presented in (Lesage et al., 2013). But the proposed method is capable of ranking the students even when the score values are equal. The example described in Section 4 illustrates that ranking of the student (T_3) is higher than the rank of the student (T_2) since the actual score (0.80) of the student (T_3) is more than that of the student (T_2) which is (0.77).

Table 6. Score of students according to negative marks method

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Actual Score	0.50	0.63	0.63	0.74	0.78	0.85	0.46

Next multiple mark question (MMQ) (Tarasowa and Auer, 2013) method has been compared with the proposed method. In MMQ method, there are options to mark more than one right choices for a particular question for the checking the depth of knowledge of the students. But in case of mathematical problems, there's generally one correct answer. So, MMQ method is suitable for medical entrance examination but not appropriate for all types of examinations. In our proposed method with a single right answer for every question, we mainly consider the guessing tendency of the examinees and calculate the actual score based on the positive and negative answers and not attempted questions. In this method, if an examinee cleverly attempts more questions based on his assumptions and if these are found wrong, he will lag behind regarding score. Table 7 shows the score value and accuracy value of the individual PFNs associated with the seven students using the score and accuracy function (Wang et al., 2017). Comparison method of two IFNs using score and accuracy functions is mentioned in Section 2.4, where the alternatives are ranked based on their score and accuracy values. But there is no proper clarification when the alternatives have the same score value. In table 7, student T_2 and T_3 have the same score and accuracy values. Therefore we can't compare those two students. But our proposed method can compare two PFNs even if the score and accuracy values are equal. As per the proposed method, the rank of T_3 is higher than that of T_2, i.e., $T_3>T_2$ since the actual scores of the students T_3 and T_2 are 0.80 and 0.77 respectively.

Table 7. Score and accuracy value according to score and accuracy function

Students	T_1	T_2	T_3	T_4	T_5	T_6	T_7
Score value(μ_i - ν_i)	0.50	0.63	0.63	0.74	0.78	0.85	0.46
Accuracy value ($\mu_i + \eta_i + \nu_i$)	1	1	1	1	1	1	1

6. Conclusion

In this article, a new approach is presented to rank the PFNs. The new approach is different and improved from the existing score and accuracy function based approaches in the sense that we can achieve the ranking of alternatives in spite of having an equal score and accuracy values. We consider the neutral membership of the picture fuzzy sets to be the key element to determine the actual score. In the proposed method, we observe that when all neutral degrees of some picture fuzzy numbers are equal, then the score depends only on goal differences. In future, the proposed method can be used to obtain the ranking for the various extensions of fuzzy sets.

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MULTI-CRITERIA EVALUATION + THE POSITIONAL APPROACH TO CANDIDATE SELECTION IN E-VOTING

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Abstract: *E-voting is one of the most important components of e-democracy and includes interesting research topics, such as the mechanisms of participation in elections, technological solutions to e-voting and the efficient application of those in e-voting. Currently, there are numerous voting systems adopted in many countries of the world and each of those has specific advantages and problems. The paper explores the e-voting system as one of the main tools of e-democracy and analyzes its advantages and drawbacks. Voting results always lead to a broad debate in terms of candidate selection and of whether the candidate elected to a position is suitable for that position. At present, the selection of qualified personnel and their appointment to responsible positions in public administration is one of the topical issues. In the paper, multi-criteria decision-making (MCDM) is proposed for the selection of candidates in e-voting. The criteria for candidate selection are determined and the relationship of each candidate with the other candidates is assessed by using a binary matrix. The candidate rank is calculated according to all the criteria. In a numerical experiment, candidate evaluation is enabled based on the selected criteria and ranked by using a positional ranking approach. The proposed model allows for the selection of a candidate with the competencies based on the criteria set out in the e-voting process and the making of more effective decisions as well.*

Key words: *e-government, e-democracy, e-voting, MCDM, candidate selection, election, e-Government Maturity Model, governance.*

1. Introduction

The implementation of information-communication technologies (ICT) has an impact on social, economic and political life. Especially, the development of ICT and e-

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government forming has substantially changed the public governance and political processes.

E-democracy is regarded as the engagement of citizens and government bodies in political relations and processes (Lee 2010; Van der Meer et al. 2014). This stage is characterized by the level of the close participation of citizens in socio-political processes and e-citizen problems. The efficiency in governance can be achieved with the close participation of citizens, as well as civil societies, in the process of politico-administrative decision-making. E-government is forming a new environment in this regard. E-democracy is mentioned as the evolutionary stage of several developmental models of e-government (Lee 2010). According to some researchers, a transition must be made from the use of the term 'e-government' to the use of the term 'e-democracy' (Meier 2012; Taghavifard et al. 2014) because e-democracy is considered as the evolutionary stage of several developmental models of e-government (Lee 2010). The strengthening and development of democratic institutions, the use of ICT and the information infrastructure for the expansion of civil participation in public and political processes reflects the essence of e-democracy (Anttiroiko 2003; Carrizales 2008; Strielkowski et al. 2017).

Currently, the study of the role of e-voting in the countries which have adopted the formation of e-democracy as a priority is deemed as an integral part of investigations in the field of e-democracy (Musial-Karg 2014). The dynamic development of ICT and the enhancement of social media tools have resulted in significant changes in the functioning of modern countries and societies. ICT has started to play an important role in practically all fields of human life, including political processes. As one of the important components of e-democracy, e-voting encompasses interesting research topics, such as participation mechanisms in elections, the provision of legitimacy, technological solutions and the efficient application of those in the e-voting process. In this regard, e-voting can be considered as one of the forms of e-democracy (Musial-Karg 2014). In this study, approaches regarding the development of new e-voting mechanisms are analyzed.

Nowadays, human resources are considered as the main strategic resource of the government. The selection of qualified personnel at the government level and their appointment to responsible positions are important issues in economic and political processes. Candidate selection is understood as a process in which the best candidates are selected for a particular position. Different methods and technologies that help decision-makers to predict how successful a candidate will be in the future workplace are applied in the recruitment and selection processes (Dursun et al. 2010; Kabak et al. 2012; Tuan 2017; Afshari et al. 2017). In the literature, multi-criteria decision-making (MCDM) is widely used in various fields, such as the selection of appropriate personnel in the recruitment process, the choice of equipment in production, the selection of projects, etc. (Kabak et al. 2012; Kazana et al. 2015; Tuan 2017, 2018; Mukhametzyanov & Pamučar 2018). There are research studies on the comparison and review of MCDM (Stanujkic et al. 2013; Zavadskas et al. 2014; Mardani et al. 2015, Khorami et al. 2015).

A literature review highlights a few research studies on the application of MCDM for candidate selection in the election process. Royes et al. (2001) use fuzzy MCDM in election predictions. The use of a computational system was proposed as a practical means of election forecasting. According to the decision-maker (the system user), the proposed flexible system allows for a choice of fuzzy weights and fuzzy evaluation functions in respect of the selection criteria. Kazana (2015) showed that a total of 15 criteria were taken into account when selecting deputy candidates for political parties. The weight of the criteria is evaluated by the party representatives by means of the analytical hierarchy process (AHP) by using the FARE (Factor Relationship) method. Candidates are assessed based on the criteria selected by applying MCDM. An empirical assessment is carried out in the research work and the candidates to the deputies are ranked through MCDM. The recent research works of the authors Alguliyev et al. (2019) have proposed an MCDM model for the selection of candidates in e-voting. The rating of candidates is calculated based on MCDM and candidates are selected based on the importance of the criteria. The proposed approach enables us to select a candidate with more relevant competencies within the framework of selected criteria. In a numerical experiment, the five candidates selected on the three criteria (education, work experience, and professional competencies), are evaluated and the candidates are ranked according to the importance of the criteria.

Note that the effective functioning of the government is directly dependent on human resources, and the participation of qualified personnel with competencies in governance is an issue of national importance. From this point of view, the selection of a candidate with appropriate competencies for the appointment of elected candidates to administrative positions as a result of e-voting, as well as the criteria and factors that should be considered in the selection process, are referred to as topical issues. The paper considers the application of MCDM in candidate selection in e-voting.

2. The e-Voting System as an Important Component of e-Democracy

The concept of e-democracy that emerged in the 1990's has started being perceived as the evidence of changes taking place against the backdrop of democratic principles in government. The support of the application of ICT in the political arena has facilitated the emergence of e-democracy, which encompasses new methods of the governance of democratic government. Political institutions, parties and politicians utilize ICT in the three main processes in the political arena, including the issues of information, communication and voting.

E-government maturity models are constituted of a sequence starting at the base stage all the way to the advanced stage, these stages determining the e-government maturity level. The proposal for methods for determining the development level of e-government and ranking e-government portals is considered as the main advantage of mature models (Fath-Allah et al. 2014). Moreover, mature models may assist organizations in promoting the efficiency of e-government. Concha et al. have proposed that mature models of e-government should be categorized into three groups, namely: governmental models, holistic approach models, and e-government maturity models (Layne et al. 2001; Andersen et al. 2006; Concha 2012).

According to developed countries' practices, research in this three categories has shown that e-government maturity models bear large importance from the standpoint of e-democracy development. The analysis of the existing e-government maturity models in the literature shows that there are several models in place, as proposed by Layne and Lee (2001), Wescott (2001), Siau et al. (2005), Chen et al. (2011), and other researchers and numerous organizations (Fath-Allah 2014; Layne et al. 2001; Wescott, 2001; Siau et al. 2005; Andersen et al. 2006; Shahkooh et al. 2008). Among those models, the formation of e-democracy has been proposed by several authors, including Wescott (2001), Siau et al. (2005) and Shahkooh et al. (2008), as the last stage of e-government development. While exploring the above-mentioned models, it is evident that e-voting, public forums, open government, the analysis of the public opinion and the development of feedback mechanisms are demonstrated as the foundation of the formation of e-democracy, which is deemed to be the evolutionary stage of e-government development. In this regard, the development of e-democracy mechanisms and e-voting technologies is necessary in order to boost transparency and efficiency, and constitutes the basis of the open government concept.

The evolutionary stage envisions the formation of new requirements and the expansion of the degree of civil participation in processes by altering the relationships between the government and the citizen. The majority of the existing developmental models incorporate democratic processes, such as political participation, e-participation, wiki democracy, interactive democracy and digital democracy (Van der Meer et al. 2014). All of these terms pertain to the democratic processes based on the transformation of the relations between citizens and the government. E-democracy has been included in these models as the last stage of a developmental model. Logically, the government must complete the preceding information, interaction and transaction/integration stages in order to proceed to the e-democracy stage.

As a new concept, the implementation of e-voting is based on reducing errors during election processes and is oriented towards maintaining the integrity of the election process in general. In the scientific literature, e-voting is considered as the use of computers and devices connected to computers in the election process, and more precisely, this term has been adopted so as to characterize elections carried out via the Internet (Abu-Shanab 2010).

The e-voting system has offered the election process numerous advantages. For instance, the facilitation of the participation of physically disabled persons, no requests for additional employees to print the election ballot papers, and a cost-effective and efficient organization of elections. In general, cost-effectiveness, the expansion of participation and the broadening of voting options, a faster and accurate registration and calculation of votes, as well as accessibility and flexibility against deviations can be considered as the main advantages of e-voting (Abu-Shanab 2010).

Research studies on e-voting have gradually become an important issue. The reason for that is a growing number of scientific-research works conducted on the development of new voting methods via the Internet and mobile services in European countries and worldwide. As a result, the terms of e-democracy, e-participation and e-voting are frequently encountered in the context of e-democracy. In European practice, the studies in the field of e-voting are mainly represented by the empirical studies conducted by Estonia, Switzerland, Poland, Norway and other countries (Drechsler et al. 2004; Braun et al. 2006; Trechsel 2002, 2007, 2016; Musia-Karg 2012, 2014; Vassil et al. 2016).

Despite the growing number of the studies devoted to researching the impact of new technologies on democracy, there is a need for conducting comprehensive research studies in the field of e-voting. In particular, it is essential to analyze the issues such as the implementation of e-participation solutions on the example of European countries and the factors necessitating the rejection of its implementation due to various drawbacks, the application opportunities of e-voting, the existing barriers and effectiveness. Hence, the development of mechanisms for and specific technological solutions to e-voting, its effectiveness and a study of undesirable results in comparison with traditional voting are deemed to be the topical research directions.

Currently, e-voting for elections and referendums at the local, regional and country levels is rapidly developing at the global scale as a more efficient and more feasible alternative to traditional voting and it favorably affects the development of democratic government. Alongside, despite the widening international practice regarding the application of the e-voting system, several challenges are still being encountered given the national interests related to legal and social problems and its implementation.

Scientific and public discourse in the field of e-voting has been broadening in the last decade. E-voting systems are categorized as location-bounded and remote voting. In the first case, the voter is required to participate in the election due to the dependence of the voting on the location. Remote voting has been applied in various countries, such as Estonia, France, The Netherlands, Switzerland, and so on. E-voting has a great potential for the expansion of the democratic participation of the public by facilitating the participation of non-represented groups in the political life, including youth and physically disabled persons. Moreover, e-voting fosters economic effectiveness and facilitates the effective organization of elections in comparison with traditional voting (Chondros et al. 2014).

In spite of the advantages of the implementation of e-voting, transition to a new technology is accompanied by numerous social, legal and technical problems (Wang et al. 2017). Among those, equal access to voting points, privacy maintenance, fight against interventions, the verification of information, examination, alteration and other procedures, universal verification, the right to vote, the one-voter-one-vote principle, strictness against errors, etc. can be considered. The necessity of transforming legal obstacles into technical and security solutions can specifically be mentioned amongst these (Wang et al. 2017). Nowadays, broad discussions are held on holding elections from a legal standpoint; as a result, it is believed that solving legal issues plays a bridging role between the law and technology.

3. MCDM-Model-Based Candidate Selection

Voting is a fundamental tool for decision-making in any consensus-based society and democracy hinges upon the accurate governance of nationwide elections. At present, numerous voting systems are adopted all over the world and each of those has specific advantages and problems. Some countries abandoned e-voting due to its risky nature. Other countries do not accept the advantages of e-voting in comparison with traditional voting. With the rapid development of the Internet, which started in the 1990s, a larger number of politicians, researchers and journalists have started reflecting upon whether e-voting proposes better solutions to elections or a referendum or not. Through numerous scientific incentives of non-government organizations at the global scale, the governments of European countries endeavor the use of the voting methods, ICT-based solutions, the application of which constitutes the basis of democratic processes (Zetter 2008; Voting system; Trechsel et al. 2016; Meserve et al. 2017). Nowadays, the majority of countries support e-voting and a growing number of countries consider the e-voting system as useful and practically apply it in their election processes. Furthermore, it is to be mentioned that, for the largest part, those efforts are still at the stages of testing and conceptual analysis. The benchmark practice regarding the application of the e-voting system at the global scale can be characterized by the USA's practice (Zetter 2008; Voting system; Trechsel et al. 2016).

At present, new voting technologies are being implemented not only in the USA, but also in several European countries (Voting system; Trechsel et al. 2016). Surely, the efforts to implement the above-mentioned e-voting system result in various outcomes in different countries. For instance, the analysis of e-voting results from the elections to the European Parliament, Country Parliament Elections (2011) and Municipal Elections (2013) shows that the interest in the implementation of a new system has systematically been growing, which is the reason for the conclusion that citizens consider this voting method to be more comfortable and more effective

(Zetter 2008; Voting system; Trechsel et al. 2016; Meserve et al. 2017). Note that the ratio of the Internet voters has grown from 1% in 2005 to 11.4% in 2014 (Mona et al. 2013; Musia-Karg 2014; Trechsel et al. 2016; McCormack 2016).

The participation of citizens in political processes and the facilitation of voting during the adoption of important decisions, as well as the provision of their direct participation, are considered as the basis of democracy. In spite of the broad implementation of ICT in business, various fields of the activity, education, public administration and government entities, the use of ICT in the voting process is treated with cautiousness in many countries. In addition, one of the main causes for the postponed implementation of advanced voting technologies is the differences in opinions and skeptical thinking when the Internet-based voting in societies is concerned (Mona et al. 2013; Musial-Karg 2014; McCormack 2016).

Despite the progress made towards a better development of e-voting systems, there is no classification for the purpose of understanding the general characteristics, aims and limitations of these approaches. Hence the absence of comparative research or the inaccurate determination of directions for selecting methods appropriate for specific requirements can be shown as the main drawbacks. In this regard, it is topical to develop efficient methods and mechanisms of e-voting by taking democratic processes into consideration.

The ability of e-democracy to overcome barriers causing the deterrence or limitation of citizens' participation in direct decision-making is considered as the main advantage of the development of effective e-voting mechanisms. From this point of view, e-voting is gaining the attention of government entities, political parties and politicians, and is deemed to be a powerful tool for sustaining democratic principles. The conducted research shows that e-voting has become one of the main tools of e-democracy by attaining greater importance (Musial-Karg 2014). In this regard, the development of e-voting technologies and the study of the implementation opportunities of new technologies are considered as important research topics.

The proposed approach to the research is based on the multi-criteria evaluation of the candidates, taking into account the relationship of each candidate with another candidate. Assume that, as a result of e-voting, the candidates are elected to be appointed to a relevant position. The intelligence quotient (IQ), age, education, work experience, health, conviction, etc. can be attributed to the criteria for the selection of competent candidates. A binary matrix is used for the evaluation of the candidates in the study.

The MCDM approach to candidate selection consists of the following stages:

Let $A = (A_1, A_2, ..., A_n)$ be the candidates and $C = (C_1, C_2, ..., C_n)$ be the criteria set.

Step 1. Each candidate constructs an evaluation matrix for the evaluation of the other candidates based on each criterion:

$$P_l^k = (p_{ij})_l^k = \begin{Vmatrix} (p_{11})_l^k & (p_{12})_l^k & \dots (p_{in})_l^k \\ \dots & \dots & \dots \; \dots \\ (p_{n1})_l^k & (p_{n2})_l^k & \dots (p_{nn})_l^k \end{Vmatrix} \tag{1}$$

where,

$$(p_{ij})_l^k = \begin{cases} 1, \text{ if according to the opinion of } A_k \text{ candidate, } A_i \text{ is superior to } A_j \\ \quad \text{according to } C_l \text{ criterion} \\ 0, \text{ otherwise} \end{cases}$$

The principal diagonal of the P_l^k matrix consists of zeros, $(p_{ii})_l^k = 0$, $(p_{ij})_l^k = (\overline{p_{ij}})_l^k$, if $i \neq j$, $\bar{0} = 1, \bar{1} = 0$.

Step 2. Thereafter, the $Q_l = (q_{ik})_l$ outcome matrix is entered and the elements are calculated as below:

$$(q_{ik})_l = \sum_{j=1}^{n} (p_{ij})_l^k, \quad i = 1,2,...,n; \; k = 1,2,...,n; \; l = 1,2,...,m, \tag{2}$$

$(q_{ik})_l$ - reflects the final opinion of the candidate A_k on the candidate A_j, based on the criterion C_l (in comparison with all the candidates):

$$Q_l = \begin{Vmatrix} (q_{11})_l & \cdots & (q_{1n})_l \\ \cdots & \cdots & \cdots \\ (q_{n1})_l & \cdots & (q_{nn})_l \end{Vmatrix} \tag{3}$$

Step 3. The overall opinion of the candidate A_k on all the candidates is based on the criterion C_l and is calculated as follows:

$$O_l^k = \sum_{i=1}^{n} (q_{ik})_l, \; k = 1,2,...,n; \; l = 1,2,...,m \tag{4}$$

Step 4. The ranking of the candidate A_i based on the criterion C_l is determined by applying the following formula:

$$R_i^l = \sum_{k=1}^{n} (q_{ik})_l \quad i = 1,2,...,n; \;\; l = 1,2,...,m \tag{5}$$

The last relationship expresses the final opinion of all the candidates on the candidate A_i based on the criterion C_l.

Step 5. In order to obtain the resulting rank of the alternatives, the resultant rank computed by means of the following formula is used (Aliguliyev 2009):

$$\text{Resultant rank} = \sum_{s=1}^{\eta} \frac{(\eta - s + 1) r_s}{\eta} \tag{6}$$

where r_s denotes the number of the times the method appears in the s-th rank and η is the number of the alternatives.

4. A Numerical Experiment

Assume that a total of six candidates are presented based on four criteria (for example, education (C_1), work experience (C_2), age (C_3) and professional competencies (C_4)). Based on the formula (1), the evaluation of the candidates according to each criterion is given in Tables 1-6.

Based on the formulas (2) and (3), the final opinion of the candidate A_k on the candidate A_j is calculated according to the criterion C_l (in comparison with all the candidates) and is given in Table 7.

Table 1. The criteria-based evaluation of the candidate A_1

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	0	0	0	1	1	0	1	0	1	0	1
A2	1	0	1	0	0	0	0	0	0	1	1	1
A3	0	0	0	1	0	1	1	1	0	0	0	1
A4	1	0	0	0	0	1	0	0	1	0	0	1
A5	0	0	1	1	0	1	1	0	1	1	0	0
A6	0	0	0	0	0	0	0	0	0	0	1	0
	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	0	1	0	0	0	0	0	1	0	0	1
A2	1	0	1	0	1	1	1	0	0	1	0	0
A3	0	0	0	1	1	0	0	1	0	1	0	0
A4	1	1	0	0	1	1	0	0	0	0	0	1
A5	1	0	0	0	0	0	0	0	1	1	0	0
A6	1	0	1	0	1	0	0	1	1	0	1	0

Table 2. The criteria-based evaluation of the candidate A_2

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	0	0	1	0	1	0	1	0	1	0	1
A2	1	0	1	0	0	0	0	0	0	1	0	0
A3	1	0	0	0	0	0	1	1	0	0	1	0
A4	0	0	1	0	0	1	0	0	0	0	0	1
A5	0	0	1	0	0	1	1	1	0	1	0	0
A6	0	0	1	0	0	0	0	1	1	0	1	0
	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	1	1	0	0	1	0	1	0	0	0	1
A2	0	0	1	0	0	0	0	0	0	1	0	0
A3	0	0	0	1	1	1	0	1	0	0	0	1
A4	1	0	0	0	0	1	0	0	1	0	0	1
A5	1	0	0	1	0	0	0	0	1	1	0	0
A6	0	1	0	0	1	0	0	1	1	0	1	0

Table 3. The criteria-based evaluation of the candidate A_3

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	1	0	1	0	0	0	1	1	0	1	0
A2	0	0	1	0	0	1	0	0	1	0	0	1
A3	1	0	0	1	1	0	0	0	0	1	1	0
A4	0	1	0	0	0	0	1	1	0	0	0	0
A5	0	0	0	1	0	1	0	1	0	1	0	1
A6	0	0	1	1	0	0	1	0	0	1	0	0
	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	1	0	1	0	0	0	0	0	0	0	1
A2	0	0	1	0	1	0	1	0	1	1	0	0
A3	1	0	0	1	0	1	0	0	0	0	1	1
A4	0	0	0	0	1	1	0	0	1	0	0	1
A5	1	0	1	0	0	0	0	0	0	1	0	0
A6	1	1	0	0	1	0	0	1	0	0	1	0

Table 4. The criteria-based evaluation of the candidate A_4

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	0	0	0	1	1	0	1	1	0	1	0
A2	1	0	0	0	0	0	0	0	0	0	1	1
A3	1	1	0	0	1	0	0	1	0	1	0	0
A4	1	0	1	0	0	1	1	1	0	0	1	0
A5	0	0	0	1	0	1	0	0	0	0	0	1
A6	0	0	1	0	0	0	0	0	0	1	0	0
	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	0	0	0	0	0	0	0	1	1	1	0
A2	1	0	1	0	1	0	0	0	0	0	1	0
A3	1	0	0	0	1	1	0	1	0	1	0	1
A4	1	1	1	0	0	1	0	0	1	0	0	1
A5	1	0	0	1	0	0	0	0	1	1	0	0
A6	1	1	0	0	1	0	1	1	0	0	1	0

Table 5. The criteria-based evaluation of the candidate A_5

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	1	0	0	0	1	0	1	0	0	1	1
A2	0	0	0	0	1	0	0	0	1	0	1	1
A3	1	1	0	1	0	1	1	0	0	1	1	1
A4	1	1	0	0	1	1	1	1	0	0	0	0
A5	1	0	0	0	0	0	0	0	0	1	0	1
A6	0	1	0	0	1	0	0	0	0	1	0	0
	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	1	0	0	0	0	0	0	1	0	0	1
A2	0	0	0	0	1	0	1	0	0	0	0	0
A3	1	1	0	0	0	0	0	1	0	1	0	0
A4	0	1	1	0	0	1	1	1	0	0	0	1
A5	0	0	1	1	0	0	0	0	1	1	0	0
A6	0	1	1	0	1	0	0	1	1	0	1	0

Table 6. The criteria-based evaluation of the candidate A_6

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	1	0	1	0	0	0	0	0	1	0	1
A2	0	0	0	0	1	0	1	0	1	0	0	0
A3	1	1	0	1	0	1	1	0	0	0	0	0
A4	0	1	0	0	0	1	0	0	0	0	0	0
A5	1	0	1	1	0	1	1	0	1	0	0	1
A6	1	0	0	0	0	0	0	1	0	0	0	0
	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	0	0	1	1	0	1	0	1	0	0	1	1
A2	1	0	0	0	1	0	0	0	0	0	1	0
A3	0	1	0	0	0	1	0	1	0	0	0	1
A4	0	1	1	0	0	1	0	1	1	0	0	1
A5	0	0	1	1	0	0	0	0	1	1	0	0
A6	0	1	0	0	1	0	0	1	0	0	1	0

Table 7. The final opinion of the candidates based on the four criteria

	C1						C2					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	2	2	2	2	2	2	3	3	3	3	3	2
A2	2	2	2	1	1	1	3	1	2	2	3	2
A3	2	1	3	3	4	4	3	3	2	2	4	1
A4	2	2	1	3	4	2	2	1	2	3	2	0
A5	3	2	2	2	1	4	3	3	3	1	2	3
A6	0	1	2	1	2	1	1	3	2	1	1	1

	C3						C4					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
A1	1	3	2	0	1	3	2	2	1	3	2	1
A2	4	1	2	3	1	2	2	1	3	1	1	4
A3	2	3	3	3	2	2	2	2	2	3	2	2
A4	4	2	2	4	3	3	1	2	2	2	3	4
A5	1	2	2	2	2	2	2	2	1	2	2	1
A6	3	2	3	3	3	2	3	3	2	3	3	3

Based on the formula (4), the overall opinion of the candidate A_k on all the candidates is calculated based on the criterion C_l, and is given in Table 8.

Table 8. The criteria-based opinion of each candidate (the final opinion)

O_1	11	10	12	12	14	14
O_2	15	14	14	12	15	9
O_3	15	13	14	15	12	14
O_4	12	12	11	14	13	13

The ranking of the candidate A_i is calculated based on the formula (5), respectively based on the criterion C_l and presented in Table 9.

Table 9. The ranking of the candidates based on each criterion

R1	Rank	R2	Rank	R3	Rank	R4	Rank
12	4	17	1	10	6	13	2
9	5	13	4	13	4	9	6
17	1	15	2	15	3	13	2
14	2	10	5	18	1	13	2
14	2	15	2	11	5	11	5
7	6	9	6	16	2	16	1

Using the positional ranking approach, the resultant rank in Table 10 was calculated by means of the formula 6. For example, the rank of the alternative A_1 is calculated as follows:

$$\text{Resultant rank } (A_1) = \sum_{s=1}^{6} \frac{(6-s+1)r_s}{6} = \frac{(6-1+1)\cdot 1}{6} + \frac{(6-2+1)\cdot 1}{6} + \frac{(6-3+1)\cdot 0}{6} + $$

$$+ \frac{(6-4+1)\cdot 1}{6} + \frac{(6-5+1)\cdot 0}{6} + \frac{(6-6+1)\cdot 1}{6} = 2.500$$

Table 10. The resultant rank of the candidates

Candidate	Resultant rank	Rank No
A1	2.500	3
A2	1.500	6
A3	3.333	1
A4	3.000	2
A5	2.333	4
A6	2.167	5

As described in Table 10, the candidates are ranked in accordance with the A_3, A_4, A_1, A_5, A_6 and A_2 sequence. As the result shows in this case, the candidate A_3 has more appropriate competencies needed for the appointment to the position, according to the multi-criteria evaluation of the candidates.

The ranking results can be improved by employing the importance of criteria and the fuzzy hybrid approach for the purpose of computing the weights of the criteria (Lin 2010; Chang et al. 2013; Sakthivel et al. 2015). In practice, a different evaluation scale for the multi-criteria selection of candidates in the e-voting process can be used in the proposed model. The tools that enable the selection of a candidate with more relevant competencies within the framework of certain criteria among the candidates can be created by implementing the proposed model.

5. Conclusion

The paper investigates the approaches, tools, and mechanisms pertaining to the formation of e-democracy as the last stage of the development of e-government. The research results show that e-voting is gradually gaining greater importance and becoming one of the main components of e-democracy. The selection of qualified personnel at the government level and their appointment to responsible positions are important issues in economic and political processes. The candidates who are the best for the vacancy are selected for a particular position. It is worth noting that the effective functioning of the government directly depends on human resources, and the participation of qualified personnel with competencies in governance is an issue of national importance. From this point of view, the selection of candidates with appropriate competencies, the appointment of elected candidates to administrative positions as a result of e-voting, and the criteria and factors to be considered in the selection process are referred to as topical issues. The paper considers the application of the MCDM model in candidate selection in e-voting.

The approach proposed in the paper is based on candidate evaluation given each candidate's attitude towards another candidate. The rank of the candidates is calculated based on the MCDM model and the candidates are selected based on the positional ranking approach. The proposed approach enables us to select a candidate with more relevant competencies within the framework of the selected criteria. In the numerical experiment, a total of the six candidates selected based on the four criteria (education, work experience, age, and professional competencies) are evaluated and the candidates are ranked according to the resultant ranking method. The proposed model allows for the selection of the candidate with the competencies based on the criteria set out in the e-voting process and the making of more effective decisions as well.

Note that, alongside the rapid development of technologies and the enhancement of the implementation of the same in political processes, there is a need for a detailed analysis of the existing practice and for conducting studies oriented towards supporting citizen participation in political processes by applying these technologies. The development of e-voting methods will allow for the creation of a new e-democracy maturity model by facilitating the direct participation of citizens in democratic processes. Future studies will examine the application of the fuzzy hybrid approach to candidate selection by taking the importance of criteria into consideration.

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FINANCIAL PERFORMANCE EVALUATION OF SEVEN INDIAN CHEMICAL COMPANIES

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Abstract: *Financial appraises create a prominent media for giving advice in the expansion, development of any society as well as its role in forbearance and stamina in depletion and recession. Obviously, manufacturing units have a main role in the development and progress of modern India. Indian economic relied on agricultural activities but industries also provide a prominent booster for the economic cycle. The current empirical study investigated the 7 Indian chemical companies in terms of financial aspect using ratio analysis, Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), COmplex PRoportional ASsessment (COPRAS) and Data Envelopment Analysis (DEA) along with weighing systems of equal weighing, Entropy Shannon and Friedman test as the objective of research during 2010 to 2018. By the way, present research resulted in weighing and ranking of above-named industries in three classes. The weighing systems of Friedman test and Entropy Shannon were revealed a relatively linear scatter plot with no significant differences between values. DEA model had distinguished and classified the efficient companies based on rank values.*

Key words: *Financial performance, Companies, DEA, Ratio analysis.*

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1. Introduction

Financial estimations create a prominent media in the expansion, development of any society as well as its role in forbearance and stamina in depression and depletion. At the micro and macro positions, the financial viability of any industrial sector presents the economic achievements and progresses. To figure out the development trend and also any fall or rise parallel with revolution towards sustainability of companies, the financial outcomes are posed as a level of judge.

Obviously, industries, companies and manufacturing units have a main role in flourish and growth of modern India. Indian economic relies on agricultural activities but industries also provide a prominent booster for the economic cycle. The preliminary activities to set up the industries started after British rule in India. The industrial sector encompassed 3 major sectors such as (1) primary sector devoted to the exploitation of raw materials using agricultural activities or mining and aggregate extractions. (2) The second sector included refining, building and construction and manufacturing developments. (3) The third sector is related to distribution, delivery of commodities and marketing purposes (Arab et al 2015; Kettiramalingam et al 2017).

By the 1938 Indian chemical council was found in order to further development of companies in this regard. This sector placed the third greatest producer in Asia and 12th in the world because of marketing expansion. It has been forecasted the growth rate around 14% per year from \$ 160 billion in 2013 to \$ 350 billion by 2021. The majority of Indian chemical products encompassed based chemicals, which include the petrochemicals, man-made fibres, industrial gases, fertilizers, chlor-alkali, and other organic and inorganic chemicals etc. over 70000 commercial products. Also, this sector included 12.5% of the total industrial output and approximately 16.2% of the total exports in India.

Financial analysis refers to the process of evaluating companies, businesses or projects in terms of budgeting and other financial aspects of these institutions, which is used to determine the suitability of these institutions for investing through financial statements. Financial analysis is often used to assess the strength of an institution and its ability to pay debts, as well as its liquidity and profitability. Financial analysis often focuses on the profit and loss account, balance sheet and cash flow, which, based on the firm's past, estimates its future performance (Kumar and Bhatia 2014).

Many scholars recognize decision making as an essential factor in management. Decision-making is the result of a process that ultimately leads to a decision, while those who are not in the decision-making process. Only see the result of the decision. In recent years, the attention of academic assemblies has attracted more decision making science in the country and relatively comprehensive research has been done in order to choose the best option in the fields of industry, commerce, trade, mining and so on. Among different decision-making methods, depending on the data of this study, COPRAS and TOPSIS methods have been selected as the ranking and weighing systems. Weighing systems have been used for data recording. In this study, Entropy Shannon and Friedman have been used for this purpose (Bulgurcu 2012; Zavadskas et al., 2008).

Data Envelopment Analysis (DEA) has been empirically declared for evaluation of relative efficiency and inefficiency of various companies and industries etc. The main purpose to figure out the DEA in industries refers to the sustainability of industries and companies. DEA can be calculated via the ratio of output costs to inputs costs. Therefore, financial data of input and output from industries are the main information to investigate the performance of industries. So, in parallel with distinguish input and outputs outlay to estimate DEA, we tried to find out both financial items of profit and loss of industries. Many kinds of research have completed based on limited criteria of industries and they focused on some single group industries or single industry during a certain period. Also, they tried to represent their results based on one methodology either DEA or financial analysis (Sinha 2015). The current study was conducted to an analysis of financial performance of selected companies with respect to liquidity ratios, turn over ratios, solvency ratio, and profitability ratios along with efficiency classification of companies' based on DEA and weighing additive models. The final achievement of the present study includes the sustainability progresses of industries and companies.

2. Literature review

The financial performance of many companies such as Tata Steel Ltd., Jindal Steel & Power Ltd., J S W Steel Ltd., Bhushan Steel Ltd. and Steel Authority of India Ltd evaluated based on Liquidity, Solvency, Activity and Profitability ratios in India (Arab et al 2015). Kettiramalingam et al (2017) estimated the financial performance using productivity and efficiency relationships as a case study industry in India. The obtained results revealed a rise in the performance of the industry in a period of 20 years. To investigate the interplay between executive compensation and companies performance has been used the ratios analysis as main and important variables by Raithatha and Komera (2016) in Indian companies. 50 listed non-financial companies on Pakistani Stock Market investigated for financial performance via working capital management, inventory turnover, cash conversion cycle, average collection period, and average payment period, return on asset, return on equity and earning per share in a period ranging from 2005 to 2014 (Bagh et al 2016).

Lots of methods have been posed for weighting and ranking systems based on multi-criteria networks and financial ratios analysis such as TOPSIS, VIKOR, WASPAS, COPRAS, EDAS, and ARAS etc. Yalcin et al (2012) set up a weighing system in the hierarchical financial performance system and ranked the criteria in the TOPSIS and VIKOR models. To compare the financial situation of 13 technology companies has been utilized ratios analysis along with the TOPSIS method in the Istanbul Stock Exchange. The results were used to rank the firm during 2009-2011 (Bulgurcu 2012). Anderkinda and Rakhmetova (2013) surveyed the financial outcomes of industries holding an adverse relationship between them such as liquidity decline, profitability loss, financial instability, raise in expenses and etc. By the way, some economic and financial models have released to further studies. The inventory turnover ratio, debtor turnover ratio, investment turnover ratio, fixed assets turnover ratio and total assets turnover ratio were studied to measure the financial performance of a case study steel industry in India (Pinku Paul and Mukherjee 2013). Kumar and Bhatia (2014) evaluated the financial performance of Tata Motors and Maruti Suzuki using ratios analysis including the liquidity, assets, profitability etc. A study by Margineana et al (2015) included ratios analysis and the existing relationship among various kinds of ratios, expenses paid for around 700 staff and raw material flow based on real data during 2006 to 2013.

Fenyves et al (2015) implemented a benchmarking method to evaluate the performance of companies based on financial analysis. So the study pointed out that the DEA procedure was a dominant method to investigate the profit-making trend comparison of companies. Rezaee and Ghanbarpour (2017) carried out research on the DEA model for investigating 59 Iranian manufacturing units based on linear multi-group relations. By the way, it was developed a score based on DEA performance model for industries individually. Rahimi et al (2013) applied a DEA model for figuring the performance out for around 22 poultry companies in Iran. It was matured the efficiency score in DEA solver. DEA model has been used for financial performance analysis (liquidity, activities, leverage) profitability (output) to find efficient and inefficient industries for around 36 companies in a period of 5 years. Findings paved the way for the classification of companies and figure out the reasons for weakness and strong points among 9 efficient and 27 inefficient units in the group (Tehrani et al 2012). Some attempts done resulted in figure out the financial analysis of around 85 Spanish industries using DEA model (Rodríguez-Pérez et al 2011). According to discussions outlined the DEA model is a dominant method for traditional ratio analysis and it also able to measure a prominent procedure to determine the operational and managerial efficiencies of companies and industries etc (Feroz et al 2017). DEA model used to measure the efficiency level of 15 insurance companies from 2005 to 2012. So, despite demystifying the efficient companies, it has been reported significant fluctuations between the technical efficiency levels obtained in the distinguished time interval (Sinha 2015). Saranga and Nagpal (2016) used a model of DEA to distinguish the efficient and inefficient Indian airline companies in terms of operational efficiency of drivers. On the other hands, the efficiency of airline companies was obtained in a high relationship with prices and cost efficiency relied on the technical aspect. A study targeted to evaluate the performance of manufacturing 744 small and medium enterprises based on input and output variables in Turkey. By the way, it has been reported to exist around 94 efficient units (Bulak and Turkyilmaz 2014). A study estimated the efficiency score (relies on value-added amounts) of manufacturing companies of both China and Turkey via the DEA model. The canonical correlation analysis used to figure out the weight values. The t-test analysis has been selected to compare the significant differences between the efficiency values of two groups of companies. The statistical analysis has been manifested the highest efficiency level to Chinese companies (Bayyurt and Duzu 2008). Amini and Alinezhad (2016) carried out his research using the DEA method for ranking 15 Iranian industries. In the following steps, it was found around 8 efficient industries with a score of 1. The research conducted by Lu et al (2014) used a similar procedure close to DEA to figure out the efficiency of industries. The results appeared with the efficiency scores about 0.905 to 0.973 for 34 Chinese life insurance companies from 2006 to 2010. An article devoted to assessing the efficiency and performance of around 40 retail workshops via DEA method in the Portuguese in the period of 2010 to 2013. It has been reported that the technical efficiency complied from a failure. Therefore the authors tried to offer some improvement steps of marketing and selling trends (Xavier et al 2015). Ahmadi and Ahmadi (2012) revealed that DEA models can provide efficiency scores scaled to a maximum value of 1 to evaluate efficiency and inefficiency of industries (case study conducted among 23 main industries). So, obtained results revealed amounts of around 0.591, 0.418 and 0.484 for Iranian recycling industries at efficiency scale, while values were about 1, 1, and 1 at pure technical efficiency during 2005, 2006 and 2007 respectively. Also, results asserted that there are 3 major manufacturing industries and two provinces which are identified as the best performers, namely tobacco, transport equipment and coal coke. Among 30 provinces, Bushehr and North Khorasan provinces have the utmost performance. Keramidou et al (2011) evaluated the purely technical and scale efficiency of the Greek meat products industry from 1994 to 2007 via DEA. The results presented the presence of inefficiencies in firms as well as a waning trend the efficiencies due to mismanagement and wastage of capital.

Rahmani (2017) used the DEA model for estimating the industrial productivity of a country.

3. Methodology

This study has relied on secondary data obtained from valuable resources (website) and then secondary data came through the following procedures. Seven Indian large chemical companies were chosen as case studies in a period from 2010 to 2018. Companies have been chosen from around the top 10 chemical companies in India. An appropriate performance analysis demands a reliable procedure to measure the availability in the best possible situation. It requires a procedure to conduct the empirical methods and practices such as DEA, ratios analysis (turn over ratios, liquidity, profitability and solvency). In order to analyze the collected data, the IBM SPSS statistics 20 and EXCEL package were used. Companies were ranked by the TOPSIS, COPRAS and DEA models.

3.1. Financial ratio analysis

To conduct the financial ratios analysis below equations were used to get the results. Below displays the applied equations.

Current Ratio	(Current Assets/Current Liabilities)	(1)
Acid Test Ratio	(Quick Assets) / (Current Liabilities)	(2)
Absolute liquid ratio	(Absolute liquid assets) / (Current liabilities)	(3)
Debtor Turnover Ratio	(Net Credit Sales) / (Average Trade Debtors)	(4)
Total asset turnover	(Total Sales / (Total Assets)	(5)
Inventory Turnover Ratio	(Cost of goods sold) / (Average Inventory)	(6)
Equity Ratio	(Shareholder funds) / (Total assets)	(7)
Debt equity ratio	Outsider Funds (Total Debts)/ (Shareholder Funds or Equity)	(8)
Debt to total capital ratio	(Total Debts) / (Total Assets)	(9)
Fixed assets to net worth ratio	(Fixed Assets × 100) / (Net Worth)	(10)
Net profit margin or ratio	(Earnings after tax × 100) / (Net Sales)	(11)
Return on net capital	(Earnings Before Interest & Tax (EBIT) × 100) / Net capital	(12)

Then TOPSIS procedure was assigned for ranking of companies and determining the performance values based on ratio analysis values (Bulgurcu 2012).

3.2. Friedman test

The current empirical study of seven Indian chemical industries was accomplished to determine the performance of industries. In the SPSS software structure, there is a test defined as the Friedman test. The Friedman test was selected to estimate weight values. This test is used by Equations 13 to 17 to estimate the weight of criteria and factors in separate columns. The test structure is formatted so that all values in the columns form a matrix with various rows and columns. The weight of each column is then estimated by comparing the values in the columns. In this estimation, higher weights are assigned to columns of higher values and medium weights for average values and vice versa. Therefore, the Friedman test is used as a highly valid test in estimating the weight of numbers with a variety of values. In the matrix of [rij] n×k the entry rij is the estimated weight of Xij within the block of I individually. The test statistic is calculated by equation 17 (Eisinga et al 2017).

$$\hat{\mathrm{r}}.\mathrm{j} = \frac{1}{\mathrm{n}} \sum_{i=1}^{n} rij \quad (13)$$

$$\hat{\mathrm{r}} = \frac{1}{\mathrm{nk}} \sum_{i=1}^{n} \sum_{j=1}^{k} rij \quad (14)$$

$$\mathrm{SSt} = \mathrm{n} \sum_{j=1}^{\cdot} (\hat{\mathrm{r}}.j - \hat{\mathrm{r}})^2 \quad (15)$$

$$\mathrm{SSe} = \frac{1}{\mathrm{n(k-1)}} \sum_{i=1}^{n} \sum_{j=1}^{k} (rij - \hat{\mathrm{r}})^2 \quad (16)$$

$$\mathrm{Q} = \frac{\mathrm{SSt}}{\mathrm{SSe}} \quad (17)$$

3.3. TOPSIS Method

TOPSIS method has been defined pertaining to the smallest distance best possible and ideal solution value and largest distance from the negative on unreliable solution value. So the findings based on the present procedure provide a steady rise and fall in the values. The important stages posed in running the process include (1) set up the matrix of data (2) weight estimation base on Hwang's rule (3) set up the non-scale matrix (4) figure out the best solutions values (5) finding the relative proximity and ranking the alternatives. To set up the non-dimension matrix was used the equation 18. In this equation, aij is the numerical value of each industry i, according to the index j. The equal weights were assumed about 0.0715 for 15 criteria individually as they provide the same significance (∑wi=1). The symbol of Wi is the weight for each ratio or criterion. Then, according to equation 19 the weights assigned to the rows of the matrix as a special vector. The special vector has collected the values in the non-scaled matrix. To find the best ideal values (A+) and (A-) were applied the equations of 20 and 21. The largest and smallest values were assumed as the best ideals solutions in the columns individually. Then Euclidean distance was employed to find the positive and negative ideal solutions for each company. The distances were calculated regarding the equations of 22 to 24. The higher the cli+, the higher the weighting value will be provided (Bulgurcu 2012).

$$\mathrm{Nd} = \frac{\mathrm{aij}}{\sqrt{\sum_{i=1}^{m} (aij)^2}} \quad (18)$$

$$\mathrm{V} = \mathrm{Nd} \times \mathrm{Wn.n} \quad (19)$$

$$\mathrm{A+} = \{(\max Vij | j \in J), (\min Vij | j \in j') | i = 1,2,\dots,m\} = \{\mathrm{V_1^+, V_2^+, ..V_j^+, V_n^+}\} \quad (20)$$

$$\mathrm{A-} = \{(\min \mathrm{i}\, Vij | j \in J), (\max Vij | j \in j') | i = 1,2,\dots,m\} = \{\mathrm{V_1^-, V_2^-, ..V_j^-, V_n^-}\} \quad (21)$$

$$\mathrm{di+} = \left\{\sum_{j=1}^{n} (Vij - Vj +)^2\right\}^{0.5} ; i, = 1,2,3, \dots m \quad (22)$$

$$\text{di}-= \{\sum_{j=1}^{n}(Vij - Vj -)^{2}\}^{0.5} \quad ; i, = 1,2,3, \dots m \tag{23}$$

$$\text{cli}+= \frac{\text{di}-}{\text{di}(+)+(di-)} \quad i = 1,2, \dots, m \tag{24}$$

3.4. Entropy Shannon weighing system

This method like other methods needs to compose a matrix for the existing data. To normalize the existing data was employed equation 25, and 26 and 27 for entropy values. The distance between each of the options was obtained from the entropy value using equation 28. It was used the equation of 29 to release the weight of each indicator by Excel 2013.

$$\text{Pij} = \frac{\text{Xij}}{\sum_{i=1}^{m} Xij} \qquad j = 1, \dots, n \tag{25}$$

$$\text{Ej} = -\text{k} \sum_{i=1}^{m} Pij \times Ln\, Pij \qquad i = 1,2, \dots, m \tag{26}$$

$$\text{k} = \frac{1}{\text{Ln}\, m} \tag{27}$$

$$\text{dj} = 1 - \text{Ej} \tag{28}$$

$$\text{Wj} = \frac{\text{dj}}{\sum dj} \tag{29}$$

3.5. DEA

Determining the performance of each company is done using the DEA method. In this method, the ranking of each option is done according to the weight assigned to it. In this study, the weight of each column was obtained by the Friedman test. Then the data was sorted by input and output and according to formulas 30 to 34, and the efficiency of the companies was estimated (Xavier et al 2015).

$$\text{DEA} = 0 \leq \frac{\sum_{r=1}^{S} Ur\, Yrj}{\sum_{i=1}^{m} Vi\, Xij} \leq 1 \tag{30}$$

$$\text{Max Z} = \frac{\sum_{r=1}^{S} Ur\, Yrj}{\sum_{i=1}^{m} Vi\, Xij} \leq 1 \quad , \quad j = 1,2,3, \dots . n \tag{31}$$

$$\text{Ur, Vi} \geq 0 \tag{32}$$

$$DEA = \frac{\text{Output (1)Weight (1)} + \text{Output (2)Weight (2)} + \dots + \text{Output (s)Weight (s)}}{\text{Input (1) Weight (1)} + \text{Input (2)Weight (2)} + \dots + \text{Input (m)Weight (m)}} \tag{33}$$

3.6. Ranking system based on COPRAS

COPRAS method is a dominant procedure to rank the alternatives that it was introduced in 1996 firstly. The procedure makes it easy for the decision making processes for multi-criteria options. It follows some steps to complete the ranking operation. Equation 35 was employed to normalize the decision matrix. By the way, the Xij and W are the values and weighted values respectively. To sum the normalized values, figure out the relative importance of alternatives and the greatest value of relative importance (Qmax) were used the equation of 36 to 39 respectively. The S-min (minimum value of S-i) and Nj (ranking amount), S+j, (maximizing criterion of j-th alternative) S-I (minimum value of the sum of minimizing criteria of the j-th option) and S-i (minimizing criteria of the j-th option) were distinguished respectively (Zavadskas et al., 2008).

$$\text{Pij} = \frac{\text{Xij .W}}{\sum_{i=1}^{n} Xij} \qquad i = \Gamma, m; \;\; j = \Gamma, n \tag{34}$$

$$\text{S + j} = \sum_{i=1}^{m} +Pij \qquad i = \Gamma, m; \;\; j = \Gamma, n \tag{35}$$

$$\text{S − j} = \sum_{i=1}^{m} -Pij \qquad i = \Gamma, m; \;\; j = \Gamma, n \tag{36}$$

$$\text{Qj} = \text{Sj+}, + \frac{\text{S−min} \times \sum_{I=1}^{n} sj-}{\text{Sj−}, \sum_{i=1}^{n} \left(\frac{S-min}{Sj-}\right)} \;\; = \text{Sj+}, + \frac{\sum_{I=1}^{n} Sj-}{\text{Sj−}, \sum_{i=1}^{n} \left(\frac{1}{Sj-}\right)} \tag{37}$$

$$\text{Nj} = \frac{\text{Qj}}{Qmax} * 100 \tag{38}$$

4. Results and discussion

4.1. Financial data analysis

Financial Statements (FS) are summaries of the operating, financing, and investment activities of a business. FS should present useful data to both investors and creditors in making credit, investment, and other business decisions. This usefulness means that investors and creditors can use these statements to predict, compare, and evaluate the amount, timing, and uncertainty of potential cash flows. In other words, FS provides the information needed to assess a company's future earnings and therefore the cash flows expected to result from those earnings. By this study, the financial data of 7 Indian industries were collected according to Table 1.

Table 1. Financial data of industries during 2010-2018 **(**Profit & Loss account in Rs, Cr**)**

					Tata Chemicals (A)				
(1)	3,447.99	3,591.36	8,170.30	9,984.39	8,590.23	8,440.93	7,912.63	6,225.27	5,411.70
(2)	3,466.01	3,606.80	8,220.86	10,082.06	8,689.64	8,529.87	7,996.25	6,332.86	5,411.70
(3)	194.49	176.92	164.37	194.75	202.92	365.6	308.57	108.03	88.35
(4)	531.39	479.95	2,041.14	3,778.55	3,194.24	2,988.79	2,864.91	2,198.87	2,724.92
(5)	-19.7	39.95	591.34	-850.84	130.19	273.78	-409.36	-10.07	171.17
(6)	258.03	266.66	286.27	330.17	267.05	273.56	239.75	207.38	204.66
(7)	86.51	100.98	215.16	186.78	185.32	203.25	210.19	201.49	189.71
(8)	126.55	129.6	153.5	192.71	158.82	214.29	224.68	204.46	187.19
(9)	1,537.82	1,513.61	2,031.18	3,072.81	2,556.19	2,542.98	2,109.54	1,744.50	717.95
					Gujarat Fluorochemicals (B)				
(1)	2,044.48	1,417.22	1,319.08	1,309.21	1,134.87	1,504.16	2,065.56	978.97	985.57
(2)	2,050.46	1,421.52	1,338.31	1,320.97	1,140.94	1,596.08	2,069.00	982.85	985.57
(3)	103.02	71.12	52.36	56.19	65.06	56.9	57.64	99.53	49.23
(4)	539.38	374.41	335.54	410.09	320.84	303.47	252.35	212.16	377.57
(5)	38.42	1.19	50.63	-47.05	41.05	-75.08	-94.3	39.66	-9.2
(6)	138.35	120.06	103.04	96.16	80.69	74.53	66.53	55.63	56.97
(7)	47.62	35.18	47.73	51.98	55.28	68.95	57.13	29.87	48.03
(8)	152.14	148.84	144.15	123.85	101.7	96.38	77.82	44.86	57.03
(9)	755.3	615.38	559.59	581.94	507.66	588.8	760.65	350.71	83.35
					Solar Industries India (C)				
(1)	1,230.54	1,094.29	1,084.25	1,009.18	896.76	884.56	722.62	531.21	480.21
(2)	1,273.27	1,137.31	1,089.50	1,014.75	904.03	886.99	723.75	534.01	480.21
(3)	18.23	13.38	10.19	19.83	17.1	17.64	24.97	24.81	20.09
(4)	750.02	678.57	640.97	599.86	489.22	509.02	393	261.62	218.92
(5)	-19.46	-1.79	-2.98	2.37	-3.81	-1.61	-1.87	-0.43	0.19
(6)	69	54.35	43.41	40.42	38.69	32.24	24.15	18.88	16.83
(7)	14.23	13.79	7.92	7.24	14.48	21.91	20.09	11.45	8.27
(8)	26.09	19.28	17.72	17.66	12.57	10.31	8.05	6.64	6.32
(9)	154.81	113.27	164.54	161.13	206.44	162.45	109.63	102.62	95.62

Gujarat Alkalies & Chemicals (D)									
(1)	2,420.13	2,023.04	1,955.67	1,931.81	1,882.85	1,794.31	1,698.22	1,423.17	1,280.47
(2)	2,454.50	2,070.21	1,995.45	1,948.12	1,896.06	1,814.60	1,710.97	1,434.68	1,280.47
(3)	105.74	55.92	46.23	48.95	30.27	18.7	11.77	12.01	49.26
(4)	1,177.41	1,132.21	1,219.66	675.57	717.22	714.75	720.8	615.79	807.34
(5)	1.47	5.68	-1.95	25.04	-4.47	3.71	-30.57	-4.66	6.2
(6)	201.39	169.76	162.1	167	151.44	118.9	118.13	114.93	119.91
(7)	14.9	12.83	9.93	9.34	6.36	8.34	20.53	21.17	17.48
(8)	127.32	110.92	107.44	98.06	150.65	151.52	138.95	133.12	121.55
(9)	285.78	308.67	268.82	800.46	658.4	481.93	506.43	423.31	64.89
Phillips Carbon Black (E)									
(1)	2,542.63	1,924.04	1,892.03	2,467.24	2,276.10	2,280.72	2,180.65	1,690.14	1,232.57
(2)	2,546.98	1,926.95	1,894.10	2,470.19	2,277.46	2,284.91	2,186.78	1,695.72	1,232.57
(3)	19.73	18.94	16.76	14.42	20.51	9.27	10.44	20.91	25.62
(4)	1,650.89	1,221.26	1,291.46	1,864.41	1,856.05	1,889.63	1,701.80	1,228.17	937.15
(5)	8.42	15.28	35.61	43.09	-25.54	-26.03	-43.57	11.24	-15.06
(6)	97.18	81.8	72.61	70.16	62.91	58.43	52.35	47.7	36.66
(7)	41.44	51.45	72.1	94.8	80.23	72.13	67.63	43.75	31.21
(8)	60.52	60.62	62.15	57.53	53.74	50.79	48.59	38.58	31.15
(9)	404.44	349.96	311.06	307.34	358.48	289.27	267.01	182.83	100.8
Gujarat Heavy Chemicals (F)									
(1)	2,905.65	2,780.70	2,532.19	2,361.58	2,210.82	2,106.28	1,868.88	1,469.11	1,215.87
(2)	2,905.65	2,780.70	2,532.19	2,373.61	2,224.21	2,124.95	1,896.73	1,498.17	1,215.87
(3)	35.75	10.78	7.47	11.26	5	2.98	9.63	13.3	14.28
(4)	1,100.08	1,069.91	900.42	903.92	888.6	790.91	770.75	593.61	655.15
(5)	23.62	-43.53	-5.33	-12.77	-10.13	5.58	-24.35	-25.15	11.41
(6)	176.37	158.13	133.24	125.87	121.99	111.03	99.93	95.67	82.98
(7)	124.16	133.77	162.82	163.84	170.53	157.96	184.96	110.43	103.39
(8)	109.53	85.69	81.74	84.45	81.57	81.97	80.85	84.4	76.11
(9)	866.62	790.79	806.23	772.06	762.08	739.55	583.56	479.04	131.13
UPL (G)									
(1)	7,091.00	6,794.00	5,821.76	5,226.20	4,814.85	3,826.27	3,216.99	2,822.46	2,699.10
(2)	7,263.00	6,939.00	5,982.53	5,334.99	4,968.27	3,939.44	3,308.00	2,911.09	2,699.10
(3)	435	325	458.78	240.47	317.84	134.32	151.49	153.59	103.88
(4)	3,517.00	3,029.00	2,833.75	2,438.76	2,014.58	1,838.39	1,557.89	1,270.96	1,415.03
(5)	2	-108	-66.28	-207.37	-153.99	-38.2	-116.85	-51.05	108.57
(6)	486	445	390.41	317.8	257.87	237.46	184.65	153.12	127.36
(7)	135	149	192.61	35.27	243.29	105.99	164.37	293.64	108.34
(8)	666	655	243.94	186.75	169.09	157.76	143.49	114.68	107.91
(9)	1,905.00	1,929.00	1,720.56	1,630.12	1,380.77	1,127.93	876.67	788.52	508.63

Revenue From Operations [Net] (1), Total Operating Revenues (2), Other Income (3), Cost Of Materials Consumed (4), Changes In Inventories Of FG,WIP And Stock-In Trade (5), Employee Benefit Expenses (6), Finance Costs (7), Depreciation And Amortization Expenses (8), Other Expenses (9)

Based on existing data in Table 1, one sample t-test had shown a significant difference around 0.001 among criteria such as Revenue From Operations [Net], Total Operating Revenues, Other Income, Cost Of Materials Consumed, Changes In Inventories Of FG, WIP And Stock-In Trade, Employee Benefits Expenses, Finance Costs, Depreciation and Amortization Expenses and Other Expenses. It was found the amount of around 0.806 for the Cronbach' s alpha reliability test. The distributions of revenue from operations (net), total operation revenues, distribution of other income, distribution of changes in inventories of FG, WIP, and stock-in-trade, depreciation amortization expenses and other expenses were obtained normally with mean and standard deviation of 2843.29 and 2273.20, 2877.84 and 2308.48, 2877.84 and 2306.48, 88.06 and 110.02, 64.72 and 140.32, 118.69 and 115.19, 762.99 and 702.47 based on one sample Kolmogorov-Simonov test. Therefore, the null hypothesis was retained for them respectively. The distributions of the cost of materials consumed, employee benefit expenses and finance cost with the mean and standard deviation of 1229.84 and 912.45, 144.00 and 104.08, 88.18 and 74.21 were also achieved normally based on the same test but null hypothesis was rejected for them respectively. Chi-square test had revealed a value of 0.000 for all criteria such as revenue from operation (net), total operating revenues, other income, cost of materials consumed, changes in inventories of FG, WIP and stock-in-trade, employee benefit expenses, finance costs, depreciation and amortization expenses and other expenses. The Friedman test was revealed the mean weights around 8.08, 8.92, 2.68, 6.83, 1.71, 4.37, 2.89, 3.38 and 6.14 for the revenue from operation (net), total operating revenues, other income, cost of materials consumed, changes in inventories of FG, WIP and stock-in-trade, employee benefit expenses, finance costs, depreciation and amortization expenses and other expenses respectively (with a chi-square value around 446.966).

4.2. Performance ranking by TOPSIS based on financial ratio analysis

Tables 2 and 3 present the data associated with financial ratio analysis from 2010 to 2018 for 7 Indian industries and weighted matrix respectively. The columns of tables were composed with the following layout. Liquidity ratio (current ratio (1), quick ratio (2), cash ratio (3)); Turnover ratio (debt turnover ratio (4), assets turnover ratio (5), inventory turnover ratio (6)); Solvency ratio (equity ratio (7), debt-equity ratio (8), debt to total capital ratio (9), (fixed assets/net worth ratio (10))); Profitability ratio (net profit margin ratio (11), (return on net worth/equity ratio (12)), return on capital employed ratio (13), return on assets ratio (14), (total debt/equity ratio (15))).

The vector of A+=0.036499171, 0.03880029, 0.0314006, 0.03763651, 0.041673431, 0.047213935, 0.042005393, 0.061909557, 0.041915856, 0.047567033, 0.04991575, 0.017316467, 0.042615593, 0.042736267, 0.047403448. The vector of A-= 0.01609707, 0.012867944, 0.002692456, 0.013447954, 0.014037746, 0.014127247, 0.00378427, 0.003621045, 0.008111058, 0.007857528, 0.011726501, 0.007936345, 0.014754619, 0.011453475, 0.004778268. Table 4 displays the TOPSIS ranking system results.

Table 2. Data of financial ratio analysis from 2010 to 2018 for seven companies

Co	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	1.39	1.09	7.88	732.5	61.76	7.88	2.25	953.2	21.93	3207.36	13.22	10.3	8.06	5.84	0.43
B	1.4	0.93	89	527.75	38.94	4.59	0.3	5687.71	17.32	9256.13	20.43	12.42	10.69	8.12	0.27
C	1.49	1.2	76.42	764.77	115.6	15.34	2.51	914.69	22.67	6203.37	9.47	19.55	16.03	10.97	0.44
D	1.95	1.58	91.9	608.65	55.87	10.58	2.3	332.67	7.75	8631.25	12.5	10.37	8.97	6.86	0.126
E	0.86	0.58	91	441.6	93.38	6.79	1.59	2275.94	35.89	15368.77	30.35	8.96	5.55	2.94	1.25
F	0.954	0.524	90.78	889.98	71.54	5.21	3.33	1204.13	40.05	19416.36	8.94	17.76	11.52	6.4	1.15
G	1.74	1.37	70.7	318	60.7	6	1.33	1520.95	20.5	5445.96	7.13	9.16	6.57	4.45	0.45

Table 3. Weighted matrix

Co	1	2	3	4	5	6	7	8
A	0.026017358	0.026767288	0.002692456	0.030976813	0.022264283	0.024253312	0.028382022	0.010375387
B	0.026204533	0.022838145	0.030409722	0.022318106	0.014037746	0.014127247	0.00378427	0.061909557
C	0.02788911	0.029468574	0.026111359	0.032341484	0.041673431	0.047213935	0.031661723	0.009956213
D	0.036499171	0.03880029	0.0314006	0.0257393	0.020140957	0.032563457	0.029012734	0.003621045
E	0.01609707	0.014243144	0.031093086	0.018674895	0.033663192	0.020898476	0.020056629	0.02477314
F	0.017856518	0.012867944	0.031017916	0.03763651	0.025789942	0.016035502	0.042005393	0.013106708
G	0.032568491	0.033643289	0.024156936	0.013447954	0.021882156	0.018466989	0.016776929	0.016555229

Co	9	10	11	12	13	14	15
A	0.022951679	0.007857528	0.021742544	0.009123253	0.021427428	0.022751121	0.016306786
B	0.018126907	0.022676065	0.033600619	0.011001049	0.028419257	0.031633408	0.010239145
C	0.023726154	0.015197282	0.01557503	0.017316467	0.042615593	0.042736267	0.016686014
D	0.008111058	0.021145207	0.020558381	0.009185256	0.023846654	0.026724776	0.004778268
E	0.037562049	0.037651073	0.04991575	0.007936345	0.014754619	0.011453475	0.047403448
F	0.041915856	0.047567033	0.014703354	0.015730969	0.030625803	0.024932736	0.043611172
G	0.021455057	0.013341747	0.011726501	0.008113495	0.017466278	0.017336043	0.017065241

Table 4. TOPSIS ranking system results

Co.	di+	di-	(di+)+(di-)	cli+	Ranks
A	0.097371108	0.045241488	0.142612596	0.317233465	6
B	0.084340124	0.076665453	0.161005577	0.476166445	4
C	0.080776734	0.079328008	0.160104742	0.495475692	3
D	0.097880454	0.060411816	0.158292269	0.38164729	5
E	0.07717889	0.083721904	0.160900794	0.520332447	2
F	0.079648027	0.08839631	0.168044337	0.526029684	1
G	0.099611083	0.044510839	0.144121922	0.308841556	7

4.3. Performance analysis based on financial data using DEA method

In many studies the financial performance evaluation ratios have been defined as asset turnover ratio (input/output), inventory turnover ratio (input/output), receivable accounts turnover ratio (input), quick ratio (input), current ratio (input), cash earned from set activities to company earning ratio (input), interest coverage ratio (input), total debt to equity ratio (input), debt ratio (input/output), earning per share ratio (output), return on assets ratio (output), net profit margin ratio (output), economic value added (output), growth rate of sales (output), growth rate of earnings per share (output), sustainable growth rate (output), price to earnings ratio (input/output), Tobin Q ratio (output). A study determined the universe of input/output parameters of introduced into DEA equations including return on equity, return on assets, net profit margin, earnings/share, receivables turnover, inventory turnover, current ratio, quick ratio, debt to equity ratio, leverage ratio, solvency ratios, price to earnings ratio, price to book ratio, revenue growth rate, net income growth rate and EPS growth rate (Edirisinghe and Zhang 2010). DEA is a non-statistical method methodology is used to measure performance in a relative manner and each producer unit or decision maker is compared to the best unit in that industry. Of course, the higher the number of units, the better the comparison and the more realistic results. Simple ratios do not lead to ranking and comparison of companies' performance, and multiple inputs and outputs in this field should be used. Also, through the method of DEA, there is no need for a definite form of production function as it is in the economy, and this technique can be used with minimal data. According to our knowledge, financial ratios and indicators make an ad hoc and a relative appraise of corporate performance, however, we know DEA can be employed to develop very complex investigations (Fenyves et al 2015). Table 5 shows the DEA score for the seven Indian chemical companies [This study].

Table 5. DEA score for the seven Indian chemical companies

Input	Weights	Output	Weights	Productivity	Co.	DEA
Cost Of Materials Consumed	6.83	Revenue From Operations [Net]	8.08	3.84167709	A	1
Changes In Inventories Of FG,WIP And Stock-In Trade	1.71	Total operating revenue	8.92	3.697579817	B	0.963
Employee Benefit Expenses	4.37	Other Income	2.68	3.314725349	C	0.863
Finance Costs	2.89			3.136225512	D	0.817
Depreciation and Amortization Expenses	3.38			2.737699477	E	0.713
Other Expenses	6.14			3.308276204	F	0.862
Weights values based on Friedman test				3.138690474	G	0.818

4.4. Performance analysis based on financial data using COPRAS method

The criteria used for weighing by Entropy Shannon were encompassed; Revenue From Operations [Net] (1), Total Operating Revenues (2), Other Income (3), Cost Of Materials Consumed (4), Changes In Inventories Of FG,WIP And Stock-In Trade (5), Employee Benefit Expenses (6), Finance Costs (7), Depreciation and Amortization Expenses (8), Other Expenses (9). There are negative and positive relations among 9 aforementioned criteria. Therefore, the weighting and ranking systems were selected Entropy Shannon and COPRAS. Table 6 includes weighted values based on Entropy Shannon procedure.

Table 6. Weighted values based on Entropy Shannon procedure

Criteria	E	dj=1-Ej	Wj	$\sum dj$	K
1	1.995278628	-0.99527863	0.133719351	-7.44304112	0.5139
2	1.994522361	-0.99452236	0.133617744		
3	1.817204902	-0.8172049	0.10979449		
4	2.001968116	-1.00196812	0.134618108		
5	0.776434672	0.223565328	-0.03003683		
6	2.008943625	-1.00894363	0.135555294		
7	1.946715084	-0.94671508	0.12719466		
8	1.959818549	-0.95981855	0.128955159		
9	1.942155183	-0.94215518	0.12658202		

Table 7. The ranking system developed in COPRAS method

Co.	Total revenue	Total expenses	Rank based on revenue score	Rank based on expenses score
A	35.44	131.94	3	2
B	13.75	44.2	6	4
C	57.04	291.766	2	1
D	14.3	29.67	5	7
E	15.12	38.7	4	5
F	11.44	31.23	7	6
G	84.089	53.883	1	3

It was found a significant difference about 0.012 between total revenue and total expenses values (between seven industries) in Table 7 according to the t-test analysis.

4.5. The relationship between the weights values obtained from the Friedman test and Entropy Shannon

It was conducted a scatter plot for the data of weights values obtained from the Friedman test and Entropy Shannon base on the results of profit & loss accounts according to Figure 1.

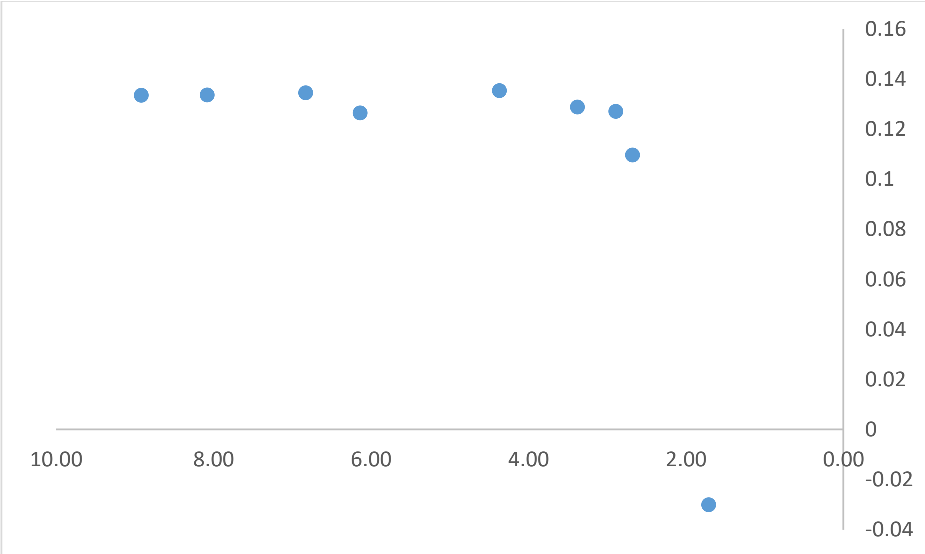

Figure 1. Scatter plot developed for the weights values obtained from the Friedman test and Entropy Shannon

According to the t-teat analysis, there is no significant difference between the weights values obtained from the Friedman test and Entropy Shannon. Moreover, the scatter plot is representing that there is a relatively linear relationship between both weight values obtained from Friedman test and Entropy Shannon with receding the weight values associated to a criterion of changes in inventories of FG, WIP and stock-in-trade.

5. Conclusion

By the present study, we tried to figure out the efficiency of seven Indian industries. The obtained results for the efficiency of industries were approached to full efficiency of industries in most cases. The statistical analysis revealed significant differences among the data of industries. The Friedman test has provided valuable weights for raw values. The Entropy Shannon weighting system has provided the positive and negative weights for existing values and also sought the highest consistency with the COPRAS ranking system. By the way, the COPRAS ranking system had classified industries based on negative and positive criteria (expenses and revenues). The TOPSIS procedure ranked the industries based on the available ratio analysis and it has emerged a good agreement among the industries ratio values. The profit and loss analysis made clear the output incomes and input expenses. Also, it resulted in output and input criteria for introducing into the DEA model. The findings based on the COPRAS model predict the situation of industries for the further financial statement concept. With regard to a rise in the expenses, the ranking system for the income will be taken lots of fluctuations.

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A NOVEL MEMETIC GENETIC ALGORITHM FOR SOLVING TRAVELING SALESMAN PROBLEM BASED ON MULTI-PARENT CROSSOVER TECHNIQUE

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Abstract: *In the present study, a Novel Memetic Genetic Algorithm (NMGA) is developed to solve the Traveling Salesman Problem (TSP). The proposed NMGA is the combination of Boltzmann probability selection and a multi-parent crossover technique with known random mutation. In the proposed multi-parent crossover parents and common crossing point are selected randomly. After comparing the cost/distance with the adjacent nodes (genes) of participated parents, two offspring's are produced. To establish the efficiency of the developed algorithm standard benchmarks are solved from TSPLIB against classical genetic algorithm (GA) and the fruitfulness of the proposed algorithm is recognized. Some statistical test has been done and the parameters are studied.*

Key words: *TSP, Memetic GA, multi-parent crossover.*

1. Introduction

One best example of a well known intensively studied the combinatorial optimization problem is TSP. TSP is also too much related to different type of transportation problem (Kundu, 2017; Kar, 2018) with vehicle convenience. It is also an example of NP-hard problem (Lawler & Lenstra, 1985; Das et al., 2010; Das et al., 2011). Many researchers are trying to solve TSP with reasonable time and space. But still, there have lacunae to solve such kind of NP-hard problems. Presently two ways are implemented such as direct method (Lin-Kernighan Helsgaun, Scant-method, Sant-cycle method) and indirect such as heuristic or metaheuristics. The classical TSP involves finding the shortest path/minimum cost with the visit of all cities exactly ones except the starting one. The probe on the efficient algorithm for TSP is an open problem.

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Moscato (1989) introduced the word Memetic Algorithm (MA) as a combination of population-depended global search and the heuristic local search based on every of the individuals. From the different context of view, MAs are recently used with different names like hybrid evolutionary algorithms (Martinez-Estudillo, 2005), many kinds of literature found in (Jampani, 2010; Li & Feng, 2013; Silberholz, 2013; Hiremath & Hill, 2013; Nesmachnow, 2014; Skinner, 2015), a different Lamarckian Evolutionary Algorithms are studied in (Omran, 2016), very recently a Cultural Algorithms developed found in (Reynolds & Peng, 2004). In the case of colonial optimization, the various number of instances of MA have been notifiable across a broad scope of application realm, generally merging to better-tone solutions much expeditious than the established affected counterparts. Real-world complex problems have been successfully satisfied with memetic algorithms. Although various researcher avail procedures nearly bound up to MA, through other names like hybrid genetic algorithms are also exploited. Nowadays MAs are applied in different research areas included pattern recognition (Aguilar & Colmenares, 1998), artificial neural networks (Ichimura & Kuriyama, 1998), circuit design (Harris & Ifeachor, 1998), robotic motion planning, beam orientation (Haas et al., 1998), electric service restoration (Kumar et al., 2006), medical expert systems (Wehrens, 1993), single machine scheduling (Chyu & Chang, 2010), etc. A study on multi-parent MA is found in (Wang et al., 2010) and they concluded that different combination got better results from others. Ye et al., (2014) developed a multi-parent recombination operator for solving Linear Ordering Problem but they do not restrict to chosen the parent because it increases the computational complexity. At the present investigation, only four parents are selected from the mating pool and randomly a common crossing point is chosen.

Genetic Algorithms (GAs) were first proposed by Holland (1992) whose ideas were applied and expanded on by Goldberg (1998). The classical GA has three operators, such as selection, crossover and mutation. Different kinds of selection operators (RW, Ranking, Tournament, etc.) and cyclic, partial-map, ordered based, etc. crossovers are available with a random mutation to solve the discrete optimization problem by GA.

In our proposed method (NMGA) are the combinations of probabilistic selection and adaptive four- parents crossover with the classical ergodic mutation. Now the crossover is taken from the realistic social observations. We see that some child born with legal parents but they adapted by other parents and grown up under them. In third world countries, it is very common. In the proposed crossover methods four parents are used and modified them finally comparing the costs/distance, the offspring is created.

The proposed algorithm has the following key features:

- Boltzmann probabilistic selection,
- Multi-parent adaptive crossover,
- Four parents,
- Random crossing point,
- Comparing the cost/distance genes are selected,
- Test on standard TSPLIB problems.

The remaining part of this paper is presented as follows: section 1, a short introduction is presented. In section 2, We describe mathematical pre-requisite. In section 3, the proposed modified memetic algorithm is presented. In section 4, some numerical experiments are done. Again in section 5, a brief discussion is given. Finally, in section 6, a conclusion with future scope is studied.

2. Classical Definition of TSP

The goal of a Traveling Salesman Problem (TSP) is that a salesperson would create a path. This path should be an ideal path. Ideal path means path would be the shortest while salesman completes his visit across a finite number of cities, visiting each city only once and finished at the starting city. Let G=(V, A) is a graph. G has n vertices and V is a set of this n vertices. A is also a set of arcs or edges of this G. Then G=(V, A). Let $C = c(i,j)$ be a distance (or cost) matrix associated with A. The intent of TSP is determining a minimum distance or cost circuit passing through each and every vertex only once except starting node. This type of circuit is familiar as a tour or Hamiltonian circuit or cycle. In case of symmetric TSP $c(i,j) = c(j,i)$ for all $i,j\in$ V. (n-1)! path will be generated for symmetric TSP and (n-1)!/2 path will be generated for asymmetric TSP. Now mathematically TSP defined as below.

Minimize $Z = \sum_{i \neq j} c(i, j) x_{ij}$

subject to

$$\sum_{i=1}^{N} x_{ij} = 1 \quad \text{for} \quad j = 1, 2, \ldots, N;$$

$$\sum_{j=1}^{N} x_{ij} = 1 \text{ for }, i = 1, 2, \ldots, N;$$

$$\sum_{i \in S}^{N} \sum_{j \in S}^{N} x_{ij} \leq |S| - 1, \forall S \subset Q;$$

Where $x_{ij} \in \{0,1\}, i, j = 1, 2.., N$.

Now,

$Q = \{1, 2, 3,.., N\}$ set of nodes

x_{ij} = the decision variable,

$x_{ij} = 1$ if thesalesman visits from city-i to city-j,

$x_{ij} = 0$ otherwise.

Then the above TSP reduces todetermine

a complete tour $(x_1, x_2, \ldots, x_N, x_1)$;

$$Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}) + c(x_N, x_1)$$

Where, $x_i \neq x_j, i, j = 1, 2 \ldots, N$.

along with sub tour elimination criteria

$$\sum_{i\in S}^{N}\sum_{j\in S}^{N} x_{ij} \le |S|-1, \forall S \subset Q \text{, where, } \quad x_{ij} \in \{0,1\}, i, j = 1, 2.., N.$$

3. Proposed Memetic Genetic Algorithm

Here we propose a probabilistic selection, multi-parent crossover with the simple random mutation for solving the TSP.

3.1. Representation

Considering N cities available to make a complete tour which stands for a solution. Say an integer vector X_i of N-dimensional. Where $X_i = (x_{i1}, x_{i2}, \cdots, x_{iN})$ is used as cities, and x_{i1}, x_{i2}, ⋯, $x_N$stand for N successive cities in a tour. At the beginning need a group of paths (tours) for the salesman. These paths are randomly generated for GA. These initial paths is a group of possible solutions for the GA part of this algorithm.

3.2 Selection

3.2.1. Probabilistic Selection

The main objective of TSP that minimizing the path cost/distance. So here minimum fitness value (say f_{min}) of the choromosome play a vital role. Matting pool is formed using the Boltzmann-Probability (Roy et al., 2018) of all chromosome from the initial population.

Now $p_B = e^{((f_{min} - f(X_j))/T)}$; $T=T_0(1-a)^k$, $k=(1+100*(g/G))$, g=ongoing generation number, G= maximum generation, T_0= rand[10,150], $f(X_i)$ corresponding fitness/objectives of chromosome corresponding to X_i, a=rand[0,1], i=chromosome number.

3.3. Multi-Parent Crossover

Nowadays in our society adaptation is very common matter from the different practical situation. Here except original parents (father, mother), there are one more parents (father, mother) taken as a part. Inspiring this realistic happening here a new approach with four parents (first two are original parents and the other two are adoptive parents) are used to produce offspring. This urged crossover method choose four individuals or parents in an ergodic manner to create offspring. To collect optimum results of a TSP, we make a journey from one node to next node maintaining minimum traveling cost based on TSP condition. Following the above conception, we make the crossover procedure in the following condition. At first select (randomly) four individuals (parents) from the mating pool. Give an example here. PR_1, PR_2, PR_3, PR_4 are the parents and CH_1, CH_2 are the offspring.

PR_1:

1	2	0	3	4

PR_2:

0	2	4	3	1

PR_3:

4	0	3	1	2

PR_4:

3	4	1	2	0

Generate random number between 0 and 4. Suppose it is 2. Then according to our proposed algorithm it would be the starting node of a new offspring (CH_1).

CH_1: | 2 | | | | |

Now we have to comparing minimum traveling cost between

| 2 | node··· (1) (1st node of Parent1 [PR_1])
| 2 | node··· (0) (1st node of Parent2 [PR_2])
| 2 | node··· (4) (1st node of Parent3 [PR_3])
| 2 | node··· (3) (1st node of Parent4 [PR_4])

and if say the traveling cost node (2) to node (4) is minimum from rest three paths, then next node of the new offspring (CH_1) would be 4. So it should be like as-

CH_1: | 2 | 4 | | | |

The above process will continue until the new offspring (CH_1) gets its all nodes maintaining the condition of TSP. Similarly, generate the second offspring but in reverse order than another. Here R_1 and R_2 are two randomly generated variable two nodes from the given set of nodes.

3.4. Random Mutation

An ergodic number r is created for every solution of P(t). Here r is generated between a range [0,1] and $r < p_m$ is a condition, if the condition is true, then a solution is selected for mutation. Two nodes are selected ergodic manner from each chromosome and they interchange their positions and replace it in the offspring set.

3.5. Proposed Novel Memetic Algorithm (NMGA)

NMGA Algorithm:

1. Input: for Crossover procedure (p_c), $Maximum_{gen}(S_0)$, (pop-size) and for Mutation procedure (p_m).
2. Output: The best solution.
3. Begin
4. Approve generation $t = 0$.
5. (Initialize) ergodic manner and generate approve population $p(t)$, here $f(x_i), i= 1,2,\cdots,(pop - size)$ state the all chromosomes.
6. All solution will be judged it's efficiency one by one from the approve population $p(t)$
7. Repeat up to (18) till ($t < S_0$)
8. Modify the current generation such as $t = t + 1$
9. Decide (p_B) for all chromosome over $p(t)$ to subsection (3.2.1)
10. Construct the mating pool on the basis $p_s$and p_B
11. For crossover parents will be chosen based on p_c over mating pool
12. According to subsection (3.3) the crossover operation will be conducted based on exclusive chromosomes/solutions
13. Produce offspring and the parents will be replaced
14. Repeat (9) to (11) based on p_c
15. Followed by the subsection (3.4) mutation process will be executed
16. Offspring will be selected for mutation depend on p_m
17. Interchange the position between selected nodes
18. Replace offspring
19. Determine the effectiveness and save the local best and near best solutions

20. Repeat (5) to (18)
21. (for best result) Store the global best and near best results
22. Stop

Proposed NMGA pseudo code:

```
Begin generation t = 0;
    for (i=1 to pop-size)
     Produce chromosome ergodic manner; end for;
     for (i=1 to pop-size)
     Judge fitness;
     end for;
     for (gen=1 to max-gen)
       {
       for (j=1 to pop - size) r=rand[0,1]; T0= rand[5,100];
         a=rand[0,1]; k=(1+100*(i/G));
           T=T0(1-a)^k;
           p_B = e^(((f_min - f(X_j))/T));
       if ( r < p_s )
             select the current chromosome; j++;
         else if (r < p_B )
             select f(X_j ) ; j++;
         else
            Select the corresponding chromosome of f_min ; j++;
       end for;
        end for;
for (s=1 to (noc * p_c))
     R1=rand[0, N-1];
     R2=rand[0, N-1];
     PR1=rand[0, pv-1];
     PR2=rand[0, pv-1];
     PR3=rand[0, pv-1];
     PR4=rand[0,
     pv-1];
     CH1[0]=R1;
     i=1; do{
CH1[i] = min {c(R1,PR1[0]),c(R1,PR2[0]),c(R1,PR3[0]),c(R1,PR4[0])}; i=i+1;}
while(CH1[ N-1]);
 CH2[n-1]=R2;
 i = n − 2 ;
do{
CH2[i] = min {(R2,PR1[N−1]),c(R2,PR2[N−1]),c(R2,PR3[N−1]),c(R2,PR4[N−
1])}};
i = i − 1 ;
   }
  while(CH2[0]) ;
  End for
 for(a=0 to noc)
{
```

If (*rand*[0,1] < p_m)
mutate;
}
for (i=1 to noc)
Evaluate fitness; *end for*
}
Stop

3.6. Termination Criteria

The proposed algorithm is terminated if it finishes the user-defined maximum number of generations or iterations, or the difference between consecutive iterations less than some predefined values which are earlier.

4. Numerical Experiments

4.1. Test Results of NMGA

We have taken benchmarks from TSPLIB (Reinelt, 1991) and select 53 standard instances form 7 city to 318 cities to test the performance of our proposed algorithm NMGA. Table 1 shows the comparison of performance between proposed NMGA and standard classical GA through the results presented in the form of percentage error. The total comparison held the basis of total cost. We have taken the best, average and the worst outcome of both NMGA and classical GA under 100 independent runs and the best solution is presented with relative percentage error. The parameters of the NMGA given in Table 2 for the same nodes of the benchmarks instance kora100 with 100 cities problem. We have increases pop-size, Maxgen, p_c and p_m of an instance as a parameter.

4.1.1. Sensitivity of CPU-time w.r.t. p_c and p_m

Sensitivity analysis is performed for CPU-time on the basis of concerned values of the key parameters p_m and p_c and outcomes are projected in Figure1 (three dimensions linear graph using STATISTICA). It is observed that for fixed value of p_c, as p_m increases, CPU-time increases. Also, it is observed that for a fixed value of p_m, as p_c increases, CPU-time also increases.

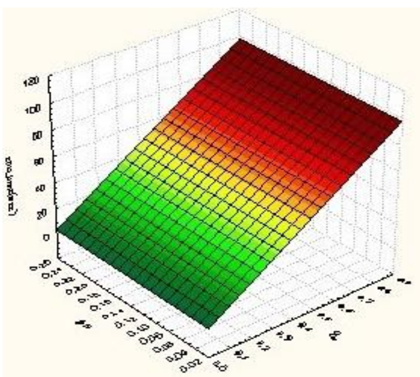

Figure 1. Sensitivity Analysis

5. Discussion

The superiority of the developed algorithm is established by solving standard benchmarks from TSPLIB which given in Table 1. This proposed algorithm NMGA is coded in C++ based on few keys like the maximum number of chromosomes (100) and a maximum number of iterations (5000). Table 1 is used to show the differences between NMGA and GA for few benchmark TSP references in TSPLIB. It is observed that the percentage of error is lesser in NMGA than the classical standard GA. Here, 53 standard instances from 7 to 318 cities are studied and most of the cases NMGA produced better results. A parameter analysis is done which is given in Table 2 for the standard benchmark kora100 with 100 cities problem.

Table 1. Performance (relative error) of benchmarks from

Instances	NMGA			Classical GA		
	Best	Worst	Average	Best	Worst	Average
sh-07	0	0	0	0	0	0
sp11	0	0	0	0	0.15	0.07
uk12	0	0	0	0.04	0.22	0.15
lau15	0	0	0	0.29	0.55	0.44
gr17	0	0	0	0.22	0.52	0.39
wg22	0	0.02	0.01	0.65	1.07	0.94
fri26	0	0.02	0.01	0.76	1.09	0.98
bay29	0	0.06	0.01	1.01	1.25	1.12
bayg29	-0.02	0.04	0.02	0.92	1.22	1.10
wi29	-0.00	0.06	0.02	1.29	1.77	1.59
ha30	0	0.08	0.02	1.14	1.53	1.37
dj38	0.00	0.04	0.01	1.90	2.24	2.12
dantzig42	0.00	0.11	0.03	1.98	2.45	2.28
swiss42	0.00	0.11	0.03	1.75	2.04	1.91
att48	2.16	2.40	2.25	9.07	10.60	10.11
eil51	0.00	0.06	0.03	1.94	2.22	2.11
berlin52	0.00	0.14	0.05	1.90	2.24	2.14
wg59	0.00	0.18	0.07	2.73	3.17	2.99
st70	0.00	0.14	0.04	3.18	3.60	3.48
eil76	0.01	0.09	0.06	3.04	3.59	3.39
pr76	0.01	0.26	0.11	3.20	3.59	3.45
rat99	0.01	0.12	0.06	14.68	16.28	15.80
kroA100	0.00	0.32	0.12	5.25	6.00	5.76
kroB100	0.02	0.27	0.10	5.05	5.64	5.44
kroC100	0.02	0.22	0.10	5.19	6.14	5.90
kroD100	0.03	0.21	0.10	5.08	5.70	5.50
kroE100	0.02	0.25	0.09	5.24	5.87	5.63
rd100	0.01	0.20	0.11	4.65	5.25	5.07
eil101	0.04	0.15	0.09	3.51	3.85	3.71
lin105	0.01	0.26	0.13	5.95	6.50	6.28
pr107	0.01	0.20	0.09	9.28	10.36	9.95
pr124	0.01	0.41	0.14	8.74	9.59	9.26
bier127	0.07	0.39	0.20	3.60	3.83	3.74
ch130	0.03	0.20	0.12	5.37	5.85	5.70
pr136	0.09	0.36	0.20	6.18	6.70	6.49
pr144	0.01	0.39	0.15	10.71	11.60	11.24

kroA150	0.06	0.37	0.18	6.91	7.80	7.52
kroB150	0.08	0.34	0.20	6.92	7.88	7.58
pr152	0.14	21.88	13.26	11.31	11.94	11.67
u159	0.03	0.27	0.16	8.13	8.80	8.45
qa194	0.09	0.27	0.17	2.28	3.20	2.23
rat195	0.06	0.25	0.16	32.79	35.82	34.81
d198	0.10	0.29	0.20	9.20	10.05	9.66
kroA200	0.13	0.45	0.26	8.95	9.73	9.37
kroB200	0.16	0.42	0.28	8.73	9.43	9.14
ts225	0.08	0.35	0.19	10.13	10.75	10.49
tsp225	0.07	0.27	0.17	8.30	8.87	8.61
pr226	0.11	0.84	0.23	16.82	18.62	18.12
gil262	0.12	0.37	0.24	9.03	9.58	9.33
pr264	0.20	0.41	0.30	17.90	20.19	19.37
a280	0.15	0.37	0.27	10.61	11.33	11.00
pr299	0.12	0.37	0.24	12.85	13.74	13.35
lin318	0.17	0.40	0.29	11.57	12.20	11.94

In Table 2, it is used to calculate the goodness of parameter of selection (p_s) in NMGA. It shows that to get the optimal solution of the standard TSP kroA100, $p_s$indicates the given space better for p_s= 0.34.

Table 2. Parameters for NMGA of kroA100 instance

Instance	p_c	pm	pop_{size}	Gen	result	cpu-time(sec)
	0.34	0.01		4673	21417	5526
		0.02		4065	21344	5733
		0.001		3407	21322	5405
		0.003		4957	21298	5373
		0.005		2427	21294	5380
		0.007		2173	21316	4810
		0.008		3376	21285	4470
		0.009		3068	21384	4214
kroA100	0.2	0.01	70	2384	21322	3331
	0.25			2787	21386	3232
	0.30			4958	21333	3916
	0.35			2612	21285	5574
	0.40			4868	21294	6164
	0.45			3883	21412	6256
	0.50			3708	21349	6149
	0.55			3505	21365	4407
	0.60			4406	21316	4438
	0.70			4467	21535	6640
	0.75			4320	21334	6874
	0.80			4975	21831	9982
	0.85			3905	21335	9840
	0.34	0.01	50	3438	21390	3175
			60	4638	21474	8382

Instance	p_c	pm	pop_{size}	Gen	result	cpu-time(sec)
			72	4453	21322	4470
			85	3460	21322	4952
			110	2706	21285	7664
			150	4700	21294	8974

It is evident from Table 2 that, for standard three parameters p_c, p_m and $pop - size$, our proposed algorithm NMGA give us optimum or near optimum result easily. Thus the importance of the parameters is discussed in Table 2.

6. Conclusion

In this paper, a novelty introduced in GA regarding selection and crossover, called Novel Memetic Genetic Algorithm (NMGA). NMGA is tested with few test references from TSPLIB and examined with classical GA. In NMGA, Boltzmann probabilistic selection and a new four parents crossover are worked with ergodic mutation. The concept of MA is not new in the area of TSPs, but the idea of multi-parent(four) crossover on the basis of the memetic concept is new, establishes our proposed algorithm as highly NP-hard combinatorial optimization problem. Realistically, it is true that multi-parent crossover especially four parent crossover may not be so much truthful than two parent crossover for a specific problem or the complexity may be high than other. But the numerical analysis proves the efficiency of our proposed algorithm. The improvement of developed NMGA is in natural form and it is also applicable to solve another discrete problem like network optimization, well-known Graph Theory, popular Standard Transportation Problems, vehicle routing problem, and electronics manufacturing units, etc. Although we have to get the much superior results by NMGA, there is also have a huge scope for improvement also.

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A 2-OPT GUIDED DISCRETE ANTLION OPTIMIZATION ALGORITHM FOR MULTI-DEPOT VEHICLE ROUTING PROBLEM

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Abstract: *The Multi-depot vehicle routing problem (MDVRP) is a real-world variant of the vehicle routing problem (VRP) where the customers are getting service from some depots. The main target of MDVRP is to find the route plan of each vehicle for all the depots to fulfill the demands of all the customers, as well as that, needs the least distance to travel. Here all the vehicles start from different depots and return to the same after serving the customers in its route. In MDVRP each customer node must be served by only one vehicle which starts from any of the depots. In this paper, we have considered a homogeneous fleet of vehicles. Here a bio-inspired meta-heuristic method named Discrete Antli-on Optimization algorithm (DALO) followed by the 2-opt algorithm for local searching is used to minimize the total routing distance of the MDVRP. The comparison with the Genetic Algorithm, Ant colony optimization, and known best solutions is also discussed and analyzed.*

Key words*: Multi depot vehicle routing problem, Antlion Optimization (ALO), Bio-inspired Algorithm, Combinatorial Optimization.*

1. Introduction

Supply of goods from source to destination is a challenging operational process in the logistic distribution system. The products can be delivered either directly from the production center or from the stock points located nearby the production site or via distribution warehouses. Such kind of problems can be mathematically modeled as a particular type of VRP which belongs to the set of NP-hard problems. It consists of a single depot or warehouse to service the demands of different cities, but most of the cases the different company has more than one warehouse to serve the demands.

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In such a scenario the problem can be formulated using more than one depot that is called Multi-Depot Vehicle Routing Problem, in short MDVRP. MDVRP deals with the delivery of items to all the customers with minimum cost or distance. VRP can be used to manage such kind of scenario efficiently.

The main task of the basic form of vehicle routing problem is to search the collection of paths to serve customers with some similar vehicles. In the classic form of VRP, a set of customer node is present, the demands of each node and other primary information such as the distance between all pair of nodes, the distance between nodes and depots, number of vehicles and vehicle capacity are known a priory. The VRP can be closed or open. In closed VRP (Laporte et al. 1987) vehicles move from a central point called depot, serves each customer and back to the central position such that the total demand served by one conveyance is less than the vehicle capacity. Whereas in the case of open VRP (Li et al. 2007) after serving the customer the vehicle does not return to the depot.

There are many variants of VRP found in the literature; some of them are capacitated VRP (CVRP), VRP with time window (VRPTW), VRP that includes pickup and delivery, multi-depot VRP, stochastic VRP, etc. In this paper, we have focused on Multi-depot VRP (MDVRP). The pictorial representation of MDVRP is presented in figure 1. In MDVRP, there will be more than one depot.

For solving MDVRP, the following two steps can be used:

I Clustering: Allocation of cities to a depot.

II Routing: Finding the optimum routes for each depot. This sub-problem is similar to VRP.

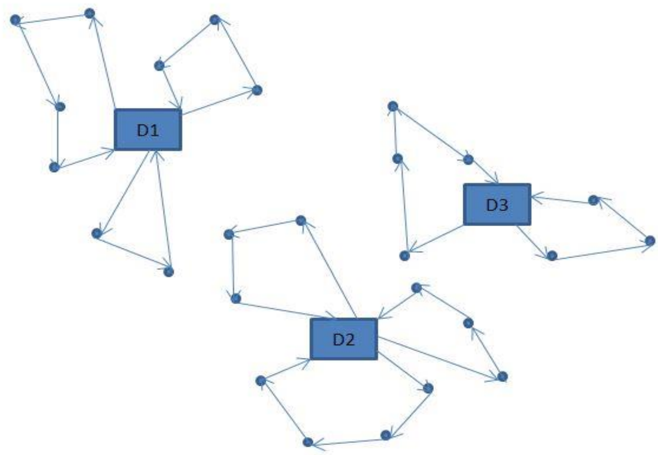

Figure1. Pictorial representation of MDVRP

MDVRP can be solved in two ways considering the two sub-problems, one is route first cluster second, and another is cluster first route second. Here we have discretized the Ant lion optimization (ALO) algorithm to solve the MDVRP. For local searching of routes, the 3-opt algorithm is used. The main contribution of this article is as follows: (1) An improved discrete ALO has been proposed to fit the MDVRP; (2) A new encoding scheme to form a solution (ant or antlion) and (3) A hybridization of ALO and 2 opt algorithm.

The paper is arranged as per the below sections. The literature review presents in section 2. The motivation behind this work is explained in section 3. Section 4 describes the mathematical model for the MDVRP. Section 5 deals with the proposed Discrete ALO. The result and discussion are presented in section 6. The conclusion is in section 7.

2. Literature Review

Some of the solving techniques for single depot VRP are exact algorithms like brunch and bound, branch and cut proposed by Fisher et al., (1994) and Lada et al. in 2001. Many heuristic algorithms like cluster first route second (Taillard, 1993), savings algorithm (Clarke & Wright, 1964) also found in the literature. Meta-heuristic like GA (Berger & Barkaoui, 2003), PSO (Chen et al., 2006), ACO (Reimann et al., 2004) are also used by many researchers to solve single depot VRP.

Laporte et al., (1984, 1988) formulated the integer linear programs for MDVRP containing degree constraints, sub-tour elimination constraints, chain-barring constraints, and integrality constraints and presented an exact solution. Renaud et al., (1996) presents a Tabu search heuristic for MDVRP. Chao et al. solved the MDVRP using a multi-phase heuristic approach. Ombuki-Berman & Hanshar (2009) applied a genetic algorithm to MDVRP. Vianna et al., (1999) proposed an evolutionary algorithm coupled with local search heuristic to minimize the total cost. Matos and Oliveira (2004) have to use ant colony optimization (ACO) to solve MDVRP. Guimarães et al., (2019) have published a paper on the multi-depot inventory-routing problem with the application on a two-echelon (2E) supply chain. It is also showing a stricter policy for inventory management. In 2017, a different version of MDVRP was developed that deals with hazardous materials by Yuan et al., (2017). It was solved using a two-stage heuristic method. In the same year, Rabbouch et al., (2017) have published a survey paper on MDVRP for heterogeneous vehicles. It also considered the time windows concept. Very recently Lalla-Ruiz & Vob (2019) have developed multi-depot cumulative capacitated VRP. It also designed a meta-heuristic approach (POPMUSIC) to solve it. In 2018, one more paper has also been published on MDVRP, and it has been solved using general variable neighborhood search meta-heuristic (Bezerra et al., 2018). It uses a local search method named randomized variable neighborhood descent. Li et al., (2018) have presented a paper on MDVRP with fuel consumption to make the benefits analysis. It finds the factors that affect the benefit ratio. In the same year, one more paper on MDVRP has also been published that deals with multi-compartment vehicles. It uses the hybrid adaptive large neighborhood search (Alinaghian & Shokouhi, 2018) to solve the problem. One more new variety of MDVRP has been proposed by Zhou et al., (2018). They have developed two –Echelon MDVRP that introduces the last mile distribution in the city logistics problem. It has been solved using a hybrid multi-population genetic algorithm. Silva et al., (2018) have presented a paper on multi-depot online vehicle routing with a soft boundary. Recently Zhang et al., (2019) have published an article on MDVRP for routing alternate fuel vehicles. They have used the ant colony method. Very recently Dutta et al., (2019) have designed a modified version of Kruskal's algorithm over the GA to solve OVRP for a single depot problem. Mukherjee et al., (2019) have developed a special version of the TSP problem that can be mapped on several real-life scenarios.

3. Motivation

There are several works that have already been published in the field of VRP using the exact method and meta-heuristics algorithms. But most of the real-life problems fit with the MDVRP. e.g., newspaper distribution, courier services, emergency services, taxi services, and refuse-collection management, etc. In literature, there are some works on MDVRP but in most of the cases they used meta-heuristic algorithms, and in few cases, exact algorithms were used. Exact algorithms give better result but take longer computational time. Meta-heuristic algorithms take less computational time but will not provide the best solution always. So finding good meta-heuristic to address the real-life problem which will give better result in reasonable computational time is a tough job. So here we try to find a hybrid algorithm which will combine an exact algorithm and one meta-heuristic algorithm to address MDVRP. Two competitive firms produce two substitute products and sell their products separately in the market.

4. Mathematical Model

The MDVRP can be represented using a graph G = (V, E) where V is the union of two subsets namely, *Vc = {V1, . . . ,Vn}* the set of city or customer and *Vd = {Vn+1,..., Vn+m}* the set of depots, and E is the edge set. A cost or distance matrix *C= {cij}* is the cost of traveling from city *i* to city *j*. Each city vi has a demand qi. In this paper symmetric cost or distance matrix is considered and triangular inequality also satisfied in C. Here all depots have a finite set of homogeneous vehicles with capacity Q. The solution to an MDVRP consists of a set of vehicle routes each starts and ends at the same depot, and each customer node is visited exactly once by only one vehicle. The total demand of customers in each route must not exceed the vehicle capacity Q. Here the goal is to minimize the total routing cost.

In this problem, n nodes are grouped into m cluster where each cluster contain ni: i = 1,2,…,m number of node and each ni clusters are again group by kj groups depending on the vehicle capacity.

The mathematical model for MDVRP proposed by Lang is given below.

$$Min\ Z = \sum_{p=1}^{m} \sum_{q=1}^{k_p} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ijpq} \tag{1}$$

Subject to

$$\sum_{j=1}^{n} q_i y_{iqp} \leq Q \tag{2}$$

$$0 \leq n_{jq} \leq n_j \tag{3}$$

$$\sum_{q=1}^{k_p} n_{jq} = \ n_j \quad \forall\, j = 1\ tom \tag{4}$$

$$\mathrm{a}\textstyle\sum_{j=1}^{m} n_j = \mathrm{n} \tag{5}$$

$$\sum_{p=1}^{m}\sum_{q=1}^{k_p} y_{iqp} = 1 \tag{6}$$

$$x_{ijqp} = \begin{cases} 1 \text{ if vehicle p in depot q travels from customer i to customer j} \\ 0 \;\; otherwise \end{cases} \tag{7}$$

$$y_{ikm} = \begin{cases} 1 \text{ if vehicle k of depot m serves customer i} \\ 0 \;\; otherwise \end{cases} \tag{8}$$

The equation (1) is the objective function, minimizes the total traveling distance or cost. Equation (2) ensures the capacity constraint of a vehicle. Equation (3) guarantees that the vehicles serving the number of customers must not exceed the number of customers in a depot. Equation (4) shows that the total number of customers served by the entire route must be equal to the sum of customers served by depot m. Each customer must be served from a single depot is ensured in Equation (5). Equation (6) shows that each customer is serviced not more than once. Equation (7) and (8) represents that the decision variables are binary.

5. Proposed Discrete ALO Algorithm

In this paper, we have used the Ant Lion Optimization algorithm proposed by Mirjalili (2015). ALO is a bio-inspired algorithm that mimics the foraging behavior of antlion. The steps of ALO are given below:

- Initialization of ant and antlions
- Random walk of ants
- Building traps by antlions
- Entrapment of ants in traps prepared by the antlion
- Catching preys by antlion
- Re-building traps.
- Elitism
- Here 2-opt algorithm is used to optimize each route covered by one vehicle.

5.1. Encoding Scheme

An MDVRP contains n cities and m depots. We have used cluster first, route second approach. So to represent an ant or ant lion one integer array A of size n is considered, and the array elements will be ranging from 1 to m. An element *A[i]* represents that ith city will be served from depot *A[i]*. As an example consider n as 10 and m as 3 then an ant or an antlion will be as in figure 2.

2	1	1	3	3	2	2	1	3	2
1	2	3	4	5	6	7	8	9	10

Figure 2. Encoding of an ant

From Figure 2 it is clear that depot 1 will serve city 2, city 3 and city 8, depot 2 will serve city 1, city 6 and city 7 and depot 3 will serve city 4, city 5 and city 9.

5.2. Fitness Evaluation

In this paper, the fitness function is considered as same as the objective function. Now to evaluate the value of fitness function we have to find the depot corresponding to each city and the vehicle which will serve the city. From the encoding scheme stated above, it is clear that which city will be served from which depot. Then we have to find the vehicle routes starting from each depot. Here we have applied a very well-known 2-opt algorithm to find the shortest path starts and end in the same depot after serving all the cities in the route.

Therefore, Total fitness value = the total distances traveled by all the vehicles from all depots. Consider an ant A as follows.

3	1	3	1	2	1	2	3	2	2

Then Depot 1 will serve city 2, 4, 6; depot 2 will serve city 5, 7, 9, 10 and depot 3 will serve city 1, 3, 8. Now according to the vehicle capacity routes are to be decided from each vehicle from the depot. Assume one vehicle is required for depot 1. Then the initial route will be as {0, 2, 4, 6, 0} for depot 1. Now, this is very similar to the traveling salesman problem. Here we have used a 2-opt algorithm for local search to optimize the route length. A similar approach is taken for all the routes from the different depot, and finally, all the route lengths are added to get the fitness value.

5.3. Operators of ALO

The Antlion Optimizer does a mimic of the relationship of antlions and ants. The ants will move on the search space, and the antlions are building traps to hunt ants. After capturing an ant, the position of the Antlion is updated if it becomes fitter. The movement of ant for searching food is stochastic therefore a random walk is as follows

$$x(t) = [0, cumsum(2r(t_1) - 1), cumsum(2r(t_2) - 1), \dots, cumsum(2r(t_n) - 1)] \quad (9)$$

Where cumsum represents the cumulative sum where n represents the maximum iteration number and t, gives the step of random walk and r(t) is a random function given by:

$$r(t) = \begin{cases} 1\ if rand > 0.5 \\ 0\ if rand \leq 0.5 \end{cases} \quad (10)$$

The position of ant and antlions are stored in the following matrix respectively

$$M_{Ant} = \begin{bmatrix} A_{1,1} & \cdots & A_{1,d} \\ \vdots & \ddots & \vdots \\ A_{n,1} & \cdots & A_{n,d} \end{bmatrix} \quad (11)$$

$$M_{Antlion} = \begin{bmatrix} Al_{1,1} & \cdots & Al_{1,d} \\ \vdots & \ddots & \vdots \\ Al_{n,1} & \cdots & Al_{n,d} \end{bmatrix} \quad (12)$$

A fitness function is used to identify the quality of ant and antlion during the optimization process. Two different matrices MOA and MOAL are used to store the fitness of all ant and antlion respectively. The matrices are as follows.

$$M_{OA} = \begin{bmatrix} f([A_{1,1}, A_{1,2}, \dots\dots, A_{1,\mathrm{d}}]) \\ f([A_{2,1}, A_{2,2}, \dots\dots, A_{2,\mathrm{d}}]) \\ \vdots \\ \vdots \\ f([A_{\mathrm{n},1}, A_{\mathrm{n},2}, \dots\dots, A_{\mathrm{n},\mathrm{d}}]) \end{bmatrix} \tag{13}$$

$$M_{OAL} = \begin{bmatrix} f([Al_{1,1}, Al_{1,2}, \dots\dots, Al_{1,\mathrm{d}}]) \\ f([Al_{2,1}, A_{2,2}, \dots\dots, Al_{2,\mathrm{d}}]) \\ \vdots \\ \vdots \\ f([Al_{\mathrm{n},1}, Al_{\mathrm{n},2}, \dots\dots, Al_{\mathrm{n},\mathrm{d}}]) \end{bmatrix} \tag{14}$$

Where *f* is the objective function. *Ai,j* gives the value of the *j*th dimension of *i*th ant, *n* represents the total number of ants and is similar for antlions.

The ALO (Mirjalili, 2015) was designed to solve continuous problems. In this paper, we are focused on solving MDVRP which is one combinatorial optimization problem. So the operators used in original ALO may not work as desired hence we have customized the operators according to our requirement.

Initialization

In this step, two populations of size N for ant and antlion are formed randomly. Let us assume n number of customers and m number of depots is present. Assume (*Al1, Al2,……, AlN*) and (*A1, A2,……, AN*) are the populations of antlion and ant respectively. Then each Alj and Aj represents the *j*th antlion and ant respectively. Both *Alj* and *Aj* are a one-dimensional array of size *n*, and the array elements will range from 1 to *m*.

Random walks of ants

In case of discrete problem random walk of an ant is implemented by inverting the entities of a randomly selected part of the string. The operation is demonstrated in Figure 3.

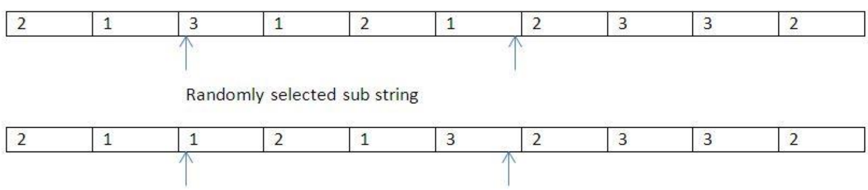

Figure 3. Random Walk of an Ant

Building traps by Antlion

In ALO, each antlion builds a trap to catch one ant. To implement this mechanism, we have used the Roulette-wheel selection mechanism to select Antlion. Roulette wheel selection chooses the fitter Antlions for catching ants with higher probability.

Entrapment of ants in traps

Ants are moving randomly in search of food while antlions build traps. The higher the fitness, the bigger the trap is. When an ant falls in the trap antlion shoot sand on it; as a result, the ant slides down towards the trap. To realize this scenario crossover operator of GA is used. In this step crossover between one selected antlion and one ant is performed. The operation is pictorially represented in figure 4. One sub-string of an ant is selected randomly, and that substring is copied into the corresponding antlion.

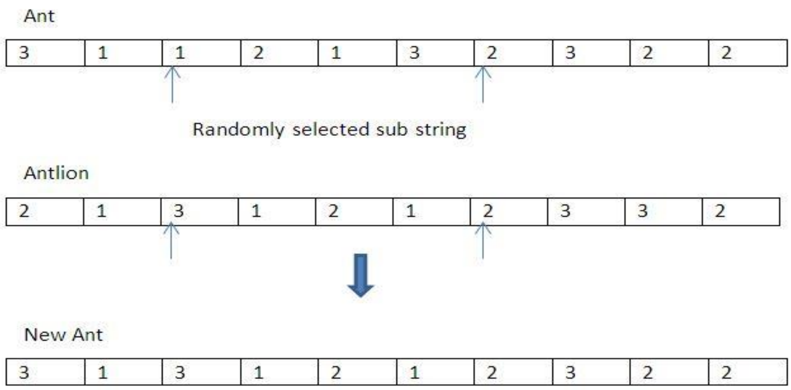

Figure 4. Representation of crossover operation

Catching of prey, re-construction of pit

The final step of ALO reaches after an antlion catches the prey. To mimic the step, it is considered that catching of ant happens when prey is going to be fitter than the corresponding antlion. Then the antlion will change the location to the corresponding ant to increase the chance of catching a new pre. The above scenario is mathematically represented by the equation (15).

$$Antlion_j^t = Ant_i^t iff(Ant_i^t) > f\left(Antlion_j^t\right) \tag{15}$$

where t shows the current iteration, Antlion $_j{}^t$ shows the position of selected j^{th} antlion at t^{th} iteration, and $Ant^t{}_j$ indicates the position of i^{th} ant at t^{th} iteration.

Elitism is one of the most important properties of evolutionary algorithms. Elitism allows preservation of one or more good solution(s) in one generation for the next generation. In continuous ALO it is assumed that the elite solution will influence random walk of every ant. In this paper, we have chosen 5% solutions from the population of Antlion as elite, and they replace the worst antlions after the selection for the next generation.

5.4. Pseudo codes the 2-opt algorithm

Croes et al., (1958) have developed the 2-opt technique to solve the TSP. It is a local search algorithm. The pseudo code for the 2-opt is given below.

```
Input: cost matrix C, number of city Nc
do {
minchange = 0;
 for (i = 0; i< Nc-2; i++)
     {
     for (j = i+2; j <Nc; j++)
            {
            change= C(i,j)+C(i+1,j+1)-C(i,i+1)-C(j,j+1);
            if (minchange> change)
                   {
                   minchange = change;
                   mini = i; minj = j;
                   }
            }
     }
```

} while (minchange< 0);

5.5. Pseudo codes the Discrete ALO algorithm

Input: Number of city n, Number of depot m, Cost matrix C, Number of vehicles available in each depot, Vehicle capacity Q.

- Perform a random Initialization of ant's population and antlions' population.
- Find the ant's fitness and the antlions' fitness
- Search the best antlion to make it elite
- while the termination condition is not satisfied
- for every ant in the population
- Select an antlion using Roulette wheel selection
- perform a random walk
- Update the position of the ant
- end for
- Calculate the fitness of all ants
- Replace an antlion with its corresponding ant if it becomes fitter using equation 15.
- Update elite if an antlion becomes fitter than the elite
- end while
- Return elite

6. Result and Discussion

The discrete ALO is implemented in C language on Intel Core i5 CPU (2.30 GHz), 4GB RAM. The performance of the MDVRP is evaluated using some of the benchmark problems proposed by Creviera et al., (2007) taken from http://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances/ online resource of University of Malaga, Spain. The specifica-tion of some of the benchmark problems is given in Table 1.

Table 1. Specification of benchmark instances

Instance	P01	P02	P03	P04	P06
Total Number of customer	50	50	75	100	100
Total Number of depots	4	4	5	2	3
Number of the vehicle in each depot	8	5	7	12	10
Vehicle capacity	80	100	140	100	100

The parameters for the proposed Discrete ALO are given in Table 2.

Table 2. Parameters of Discrete ALO

Parameter	Value
Population Size	70 if total customer<50 else 100
Iteration	2500 to 4000
Selection	Roulette wheel
Elitism	5% of total population size, i.e., 5

The solutions of instance p1 are given in table 3.

Table 3. The solution of Instance P01

Depot	Routes
1	Vehicle 1: 0 25 18 4 0 Vehicle 2: 0 44 45 33 15 37 17 0 Vehicle 3: 0 42 19 40 41 13 0
2	Vehicle 1: 0 48 8 26 31 28 22 0 Vehicle 2: 0 6 27 1 32 11 46 0 Vehicle 3: 0 12 47 0 Vehicle 4: 0 23 7 43 24 14 0
3	Vehicle 1: 0 49 5 38 0 Vehicle 2: 0 9 34 30 39 10 0
4	Vehicle 1: 0 21 50 16 2 29 0 Vehicle 2: 0 35 36 3 20 0

The results of MDVRP instances using discrete ALO guided with 2-opt are compared with the exact solution, solution using Discrete ALO, GA and ACO are presented in Table 4.

Table 4. Comparison of solutions of MDVRP using discrete ALO with GA, ACO and exact solution

Instance	Exact Solution	Discrete ALO guided with 2-opt	Discrete ALO	GA	ACO
p01	576.87	576.87	591.45	598.45	576.87
p02	473.53	473.53	483.15	473.53	473.53
p03	641.15	641.15	694.49	641.18	645.15
p04	1001.04	1003.86	1011.36	1006.66	1001.04
p05	750.03	750.03	750.72	752.39	750.11
p06	876.5	876.5	882.48	877.84	876.5
p07	885.8	885.8	907.55	893.36	888.41
p08	4437.68	4449.65	4450.37	4474.23	4437.68
p09	3895.7	3895.7	4085.51	3900.22	3904.92
p10	3663.02	3663.02	3825.73	3680.02	3666.35
p11	3554.18	3554.18	3732.36	3593.37	3569.68
p12	1318.95	1318.95	1318.95	1318.95	1318.95
p13	1318.95	1318.95	1318.95	1318.95	1318.95
p14	1360.12	1360.12	1365.69	1365.69	1360.12
p15	2505.42	2505.42	2554.12	2549.65	2526.06
p16	2572.23	2572.23	2606.22	2606.22	2572.23
p17	2709.09	2709.09	2733.8	2733.8	2709.09
p18	3702.85	3702.85	3871.01	3781.66	3771.35
p19	3827.06	3827.06	3884.81	3884.81	3827.06
p20	4058.07	4058.07	4058.07	4094.86	4058.07
p21	5474.84	5474.84	5824.58	5668.97	5608.26
p22	5702.16	5702.16	5873.41	5873.41	5708.78
p23	6095.46	6095.46	6129.99	6159.9	6124.67

The percentage of the gap in the result found in the proposed method with the other method in the literature is given in table 5. The gap is calculated using the following formula.

$$Gap = \frac{(Z_l - Z_p)}{Z_p} * 100 \quad (26)$$

Where Z_p represents the objective value obtained by the proposed method, and Z_l is the objective value of the problem by the others method. Therefore, the posi tive gap represents the better performance of the proposed algorithm compared to others. Whereas negative gap represents the opposite fact.

Table 5. The percentage of Gap in the result in comparison with other methods

Instance	Exact Solution	Discrete ALO	GA	ACO
p01	0	2.527433	3.740877	0
p02	0	2.03155	0	0
p03	0	8.319426	0.004679	0.623879
p04	-0.28092	0.747116	0.278923	-0.28092
p05	0	0.091996	0.314654	0.010666
p06	0	0.682259	0.152881	0
p07	0	2.455408	0.853466	0.294649
p08	-0.26901	0.016181	0.552403	-0.26901
p09	0	4.872295	0.116025	0.236671
p10	0	4.441963	0.464098	0.090909
p11	0	5.013252	1.102645	0.436106
p12	0	0	0	0
p13	0	0	0	0
p14	0	0.409523	0.409523	0
p15	0	1.943786	1.765373	0.823814
p16	0	1.321421	1.321421	0
p17	0	0.912114	0.912114	0
p18	0	4.541367	2.128361	1.849926
p19	0	1.508991	1.508991	0
p20	0	0	0.906589	0
p21	0	6.388132	3.545857	2.436966
p22	0	3.003248	3.003248	0.116096
p23	0	0.566487	1.05718	0.479209
Average Gap %	-0.02391	2.251911	1.049535	0.297781

From the above table, we observe that 2-opt guided discrete ALO gives a better result than discrete ALO, GA, and ACO in most of the case. It is also found that the proposed algorithm fails to yield the exact solution always. The ACO gives a better result than Discrete ALO guided with the 2-opt technique in case of instance p04, p08.

7. Conclusion

In distribution logistics, two main decision problems are routing and scheduling. The cost of delivering an item from source to the destination is optimized only by efficient routing. Single depot VRP often fails to solve real-life scenario because there exists more than one depot. As an NP-hard problem, MDVRP is very difficult to solve and to find exact solutions by exact methods. In this paper, we proposed a 2-opt local exchange guided discrete antlion optimization algorithm to solve MDVRP. This amalgamation of heuristics with local search gives good result in case of MDVRP. Moreover, the algorithm can be applied to solve similar kind of problem like multi-depot location routing problem, waste collection problem, etc.

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NEW MODEL FOR DETERMINING CRITERIA WEIGHTS: LEVEL BASED WEIGHT ASSESSMENT (LBWA) MODEL

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Abstract: *This paper presents new subjective model for determining weight coefficients in multi-criteria decision-making models. The new Level Based Weight Assessment (LBWA) model enables the involvement of experts from different fields with the purpose of defining the relations between criteria and providing rational decision making. The method can be applied in practical cases in specialized decision-making support systems, as well as in alternative dispute resolutions in virtual environment. The LBWA model has several key advantages over other subjective models based on mutual comparison of criteria, which include the following: (1) the LBWA model allows the calculation of weight coefficients with small number of criteria comparisons, only n-1 comparison; (2) The algorithm of the LBWA model does not become more complex with the increase of the number of criteria, which makes it suitable for use in complex multi-criteria (MCDM) models with a large number of criteria; (3) By applying the LBWA model, optimal values of weight coefficients are obtained with simple mathematical apparatus that eliminates inconsistencies in expert preferences, which are tolerated in certain subjective models (Best Worst Method - BWM and Analytic Hierarchy Process - AHP); (4) The elasticity coefficient of the LBWA model enables, after comparing the criteria, additional corrections of the values of the weight coefficients depending on the preferences of the decision makers. This feature of the LBWA model enables sensitivity analysis of the MCDM model by analyzing the effects of variations in the values of the weights of criteria on final decision.*

Key words: *multi-criteria decision making, criteria weights; LBWA model.*

1. Introduction

Determining weights of criteria is one of the key problems arising in the models of multi-criteria analysis to which the problem being solved in this paper belongs to. The

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absence of unique precise definition of the notion of the weight of criteria and the problem of selecting appropriate method for determining weights of criteria in specific decision-making situation are among the most important factors that make the problem of determining weights of criteria significantly more complex. Taking into account the fact that the weights of criteria can significantly influence the outcome of the decision-making process, it is clear that special attention must be paid to the models for determining weights of criteria. Most authors suggest the division of models for determining weights of criteria on subjective and objective models (Zhu et al, 2015).

Subjective approaches reflect subjective thinking and intuition of a decision maker. In such approach the weights of criteria are determined on the basis of information obtained from the decision makers or from the experts involved in the decision-making process. Traditional methods of determining weights of criteria include tradeoff method (Keeney and Raiffa, 1976), proportional (ratio) method, Swing method (Weber, 1988) and conjoint method (Green and Srinivasan, 1990), Analytic Hierarchy Process model (AHP) (Saaty, 1980), SMART method (the Simple Multi Attribute Rating Technique) (Edwards & Barron, 1994), MACBETH method (Measuring Attractiveness by Categorical Based Evaluation Technique) (Bana e Costa & Vansnick, 1994), Direct point allocation method (Poyhonen & Hamalainen, 2001), Ratio or direct significance weighting method (Weber & Borcherding, 1993), Resistance to change method (Rogers & Bruen, 1998), AHP method (Saaty, 1980), WLS method (Weighted Lest Square) (Graham, 1987) and FPP method (the Fuzzy Preference Programming method) (Mikhailov, 2000). Recent subjective methods include multipurpose linear programming (Costa and Climaco, 1999), linear programming (Mousseau et al, 2000), DEMATEL (DEcision MAking Trial and Evaluation Laboratory) method (Gabus i Fontela, 1972), SWARA (Step-wise Weight Assessment Ratio Analysis) method (Valipour et al. 2017), BWM (Best Worst Method) (Rezai, 2015) and FUCOM (FUll COnsistency Method) (Pamucar et al., 2018).

On the other hand, objective approaches ignore decision makers' opinion and are established on determining weights of criteria based on the information contained in decision-making matrix using certain mathematical models. Among the most known objective methods are the following: entropy method (Shannon and Weaver, 1947), CRITIC method (CRiteria significance Through Intercriteria Correlation), (Diakoulaki, et al, 1995) and FANMA method whose name was derived from the names of the authors of the method (Srđević et al, 2003). According to Zhu et al (2015) the most commonly used models for determining weight coefficients of criteria are subjective models with pair comparisons of criteria. In the models with pair comparisons, decision makers compare each criterion with other criteria and determine the level of preferences for each pair of criteria. As a support in determining the size of the preference of a criterion over another one it is used the ordinal scale. The most commonly used methods based on pair comparisons include (Zavadskas et al, 2016) AHP method, BWM and DEMATEL method. Zavadskas et al (2016) have shown in their research that the AHP method is the most commonly used method for determining weights of criteria in the literature. However, in the AHP method needs to be performed $n(n-1)/2$ comparison in pairs of criteria. A large number of comparisons makes the application of the model more complex, especially in cases of a large number of criteria. According to Zhu et al (2015) in the AHP method it is almost impossible to perform fully consistent comparisons in pairs with over nine criteria. This problem is often overcome by dividing the criteria into subcriteria, which further makes the model more complex.

The DEMATEL method is also used in numerous studies, but its main disadvantage is a large number of comparisons in pairs which is $n(n-1)$. Therefore, the DEMATEL method is mostly used to determine the interaction between the criteria and the relationship diagram (Parezanovic et al., 2019).

The method that has become widely used in a short time is the BWM method. Its biggest advantage compared to the AHP model is smaller number of pair comparisons ($2n-3$). However, a large number of comparisons in pairs of criteria, defining the limitations for solving nonlinear model and solving non-linear model make the application of the BWM significantly more complex. Therefore, this model is still unacceptable to a large number of researchers.

Taking into consideration the stated deficiencies of the presented models, the need arises to provide for a method whose algorithm requires small number of comparisons in pairs of criteria and which has rational and logical mathematical algorithm. Starting from this point, a Level Based Weight Assessment model (LBWA) has been developed. The first goal of the paper is to present the new model for determining weights of criteria which requires small number of criteria comparisons, just $n-1$ comparison. The second goal of the paper is to present practical model for solving complex MCDM models, regardless of the number of evaluation criteria. One of significant characteristics of the LBWA model is to maintain simple algorithm regardless of the complexity of the model. The third goal is to define a model which allows the calculation of reliable values of weight coefficients of criteria that contribute to rational judgment. The fourth goal of the paper is the development of a model that can be easily presented/explained to decision-makers, and therefore easily implemented in solving practical problems.

The remaining part of the paper is organized in the following way. In the second section of the paper, the LBWA model algorithm is presented. In the third section of the paper, the LBWA model is tested with two examples from the literature. The fourth chapter provides concluding observations and directions for future research.

2. Level Based Weight Assessment (LBWA) model

Let us consider a multi-criteria model with *n* criteria $S=\{C_1,C_2,\ldots,C_n\}$. Suppose that weight coefficients associated to these criteria are to be determined, i.e., they are not given in advance. In the following part it is presented the process of obtaining the weight coefficients of criteria by applying the LBWA model:

Step 1. Determining the most important criterion from the set of criteria $S=\{C_1,C_2,\ldots,C_n\}$. Let the decision maker determine the most important criterion, i.e., let the criterion C_1 be the criterion in the set of criteria $S=\{C_1,C_2,\ldots,C_n\}$ that is the most significant for the decision-making process.

Step 2. Grouping criteria by levels of significance. Let the decision maker establish subsets of criteria in the following way:

Level S_1*:* At the level S_1 group the criteria from the set *S* whose significance is equal to the significance of the criterion C_1 or up to twice as less as the significance of the criterion C_1;

Level S_2*:* At the level S_2 group the criteria from the set S whose significance is exactly twice as less as the significance of the criterion C_1 or up to three times as less as the significance of the criterion C_1;

Level S_3 *:* At the level S_3 group the criteria from the set *S* whose significance is exactly three times as less as the significance of the criterion C_1 or up to four times as less as the significance of the criterion C_1 ;

...

Level S_k *:* At the level S_k group the criteria from the set *S* whose significance is exactly k times as less as the significance of the criterion C_1 or up to $k+1$ as less as the significance of the criterion C_1 .

By applying the rules presented above, the decision maker establishes rough classification of the observed criteria, i.e., groups the criteria according to the levels of significance. If the significance of the criterion C_j is denoted by $s(C_j)$, where $j \in \{1,2,\ldots,n\}$, then we have $S = S_1 \cup S_2 \cup \cdots \cup S_k$, where for every level $i \in \{1,2,\ldots,k\}$, the following applies

$$S_i = \left\{C_{i_1}, C_{i_2}, \ldots, C_{i_s}\right\} = \left\{C_j \in S : i \le s(C_j) < i+1\right\} \tag{1}$$

Also, for each $p,q \in \{1,2,\ldots,k\}$ such that $p \ne q$ holds $S_p \cap S_q = \varnothing$. Thus, in this way is well defined partition of the set of criteria *S*.

Step 3. Within the formed subsets (levels) of the influence of the criteria it is performed the comparison of criteria by their significance. Each criterion $C_{i_p} \in S_i$ in the subset $S_i = \left\{C_{i_1}, C_{i_2}, \ldots, C_{i_s}\right\}$ is assigned with an integer $I_{i_p} \in \{0,1,\ldots,r\}$ such that the most important criterion C_1 is assigned with $I_1 = 0$, and if C_{i_p} is more significant than C_{i_q} then $I_p < I_q$, and if C_{i_p} is equivalent to C_{i_q} then $I_p = I_q$. Where the maximum value on the scale for the comparison of criteria is defined by applying the expression (2)

$$r = \max\left\{|S_1|, |S_2|, \ldots, |S_k|\right\} \tag{2}$$

Step 4. Based on the defined maximum value of the scale for the comparison of criteria (*r*), the equation (2), it is defined the elasticity coefficient $r_0 \in N$ (where *N* presents the set of real numbers) which should meet the requirement where $r_0 > r$, $r = \max\left\{|S_1|, |S_2|, \ldots, |S_k|\right\}$.

Step 5. Calculation of the influence function of the criteria. The influence function $f : S \to R$ is defined in the following way. For every criterion $C_{i_p} \in S_i$ can be defined the influence function of the criterion

$$f(C_{i_p}) = \frac{r_0}{i \cdot r_0 + I_{i_p}} \tag{3}$$

where *i* presents the number of the level/subset in which is classified the criterion, r_0 presents the elasticity coefficient, while $I_{i_p} \in \{0,1,\ldots,r\}$ presents the value assigned to the criterion C_{i_p} within the observed level.

Step 6. Calculation of the optimum values of the weight coefficients of criteria. By applying the equation (4) it is calculated the weight coefficient of the most significant criterion:

$$w_1 = \frac{1}{1 + f(C_2) + \cdots + f(C_n)} \tag{4}$$

The values of the weight coefficients of the remaining criteria are obtained by applying the expression (5)

$$w_j = f(C_j) \cdot w_1 \tag{5}$$

where $j = 2,3,\ldots,n$, and n present total number of criteria.

3. Application of the LBWA model

In the following section it is presented the application of the LBWA model in determining weight coefficients of criteria in the multi-criteria problems discussed in the literature. In the first section, the multicriteria problem of prioritizing railway level crossings for safety improvements is presented (Pamucar et al., 2013), while in the second section the problem of determining weight coefficients in evaluating the performance of suppliers is considered (Chatterjee et a., 2018).

Example 1. Determination of the weight coefficients of criteria for the evaluation of level crossings

In the research conducted by Pamucar et al (2013), eight criteria were identified that influence the selection of the level crossings for the installation of necessary equipment for increasing traffic safety at the observed crossing: C1 - Rail traffic frequency at the observed crossing, C2 - Road traffic frequency at the observed crossing, C3 - Number of tracks at the observed crossing, C4 - Maximum allowed train speeds at the crossing chainage, C5 - Rail and road crossing angle, C6 - Number of extraordinary events at the observed crossing in the past year, C7 - Sight distance of the observed crossing from the aspect of road traffic and C8 - Investment value of the activities in terms of the width of the crossing.

The following section presents the application of the LBWA model in calculating the weight coefficients of criteria for the evaluation of level crossings:

Step 1. Determining the most important criterion from the set of criteria $S = \{C_1, C_2, \ldots, C_8\}$. In the defined problem, the criterion C_2 is selected as the most important/influential criterion.

Step 2. Grouping criteria by levels of significance. In accordance with the preferences of the decision makers, the criteria are grouped in the following subsets/levels:

Level S_1: the criteria C_1, C_3, C_5, C_6 and C_7 are up to twice as less significant as the criterion C_2 and

Level S_2: (2) the criteria C_4 and C_8 are between twice and three times less significant than the criterion C_2. Than, based on the preferences mentioned the criteria can be grouped in the following subsets/levels:

$$S_1 = \{C_2, C_1, C_3, C_5, C_6, C_7\},$$
$$S_2 = \{C_4, C_8\}.$$

Step 3. Within the formed subsets/levels of criteria influence, a comparison of the criteria with respect to their significance is made. Based on the equation (2), it is defined the maximum value of the scale for comparing the criteria

$$\left.\begin{array}{l} S_1 = \{C_2, C_1, C_3, C_5, C_6, C_7\} \\ S_2 = \{C_4, C_8\} \end{array}\right\} \Rightarrow r = \max\{|S_1|, |S_2|\} = 6$$

On the basis of the obtained value can be concluded that the scale for comparing the criteria ranges in the interval $I_{i_p} \in \{0,1,\ldots,6\}$. Applying previously defined relations can be performed the comparison of criteria within each individual set of criteria:

Level S_1: Based on the preferences of the decision makers, the following relations can be defined: $I_2 = 0$, $I_5 = 2$, $I_7 = 3$, $I_6 = 4$, $I_1 = 4$, $I_3 = 5$.

Considering that the criterion C_2 has the largest influence, its value assigned is $I_1 = 0$. To the remaining criteria are assigned the values from the predefined scale $I_{i_p} \in \{0,1,\ldots,6\}$, under the condition where if the criterion C_{i_p} has higher weight coefficient than the criterion C_{i_q}, then the condition $I_p < I_q$ is met.

Level S_2: Based on the preferences of the decision makers, the following relations can be defined: $I_8 = 1$ i $I_4 = 2$.

Step 4. Based on the defined maximum value of the scale for comparing the criteria $r = 6$, it is defined the elasticity coefficient where $r_0 > r$, respectively, $r_0 > 7$.

Step 5. Defining the influence function of the criteria. If it is known that $r_0 > 6$, it is arbitrarily determined the value $r_0 = 7$. By applying the equation (3) the influence functions of the criteria are calculated.

$$f(C_2) = \frac{7}{1\cdot 7+0} = \frac{7}{7} = 1;\ f(C_5) = \frac{7}{1\cdot 7+2} = \frac{7}{9};\ f(C_7) = \frac{7}{1\cdot 7+3} = \frac{7}{10};\ f(C_6) = \frac{7}{1\cdot 7+4} = \frac{7}{11};$$

$$f(C_1) = \frac{7}{1\cdot 7+4} = \frac{7}{11};\ f(C_3) = \frac{7}{1\cdot 7+5} = \frac{7}{12};\ f(C_8) = \frac{7}{2\cdot 7+1} = \frac{7}{15};\ f(C_4) = \frac{7}{2\cdot 7+2} = \frac{7}{16}.$$

Step 6. Calculation of the optimum values of the weight coefficients of criteria. By applying the equation (4) it is calculated the weight coefficient of the most influential criterion

$$w_2 = \frac{1}{1+0.778+0.700+\ldots+0.438} = 0.191$$

The values of the weight coefficients of the remaining criteria are obtained by applying the equation (5). Therefore, for the criterion C_1 it is obtained the weight coefficient $w_1 = f(C_1)\cdot w_2 = 0.636\cdot 0.191 = 0.121$. In the similar way are obtained the values of the weight coefficients of the remaining criteria which meet the condition where $\sum_{j=1}^{n} w_j = 1$.

$$w_3 = f(C_3)\cdot w_2 = 0.583\cdot 0.191 = 0.111;$$
$$w_4 = f(C_4)\cdot w_2 = 0.438\cdot 0.191 = 0.084;$$
$$w_5 = f(C_5)\cdot w_2 = 0.778\cdot 0.191 = 0.148;$$
$$w_7 = f(C_7)\cdot w_2 = 0.700\cdot 0.191 = 0.134;$$
$$w_6 = f(C_6)\cdot w_2 = 0.636\cdot 0.191 = 0.121;$$
$$w_8 = f(C_8)\cdot w_2 = 0.467\cdot 0.191 = 0.089.$$

Finally, it is obtained the vector of the weight coefficients

$$w_j = (0.121, 0.191, 0.111, 0.084, 0.148, 0.121, 0.134, 0.089)^T.$$

By comparing the values of the weight coefficients obtained using the LBWA model with the weight coefficient values from the study made by Pamucar et al (2013), it can be noted that almost identical weight values are obtained, which confirms successful validation of the LBWA model.

Considering that the value of the elasticity coefficient r_0 in this example is defined arbitrarily as $r_0 = 7$, in the following part (Figure 1) is presented the influence of the value r_0 to the change of the values of the weight coefficients of criteria.

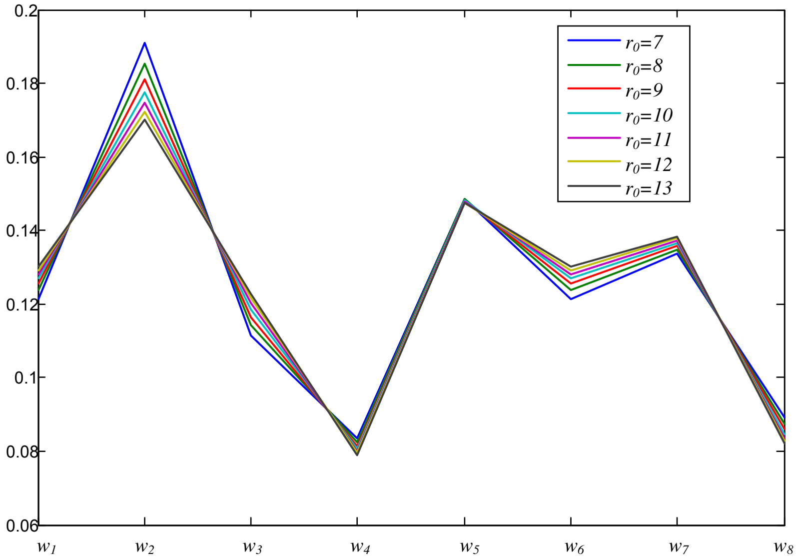

Figure 1. Influence of the value of r_0 to the change of the weight coefficients values

From the Figure 1 it can be noted that the parameter r_0 in certain measure can cause smaller changes of the weight coefficients values. The parameter r_0 allows decision makers to make fine adjustments of the weight coefficients values in accordance with their own preferences. The authors recommend the initial values of the weight coefficients to be defined on the basis of the value of the parameter $r_0 = r + 1$. After the definition of initial values, decision makers can make additional adjustment of weight coefficients by changing the parameter r_0.

Example 2. Determination of the weight coefficients of criteria in the evaluation of the work of advisors in the transport of dangerous goods

In the research carried out by Pamucar et al (2019), nine criteria were identified for the evaluation of the work of advisors in the transport of dangerous goods: C1 - Knowledge of regulations and professional development, C2 - Analytic processing of established requirements, C3 - Quality of proposed measures, C4 - Level of realization of the proposed measures, C5 - Quality of professional training of employees, C6 - Response method in emergency situations, C7 - Document preparation, C8 - Method of solving professional questions and C9 - Activity in professional institutions.

The weight coefficients of the criteria for evaluating the work of advisors in the transport of dangerous goods are defined using the LBWA model:

Step 1. Determining the most important criterion from the set of criteria $S = \{C_1, C_2, \ldots, C_9\}$. As the most significant/influential criterion, it is selected the criterion C_5 within the defined problem.

Step 2. Grouping criteria by levels of significance. In accordance with the preferences of the decision makers, the criteria are grouped in the following subsets/levels:

Level S_1: The criteria C_1, C_8 and C_9 are up to twice as less important as the criterion C_5,

Level S_2: The criteria C_3 and C_4 are between twice or three times as less important as the criterion C_5

Level S_4: The criterion C_6 is between four or five times as less important as the criterion C_5,

Level S_7: The criteria C_2 and C_7 are between seven or eight times as less important as the criterion C_5

Then, based on the mentioned preferences of the decision makers the criteria can be grouped in the following subsets/levels:

$S_1 = \{C_1, C_5, C_8, C_9\}$,
$S_2 = \{C_3, C_4\}$,
$S_3 = \varnothing$,
$S_4 = \{C_6\}$,
$S_5 = S_6 = \varnothing$,
$S_7 = \{C_2, C_7\}$.

Step 3. Based on the equation (2), it is defined the maximum value of the scale for the comparison of criteria

$$\left.\begin{array}{l} S_1 = \{C_1, C_5, C_8, C_9\}, \\ S_2 = \{C_3, C_4\}, \\ S_3 = \varnothing, \\ S_4 = \{C_6\}, \\ S_5 = S_6 = \varnothing, \\ S_7 = \{C_2, C_7\}. \end{array}\right\} \Rightarrow r = \max\{|S_1|, |S_2|, |S_4|, |S_7|\} = 4$$

Based on the maximum value of the scale for comparison, it can be concluded that the scale for comparing the criteria ranges in the interval $I_{i_p} \in \{0, 1, \ldots, 4\}$. Based on the scale and the pre-defined set of criteria, it can be performed the comparison of criteria within each individual set:

Level S_1: Based on the preferences of the decision makers, the following relation are defined: $I_5 = 0$, $I_8 = 1$, $I_9 = 2$ and $I_1 = 4$.

Level S_2: Within the set S_2 the following relations are defined: $I_3 = 1$ and $I_4 = 2$.

Level S_4: Within the set S_4 the following relation is defined $I_6 = 2$.

Level S_7: Within the set S_7 the following relations are defined: $I_2 = 1$ and $I_7 = 3$.

Step 4. Based on the defined maximum value of the scale for the comparison of criteria $r = 4$, it is defined the elasticity coefficient such that $r_0 > r$, respectively, $r_0 > 4$.

Step 5. Defining the influence function of the criteria. If it is known that $r_0 > 4$, arbitrarily is determined the value $r_0 = 5$. By applying the equation (3) the influence functions of the criteria are calculated.

$$f(C_5)=\frac{5}{1\cdot 5+0}=1;\ f(C_8)=\frac{5}{1\cdot 5+1}=\frac{5}{6};\ f(C_9)=\frac{5}{1\cdot 5+2}=\frac{5}{7};$$
$$f(C_1)=\frac{5}{1\cdot 5+4}=\frac{5}{9};f(C_3)=\frac{5}{2\cdot 5+1}=\frac{5}{11};\ f(C_4)=\frac{5}{2\cdot 5+2}=\frac{5}{12};$$
$$f(C_6)=\frac{5}{4\cdot 5+2}=\frac{5}{22};\ f(C_2)=\frac{5}{7\cdot 5+1}=\frac{5}{36};\ f(C_7)=\frac{5}{7\cdot 5+3}=\frac{5}{38}.$$

Step 6. Calculation of the optimum values of the weight coefficients of criteria. By applying the equation (4) it is calculated the value of the weight coefficient of the most influential criterion

$$w_5=\frac{1}{1+0.833+0.714+...+0.132}=0.224$$

By applying the equation (5) are obtained the values of the weight coefficients of the remaining criteria:

$$w_8=f(C_8)\cdot w_5=0.883\cdot 0.224=0.186;\quad w_4=f(C_4)\cdot w_5=0.417\cdot 0.224=0.093;$$
$$w_9=f(C_9)\cdot w_5=0.714\cdot 0.224=0.160;\quad w_6=f(C_6)\cdot w_5=0.227\cdot 0.224=0.051;$$
$$w_1=f(C_1)\cdot w_5=0.556\cdot 0.224=0.124;\quad w_2=f(C_2)\cdot w_5=0.139\cdot 0.224=0.031;$$
$$w_3=f(C_3)\cdot w_5=0.455\cdot 0.224=0.102;\quad w_7=f(C_7)\cdot w_5=0.132\cdot 0.224=0.029.$$

Finally, the vector of the weight coefficients is obtained

$$w_j=(0.124;0.03;0.102;0.093;0.224;0.051;0.029;0.186;0.160)^T.$$

In this example, the value of the elasticity coefficient r_0 is arbitrarily defined as $r_0=5$, and in the following part (Figure 2) is presented the influence of the value r_0 to the change of the weight coefficients of criteria.

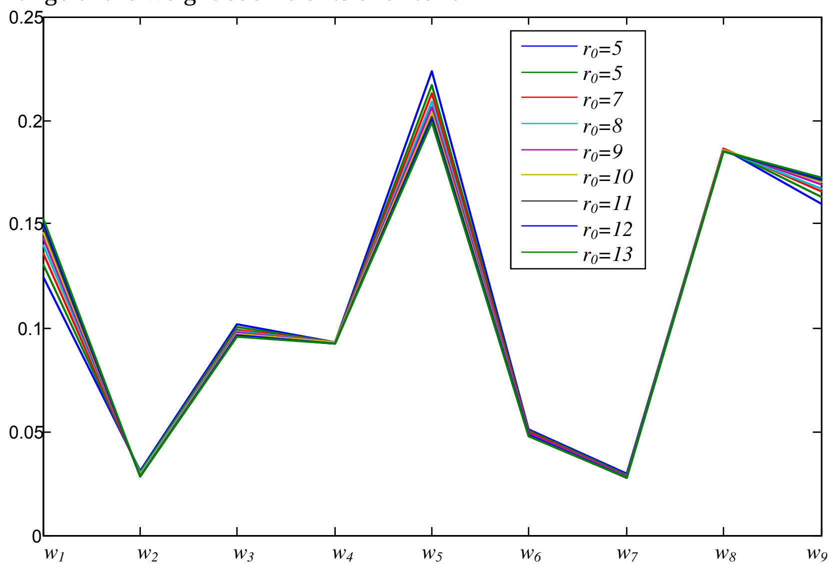

Figure 2. Influence of the value r_0 to the change of the weight coefficients

From the Figure 2 can be observed that the changes in the elasticity coefficient lead to minor changes in the weight coefficients of criteria. This feature of the LBWA model allows additional adjustment of the weight coefficients in accordance with the decision makers preferences.

4. Discussion of results and conclusion

Literature review and the analysis of the models for determining weight coefficients of criteria present in the literature so far clearly indicate the need for the development of a new credible model for determining weight coefficients of criteria. Therefore, in this paper is presented a new model, the LBWA model, which is characterized by simple and rational mathematical algorithm. The results of this study have shown that the LBWA model allows obtaining credible and reliable weight coefficients that contribute to rational judgment, and thus to obtaining credible results in decision-making process.

Based on the results presented can be outlined the following advantages of the LBWA model: (1) The LBWA model allows the calculation of weight coefficients with small number of criteria comparisons, only $n-1$ comparison; (2) The LBWA model algorithm does not become more complex with the increase of the number of criteria, which makes it suitable for use in complex MCDM models with a larger number of evaluation criteria; (3) The LBWA model allows decision makers to present their preferences through logical algorithm when prioritizing criteria. Using the LBWA model, optimal values of weight coefficients are obtained with simple mathematical apparatus that eliminates inconsistencies in expert preferences, which are tolerated in certain subjective models (BWM and AHP); (4) Flexibility of the model in terms of using all the values from the predefined scale, i.e., it is not limited to integer values from the defined interval.

In addition to the mentioned advantages, it is necessary to emphasize the flexibility of the LBWA model in terms of additional corrections of weight coefficients values by the elasticity coefficient (r_0). The elasticity coefficient allows decision makers to further adjust weight coefficients values in accordance with their own preferences. In addition, the elasticity coefficient allows the analysis of the robustness of the MCDM model by defining the effect of the change of the criteria weight coefficients on the final decision.

In order to approach users and exploit all the advantages of the LBWA model, the need for software development and implementation in real-world applications is imposed. One of the directions of future research should cover the extension of the algorithm for the application in group decision making. Also, one of the directions of future research should be the extension of the LBWA model using different uncertainty theories (neutrosophic sets, fuzzy sets, rough numbers, gray theory, etc.). The implementation of the LBWA model in uncertain environment will enable the processing of expert preferences, even in cases where the information about the considered problem are partially accessible or even very little known. This would enable more objective expression of the decision makers' preferences by respecting subjectivity and lack of information on certain phenomena.

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AN ENSEMBLE APPROACH FOR PORTFOLIO SELECTION IN A MULTI-CRITERIA DECISION MAKING FRAMEWORK

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Abstract: *Investment in Mutual Funds (MF) has generated increasing interest among the investors over last few decades as it provides an opportunity for flexible and transparent choice of funds to diversify risk while having return potential. MF are essentially a portfolio wherein investors' funds are invested in the securities traded in the capital market while sharing a common objective. However, selection and management of different asset classes pertaining to a particular MF are done by an active fund manager under regulatory supervision. Hence, for an individual investor, it is important to assess the performances of the MF before investment. Performances of MF depend on several criteria based on risk-return measures. Hence, selection of MF is subject to satisfying multiple criteria. In this paper, we have adopted an ensemble approach based on a two-stage framework. Our sample consists of the open ended equity large cap funds (direct plan) in India. In the first stage, the efficiencies of the funds are analyzed using DEA for primary selection of the funds. In order to rank the funds based on risk and return parameters for investment portfolio formulation, we have used MABAC approach in the second stage wherein criteria weights have been calculated using the Entropy method.*

Keywords: *Mutual Fund, Portfolio Selection, Multi-Criteria Decision Making, Data Envelopment Analysis (DEA), Entropy, Multi-Attribute Border Approximation Area Comparisons (MABAC).*

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1. Introduction

The aftermath of the economic liberalization, Indian Capital Market (ICM) has witnessed significant changes in investment pattern. With the development of Information and Communication Technology (ICT), there is no dearth of information regarding available investment opportunities. Moreover, with increasing efforts from the Govt. of India (GOI), MF have emerged as a preferred choice for common investors because of many reasons. First, it provides reportedly high return as compared to other popular investment options like Fixed Deposits (FD), National Savings Certificates (NSC), Public Provident Funds (PPF), and other postal savings. Alongside, with increasing awareness and available information, risk can also be brought down to an affordable level by carefully evaluating the performances of the MF for the prudent selection of funds to invest. The concept of MF resembles the selection of portfolio wherein the active fund managers invest the total amount invested in other asset classes such as stocks. The objective is to generate sizeable return out of the investment made in the portfolio while minimizing the risk through diversification i.e. by appropriate selection of the securities and allocating optimum weights dynamically (Gupta et al., 2018). MF in India have a long stint with ICM since the inception of Unit Trust of India (UTI) in 1963. Recent reports(AMFI, India, 2018)indicate a significant growth in the asset base of the Indian MF Industry (IMI) from INR 5.05 trillion (31st March, 2008) to INR 22.20 trillion (February, 2018)despite the event of bankruptcy of the renowned US bank Lehman Brothers in September, 2008. Therefore, a large number of investors have been attracted by the promising nature of IMI. However, in order to ensure the possibility of considerable return at an affordable risk, one has to select the funds apt to his/her risk appetite and financial goal.

With this preamble, in this study, we have focused on open ended equity MF(direct plans) in India belonging to the large cap segment. The equity MF segment accounts for around 50.7% share of the total asset base of the industry in February, 2018 as reported by AMFI. Unlike the close ended funds (CF), open ended funds (OF) allow the investors to buy and sell the units of the funds on a continuous basis, which means the new investors are allowed to enter at convenience and so as the existing investors can exit whenever needed. Moreover, for CF the unit capital is fixed and also there is a limit on sales. In other words, OF allow greater flexibility for the investors than CF. Large cap funds show relatively stable movements in the return as compared to the mid-cap and small cap funds. We have considered direct plans only since, unlike regular plans, they impose less pressure on expense ratio as no intermediate commission is involved. In essence, we like to perceive the performance of MFs from a common investors' point of view without imposing significant burden on Net Assets Value (NAV). In our approach, we have used Non-Parametric Methods (NPM). Literatures manifest comparatively less evidence of such methods than their traditional parametric counterparts (Babalos et al., 2011). Selection of MF needs to satisfy the objectives pertaining to several criteria based on risk and return and time horizon, etc. Therefore, among NPM, DEA has been a popular method, though, moderate evidences of the use of MCDM techniques have been found in the state-of-the art. The reason lies in the fundamental use of DEA to assess and differentiate between the efficient and non-efficient Decision Making Units (DMU). MCDM techniques allow to rank the DMU based on a number of criteria. Hence, for identifying efficient DMU or MF while ranking them based on performance parameters, we propose a two-stage assessment framework. The rest of the paper proceeds as follows. Section 2 highlights some of the related work while in the section 3 we discuss the research methodology. Section 4 summarizes the results and put forward necessary discussions on the same. Finally, section 5 concludes the paper and posits some future scope of research.

2. Related Work

A plethora of research has been conducted on MF for understanding the nature and setting performance measurement framework with an objective to inflate the expected utility while reducing the risk level. In one of the seminal works in the stated field, Markowitz postulated the mean-variance model related to efficiently diversified portfolio (Markowitz, 1952). In the following works, the researchers (Tobin, 1958; Markowitz, 1959; Sharpe, 1966; Jensen, 1968; Treynor, 1965) further explained and extended the framework by introducing new risk measures such as semi-variance and risk adjusted performance metrics such as reward to volatility ratio, alpha and reward to variability ratio based on the Capital Assets Pricing Model (CAPM). The objective was to assess portfolio performance with respect to the benchmark with an objective to minimize the systematic risk which is represented by beta. The authors exhibited that unsystematic risk can be reduced through diversification of the portfolio. Based on these measures, several researchers and practitioners worked on evaluating the performances of the MF (Kacperczyk et al., 2005; Pedersen &Rudholm-Alfvin, 2003; Eling & Schuhmacher, 2007; Plantinga & de Groot, 2002; Redman et al., 2000). In the Indian context also, across different periods, many researchers (Barua&Verma, 1991; Jaydev, 1996; Gupta, 2000; Sehgal&Jhanwar, 2008; Tripathy, 2004; Anand & Murugaiah, 2006; Anitha et al., 2011; Arora, 2015; Kundu, 2009) worked on selection of funds based the criteria like Sharpe ratio, Jensen ratio, and Sortino ratio, alpha, beta, NAV, timing to market, and selectivity skill following traditional statistical approaches.

Over the years apart from the traditional parametric approaches, applied operations research techniques have also been adopted by the researchers. The authors (Pendaraki et al., 2004; Sharma & Sharma, 2006) have applied goal programming to evaluate the performances of MF for formulating the portfolio. DEA has been a widely accepted method by the researchers and practitioners (Murthi& Choi, 2001; Murthi et al., 1997; Anderson et al., 2004; Sengupta, 2003; Daraio & Simar, 2006; Babalos et al., 2015; McMullen & Strong, 1998; Wilkens & Zhu, 2001; Tarim& Karan, 2001; Galagedera & Silvapulle, 2002; Chang, 2004; Carlos Matallín et al., 2014; Nguyen-Thi-Thanh, 2006; Chu et al., 2010; Tsolas, 2011; Morey & Morey, 1999; Basso & Funari, 2001; Briec et al., 2004; Zhao et al., 2011; Jaro & Na, 2006; Kooli et al., 2005; Haslem & Scheraga, 2003)among the non-parametric applied operations research techniques. The researchers have considered the variables like standard deviation, expense ratio, loads, turnover, beta, costs, fund size, variance, percentage of periods with negative return, lower semi-variance, sales charges, operating expenses, cash percentage, P/E ratio, P/B ratio, total assets, lower mean, lower semi-skewness, and excess kurtosis as input while considering the variables like return, deviations from median return, capital flow, skewness, Sharpe ratio, upper semi-variance, upper semi-skewness, and Jensen's α as output in assessing the performances of the funds under study. There has been another string of the literature in which MCDM techniques are applied for selection of MF (Pendaraki et al., 2005; Lin et al., 2007; Gladish et al., 2007; Chang et al., 2010; Babalos et al., 2011; Alptekin, 2009; Karmakar et al., 2018; Pendaraki & Zopounidis, 2003; Sielska, 2010). Attribute based classification approaches like UTADIS (UTilités Additives DIScriminantes) (Pendaraki et al., 2005; Lin et al., 2007), fuzzy MCDM techniques (Gladish et al., 2007), distance based MCDM methods like TOPSIS (Lin et al., 2007; Chang et al., 2010; Alptekin, 2009; Karmakar et al., 2018; Sielska, 2010) and EDAS (Karmakar et al., 2018), outranking methods such as PROMETHEE (Sielska, 2010) and PROMETHEE II (Pendaraki&Zopounidis, 2003) have been selected for portfolio selection issue based on the parameters like Sharpe ratio, Sortinoratio, Treynorratio, Jensen's α, AUM, beta, standard deviation, NAV, annualized return, average return, Information ratio, and R-squared. In this context, in paper (Babalos et al., 2011) the authors used Stochastic Multi-criteria Acceptability Analysis (SMAA-2) framework for assessing performances of MF. Predominantly, objective weight method using Euclidean Distance has been applied for calculating criteria weight. However, some authors like Chang et al. (2010) experimented with different distance measures for examining performances of the MF.

3. Data and Methodology

3.1. Development of the framework

In this study, we have followed a non-parametric two stage framework wherein we have ranked the funds under study. In the first stage, we have applied DEA to appraise the efficiencies of the funds for a primary level selection. For a refined selection in a common setting for investment choice among the relatively efficient portfolios, at stage two the MCDM technique like MABAC has been used. The entropy method has been employed for calculation of criteria weight in this regard. Figure 1 depicts the framework followed in this study. We have started our analysis with total 48 number of open ended equity large cap funds under direct plan.

In the seminal works of Markowitz (1952, 1959), the underline assumption considered first two central moments of the utility function of the return which is treated as a normal distribution. However, the studies (Lau et al., 1990; Cambell & Hentsche, 1992) arguably reported that portfolio returns are always not normally distributed in practice. Hence, it is imperative to consider higher moments. Average investors prefer lower value of Kurtosis (Scott & Horvath, 1980) as it entails a higher degree of sensitivity of the funds with respect to non-favorable market condition. In view of this, for filling the gap in the literature in Indian context, this study considers Kurtosis as one of the inputs. Expense ratio puts load on the profitability as it covers the management fees and operating expense.

The third quartile return has a typical significance in the sense that it indicates relative closeness to the highest value than the average return. The ratio PSV to NSV signifies inclination of the deviation of the return from the mean towards the higher side which essentially acts as a favorable proposition for the investors. We have used standard Variable Returns to Scale (VRS) model as it resembles real life situation compared to Constant Returns to Scale (CRS) which reflects the proportionate change in the output with respect to the input (Ali & Seiford, 1990). Further, we have calculated super efficiency in order to discriminate the funds to a considerable extent.

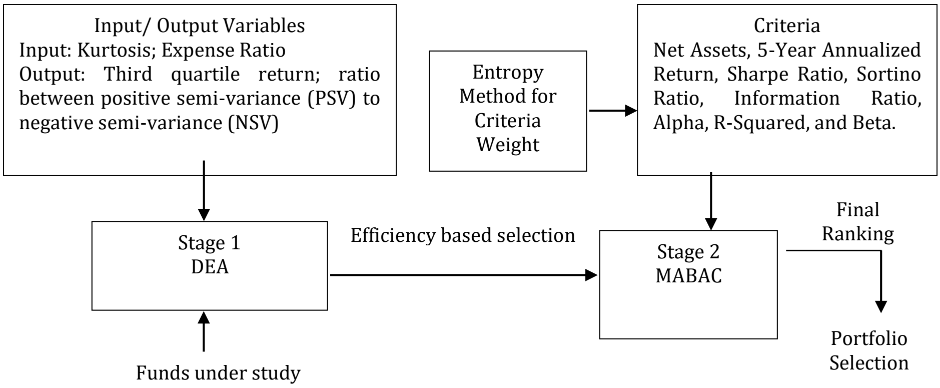

Figure 1. Research Framework

The higher value of net assets of a fund indicates the greater possibility of a good return. The Sharpe ratio, α, and Sortino ratio specify the excess return with respect to risk, i.e. risk-adjusted performance while beta captures the systematic risk. To what extent, the portfolio return with respect to the benchmark, i.e. index, adjusts the volatility of the return is reflected in the value of Information ratio which stands beneficial for the investors. The value of R-squared manifests the nature of diversification of the portfolio, which leads to reduction of unsystematic risk.

3.2. Sample

In our study, we focus on open ended and large cap equity mutual funds under direct plan. In this context, we have excluded the fixed maturity plan, and plans suspended for sales which otherwise leaves lesser chance to gain more at calculated risk. For spotting the funds, we have referred the Value research online data base and subsequently for collecting information on the performance criteria. Appendix 1 shows the descriptive statistics of the total 48 funds selected initially for the study. Returns of the last 12 quarters (i.e. Sep 2015 to Jun 2018) have been considered for calculating distribution based parameters used in DEA.

3.3. Data Envelopment Analysis (DEA)

DEA evaluates a set of peer entities (homogenous) i.e. Decision Making Units (DMUs) having multiple inputs or outputs. As introduced by Charnes et al. (1978), this technique has gained extensive importance by the researchers in measuring performance efficiency in terms of the frontiers or envelop rather than the central tendency as in the case of fitting a regression model. In DEA, the efficiency of the homogenous entities is calculated by using the linear programming method. Calculations for input oriented, constant return to scale (CRS) are as follows:

$\min \theta$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \lambda_j \le \theta x_{it} \quad i = 1,2,\dots,m; \quad \text{Input Constraint} \tag{1}$$

$$\sum_{j=1}^{n} y_{rj} \lambda_j \ge y_{rt} \quad r = 1,2,\dots,s; \quad \text{Output Constraint} \tag{2}$$

Where, $\lambda_j \ge 0 \ \forall \ i, j, r$

For variable return to scale (VRS) the sets of equations are:

$\min \theta$

Subject to:

$$\sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta x_{it} \quad i = 1,2,\dots,m; \quad \text{Input Constraint} \tag{3}$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rt} \quad r = 1,2,\dots,s; \quad \text{Output Constraint} \tag{4}$$

Where, $\sum_{j=1}^{n}\lambda_j = 1$; $\lambda_j \geq 0 \ \forall \ i, j, r$

If two or more DMUs are found to be efficient (i.e. θ = 1 or 100%) then the Super-efficiency value is calculated to discriminate them. For VRS the super efficiency is calculated as below:

min θ

Subject to

$$\sum_{j=1}^{n} x_{ij}\lambda_j \leq \theta x_{it} \quad i = 1,2,\dots,m; \quad \text{Input Constraint} \tag{5}$$

$$\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rt} \quad r = 1,2,\dots,s; \quad \text{Output Constraint} \tag{6}$$

Where, $\sum_{j=1}^{n}\lambda_j = 1; \lambda_j \geq 0 \ \forall \ i, j, r \ ; j \neq t$.

3.4. Entropy Method

It is an objective method for calculating criteria weights based on relative information content wherein, higher value of the entropy (degree of disorder) indicates more information content for the respective criterion (Shannon, 1948). The steps are as given below where,

a_i: i^{th} alternative where i = 1,2,3,…..m;

c_j: j^{th} criterion where j = 1,2,3,…..n;

x_{ij}: j^{th} criterion value for the i^{th} alternative;

Step1. Normalization of the criteria.

For this purpose, we have used the enhanced accuracy method of normalization Zeng et al. (2013) as mentioned in (Jahan & Edwards, 2015). Accordingly, the normalized matrix R = $[r_{ij}]_{m \times n}$ is given by:

$$r_{ij} = 1 - \frac{x_j^{max} - x_{ij}}{\sum_{i=1}^{m}\left(x_j^{max} - x_{ij}\right)} \quad \text{(Beneficial Criteria)} \tag{7}$$

$$r_{ij} = 1 - \frac{x_{ij} - x_j^{min}}{\sum_{i=1}^{m}\left(x_{ij} - x_j^{min}\right)} \quad \text{(Non-beneficial Criteria)} \tag{8}$$

Step 2. Entropy calculation for the criterion

Entropy of the j^{th} criterion is given by:

$$H_j = -\frac{\sum_{i=1}^{m} f_{ij} \ln f_{ij}}{\ln m}, i = 1,2,\dots m; j = 1,2,..n \tag{9}$$

Where,

$$f_{ij} = \frac{r_{ij}}{\sum_i^m r_{ij}}, i = 1, 2, \dots m; j = 1,2, \dots \tag{10}$$

Step 3. Calculation of the entropy weight for the criterion

Entropy weight of the j^{th} criterion is determined by:

$$w_j = \frac{1 - H_j}{n - \sum_{j=1}^{n} H_j}, \text{ where } \sum_{j=1}^{n} w_j = 1 \tag{11}$$

3.5. Multi-Attribute Border Approximation Area Comparisons (MABAC)

Since its proposal (Pamučar & Ćirović, 2015), this method has drawn significant attention from the researchers for its inherent computational ease and stability. Unlike TOPSIS, this method classifies the performances of the criteria into two areas such as Upper Approximation Area (UAA) for ideal solutions and Lower Approximation Area (LAA) for non-ideal solutions instead of calculating the distance of any solution from the ideal and non-ideal solutions. In a sense, this method examines the relative strength and weakness of each alternative with respect to the others pertaining to each criterion (Roy et al., 2016). This method has been widely applied by the researchers for solving multi-criteria decision making problems such as railway management (Sharma et al., 2018; Vesković et al., 2018)medical tourism site selection (Roy et al., 2018) and selection of hotels (Yu et al., 2017).

Let, D is the initial decision matrix represented by

a_i: i^{th} alternative where i = 1,2,3,.....m;

c_j: j^{th} criterion where j = 1,2,3,.....n;

x_{ij}: j^{th} criterion value for the i^{th} alternative;

The broad steps under this method are given below.

Step 1. Normalization of the criteria values

$$r_{ij} = \frac{(x_{ij} - x_i^-)}{(x_i^+ - x_i^-)} \text{; For beneficial criteria} \tag{12}$$

$$r_{ij} = \frac{(x_{ij} - x_i^+)}{(x_i^- - x_i^+)} \text{; For non-beneficial criteria} \tag{13}$$

Where, x_i^+ and x_i^- are the maximum and minimum criteria values respectively.

Step 2. Construction of weighted normalization matrix (Y)

Elements of Y are given by:

$$y_{ij} = w_j(r_{ij} + 1) \text{ ; Where, } w_j \text{ are the criteria weight.} \tag{14}$$

Step 3. Determination of the Border Approximation Area (BAA)

The elements of the Border Approximation Area (BAA) T is given by:

$$t_j = \left(\prod_{i=1}^{m} y_{ij}\right)^{1/m} \tag{15}$$

Where, m is the total number of alternatives &t_j corresponds to each criterion.

Step 4. Calculation of the matrix Q related to the separation of the alternatives from BAA

$$Q = Y\text{-}T \tag{16}$$

A particular alternative a_i is said to be belonging to the Upper Approximation Area (UAA) i.e. T^+ if $q_{ij} > 0$ or Lower Approximation Area (LAA) i.e. T^- if $q_{ij} < 0$or BAA i.e. T if $q_{ij} = 0$.

The alternative a_i is considered to be the best among the others if more numbers of criteria pertaining to it possibly belong to T^+.

Step 5. Ranking of the alternatives

It is done according to the final values of the criterion functions as given by

$$S_i = \sum_{j=1}^{n} q_{ij} \text{ for } j = 1,2, \dots n \text{ and } i = 1,2, \dots m \tag{17}$$

Higher the value, the better is the rank.

In this study for carrying out DEA, we have used Lingo (version 11) software while for MCDM related calculations, Microsoft Office excel (version 2010) is utilized.

4. Results and discussions

We have considered total 48 funds initially. However, before analyzing them using DEA, we have checked whether the condition of the required number of DMUs is satisfied or not. In this regard, though there are several studies, we have followed one of the widely accepted study conducted by Banker et al. (1989). According to the study, the rule of the thumb is $n >= \max \{p \times q, 3(p + q)\}$ where, p be the number of inputs and q is the number of outputs used in the analysis, and n is the number of DMUs to be considered. Our model has two inputs and two outputs. Hence, it satisfies the condition. The top 20 funds (i.e. DMUs) based on the result of DEA considering VRS model is given in the table 1 while the details is included in the appendix 2.

The performance criteria values of the above funds (Table 1) are given in the appendix 1. We then have used MCDM model for ranking the above mentioned funds. For calculating criteria weights, we have used a modified Entropy method. The results are listed in tables 2-3. After calculating the criteria weights, we then proceed to the stage 2 i.e. ranking of the funds (primary selection through DEA) using the MABAC technique. The results are given in tables 4-6.

From the result, it is evident that the top five funds (i.e. A27, A19, A38, A22 and A25; the names are given in appendix 1) are rated 4-star and 5-star by Value research and except one of them, and their risk grades are above average or more. On the other hand, bottom five funds (i.e. A40, A42, A21, A3 and A5; the names are given in appendix 1) are rated 2-star or below and having an overall average risk grade. Hence, this study also conforms to the market based rating of the funds by Value research.

In order to check the dependability of the result obtained from MABAC, we have also ranked the funds selected from DEA result using TOPSIS technique. In line with the method suggested by Hwang and Yoon (1981), we have obtained the rankings as given in the table 7. For checking consistency with MABAC based ranking, we have performed Spearman's Rank Correlation test using IBM SPSS 22, which is 0.952 (significant at 0.01 level). Hence, the result obtained from MABAC is acceptable.

Table 1. DEA result (Top 20 Funds)

Funds under study	Kurtosis	Kurtosis (Normalized)(Input)	Expense Ratio(Input)	PSV/NSV (Output)	Q3 (Output)	VRS Result	Rank VRS
DMU3	-0.994218297	0.228357785	0.51	0.692524206	6.325	0.277	20
DMU5	0.430052796	1	1.05	1.190759453	7.5125	0.4196	17
DMU10	0.068745875	0.804250978	2.09	1.712988573	6.065	1	7
DMU13	-1.392072451	0.012808188	0.15	1.024946667	7.3475	1.5272	2
DMU14	-1.069736073	0.187443732	0.15	0.697821479	6.6575	0.8342	11
DMU15	-0.986286229	0.232655225	1.26	1.65839919	8.5375	1	7
DMU19	-0.867681981	0.296912686	0.44	1.167185892	9.8025	1.46	3
DMU21	-1.047245089	0.199628907	0.43	0.708575851	6.5225	0.3276	18
DMU22	-0.914640359	0.271471555	0.56	1.177342789	9.7425	1.02	6
DMU24	-1.020204286	0.214279085	0.17	0.702101309	6.5925	0.7349	13
DMU25	-0.756941711	0.356909598	0.75	0.703951659	8.09	0.3169	19
DMU27	-1.340703715	0.040638764	2.94	0.703018825	12.0225	1	7
DMU32	-1.415713371	0	1.15	1.038445735	7.18	1	7
DMU36	-0.9687944	0.242131955	0.29	0.706595066	6.61	0.4586	16
DMU37	-1.414566835	0.000621171	0.29	1.02757939	7.045	3.0343	1
DMU38	-0.950082133	0.252269895	1.32	1.528358013	7.07	0.7819	12
DMU40	-1.046793817	0.199873397	0.29	0.703356945	6.6075	0.4689	15
DMU42	-1.063892456	0.190609689	0.21	0.702148768	6.685	0.6264	14
DMU43	-1.403733132	0.006490659	0.22	1.033529532	7.24	1.4103	4
DMU48	-1.072451509	0.185972562	0.12	0.691073629	6.5875	1.25	5

Table 2. Normalization Table

Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A3	0.9450	0.9338	0.9315	0.9315	0.8427	0.9350	1.0000	0.9600
A5	0.9477	0.9561	0.8910	0.8883	0.9688	0.9165	0.8951	0.9850
A10	0.9450	0.9446	0.9688	0.9694	0.9866	0.9611	0.9161	1.0000
A13	0.9453	0.9381	0.9564	0.9514	0.9839	0.9499	0.9930	0.9600
A14	0.9460	0.9406	0.9470	0.9477	0.9304	0.9453	1.0000	0.9600
A15	1.0000	0.9627	0.9315	0.9333	0.9831	0.9350	0.9510	0.8650
A19	0.9454	0.9834	0.9844	1.0000	0.9914	0.9815	0.7832	0.9800
A21	0.9453	0.9359	0.9377	0.9369	0.8585	0.9389	1.0000	0.9550
A22	0.9446	0.9809	0.9782	0.9928	0.9902	0.9784	0.7762	0.9850
A24	0.9449	0.9400	0.9470	0.9459	0.9346	0.9446	1.0000	0.9550
A25	0.9459	0.9509	0.9751	0.9712	0.9917	0.9655	0.9231	0.9550
A27	0.9446	1.0000	1.0000	0.9946	1.0000	1.0000	0.8531	0.7850
A32	0.9445	0.9314	0.9315	0.9279	0.9634	0.9363	0.9930	0.9600
A36	0.9449	0.9383	0.9377	0.9405	0.9144	0.9399	1.0000	0.9550
A37	0.9445	0.9333	0.9470	0.9441	0.9756	0.9452	0.9930	0.9700
A38	0.9841	0.9808	0.9502	0.9423	0.9848	0.9491	0.9301	0.9400
A40	0.9456	0.9371	0.9439	0.9459	0.8882	0.9435	1.0000	0.9550
A42	0.9445	0.9375	0.9439	0.9441	0.8936	0.9426	1.0000	0.9550
A43	0.9445	0.9345	0.9470	0.9441	0.9774	0.9457	0.9930	0.9600
A48	0.9478	0.9401	0.9502	0.9477	0.9405	0.9459	1.0000	0.9600

Table 3. H_j values and criteria weights

Hj	0.748	0.748	0.748	0.748	0.747	0.748	0.747	0.747
w_j	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

Table 4. Normalization Result

Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A3	0.009	0.035	0.371	0.387	0.000	0.221	1.000	0.814
A5	0.058	0.359	0.000	0.000	0.802	0.000	0.531	0.930
A10	0.009	0.193	0.714	0.726	0.915	0.534	0.625	1.000
A13	0.016	0.098	0.600	0.565	0.898	0.399	0.969	0.814
A14	0.028	0.134	0.514	0.532	0.558	0.345	1.000	0.814
A15	1.000	0.456	0.371	0.403	0.892	0.221	0.781	0.372
A19	0.017	0.759	0.857	1.000	0.945	0.779	0.031	0.907
A21	0.014	0.065	0.429	0.435	0.100	0.268	1.000	0.791
A22	0.003	0.722	0.800	0.935	0.938	0.741	0.000	0.930
A24	0.007	0.125	0.514	0.516	0.584	0.336	1.000	0.791
A25	0.026	0.284	0.771	0.742	0.947	0.587	0.656	0.791
A27	0.002	1.000	1.000	0.952	1.000	1.000	0.344	0.000
A32	0.001	0.000	0.371	0.355	0.767	0.237	0.969	0.814
A36	0.008	0.100	0.429	0.468	0.456	0.280	1.000	0.791
A37	0.000	0.028	0.514	0.500	0.845	0.344	0.969	0.860
A38	0.714	0.720	0.543	0.484	0.904	0.390	0.688	0.721

Alt.	C1	C2	C3	C4	C5	C6	C7	C8
A40	0.021	0.083	0.486	0.516	0.289	0.324	1.000	0.791
A42	0.000	0.088	0.486	0.500	0.323	0.313	1.000	0.791
A43	0.000	0.045	0.514	0.500	0.856	0.350	0.969	0.814
A48	0.061	0.126	0.543	0.532	0.622	0.351	1.000	0.814

Table 5. Border Approximation Area Matrix

BAA	0.1347	0.1552	0.1907	0.1919	0.2066	0.1730	0.2181	0.2189

Table 6. Ranking of the Funds

Fund	A3	A5	A10	A13	A14	A15	A19	A21	A22	A24
Sum (S_i)	-0.134	-0.154	0.101	0.056	0.002	0.073	0.173	-0.101	0.144	-0.005
Rank	19	20	6	8	12	7	2	18	4	13
Fund	A25	A27	A32	A36	A37	A38	A40	A42	A43	A48
Sum (S_i)	0.111	0.173	-0.050	-0.048	0.019	0.156	-0.050	-0.051	0.017	0.017
Rank	5	1	15	14	9	3	16	17	11	10

Table 7. TOPSIS Ranking

Fund	A3	A5	A10	A13	A14	A15	A19	A21	A22	A24
TOPSIS Rank	20	15	7	8	12	3	4	19	5	13
Fund	A25	A27	A32	A36	A37	A38	A40	A42	A43	A48
TOPSIS Rank	6	1	14	16	10	2	17	18	9	11

5. Conclusion

We have made an attempt to assess the funds from two perspectives such as efficiency and performance. Accordingly, we have filtrated the funds through a two stage process using DEA at stage 1 and MABAC at stage 2. The rationale behind this study lies in the selection of the funds to form an investment portfolio based on their return distribution and performance parameters encompassing risk-return tradeoff. In effect, this study not only has adjudged the funds on efficiency dimension, but also sets out to establish a ranking based on risk-return criteria. Our results conform to the market based rating of the funds. A combination of DEA-Entropy-MABAC turns this study considerably different from the existing contributions in the Indian context as far as the approach is concerned. The results of this study provide the investors a broader perspective for selection of the portfolio. However, future research shall be required to focus more clinical approach by considering fundamental parameters and stock level analysis. It is important to analyze the stocks on which the fund managers invest the amount invested by the investors both on fundamental dimension and organizational dimensions to ascertain the decision and establish a causal relationship among the stock performance and MF performance. Further, the efficiencies of the fund houses need to be examined. There are requirements to investigate the relationship between investors' sentiments and market performance of the MF. Also, a consistency between the performances of the funds belonging to different categories can be thought of. Finally, the framework used in this study can be further explored for different other applications.

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Appendix 1. Descriptive Statistics

S/L	Fund	Rating	Risk Grade	Net Assets (Cr) (C1)	5-Year annualized Ret (C2)	Sharpe Ratio (C3)	Sortino Ratio (C4)	Information Ratio (C5)	Alpha (C6)	R-Squared (C7)	Beta (C8)	SD	Expense Ratio
A1	Aditya Birla Sun Life Focused Equity Fund - Direct Plan	****	Below Average	4239.39	21.98	0.36	0.55	-0.16	-0.12	0.93	0.92	13.22	1.26
A2	Aditya Birla Sun Life Frontline Equity Fund - Direct Plan	****	Below Average	21380.42	21.39	0.38	0.60	-0.38	0.13	0.94	0.93	13.32	1.31
A3	Aditya Birla Sun Life Index Fund - Direct Plan	*	Above Average	142.91	16.19	0.31	0.50	-4.75	-1.09	1.00	0.99	13.84	0.51
A4	Axis Bluechip Fund - Direct Plan	*****	Low	2568.10	20.77	0.59	0.78	0.24	3.06	0.87	0.87	13.00	0.94
A5	BNP Paribas Large Cap Fund - Direct Plan	**	Average	892.52	19.71	0.18	0.26	-0.51	-2.52	0.85	0.94	14.22	1.05
A6	Canara Robeco Bluechip Equity Fund - Direct Plan	***	Below Average	131.66	18.72	0.40	0.61	0.09	0.49	0.92	0.97	14.07	1.48
A7	DHFL Pramerica Large Cap Fund - Direct Plan	***	Below Average	414.49	19.85	0.35	0.60	-0.34	-0.44	0.97	0.93	13.18	1.51
A8	DSP Top 100 Equity Fund - Direct Plan	*	High	3007.80	17.40	0.25	0.41	-0.33	-1.69	0.90	1.02	14.99	1.47
A9	Edelweiss Large Cap Fund - Direct Plan	***	Average	140.81	20.33	0.38	0.59	0.00	0.02	0.94	1.00	14.28	0.58
A10	Essel Large Cap Equity Fund - Direct Plan	***	Below Average	150.66	17.90	0.43	0.71	0.09	0.93	0.88	0.91	13.47	2.09
A11	Franklin India Bluechip Fund - Direct Plan	**	Average	8107.94	18.84	0.23	0.43	-0.79	-1.87	0.95	0.90	12.84	1.16
A12	Franklin India Index Fund - NSE Nifty Plan - Direct Plan	**	Average	253.21	16.64	0.32	0.52	-3.68	-0.86	1.00	0.99	13.77	0.64

A13	HDFC Index Fund - Sensex Plan - Direct Plan	* * *	Below Average	244.04	16.87	0.39	0.61	0.00	0.06	0.99	0.99	13.84	0.15
A14	HDFC Index Fund Nifty 50 Plan - Direct Plan	* * *	Average	435.89	17.26	0.36	0.59	-1.80	-0.29	1.00	0.99	13.84	0.15
A15	HDFC Top 100 Fund - Direct Plan	* * *	High	15260.79	20.76	0.31	0.51	-0.03	-1.09	0.93	1.18	16.99	1.26
A16	HSBC Large Cap Equity Fund - Direct Plan	* * *	Average	729.82	18.90	0.41	0.66	0.21	0.49	0.95	1.04	14.85	1.51
A17	ICICI Prudential Bluechip Fund - Direct Plan	* * * *	Low	18747.28	20.62	0.47	0.73	0.25	1.21	0.94	0.93	13.27	1.17
A18	ICICI Prudential Nifty Index Fund - Direct Plan	* *	Average	357.34	17.20	0.34	0.56	-3.55	-0.62	1.00	0.99	13.84	0.57
A19	ICICI Prudential Nifty Next 50 Index Fund - Direct Plan	* * * * *	Above Average	265.36	24.04	0.48	0.88	0.25	2.51	0.69	0.95	15.85	0.44
A20	IDBI India Top 100 Equity Fund - Direct Plan	* *	Average	417.49	18.94	0.16	0.28	-0.71	-2.67	0.89	0.89	13.13	1.06
A21	IDBI Nifty Index Fund - Direct Plan	* *	Above Average	224.64	16.52	0.33	0.53	-4.22	-0.79	1.00	1.00	13.87	0.43
A22	IDBI Nifty Junior Index Fund - Direct Plan	* * * * *	Above Average	55.99	23.64	0.46	0.84	0.21	2.27	0.68	0.94	15.79	0.56
A23	IDFC Large Cap Fund - Direct Plan	* * *	Below Average	373.25	17.30	0.44	0.65	0.17	0.92	0.95	0.93	13.26	1.83
A24	IDFC Nifty Fund - Direct Plan	* * *	Average	117.70	17.17	0.36	0.58	-1.66	-0.35	1.00	1.00	13.85	0.17
A25	Indiabulls Bluechip Fund - Direct Plan	* * * *	Average	401.75	18.89	0.45	0.72	0.26	1.27	0.89	1.00	14.70	0.75
A26	Invesco India Largecap Fund - Direct Plan	* * * *	Low	152.26	21.05	0.47	0.72	0.28	1.25	0.96	0.92	12.99	0.96
A27	JM Core 11 Fund - Direct Plan	* * * * *	High	36.22	26.66	0.53	0.85	0.54	3.94	0.79	1.34	20.89	2.94
A28	JM Large Cap Fund - Direct Plan	*	Below Average	2818.01	17.25	0.06	0.09	-1.31	-3.57	0.96	0.80	11.36	1.61
A29	Kotak Bluechip Fund - Direct Plan	* * *	Below Average	1423.60	20.42	0.35	0.56	-0.23	-0.41	0.96	0.95	13.52	1.15

A30	L&T India Large Cap Fund - Direct Plan	* *	Average	423.85	18.74	0.21	0.38	-0.62	-2.16	0.92	0.95	13.77	2.04
A31	LIC MF Index-Nifty Plan - Direct Plan	*	Above Average	23.77	16.39	0.30	0.49	-3.49	-1.17	1.00	1.00	13.96	0.69
A32	LIC MF Index-Sensex Plan - Direct Plan	*	Average	20.69	15.81	0.31	0.48	-0.69	-0.99	0.99	0.99	13.86	1.15
A33	LIC MF Large Cap Fund - Direct Plan	* *	Below Average	245.09	18.09	0.19	0.32	-0.77	-2.35	0.93	0.90	13.02	1.52
A34	Motilal Oswal Focused 25 Fund - Direct Plan	* * * *	Low	1170.53	21.94	0.38	0.55	-0.06	0.72	0.72	0.78	12.83	1.15
A35	Principal Nifty 100 Equal Weight Fund - Direct Plan	*	Above Average	18.17	15.84	0.23	0.39	-1.26	-2.16	0.98	0.99	13.92	0.50
A36	Reliance Index Fund - Nifty Plan - Direct Plan	* *	Above Average	132.99	16.90	0.33	0.55	-2.34	-0.71	1.00	1.00	13.97	0.29
A37	Reliance Index Fund - Sensex Plan - Direct Plan	* *	Average	7.63	16.11	0.36	0.57	-0.28	-0.30	0.99	0.97	13.55	0.29
A38	Reliance Large Cap Fund - Direct Plan	* * * *	Above Average	10897.82	23.62	0.37	0.56	0.03	0.00	0.90	1.03	15.06	1.32
A39	SBI Bluechip Fund - Direct Plan	* * * * *	Low	20283.92	22.61	0.41	0.62	-0.03	0.53	0.90	0.87	12.83	1.18
A40	SBI Nifty Index Fund - Direct Plan	* *	Average	320.99	16.71	0.35	0.58	-3.22	-0.43	1.00	1.00	13.93	0.29
A41	Sundaram Select Focus Fund - Direct Plan	* * * *	Below Average	835.70	18.62	0.43	0.66	0.05	0.69	0.95	0.90	12.82	0.55
A42	Tata Index Nifty Fund - Direct Plan	* *	Average	12.06	16.77	0.35	0.57	-3.04	-0.50	1.00	1.00	13.87	0.21
A43	Tata Index Sensex Fund - Direct Plan	* * *	Average	5.87	16.30	0.36	0.57	-0.22	-0.26	0.99	0.99	13.81	0.22
A44	Tata Large Cap Fund - Direct Plan	* * *	Below Average	803.29	18.50	0.29	0.47	-0.45	-1.15	0.95	0.95	13.53	0.74
A45	Taurus Largecap Equity Fund - Direct Plan	*	High	39.38	17.04	0.00	0.00	-1.27	-5.35	0.91	0.99	14.41	2.15
A46	Taurus Nifty Index Fund - Direct Plan	* * *	Average	19.06	17.11	0.37	0.60	-0.26	-0.20	0.99	0.98	13.63	1.22
A47	UTI Mastershare Fund - Direct Plan	* * *	Below Average	5530.66	19.57	0.30	0.53	-0.49	-0.97	0.96	0.91	12.96	1.43
A48	UTI Nifty Index Fund - Direct Plan	* * *	Average	935.94	17.18	0.37	0.59	-1.46	-0.25	1.00	0.99	13.79	0.12

Appendix 2. Calculation of efficiency using DEA

Funds under study	Kurtosis	Input		Output		VRS result	Rank VRS
		Kurtosis (normalized)	Expense Ratio	PSV/ NSV	Q3		
DMU1	-0.4904	0.5013	1.2600	1.0549	6.6575	16.07%	31
DMU2	-0.7490	0.3612	1.3100	1.0393	7.1600	13.37%	36
DMU3	-0.9942	0.2284	0.5100	0.6925	6.3250	27.70%	20
DMU4	-0.5445	0.4720	0.9400	0.5358	7.4000	16.62%	30
DMU5	0.4301	1.0000	1.0500	1.1908	7.5125	41.96%	17
DMU6	0.2381	0.8960	1.4800	0.7074	5.9725	9.31%	43
DMU7	-0.0014	0.7662	1.5100	0.7634	5.5975	9.27%	44
DMU8	-0.9583	0.2478	1.4700	0.6868	6.0125	10.06%	41
DMU9	-0.5300	0.4798	0.5800	0.6661	6.8850	22.95%	25
DMU10	0.0687	0.8043	2.0900	1.7130	6.0650	100%	7
DMU11	-0.5220	0.4842	1.1600	0.7371	4.7950	12.24%	38
DMU12	-1.0308	0.2085	0.6400	0.7048	6.3925	22.51%	26
DMU13	-1.3921	0.0128	0.1500	1.0249	7.3475	152.72%	2
DMU14	-1.0697	0.1874	0.1500	0.6978	6.6575	83.42%	11
DMU15	-0.9863	0.2327	1.2600	1.6584	8.5375	100%	7
DMU16	-0.5595	0.4639	1.5100	0.7604	6.7950	9.57%	42
DMU17	-1.2878	0.0693	1.1700	0.9866	6.7800	15.06%	32
DMU18	-1.0226	0.2130	0.5700	0.7052	6.5200	25.08%	22
DMU19	-0.8677	0.2969	0.4400	1.1672	9.8025	146.00%	3
DMU20	-0.0496	0.7401	1.0600	1.0686	6.1950	21.37%	27
DMU21	-1.0472	0.1996	0.4300	0.7086	6.5225	32.76%	18
DMU22	-0.9146	0.2715	0.5600	1.1773	9.7425	102.00%	6
DMU23	-1.0924	0.1751	1.8300	0.5602	6.7950	8.18%	46
DMU24	-1.0202	0.2143	0.1700	0.7021	6.5925	73.49%	13
DMU25	-0.7569	0.3569	0.7500	0.7040	8.0900	31.69%	19
DMU26	-0.7229	0.3754	0.9600	0.6534	6.3100	14.85%	33
DMU27	-1.3407	0.0406	2.9400	0.7030	12.0225	100%	7
DMU28	-0.6121	0.4354	1.6100	0.7645	3.8550	9.03%	45
DMU29	-0.5916	0.4465	1.1500	1.0974	5.9675	24.09%	24
DMU30	-0.3986	0.5510	2.0400	0.7255	6.5800	7.13%	47
DMU31	-1.0855	0.1789	0.6900	0.6874	6.5025	21.11%	28
DMU32	-1.4157	0.0000	1.1500	1.0384	7.1800	100%	7
DMU33	-1.3228	0.0504	1.5200	0.5927	5.8700	14.05%	35
DMU34	-0.8733	0.2939	1.1500	0.6444	7.5100	14.71%	34
DMU35	-0.7201	0.3768	0.5000	0.7821	4.9900	26.93%	21
DMU36	-0.9688	0.2421	0.2900	0.7066	6.6100	45.86%	16
DMU37	-1.4146	0.0006	0.2900	1.0276	7.0450	303.43%	1
DMU38	-0.9501	0.2523	1.3200	1.5284	7.0700	78.19%	12
DMU39	-0.6039	0.4398	1.1800	0.8370	7.2125	12.26%	37
DMU40	-1.0468	0.1999	0.2900	0.7034	6.6075	46.89%	15
DMU41	-0.1160	0.7042	0.5500	0.8305	5.9050	24.10%	23
DMU42	-1.0639	0.1906	0.2100	0.7021	6.6850	62.64%	14

Funds under study	Kurtosis	Input		Output		VRS result	Rank VRS
		Kurtosis (normalized)	Expense Ratio	PSV/ NSV	Q3		
DMU43	-1.4037	0.0065	0.2200	1.0335	7.2400	141.03%	4
DMU44	-0.8221	0.3216	0.7400	0.6315	6.6900	19.13%	29
DMU45	-1.0498	0.1983	2.1500	0.4023	6.5025	6.97%	48
DMU46	-0.6233	0.4293	1.2200	0.7896	6.4950	11.76%	39
DMU47	-0.8883	0.2857	1.4300	0.7594	5.8050	10.29%	40
DMU48	-1.0725	0.1860	0.1200	0.6911	6.5875	125.00%	5

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