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DECISION MAKING IN OPERATOR-MACHINE ASSIGNMENT PROBLEMS: AN OPTIMIZATION APPROACH IN U-SHAPED PRODUCTION LINES

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Abstract: *Decision-making is a complex problem in all fields of production, because decision makers have to find the most suitable solution alternative among the numerous, while the solution of the problem is limited by multiple, usually conflicting constraints. In the Industry 4.0 era, one of the most widely used production lines are the U-shaped production lines, which focuses on the elimination of wastes through a high utilization of workers. It means, that worker-machine assignment plays a significant role in the optimization of production processes in a U-shaped production line. The current investigation proposes a novel absorbing Markov chain (AMC) optimization approach to support the decision making regarding worker selection and assignment. The proposed approach integrates the AMC and the performance analysis of the production process. The numerical results validated the decision making model and showed that optimal worker-machine assignment can lead to about 20% production cost reduction, while the key performance indicators (KPI) of the U-shaped production line are significantly increased. The sensitivity analysis of influencing factors made it possible to identify the critical factors of this decision making problem, such as qualification of operators, time of technological and logistics operations, maintenance policy.*

Key words: *Uncertain production environment, absorbing Markov Chain, assignment problem, Decision making.*

1. Introduction

U-shaped production lines represent a special area of production systems where machines are arranged in a U-shape to allow the operators to service them as efficiently as possible. The design and operation of U-shaped production lines includes a wide range of optimization tasks including facility location, layout planning, scheduling, inventory control, operator-machine assignment and routing. The optimal solution of these tasks can lead to an efficient, flexible and available production

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system. In the case of U-shaped production lines, the workers and the operator plays an especially important role in the operation, because their performance significantly influences the efficiency of the U-shaped production line (Chutima & Khotsaenlee, 2022). In an optimized U-shaped production line the cycle time and the lead time is shorter, the required plant area for production is smaller, balancing of the line is easier than in the case of conventional production lines (Zhang et al., 2021). The U-shaped production lines can be generally not only U-shaped, O-shaped, C-shaped or W-shaped production lines can also shorter the required routes of operators while the input and output storages are close to each other. In the U-shaped production lines one-piece flow can implemented, which can lead to significant decrease of work in process (WIP) inventory. Based on all these aspects, it can be concluded that the development of u-shaped production lines is a current area of research that can lead to significant cost savings for both small and medium-sized enterprises and large companies through better use of available resources.

The objective of the present study is to propose an optimization approach to support decision making that can be an essential tool in solving problem regarding operation of U-shaped production lines. The motivation of the research is that the utilization of available human resources is important for manufacturing companies, therefore the optimal assignment of operators to machines can increase the productivity, efficiency and flexibility of U-shaped production lines, while the cost efficiency and the utilization of human resources can also increase.

This paper is organized, as follows. Section 2 presents a literature review to summarize the available research background and research results in the field of operator assignment problems in U-shaped production lines. Section 3 presents the mathematical model of the horizontal and vertical operator-machine assignment problems focusing on total cost, cycle time and lead time optimization. Section 4 presents the results of the numerical analysis of different scenarios. Conclusions, managerial impacts, limitations and future research directions are discussed in the last section.

2. Literature review

In this section, we review some of the existing literatures which are related to the current research work focusing on the optimization of operator-machine assignment problems in U-shaped production lines. Miltenburg (2001) concluded that in a U-shaped production line, the machine work and the work performed by the operators are as independent as possible. It means that the scheduling of the machine work is the input parameter of the scheduling of the operator work and from this it follows that the optimal operator-machine assignment requires multi-skilled operators, so they can be assigned to different machines to perform different technological and logistics operations. The multi-skilled operators can be characterized by different parameters, for example by scrap rate, productivity or labor cost.

Zülch & Zülch (2017) found that the design of U-shaped production lines must be based on both productivity and cost-related objectives. The design process must focus on a wide range of design tasks including efficiency, lead time, cycle time, scrap rate or ergonomics. The design process of U-shaped production lines can use both static methods (spreadsheet calculations) or dynamic methods (simulation). As Chen et al. (2016) concluded, operator-machine assignment and the working strategy of operators can significantly influence the key performance indicators of U-shaped production lines. They analyzed the different types of operator strategies of U-shaped production systems (conventional travelling production line, chasing-overtaking production line or a classic bucket brigade production line) and they found that the chosen type has great impact on production capacity, work in process inventory (WIP), utilization of operators and machines. Chen et al. (2010) suggested in their research work to focus on the dynamic design of U-shaped production systems, because in the case of just-in-time (JIT) or just-in-sequence (JIS) production, the U-cell must be balanced dynamically, depending on the demands of customers. In their approach they propose a dynamic programming methodology to solve the balancing problem of U-shaped production lines in order to improve flexibility, efficiency and quality.

As Cao & Kong (2015) concluded in their study, the skills of operators significantly influences the efficiency of the production cells, therefore it is important to find the best operator-machine assignment. It is especially important in the case of cooperative operators, where not only the individual skills and performance indicators but also the cooperative ability and the interdependent performance should be taken into consideration, to find the best cooperative operator combinations for a specific production program in the U-shaped production line. According to (Kuo et al., 2022) the balancing of U-shaped production lines is an especially complex problem in the case of high flexibility, where the production operations are exchangeable and this exchangeability lead to dynamically changing walking routes and walking times of operators, which has a great impact on the cycle time and lead time of the U-cell. I is therefore necessary to develop design methods that take the flexibility of operator-machine assignment into consideration. As Cao & Kong (2016) concluded, operator assignment problems are stochastic problems, because the technological, logistics and human parameters have uncertainties in the U-shaped production line, therefore the modelling and optimization of U-cells are generally based on either Fuzzy models or Markov chains.

Ayough & Khorshidvand (2023) proposes a novel approach which focuses on the integration of operator-machine assignment and sequencing of the work process. In this approach, the skill level of workers is also taken into consideration. Their proposed model was solved using a commercial solver, which shows, that in the case of small and medium sized problems commercial solvers can be used to optimize complex models. In the Industry 4.0 era, the application of emerging technologies can lead to increased flexibility and productivity. Gil-Vila et al. (2017) analyzed the impact of the integration of collaborative robots into U-shaped production lines. Their study shows that the application of collaborative robots in U-shaped production lines can result better productivity than in the case of conventional human operators or conventional robotized lines.

As Nakade & Ohno concluded, the cycle time and the lead time depends on the machines and the technology; in the case of the operators not only the processing time of each operator at each machine but also the related processing time and walking time must be taken into consideration. They compared two different types of operator allocation strategy (carousel allocation and separate allocation), and they concluded that the allocation and assignment strategy of operators significantly influences the key performance indicators of the U-shapes production line. The operation of U-shaped production lines can be analyzed from energy efficiency point of view. As Zhang et al. (2021a) suggested in their research work focusing on the optimization of U-shaped robotic assembly lines, the material handling processes have great impact on both the logistics-related performance indicators and the energy consumption. This approach shows the conflict between energy efficiency, layout of the U-shaped production line and the make-span of the robots. These aspects can also be used in the case of conventional human operator-based or cobot-based U-cells.

The above-mentioned research results indicate the scientific potential of the optimization of worker allocation and assignment problems in U-shaped production lines. The articles that addressed the operator-worker assignment problem are generally used deterministic parameters to describe the production process and the operators, and only a few of them uses stochastic parameters and takes the uncertainties into consideration. According to that, the focus of this research is on the analyses of the impact of operator-machine assignment on the total production cost, component cost, cycle time, lead time and scrap rate in U-shaped production lines.

As a consequence, the main contributions of this article are the following: (1) mathematical description of technological and logistics processes in U-shaped production lines using absorbing Markov chains; (2) optimization of operator-machine assignment from cost efficiency, cycle time, lead time and scrap rate point of view; (3) numerical analysis of the impact of different optimization aspects on key performance indicators.

3. Materials and methods

The production process of an U-shaped production line can be described as an absorbing Markov chain. In this approach, the Markov chain–based approach to describe general processes in production systems of Pillai & Chandrasekharan (2008) is applied for U-shaped production lines. Markov chain can be described using a transition probability matrix $P = [p_{i,j}]$, where $p_{i,j}$ is for the probability that product is moving from state *i* to state *j*. The transition probability matrix describes the relations among transient and absorbing states. Generally, in a production system production cells, assembly stations, repair stations, quality control stations represent transient states, because product can leave these states and move to a next one, while storages and warehouses are absorbing states, because products from these states do not move back to the production system. Based on this, the transition probability matrix has four main parts. The first part describes the transition probabilities between transient states:

$$P^{TT} = [p_{i,j}^{TT}] = [p_{i,j}], i, j = 1 \dots m, \quad (1)$$

where $p_{i,j}^{TT}$ is for the transition probability between transient states *i* and j, and *m* is the number of transient states. In our approach the transient states are numbered as follows: state 1 is the input storage of the U-shaped production cell, states 2 to m_1 represent the machines and assembly stations and states $m_1 + 1$ to m_2 represent the repair stations, so $m = m_1 + m_2$.

The second part describes the transition probabilities between transient and absorbing states:

$$P^{TA} = [p_{i,j}^{TA}] = [p_{i,m+j}], i = 1 \dots m, j = 1 \dots n, \quad (2)$$

where $p_{i,j}^{TA}$ is for the transition probability between transient stage *i* and absorbing state *j*, and *n* is the number of absorbing states.

The third part describes the transition probabilities between absorbing and transient states:

$$P^{AT} = [p_{i,j}^{AT}] = [p_{m+i,j}], i = 1 \dots n, j = 1 \dots m, \quad (3)$$

where $p_{i,j}^{AT}$ is for the transition probability between absorbing state i and transient state j, and based on the definition of absorbing states

$$\forall, i = 1 \dots n, j = 1 \dots m: p_{i,j}^{AT} = 0. \tag{4}$$

The fourth part describes the transition probabilities between absorbing states:

$$P^{AA} = \left[p_{i,j}^{AA}\right] = \left[p_{m+i,m+j}\right], i,j = 1 \dots n, \tag{5}$$

where $p_{i,j}^{AA}$ is for the transition probability between absorbing state i and j, and based on the definition of absorbing states

$$\forall\, i,j = 1 \dots n: p_{i,j}^{AA} = 1 \text{ if } i = j \text{ otherwise } p_{i,j}^{AA} = 0. \tag{6}$$

These transition probabilities are influenced by technological, logistics and human resource parameters. The transition probabilities can be calculated based on the scrap rate of the assigned operator, because scrap rate determine the potential ways of the product in the U-cell.

The qualification of the operators can be characterized by the machines which can be operated by themselves and by their scrap rate. The scrap rate describes what percentage of the products produced will be scrapped either as repairable or non-repairable scrap. In the case of U-shaped production lines, we can define two different types of scrap rates depending on the type of the state. In the case of manufacturing and assembly cells we can define both repairable and non-repairable scrap rates, while in the case of repair we can define a simple scrap rate (Figure 1):

$$SR^{R} = \left[sr_{h_i,i}^{R}\right], h_i = 1 \dots o^{max}, i = 1 \dots m, \tag{7}$$

$$SR^{NR} = \left[sr_{h_i,i}^{NR}\right], h_i = 1 \dots o^{max}, i = 1 \dots m+n. \tag{8}$$

where h_i is the operator assigned to state i and o^{max} is the number of available operators to be assigned to machines. The decision variable of the optimization problem is h_i.

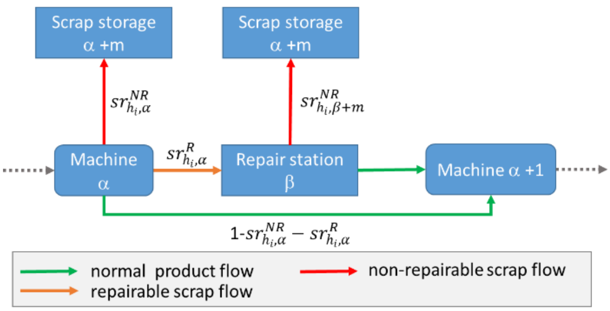


Figure 1. The links of repairable and non-repairable scrap rates of the operators

Within the frame of this approach, the impact of human resources (machine operators) are analyzed, therefore

$$\forall\, p_{i,j} \in \left(p_{i,j}^{TT}, P^{TA}\right): p_{i,j} = p_{i,j}\left(sr_{h_i,i}^{R}, sr_{h_i,i}^{NR}\right). \tag{9}$$

3.1. Optimization of production cost

In the first model, the objective function of the operator-machine assignment problem is the total production cost, which can be written as follows:

$$C^{PROD} = \frac{q}{\varphi_{1,n}} \cdot \sum_{i=1}^{m} \theta_{1,i} \cdot k_i^{PROD} \to min., \tag{10}$$

In Eq (1), q is the required demand to be produced, $\varphi_{1,n}$ is the overall probability distribution from the first transient state to the last absorbing state. In U-shaped production cells, the first transient state is the input storage of the U-cell, while the last absorbing state is the output storage for final products:

$$\varphi_{1,n} \in \Phi = (P^{AA} - P^{TT})^{-1} \cdot P^{AT}. \tag{11}$$

In Eq (1), $\theta_{1,i}$ is the number of operations performed in state *i,* if the initial state of the product is the first state, which is the input storage of the U-shaped production cell:

$$\theta_{1,i} \in \Theta = (P^{AA} - P^{TT})^{-1}. \tag{12}$$

In the objective function, the decision variable seems to be hidden, but it is integrated into matrices P^{AA} and P^{TT}, because the assigned operators influence the transition probabilities:

$$p_{1,2}^{TT} = 1 - sr^{IS}, \tag{13}$$

$$\forall i = 2 \ldots m_1 - 1 : p_{i,i+1}^{TT} = 1 - sr_{h_i,i}^{R} - sr_{h_i,i}^{NR}, \tag{14}$$

$$\forall i = m_1 + 1 \ldots m - 1 : p_{i-m_1+2}^{TT} = 1 - sr_{h_i,i}^{NR}, \tag{15}$$

$$\forall i = 2 \ldots m_1 : p_{i,i+m_1-1}^{TT} = sr_{h_i,i}^{R}, \tag{16}$$

$$\forall i = 1 \ldots m : p_{i,i+m}^{TA} = sr_{h_i,i}^{N}, \tag{17}$$

$$p_{m_1,m+n}^{TT} = 1 - sr_{h_i,i}^{R} - sr_{h_i,i}^{N}, \tag{18}$$

$$p_{m,m+n}^{TT} = 1 - sr_{h_i,i}^{N}, \tag{19}$$

where sr^{IS} is the scrap rate at the input storage of the U-shaped production cell.

The solutions of the above-described operator-machine assignment problem are limited by the following constraints:

Constraint 1: Each machine in the U-shaped production cell must be assigned to exactly one operator:

$$\forall i = 1 \ldots m + n : h_i \in (1,2, \ldots o^{max}). \tag{20}$$

Constraint 2: It is possible to assign one operator to more machines. If $h_i = h_j$ then the same operators are assigned to machine *i* (represented by state *i*) and machine *j* (represented by state *j*). If $h_i \neq h_j$ then different operators are assigned to machine *i* and *j*. Figure 2 and Figure 3 shows two potential assignment of a two-operators U-shaped production cell. In the case of vertical assignment (see Figure 2), the operators are assigned to vertical clusters of machines and the required materials handling routes among machines are significantly shorter, than in the case of horizontal assignment. The disadvantage of this operator-machine assignment is that the operations assigned to the same operator are non-consecutive operations.

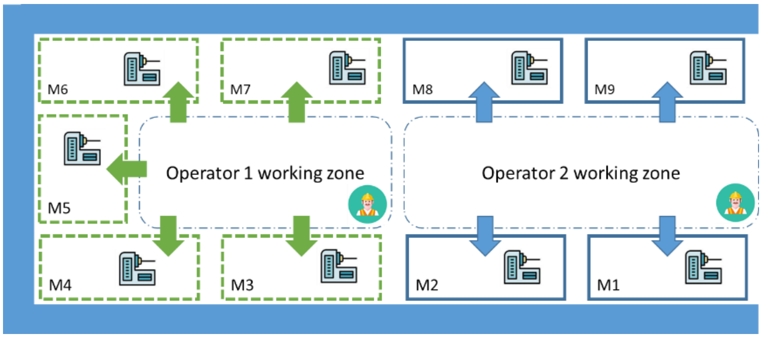


Figure 2. Assignment of operators to vertical machine clusters

In the case of horizontal assignment (see Figure 3), the assigned machines belong to horizontal machine clusters. The average distance between the required material handling operations is longer, than in the case of vertical assignment, but the operators can follow the value chain in the U-shaped production cell.

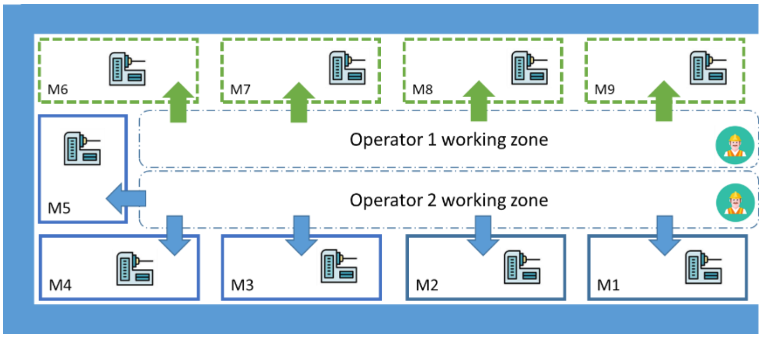


Figure 3. Assignment of operators to horizontal machine clusters

3.2. Optimization of cycle time and lead time

In the second model, the objective function of the operator-machine assignment problem is either the minimization of the cycle time or the minimization of lead time. The cycle time can be written as follows:

$$CT = \max_{i=1\ldots m} \frac{\theta_{1,i} \cdot \tau_i}{\varphi_{1,n}} \rightarrow min., \tag{21}$$

where τ_i is the specific production time at machine i, which is represented by state i and $\forall i = 1 \ldots m: \theta_{1,i} \in \Theta = (P^{AA} - P^{TT})^{-1}$.

In this cycle time related objective function, the decision variable is integrated into $\theta_{1,i}$ and $\varphi_{1,n}$ parameters defined by the transition probability matrix as shown in Eq. (13-19). In this approach the constraints are the same as in the case of optimization of total production cost.

In this approach, the cycle time includes only the technological time, which means, that the required time for logistics operations (transportation between states) is more less, than technological time, and the lead time can be defined also as an objective function as follows:

$$LT = (q-1) \cdot \max_{i=1...m} \frac{\theta_{1,i} \cdot \tau_i}{\varphi_{1,n}} \rightarrow min. \quad (22)$$

In this approach, the constraints are the same as in the case of optimization of total production cost. The above-mentioned assignment problems focusing on the minimization of total cost, cycle time and lead time are NP-hard optimization problems, in the case of above 200 decision variables and constraints it is possible to solve them with a general purpose solver. Within the frame of this research, the OpenSolver was used to solve the operator-machine assignment problem modelled as an absorbing Markov chain.

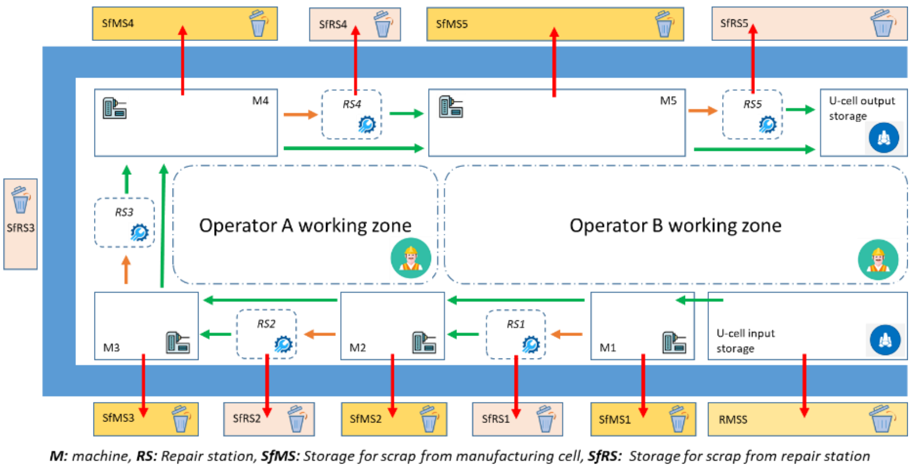
4. Results

Within the frame of this section, the main results regarding the optimization of total cost, cycle time and lead time are discussed.

4.1. Results of cost optimization

4.1.1. Scenario 1: vertical operator-machine assignment with cost optimization

Within the frame of the first scenario analysis, a U-shaped manufacturing system is analyzed including 5 machines, 5 repair stations, 1 input storage, 1 output storage for final products and each machines and repair station also has storages for scrap. In the analyzed period, 1000 final products must be produced. Figure 4 shows the layout of the analyzed U-shaped production cell.



***M:** machine, **RS:** Repair station, **SfMS:** Storage for scrap from manufacturing cell, **SfRS:** Storage for scrap from repair station*

Figure 4. Layout of the analyzed U-shaped production system with a vertical operator assignment

The U-cell is working with 2 operators, therefore in this operator-machine assignment problem the most important decision task is to choose the best operator for all 5 machines and repair stations. The assignment of the operators is given. Operator A works at machines 2,3 and 4, and repair stations 2, 3 and 4, while operator B works at machines 1 and 5, and repair stations 1 and 5. In this scenario, there are 31 operators suitable for the tasks to be performed in the U-cell, and their scrap rate is available a statistical data. The available operators and their scrap rate parameters are shown in Table A1 and Table A2. The operational cost for each machines and repair stations can be defined depending on the used technology, production time, required tools and fixtures, cost of human operators as shown in Table 1.

Table 1. Specific operational cost of transient states (machines and repair stations) in [EUR/pcs]

i	1	2		3	4	5	6	7	8	9	10	11
k_i^{PROD}	20	3.3	6.2	5.9	3.2	4.9	5.1	4.5	4.2	8	3.6	20

Table 2 shows the transition probabilities between transient states, and Table 3 between transient and absorbing states in the case of the optimal operator-machine assignment.

In the case of this optimal solution based on cost efficiency, we can calculate the transition probability matrices based on Eq. (13-19). Table 2 shows the computed transition probability matrix in the case of transient states, which are the active objects of the U-shaped production cell including machines and repair station.

As Table 2 and Table 3 demonstrates, machines have three transition probabilities, including transition from manufacturing phase i to manufacturing phase $i+1$, to repair station $i+m_1$, and to the storage of scrap from manufacturing $i+m$. The sum of the transition probabilities is for all transient and absorbing states is 1:

$$\forall i = 1 \dots m+n : \sum_{j=1}^{m+n} p_{i,j} = 1. \quad (23)$$

Table 2. Transition probability matrix of transient states (machines and repair stations)

	1	2	3	4	5	6	7	8	9	10	11
1		0.995									
2			0.997				0.000				
3				0.986				0.001			
4					0.986				0.003		
5						0.985				0.001	
6											0.006
7			0.990								
8				0.961							
9					0.952						
10						0.985					
11											

Table 3. Transition probability matrix from transient to absorbing states (from machines and repair stations to scrap storages and final product storage)

	12	13	14	15	16	17	18	19	20	21	22	23
1	0.004	0	0	0	0	0	0	0	0	0	0	0
2	0	0.002	0	0	0	0	0	0	0	0	0	0
3	0	0	0.013	0	0	0	0	0	0	0	0	0
4	0	0	0	0.011	0	0	0	0	0	0	0	0
5	0	0	0	0	0.014	0	0	0	0	0	0	0
6	0	0	0	0	0	0.020	0	0	0	0	0	0.974
7	0	0	0	0	0	0	0.01	0	0	0	0	0
8	0	0	0	0	0	0	0	0.039	0	0	0	0
9	0	0	0	0	0	0	0	0	0.048	0	0	0
10	0	0	0	0	0	0	0	0	0	0.015	0	0
11	0	0	0	0	0	0	0	0	0	0	0.1	0.9

The OpenSolver resulted a minimum total cost of $C^{PROD} = 46038\ EURO$. In this optimal solution, operator 16 was assigned to machines 2,3 and 4, and repair stations 2, 3 and 4 and operator 31 was assigned to machines 1 and 5, and repair stations 1 and 5. How OpenSolver calculates the value of the objective function in the case of this scenario? The following numerical example demonstrates the calculation of the objective function depending on the decision variable and the constraints.

As a next step, we can calculate the Φ matrix based on Eq. (11) and the Θ matrix based on Eq. (12). The results are shown in Table 4 and Table 5.

These matrices are required to compute the total production cost, as the objective function of this scenario.

Table 4. The Θ matrix in the case of the minimal total cost

	1	2	3	4	5	6	7	8	9	10	11
1	1	0.9954	0.9928	0.9798	0.9685	0.9546	0.0004	0.0009	0.0025	0.0006	0.0057
2	0	1.0000	0.9974	0.9844	0.9730	0.9590	0.0004	0.0009	0.0025	0.0006	0.0057
3	0	0.0000	1.0000	0.9869	0.9755	0.9615	0.0000	0.0010	0.0025	0.0006	0.0057
4	0	0.0000	0.0000	1.0000	0.9884	0.9742	0.0000	0.0000	0.0025	0.0006	0.0058
5	0	0.0000	0.0000	0.0000	1.0000	0.9856	0.0000	0.0000	0.0000	0.0006	0.0059
6	0	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0060
7	0	0.0000	0.9900	0.9770	0.9657	0.9518	1.0000	0.0009	0.0025	0.0006	0.0057
8	0	0.0000	0.0000	0.9610	0.9499	0.9362	0.0000	1.0000	0.0024	0.0006	0.0056
9	0	0.0000	0.0000	0.0000	0.9520	0.9383	0.0000	0.0000	1.0000	0.0006	0.0056
10	0	0.0000	0.0000	0.0000	0.0000	0.9850	0.0000	0.0000	0.0000	1.0000	0.0059
11	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Table 5. The Φ matrix in the case of the minimal total cost

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.005	0.003	0.013	0.011	0.014	0.019	0.000	0.000	0.000	0.000	0.001	0.935
2	0.000	0.003	0.013	0.011	0.014	0.019	0.000	0.000	0.000	0.000	0.001	0.939
3	0.000	0.000	0.013	0.011	0.014	0.019	0.000	0.000	0.000	0.000	0.001	0.942
4	0.000	0.000	0.000	0.011	0.014	0.020	0.000	0.000	0.000	0.000	0.001	0.954
5	0.000	0.000	0.000	0.000	0.014	0.020	0.000	0.000	0.000	0.000	0.001	0.965
6	0.000	0.000	0.000	0.000	0.000	0.020	0.000	0.000	0.000	0.000	0.001	0.979
7	0.000	0.000	0.013	0.011	0.014	0.019	0.010	0.000	0.000	0.000	0.001	0.932
8	0.000	0.000	0.000	0.011	0.014	0.019	0.000	0.039	0.000	0.000	0.001	0.917
9	0.000	0.000	0.000	0.000	0.014	0.019	0.000	0.000	0.048	0.000	0.001	0.919
10	0.000	0.000	0.000	0.000	0.000	0.020	0.000	0.000	0.000	0.015	0.001	0.965
11	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.100	0.900

Based on Eq. (10) we can now calculate the required products in the input storage of the U-shaped production cell and the total cost in the case of the optimal operator-machine assignment. In the case of this scenario, the cycle time is $CT = 75.56\ s$, while the lead time is $LT = 21.067\ hours$.

The analysis of the different operator-machines assignments shows, that it is possible to identify critical operator combinations, and we can also find critical individual operators, which cannot assign to any other operators with a suitable, acceptable total operation cost (see Figure 5a). In this scenario, the critical operator pair is operator 6 and 19, while the critical operator is operator 6. In Figure 5a, the best operator pairs from total cost optimization point of view are demonstrated by blue and green peaks, while the worst operator pairs are orange and light blue valleys.

Based on the impact of operator-machine assignment and the potential operator pairs on the total cost, it can be concluded, that the total cost based optimization can lead to a total cost saving of 13917 EUR, which is in the case of the first scenario more than 30% (see Figure 5a).

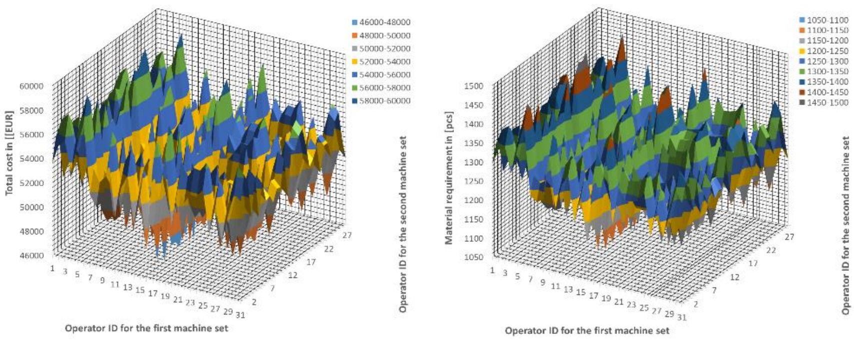


Figure 5. Impact of the operators: (a) on the total production cost and (b) on the required components in the case of vertical operator-machine assignment while optimizing total production cost

The total cost-based assignment can result 39.4% saving of required components to fulfill the defined 1000 pieces of final product demand, which means 422 pieces (see Figure 5b). The maximum cycle time without optimization is 86.11s, while the total cost based optimization leads to a cycle time of 75.56s, which means 14% shorter cycle time. The lead time can be also shortened using the total cost-based optimization. The lead time can be shortened with 2.94 hours to 21.067 hours, which means 13.9% saving in lead time. In the total cost-based optimal operator-machine assignment, the global transition probability is 0.9349 from the input storage to the final product storage, while the worst probability value is 0.6704.

4.1.2. Scenario 2: horizontal operator-machine assignment with cost optimization

Within the frame of the second scenario analysis, the same layout is analyzed, but in this case the assignment of the operators is horizontal, which means that operator A works at machines 4 and 5, and repair stations 4 and 5, while operator B works at machines 1, 2 and 3, and repair stations 1, 2 and 3 (see Figure 6). The available operators and their scrap rate parameters are the same mentioned in scenario 1 (see Table A1 and Table A2).

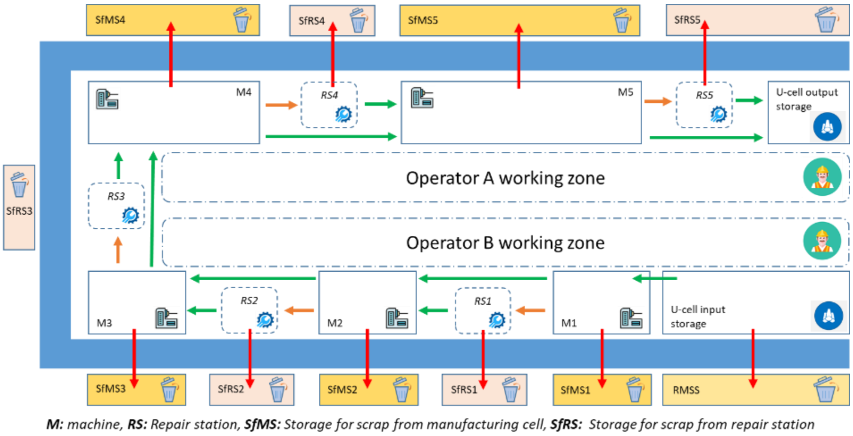


Figure 6. Layout of the analyzed U-shaped production system with a horizontal operator assignment

The OpenSolver resulted a minimum total cost of $C^{PROD} = 46644\ EURO$. In this optimal solution, operator 26 was assigned to machines 1, 2, and 3, and repair stations 1, 2, and 3 and operator 11 was assigned to machines 4 and 5, and repair stations 4 and 5. Based on Eq. (10) we can now calculate the required products in the input storage of the U-shaped production cell and the total cost in the case of the optimal operator-machine assignment. In the case of this scenario, the cycle time is $CT = 74.93\ s$, while the lead time is $LT = 20.896\ hours$.

The analysis of the different operator-machines assignments shows, that it is possible to identify critical operator combinations, and we can also find critical individual operators, which cannot assign to any other operators with a suitable, acceptable total operation cost (see Figure 7a). In this scenario, the critical operator pair is operator 3 and 10, while the critical operator is operator 10. In Figure 7a, the best operator pairs from total cost optimization point of view are demonstrated by blue and green peaks, while the worst operator pairs are orange and light blue valleys.

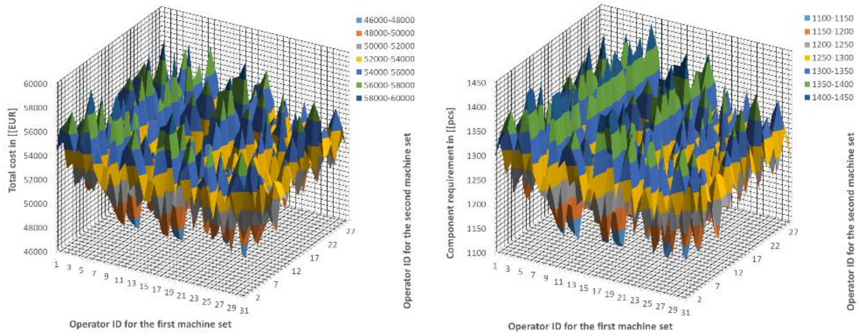


Figure 7. Impact of the operators: (a) on the total production cost and (b) on the required components in the case of horizontal operator-machine assignment while optimizing total production cost

Based on the impact of operator-machine assignment and the potential operator pairs on the total cost, it can be concluded, that the total cost based optimization can lead to a total cost saving of 13854 EUR, which is in the case of this scenario more than 29.5% (see Figure 7a). The total cost-based assignment can result 37.6% saving of required components to fulfill the defined 1000 pieces of final product demand, which means 415 pieces (see Figure 7b). The maximum cycle time without optimization is 87.65s, while the total cost based optimization leads to a cycle time of 74.93s, which means 16.9% shorter cycle time. The lead time can be also shortened using the total cost-based optimization. The lead time can be shortened with 3.54 hours to 20.896 hours, which means 16.9% saving in lead time. In the total cost-based optimal operator-machine assignment, the global transition probability is 0.9067 from the input storage to the final product storage, while the worst probability value is 0.6586.

4.1.3. Comparison of horizontal and vertical operator-machine assignment with cost optimization

Comparing the results of the horizontal and vertical assignment of operators and machines it is concluded, that the optimal operator-machine assignment resulted a better total cost in the case of vertical operator-machine assignment, which means that the vertical operator-machine assignment is more suitable for this layout than the vertical assignment. Table 6 summarizes the most important parameters of the optimal operator-machine assignment in both cases.

Table 6. Comparison of the vertical and horizontal operator-machine assignment with total cost optimization

	Vertical assignment	Horizontal assignment
Assigned operators	16 and 31	3 and 10
Transition probability between input and output storage	0.9349	0.9067
Required components [pcs]	1070	1102
Total cost [EUR]	46038	46644
Cycle time [sec]	75.56	74.93
Lead time [hours]	21.067	20.896

As the comparative analysis of the optimized vertical and horizontal operator-machine assignment shows, the vertical assignment resulted less components in the input storage of the U-cell, and the total production cost is also lower, but the cycle time and the lead time is approximately the same, no significant difference can be identified.

4.2. Results of cycle time and lead time optimization

4.2.1. Scenario 3: vertical operator-machine assignment with cycle time and lead time optimization

Within the frame of the third scenario analysis, the same U-shaped manufacturing system is analyzed from cycle time optimization point of view. As Eq. (21) shows, the cycle time can be computed based on matrix Φ, matrix Θ and the specific operation time, which is defined in this scenario as 7 shows.

Table 7. Specific operation time of transient states (machines and repair stations) in [sec]

τ_i	1	2		3	4	5	6	7	8	9	10	11
11	0	58	44	37	37	74	65	61	25	34	42	0

The OpenSolver resulted a minimum cycle time of $CT = 74.21\ s$. In this optimal solution, operator 4 was assigned to machines 2,3 and 4, and repair stations 2, 3 and 4 and operator 10 was assigned to machines 1 and 5, and repair stations 1 and 5. However the cycle time is minimized in this scenario, but the related total cost is increased to $C^{PROD} = 51955\ EURO$. In this scenario the lead time is $LT = 20.701\ hours$, which means, that no significant changes are impacted by the new objective function. The worst and best operator combinations and individual operators can be also defined in this scenario in the same way, as in the case of scenario 1.

Based on the impact of operator-machine assignment and the potential operator pairs on the total cost, it can be concluded, that the cycle time-based optimization can lead to a total cost saving of 8000 EUR, which is in the case of this scenario more than 15.4% (see Figure 6a). The cycle time-based assignment can result 17% saving of required components to fulfill the defined 1000 pieces of final product demand, which means 218 pieces (see Figure 6b). The maximum cycle time without optimization is 86.11s, while the cycle time-based optimization leads to a cycle time of 74.21, which means 16% shorter cycle time. The lead time can be also shortened using the cycle time-based optimization. The lead time can be shortened with 3.31 hours to 20.701 hours, which means 15.9% saving in lead time. In the case of the optimal operator-machine assignment, the global transition probability is 0.785 from the input storage to the final product storage, while the worst probability value is 0.6704.

4.2.2. Scenario 4: Horizontal operator-machine assignment with cycle time and lead time optimization

Within the frame of the fourth scenario analysis, the same layout is analyzed, but in this case the assignment of the operators is horizontal, which means that operator A works at machines 4 and 5, and repair stations 4 and 5, while operator B works at machines 1, 2 and 3, and repair stations 1, 2 and 3.

The OpenSolver resulted a minimum cycle time of $CT = 74.21\ s$. In this optimal solution, operator 1 was assigned to machines 1, 2, and 3, and repair stations 1, 2, and 3 and operator 10 was assigned to machines 4 and 5, and repair stations 4 and 5. However the cycle time is minimized in this scenario, but the related total cost is increased to $C^{PROD} = 49635\, EURO$. In this scenario the lead time is $LT = 20.699\, hours$, which means, that no significant changes are impacted by the new objective function. The worst and best operator combinations and individual operators can be also defined in this scenario in the same way, as in the case of scenario 2.

Based on the impact of operator-machine assignment and the potential operator pairs on the total cost, it can be concluded, that the cycle time-based optimization can lead to a total cost saving of 11057 EUR, which is in the case of this scenario more than 22.2% (see Figure 7a). The cycle time-based assignment can result 29% saving of required components to fulfill the defined 1000 pieces of final product demand, which means 341 pieces (see Figure 7b). The maximum cycle time without optimization is 87.65s, while the cycle time-based optimization leads to a cycle time of 74.21, which means 16% shorter cycle time. The lead time can be also shortened using the cycle time-based optimization. The lead time can be shortened with 3.74 hours to 20.699 hours, which means 18.1% saving in lead time. In the case of the optimal operator-machine assignment, the global transition probability is 0.8498 from the input storage to the final product storage, while the worst probability value is 0.6586.

4.2.3. Comparison of horizontal and vertical operator-machine assignment with cycle time and lead time optimization

Comparing the results of the horizontal and vertical assignment of operators and machines it is concluded, that the optimal operator-machine assignment resulted a better total cost in the case of vertical operator-machine assignment, which means that the vertical operator-machine assignment is more suitable for this layout than the vertical assignment. Table 8 summarizes the most important parameters of the optimal operator-machine assignment in both cases.

As the comparative analysis of the optimized vertical and horizontal operator-machine assignment shows, the vertical assignment resulted less components in the input storage of the U-cell, and the total production cost is also lower, but the cycle time and the lead time is approximately the same, no significant difference can be identified.

Table 8. Comparison of the vertical and horizontal operator-machine assignment with cycle time and lead time optimization

	Vertical assignment	Horizontal assignment
Assigned operators	4 and 10	1 and 10
Transition probability between input and output storage	0.785	0.8498
Required components [pcs]	1273	1176
Total cost [EUR]	51955	49635
Cycle time [sec]	74.21	74.21
Lead time [hours]	20.701	20.699

5. Conclusions

Within the frame of this research work, the author developed a novel model to analyze the impact of operator-machine assignment on the performance of U-shaped production lines. This model makes it possible to describe the influence of different worker allocation strategies on total cost, cycle time and lead time. More generally, this paper focuses on the mathematical description of transition probabilities among absorbing and transient states of the production line using absorbing Markov chain. The existing studies include the optimization of worker-cell assignment problems, but while only a few of them consider the optimization from total cost, cycle time and lead time.

The added value of the paper is in the description of the impact of the allocation strategy of operators on the key performance indicators of the U-shaped production line. The scientific contribution of this paper for researchers in this field is the mathematical modelling of relationship between operator assignment and key performance indicators. The results can be generalized because the model can be applied for different production systems.

The highlight of the research work can be summarized as follows:

- It is possible to model the relationship between human and technological resources using absorbing Markov chains. This methodology focuses on the description of skill-related parameters of operators (scrap rate) and defines the impact of assignment of operators on the transitions between machines, repair stations and storages.
- In this approach, both the horizontal and the vertical operator-machine assignment was analyzed. Based on this analyses it can be concluded, that there are no general purpose strategies suitable for all U-shaped production lines, because depending on the objective function different optimal operator machine assignments can be computed.
- In the case of the scenario analysis of horizontal assignment, it can be concluded, that the total cost-based optimization leads to 22.2% total cost saving, 29% material cost savings, 16% cycle time reduction and 18.1% lead time reduction.
- In the case of the scenario analysis of vertical assignment, it can be concluded, that the total cost-based optimization leads to 29.5% total cost saving, 37.6% material cost savings, 16.9% cycle time reduction and 16.9% lead time reduction.

Managerial decisions can be influenced by the results of this research, because the described method makes it possible to analyze the structure of the available human resources and based on the results of the operator assignment problems it is possible to improve the human resource management strategy of the company from skill and multi-skill development point of view.

However, there are also limitations of the study and the described model, which provides direction for further research. Within the frame of this model, the uncertainties are limited on the scrap rate of the workers, but in further studies, the model can be extended to a more complex model including the description of uncertainties of technology, logistics, structure and layout.

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Appendix A

Table A1. Scrap rate of the operators at the repair stations.

Operator	Repair 1	Repair 2	Repair 3	Repair 4	Repair 5
	$sr_{h_i,7}^{NR}$	$sr_{h_i,8}^{NR}$	$sr_{h_i,9}^{NR}$	$sr_{h_i,10}^{NR}$	$sr_{h_i,11}^{NR}$
1	8.10%	7.80%	2.00%	6.50%	8.40%
2	6.90%	7.90%	4.60%	1.60%	6.10%
3	7.70%	9.90%	4.20%	2.90%	4.90%
4	5.30%	7.20%	5.30%	2.90%	4.70%
5	3.50%	8.30%	8.80%	6.90%	8.70%
6	4.30%	2.60%	7.80%	7.60%	8.90%
7	2.30%	7.00%	2.00%	2.10%	0.80%
8	2.20%	4.90%	3.30%	3.40%	1.20%
9	9.60%	0.30%	8.50%	5.10%	3.80%
10	2.70%	8.00%	5.10%	0.10%	6.90%
11	3.40%	7.90%	9.40%	6.90%	8.30%
12	9.00%	5.10%	9.40%	9.10%	1.20%
13	0.10%	0.80%	3.20%	2.10%	3.70%
14	4.90%	4.20%	8.60%	9.10%	5.80%
15	3.80%	3.10%	7.40%	3.20%	0.10%
16	4.10%	3.90%	4.80%	1.50%	9.90%
17	0.60%	1.20%	4.10%	5.70%	6.40%
18	5.30%	3.10%	1.00%	6.70%	6.90%
19	8.00%	6.60%	5.60%	5.30%	0.30%
20	1.30%	6.50%	4.10%	2.40%	1.60%
21	5.60%	2.70%	1.80%	5.20%	5.30%
22	4.10%	3.50%	2.90%	8.70%	1.40%
23	2.60%	4.70%	9.20%	3.90%	4.20%
24	2.90%	3.80%	9.90%	1.60%	3.20%
25	0.80%	5.90%	2.70%	3.50%	2.90%
26	8.00%	2.40%	2.80%	7.00%	6.60%
27	7.70%	2.30%	1.70%	8.80%	9.00%
28	9.30%	4.20%	8.40%	7.90%	4.30%
29	0.10%	3.30%	0.80%	7.20%	4.70%
30	7.30%	1.80%	6.10%	10.00%	6.00%
31	1.00%	1.10%	8.40%	0.70%	10.00%

Table A2. Repairable and non-repairable scrap rate of the operators at the machines of the U-shaped production cell.

Operator	Machine 1		Machine 2		Machine 3		Machine 4		Machine 5	
	$sr_{h_i,2}^{R}$	$sr_{h_i,2}^{NR}$	$sr_{h_i,3}^{R}$	$sr_{h_i,3}^{NR}$	$sr_{h_i,4}^{R}$	$sr_{h_i,4}^{NR}$	$sr_{h_i,5}^{R}$	$sr_{h_i,5}^{NR}$	$sr_{h_i,6}^{R}$	$sr_{h_i,6}^{NR}$
1	1.26%	0.56%	2.12%	4.98%	2.43%	6.17%	0.12%	4.18%	1.00%	9.00%
2	9.93%	4.43%	1.66%	4.94%	1.76%	5.14%	0.32%	8.18%	0.15%	4.65%
3	8.58%	4.18%	1.45%	4.05%	1.82%	8.18%	1.39%	6.81%	1.59%	8.11%
4	9.39%	4.19%	1.88%	5.62%	1.23%	3.07%	1.18%	5.72%	0.97%	6.73%
5	12.16%	5.26%	0.21%	5.49%	1.08%	5.92%	0.44%	8.06%	0.18%	0.62%
6	8.90%	4.30%	0.50%	9.00%	0.21%	8.49%	1.27%	6.33%	0.80%	6.71%
7	8.09%	3.69%	0.63%	2.07%	1.52%	5.08%	0.68%	4.02%	0.30%	0.80%
8	1.37%	0.67%	0.40%	5.90%	0.41%	1.49%	0.30%	2.60%	2.39%	7.01%
9	2.48%	1.18%	0.91%	4.39%	0.46%	1.44%	0.29%	5.01%	0.28%	3.92%
10	16.27%	8.07%	0.07%	9.23%	0.03%	7.27%	0.42%	3.18%	0.01%	0.29%
11	8.67%	3.97%	1.55%	7.95%	0.10%	0.40%	0.13%	0.88%	0.05%	1.25%
12	6.65%	2.85%	1.47%	7.43%	0.97%	3.43%	0.16%	3.74%	0.95%	3.45%
13	4.59%	2.29%	1.88%	6.52%	1.39%	7.21%	0.60%	3.00%	0.04%	3.86%
14	12.69%	6.19%	0.46%	1.24%	0.52%	5.88%	0.07%	0.53%	0.47%	5.23%
15	5.52%	2.52%	1.61%	4.79%	0.09%	0.31%	1.30%	8.00%	0.46%	1.44%
16	12.31%	5.11%	0.10%	1.30%	0.25%	1.15%	0.06%	1.44%	0.04%	6.26%
17	18.97%	9.47%	0.14%	1.56%	2.27%	7.73%	0.12%	8.78%	0.98%	3.42%

18	12.77%	5.57%	0.60%	6.80%	0.54%	6.56%	0.05%	0.95%	1.89%	6.91%
19	14.14%	6.74%	0.42%	7.88%	2.46%	7.04%	1.65%	6.75%	0.97%	7.23%
20	6.35%	3.05%	1.09%	3.31%	0.73%	3.47%	0.10%	2.50%	1.44%	6.77%
21	4.71%	2.11%	0.31%	6.69%	1.42%	5.78%	0.84%	4.46%	0.01%	3.69%
22	12.72%	6.02%	1.48%	6.62%	0.14%	4.76%	0.29%	3.31%	0.34%	2.26%
23	0.39%	0.19%	0.16%	0.44%	1.38%	6.62%	2.13%	7.47%	0.37%	6.94%
24	1.15%	0.55%	1.11%	3.49%	0.21%	5.99%	2.37%	5.73%	0.84%	2.46%
25	17.68%	7.68%	1.62%	5.68%	0.43%	4.67%	1.98%	6.62%	0.01%	1.89%
26	13.71%	6.81%	0.27%	0.93%	0.02%	0.18%	0.75%	7.45%	0.48%	2.62%
27	9.52%	4.02%	1.15%	3.75%	1.95%	6.35%	0.55%	4.65%	0.05%	1.45%
28	3.49%	1.49%	0.44%	4.16%	0.89%	4.21%	0.11%	0.59%	0.88%	7.82%
29	16.91%	7.01%	1.67%	7.83%	0.06%	0.34%	0.57%	5.93%	1.95%	5.35%
30	9.16%	3.86%	0.06%	8.84%	0.18%	4.92%	0.05%	5.15%	0.65%	3.15%
31	0.56%	0.26%	0.64%	4.66%	1.63%	7.97%	0.34%	4.16%	0.60%	2.00%

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