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A COMPREHENSIVE NEUTROSOPHIC MODEL FOR EVALUATING THE EFFICIENCY OF AIRLINES BASED ON SBM MODEL OF NETWORK DEA

Reza Rasinojehdehi[1] and Hadi Bagherzadeh Valami*[2]

[1] Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
[2] Department of Applied Mathematics, Yadegar-e-Imam Khomeini (RAH), Shahr-rey Branch, Islamic Azad University, Tehran, Iran

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Abstract: *This research aims to provide an innovative framework for evaluating airline performance through the networking of decision-making units (DMUs) and neutrosophic data. As a consequence, we propose a comprehensive methodology for assessing the efficiency of these decision-making units. Network data envelopment analysis (DEA) models deal with measurements of relative efficiency of DMUs when the insight of their internal structures is available. In network models, sub-processes are connected by links or intermediate products. Links have the dual role of output from one division or sub-process and input to another one. Therefore, improving the efficiency score of one division by increasing its output may reduce the score of another division because of increasing its input. To address this conflict, we proposed a new approach in Slack-Based Measure (SBM) framework and neutrosophic logic, which provide deeper insights regarding the sources of inefficiency. Our approach is a new network model, in which the intermediate products are classified into two groups of "input-type", and "output-type" that their excesses and shortfalls directly concern with the objective function. The results of the illustrated case and analysis show that the constructed model may successfully produce performance assessments. By taking into account the number of slacks or surpluses of these products in the objective function, the proposed model gives analysts and managers a more accurate assessment of network structure efficiency. In this paper, we present a new model in the field of network data envelopment analysis with SBM approach in a neutrosphonic environment, in which for the first time the nature of intermediate products in terms of input or output is investigated. Also, the proposed framework is the first attempt to performance evaluation in neutrosophic network DEA.*

Key words*: Neutrosophic set, Network DEA, Intermediate product, SBM, Efficiency score.*

* Corresponding author.
E-mail addresses: Reza.Rasi1515@gmail.com (R. Rasinojehdehi), hadibagherzade0@gmail.com (H. Bagherzadeh)

1. Introduction

Data Envelopment Analysis (DEA) was developed to measure the relative efficiency of operational units called "Decision Making Units" — DMUs (Charnes et al., 1978) that consume multiple inputs to produce multiple outputs. In its original settings, the operations and interrelations of the processes within the DMU are neglected and only the inputs to the DMU and outputs from it are considered. In the literature of DEA this approach is called black box and the corresponding model is called black box model.

In real world problems organizations have complex internal structures and many of them consist of several divisions or sub-units/processes that are linked together by products or services. Since the Black-Box approach ignores the internal structures and operations of DMUs, A significant number of researchers and scholars have started to develop approaches which consider the internal structures of DMUs.

Regarding efficiency evaluation of multi-division organizations, in many situations, analysts considered the subunits as independent DMUs, and calculated their efficiencies separately. This approach is thus called separation approach and the corresponding model is called separation model. In separation approach the links between two sub-units have the both role of input to one sub-unit and output from another one, hence the complex structure can be divided into sub-units or divisions and for each division some benchmarks can be found. Since links are treated as discretionary inputs or outputs, the separation model takes the inefficiency associated with linking activities into account but does not keep the continuity of flows between subunits.

In the literature of DEA many researchers are interested in investigating the sources of inefficiency within the DMUs with complex structure and evaluating efficiencies of subunits as well as the efficiency of whole DMU in a unified framework. To accomplish this, researchers have developed network DEA methodologies which are more sensitive in detecting inefficiencies than traditional DEA models (Cook et al., 2010; Lewis & Sexton, 2004).

Network DEA models were introduced for the first time to the literature of DEA by Färe and Groskopf (2000). After pioneering work of Färe and Grosskopf a significant number of researchers and scholars have abandoned the black box perspective and started to look into the black box.

A network structure can be a simple two-stage process or a complex system with multiple divisions that are linked together with intermediate measures. Linking activities or intermediate measures are indispensable parts of Network DEA models. They play the dual role of input to one division and output from another division and this causes a major problem in measuring overall and divisional efficiencies of a network. Improving the efficiency of one division by expanding its outputs may reduce the efficiency of another division due to the expansion of inputs. Similarly, raising efficiency score of one division by reducing the amount of its input may diminish the score of the other division due to the output reduction (Y. Chen et al., 2014; Cook et al., 2010). Since standard DEA models do not resolve this conflict, they are not good choices for assessing the efficiency of DMUs with network structure and therefore, many scholars and researchers have proposed their own solutions (Färe et al., 2007; Kao & Hwang, 2008; H. F. Lewis et al., 2013; L. F. Lewis & Sexton, 2004; Zha et al., 2008)

For instance, Zha et al. (2008) computed the overall efficiency score by multiplication of divisions' scores. They utilized the input-oriented VRS model to evaluate the efficiency of the first division and the output-oriented VRS model to measure the efficiency of the second division.

Kao and Hwang (2008) also defined the overall efficiency score of the two-stage structure as the geometric mean of stage efficiencies. Liang et al. (2008) also use such multiplicative efficiency decomposition in their study. They use the concepts of the Stackelberg game (or leader-follower) and the centralized or cooperative game to evaluate the overall efficiency score.

Sexton and Lewis (2003) proposed a network DEA model with network structure to evaluate the efficiency of Major League Baseball (MLB) teams. Their methodology provides efficiency scores for each stage and overall efficiency. In 2004, they extended their model to a multi divisional structure and in 2013 they propose an un-oriented two-stage DEA methodology to evaluate efficiency of MLB teams during the 2009 season (Lewis et al., 2013; Lewis & Sexton, 2004).

In all studies mentioned above the researchers utilized CCR (Charnes et al., 1978) or BCC (Banker et al., 1984) models which are the basic DEA models. In other words, they apply the radial measure of efficiency that rely on the assumption that DMU’s efficiency score depends on its proportional distance to the efficiency frontier. However, in real words problems some inputs and outputs are substitutional and do not change proportionally. Non-radial models have the advantage of measuring efficiencies in the case that inputs and outputs change non-proportionally. One of the most popular non-radial models in the literature of DEA is slacks-based measure (SBM) model (Pastor et al., 1999; Tone, 2001). Non-radial SBM models deal with slacks directly and do not consider the assumption of changing inputs and outputs proportionally.

Tone and Tsutsui (2009) develop a slacks-based network DEA model applying the production possibility Sets. In their study intermediate measures are called links. They considered the component efficiency as a function of slack variables and the overall efficiency as a weighted average of the component efficiencies. In their study the component weights are determined exogenously to represent the importance of the components. They proposed two possible cases for linking activities, called fixed link and free link. In both cases the continuity of link flows between components are kept. Fukuyama and Weber (2010) proposed a measure for efficiency called network directional slacks-based measure. They normalized values of the slack variables by user defined coefficients.

Paradi et al. (2011) developed a modified slacks-based measure to evaluate efficiency of DMUs with two-stage structure. They generated a composite performance index for each unit by aggregating the stages' efficiency scores.

Lozano (2015) proposed an SBM model for measuring efficiency of networks. In his proposed approach the optimal inputs, outputs and intermediate measures of each process may be greater or smaller than their observed values. By relaxing the constraints corresponding to intermediate measures the discriminating power of the model was enhanced.

Shamsijamkhaneh et al. (2018) proposed an approach in which the intermediate measures are classified into input or output type and the continuity of linking activities are kept simultaneously. Based on their approach they proposed two models to study on direct and indirect effect of inefficiency arising from intermediate measure in efficiency measurement.

Khoshandam and Nematizadeh (2023) proposed an inverse model for the two-stage DEA in the presence of undesirable outputs.

Zhang et al. (2022) utilized intermediate approach instead of radial or non-radial approaches to construct their novel network DEA model. In their study they proposed a new framework including their model which considers the sustainability system of Chinese provinces from 2009 to 2017.

All of the models stated above are based on accurate data. However, real-world data are not always clear, and the information gathered is uncertain and indeterminate. To capture imprecise data, intuitionistic fuzzy sets (IFSs) (Atanassov, 2016), an extension of Zadeh's fuzzy sets (Zadeh, 1965), have been frequently employed to solve impreciseness and uncertainties in situations, particularly in DEA literature (Arya & Yadav, 2020; Daneshvar Rouyendegh, 2011; Davoudabadi et al., 2021; Edalatpanah, 2019; I. Eyo et al., 2021; I. J. Eyo et al., 2022).

Wen et al. (2020) in a study report on some novel studies on DEA with imprecise data applying the Hurwicz criterion, to investigate the intermediate area between extremes. They proposed, the Hurwicz ranking method to evaluate the DMUs under consideration.

Ucal Sari and Ak (2022) to illustrate how fuzzy factors can affect decision-making process in their study evaluate the efficiency of Industry 4.0 applications in the home appliance manufacturing sector using classical DEA and fuzzy DEA models.

Bagherzadeh Valami and Raeinojehdehi (2016) introduced a novel fuzzy DEA approach based on α-cuts and a new ranking method for DMUs with fuzzy data.

Tavassoli and Saen (2022) proposed a new non-radial network DEA model based on SBM to measure the efficiencies of railways with sustainability considerations. They developed the fuzzy version their model to deal with both qualitative and quantitative criteria.

Monzeli et al. (2020) proposed a model to estimate the unfavorable performance caused by undesirable inputs and outputs in DMUs. Their proposed model in addition to measuring the efficiency of DMUs with the presence of undesirable inputs and outputs, investigates the effect of undesirable components on the efficiency limit.

Although fuzzy set and IFS theories have been established and expanded, they have failed to properly cope with situations involving indeterminate and inconsistent information. To manage such issues, Smarandache (1999) pioneered the concept of neutrosophic sets that consider the truth membership, indeterminacy and falsity membership simultaneously. Neutrosophic sets are recognized as a versatile method for dealing with ambiguous, partial, and inconsistent data. However, without a detailed definition, neutrosophic sets are difficult to apply in real-world scientific and technical applications. Wang et al. (2010) defined single-valued neutrosophic sets (SVNSs), which are a special case of neutrosophic sets. Many researches have been conducted in recent years that have focused on scientific problems based on SVNSs and their expansions. (Akram et al., 2019; Z. S. Chen et al., 2021; Dhar, 2021; Edalatpanah, 2020a; H. Garg, 2022; K. Garg et al., 2015; Kumar Das, 2020; Veeramani et al., 2021; Zhan et al., 2019)

Edalatpanah (2018) was the first to expand the DEA model in the setting of a single value neutrosophic number. In order to improve the efficiency of private institutions. Kahraman et al. (2019) devised the Neutrosophic Analytic Hierarchy Process (NAHP) and introduced a hybrid NAHP-NDEA algorithm for performance evaluation. Following that, Abdelfattah (2019), Mao et al. (2020) and Yang et al. (2020) investigated and developed the neutrosophic DEA model in a variety of real-world applications; see also (Abdelfattah, 2021; Edalatpanah, 2020b; Edalatpanah & Smarandache, 2019; Mao et al., 2020). We noticed a few research gaps in this fascinating area. Neutrosophic sets, which cover more uncertainty than other fuzzy numbers and are the most acceptable form of a fuzzy number, have never been employed in network DEA with neutrosophic inputs and outputs.

In this paper we propose a new network model in SBM framework with neutrosophic environment to measure the overall and divisional efficiencies of the DMUs under consideration. The proposed model account the excesses or shortfalls of intermediate measures into the objective function. The major contribution of this paper is to resolve the conflict caused by the dual role of linking activities and incorporate their excesses and shortfalls in efficiency measurement. The rest of this paper is structured as follows; Section 2 presents some preliminaries and notation. In Section 3, we present our new network model in the slacks-based framework. To verify our proposed model, we provide a case study in Section 4. Finally, we conclude the paper in section 5 and propose some suggestion for future studies.

2. Preliminaries

In this section, we will review some fundamental backgrounds required in this paper.

2.1. Neutrosophic Logic

Definition 1 (Smarandache, 1999). Let X be a space of points with a generic element in X, denoted by x. Then, a neutrosophic set A in X is characterized by

$$A = \{< x, T_A(x), I_A(x), F_A(x) | x \in X >\}$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in]0^-, 1^+[$ represent the truth, indeterminacy and falsity-membership functions, respectively, such that

$$0^- \leq sup\, T_A\,(x) + sup\, I_A\,(x) + sup\, F_A\,(x) \leq 3^+.$$

Since neutrosophic sets cannot easily be applied to practical problems, Wang et al. (2010) reduced the neutrosophic sets of nonstandard interval numbers into SVNSs of standard interval numbers.

Definition 2 (Wang et al., 2010). Let X be a space of points with a generic element in X, denoted by x. Then, An SVNS in A in X is characterized by

$$A = \{< x, T_A(x), I_A(x), F_A(x) | x \in X >\},$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$, such that

$$0 \leq sup\, T_A\,(x) + sup\, I_A\,(x) + sup\, F_A\,(x) \leq 3.$$

For an SVNS $\{< x, T_A(x), I_A(x), F_A(x) | x \in X >\}$, the ordered triple components $< T_A(x), I_A(x), F_A(x) >$ are described as an SVNN, and each SVNN can be represented as $a =< T_a, I_a, F_a >$, where $T_a, I_a, F_a \in [0,1]$ and $0 \leq T_a + I_a + F_a \leq 3$.

Definition 3 (Edalatpanah & Smarandache, 2019). The triangular neutrosophic number (TNNs) $\widetilde{\Upsilon} = \prec (r_1, r_2, r_3), (\omega, \theta, \chi) \succ$ have the following truth, indeterminacy, and falsehood membership functions of x as presented in Eqs. (1)-(3):

$$T_{\widetilde{\Upsilon}}(x) = \begin{cases} \dfrac{(x - r_1)}{(r_2 - r_1)}\omega & r_1 \leq x < r_2, \\ \omega & x = r_2, \\ \dfrac{(r_3 - x)}{(r_3 - r_2)}\omega & r_2 \leq x < r_3, \\ 0 & otherwise. \end{cases} \tag{1}$$

$$I_{\widetilde{\Upsilon}}(x)=\begin{cases}\dfrac{(r_2-x)}{(r_2-r_1)}, & r_1\le x<r_2,\\ \theta, & x=r_2,\\ \dfrac{(x-r_3)}{(r_3-r_2)}\theta, & r_2\le x<r_3,\\ 1, & otherwise.\end{cases} \tag{2}$$

$$F_{\widetilde{\Upsilon}}(x)=\begin{cases}\dfrac{(r_2-x)}{(r_2-r_1)}, & r_1\le x<r_2,\\ \chi, & x=r_2,\\ \dfrac{(x-r_3)}{(r_3-r_2)}\chi, & r_2\le x<r_3,\\ 1, & otherwise.\end{cases} \tag{3}$$

where, $0\le T_{\widetilde{\Upsilon}}(x)+I_{\widetilde{\Upsilon}}(x)+F_{\widetilde{\Upsilon}}(x)(x)\le 3, x\in\widetilde{\Upsilon}$.

Definition 4. (Akram et al., 2019). Let $\widetilde{\Upsilon}=\prec(r_1,r_2,r_3),(\omega,\theta,\chi)\succ$ is a TNN. Then the aggregate coefficient $\varphi_{(\alpha,\beta,\gamma)}$ presented in (4).

$$\varphi_{(\alpha,\beta,\gamma)}=[\widetilde{\Upsilon}^L_{(\alpha,\beta,\gamma)},\widetilde{\Upsilon}^U_{(\alpha,\beta,\gamma)}=[r_1+(r_2-r_3)\hbar, r_3-(r_3-r_2)\hbar] \tag{4}$$

where, $\alpha\in[0,\eta],\beta\in[\lambda,1],\gamma\in[\kappa,1]$, and $\hbar=\frac{1}{4}(\frac{\alpha}{\omega}+2\frac{(1-\beta)}{1-\theta}+\frac{(1-\gamma)}{1-\chi})$ is a variation degree of the TNNs.

2.1. Network DEA Based on SBM Approach

Suppose there is a set of n DMUs indexed by $(j=1,\ldots,n)$ and each DMU consists of K divisions $(k=1,..,K)$. Assume division k $(Divk)$ consumes m_k inputs and produces r_k outputs. Let $\{X_j^k\in R_+^{m_k}\}$ and $\{Y_j^k\in R_+^{m_k}\}$, $(j=1,\ldots,n;k=1,..,K)$ be respectively, the input vector to and output vector from $Divk$. Intermediate products from $Divk$ to $Divh$ are also denoted by $\{z_j^{(k,h)}\in R_+^{l_{(k,h)}}\}$ $(j=1,\ldots,n;(k,h)\in L)$ where $l_{(k,h)}$is the number of intermediate measures from $Divk$ to $Divh$ and L denotes the set of links.

Tone and Tsutsui (2009) propose a Network DEA (NDEA) model based on the weighted slacks-based measure (WSBM) approach to measure the overall and divisional efficiencies of the network. Their model presented in Eqs. (5)-(11);

$$\rho=min\sum_{k=1}^K w_k\left[\frac{1-\frac{1}{m_k}(\sum_{i=1}^{m_k}\frac{s_{ip}^{k-}}{x_{ip}})}{1+\frac{1}{r_k}(\sum_{r=1}^{r_k}\frac{s_{rp}^{k+}}{y_{rp}})}\right] \tag{5}$$

$$s.t.\ \sum_{j=1}^n\lambda_j^k x_{ij}^k+s_{ip}^{k-}=x_{ip}^k\quad \forall k,\forall i \tag{6}$$

$$\sum_{j=1}^n\lambda_j^k y_{rj}^k-s_{rp}^{k+}=y_{rp}^k\quad \forall k,\forall r \tag{7}$$

$$\sum_{j=1}^n\lambda_j^k z_{dj}^{(k,h)}=z_{dp}^{(k,h)}\qquad \forall(k,h),\forall d \tag{8}$$

$$\sum_{j=1}^n\lambda_j^h z_{dj}^{(k,h)}=z_{dp}^{(k,h)}\qquad \forall(k,h),\forall d \tag{9}$$

$$\sum_{j=1}^{n} \lambda_j^k = 1 \quad \forall k \tag{10}$$

$$\lambda_j^k \geq 0\ (\forall j,k),\ s_{rp}^{k+} \geq 0(\forall r,k), s_{ip}^{k-}0(\forall r,k),\ s_{dp}^{(k,h)-} \geq 0\ , s_{dp}^{(k,h)+} \geq 0\ , \forall d, \forall (k,h) \tag{11}$$

where λ_j^k,($\lambda_j^k \geq 0$) is the intensity weight corresponding to *Divk* of *DMUj* ($j = 1,..,n$; $k = 1,..,K$), and w_k denotes the relative weight of *Divk* which is determined exogenously by decision maker to represent its importance and $\sum_{k=1}^{K} w_k = 1$,$w_k \geq 0(\forall k)$.

It should be noted that the model presented in Eqs. (5)-(11) computes the non-oriented overall efficiency of *DMUp* under the assumption of variable returns-to-scale (VRS) for production eliminating constraint (10) modifies the assumption of VRS to constant returns-to-scale (CRS) for production. Tone and Tsutsui (2009) proposed the input and output-oriented case of their model by minimizing the numerator and maximizing the denominator of the objective function (5), respectively.

In the presented model, linking constraints (8), (9) are kept unchanged and fixed, and the intermediate products are beyond the control of DMUs. Tone and Tsutsui (2009) called this case as "fixed" link value case.

substituting constraints (8), (9) by constraints (12), they introduced another possible case for linking activities called "free" link value case in which the linking activities can be freely determined.

$$\sum_{j=1}^{n} \lambda_j^k z_{dj}^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h z_{dj}^{(k,h)} \qquad \forall (k,h)\ \forall d \tag{12}$$

Note that continuous flow of linking activities between divisions are kept in both cases.

In the case that the linking activities are classified into input type and output type exogenously by decision maker, Tone and Tsutsui (2009) incorporate the input excesses and output shortfalls by setting the linking constraints (13), (14) and modifying the objective functions (5) to (15).

$$\begin{cases} \sum_{j=1}^{n} \lambda_j^k z_j^{(k,h)} - s_{dp}^{(k,h)+} = z_{dp}^{(k,h)} \\ \sum_{j=1}^{n} \lambda_j^k z_j^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h z_j^{(k,h)} \end{cases} \tag{13}$$

$$\begin{cases} \sum_{j=1}^{n} \lambda_j^k z_j^{(k,h)} + s_{dp}^{(k,h)-} = z_{dp}^{(k,h)} \\ \sum_{j=1}^{n} \lambda_j^k z_j^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h z_j^{(k,h)} \end{cases} \tag{14}$$

$$\eta_p^* = min \sum_{k=1}^{K} w_k \left[\frac{1 - \frac{1}{m_k + l_k} (\sum_{i=1}^{m_k} \frac{s_{pi}^{k-}}{x_{ip}} + \sum_{h \in F_k}^{h_k} \frac{s_{hp}^{(h,k)-}}{z_{hp}^{(k,f)}})}{1 + \frac{1}{r_k + t_k} (\sum_{r=1}^{r_k} \frac{s_{rp}^{k+}}{y_{rp}} + \sum_{t=1}^{t_k} \frac{s_{tp}^{(k,f)+}}{z_{tp}^{(k,f)}})} \right] \tag{15}$$

where $\sum_{k=1}^{K} w_k = 1$, $w_k \geq 0$ $(\forall k)$ and t_k (l_k) denotes the number of those intermediate products that are considered as output from (input to) *Divk*.

In real world problems there are many situations in which the intermediate measures cannot be categorized into input or output type by the decision maker. In the next section we propose a non-radial network DEA model in the framework of SBM that classifies the intermediate products into two different groups of *input-type* and *output-type*. The proposed model takes the shortfalls and excesses of intermediate products in to the account to identify the potential improvements regarding linking activities.

3. Neutrosophic-Network-DEA-SBM Model

In this section, we seek to provide neutrosophic model that has the ability to determine the role of intermediate products. This model should also be able to incorporate the inefficiencies of intermediate products into performance calculations while data are available in neutrosophic numbers. Our proposed model divides the intermediate products into two categories of *input- type* and *output-type* without violating the assumption of continuous product flow from one subunit to another. Inefficiencies due to shortages or surpluses of these intermediate products are also directly entered into the objective function.

In order to incorporate the inefficiency associated with intermediate measures in efficiency measurement, with the parameters described in Section 2 subject that $\tilde{x}_{ij}$, $\tilde{y}_{rj}^{k}$, and $\tilde{z}_{dj}^{(k,h)}$ are neutrosophic numbers, we establish the neutrosophic mixed-integer programming model presented in Eqs. (16)–(27):

$$min\, \tilde{\rho}_p = \sum_{k=1}^{K} w_k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} \left(\sum_{i=1}^{m_k} \frac{s_{ip}^{k-}}{\tilde{x}_{ip}} + \sum_{d=1}^{l(f,k)} \frac{s_{dp}^{(f,k)-}}{\tilde{z}_{dp}^{(f,k)}} \right) \right] \tag{16}$$

$$s.t. \sum_{j=1}^{n} \lambda_j^k x_{ij}^k + s_{ip}^{k-} = x_{ip}^k \;\forall k\,, \forall i \tag{17}$$

$$\sum_{j=1}^{n} \lambda_j^k y_{rj}^k - s_{rp}^{k+} = y_{rp}^k \;\; \forall k\,, \forall r \tag{18}$$

$$\sum_{j=1}^{n} \lambda_j^h \tilde{z}_{dj}^{(k,h)} + s_{dp}^{(k,h)-} = \tilde{z}_d^{(k,h)} \tag{19}$$

$$s_{dp}^{(k,h)-} \leq M g_d^{(k,h)} \tag{20}$$

$$\tilde{z}_{dp}^{(k,h)} - M(1 - g_d^{(k,h)}) \leq \tilde{z}_d^{(k,h)} \leq \tilde{z}_{dp}^{(k,h)} + M(1 - \quad g_d^{(k,h)}) \tag{21}$$

$$\sum_{j=1}^{n} \lambda_j^k \tilde{z}_{dj}^{(k,h)} - s_{dp}^{(k,h)+} = \tilde{z}_d^{(k,h)} \tag{22}$$

$$s_{dp}^{(k,h)+} \leq M(1 - g_d^{(k,h)}) \tag{23}$$

$$\tilde{z}_{dp}^{(k,h)} - Mg_{dp}^{(k,h)} \leq \tilde{z}_d^{(k,h)} \leq \tilde{z}_{dp}^{(k,h)} + Mg_{dp}^{(k,h)} \tag{24}$$

$$\sum_{j=1}^{n} \lambda_j^k \tilde{z}_{dj}^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h \tilde{z}_{dj}^{(k,h)} \tag{25}$$

$$\sum_{j=1}^{n} \lambda_j^k = 1 \tag{26}$$

$$g_d^{(k,h)} = \{0,1\}; \tilde{z}_d^{(k,h)}, \tilde{z}_d^{(k,h)}: free,\ \lambda_j^k \geq 0, s_{rp}^{k+} \geq 0,\ s_{ip}^{k-} \geq 0, s_{dp}^{(k,h)-} \geq 0, s_{dp}^{(k,h)+} \geq 0 \tag{27}$$

where M is a large positive number and $w_k \geq 0$ ($\sum_{k=1}^{K} w_k = 1$) is the relative weight of *Divk* which is determined exogenously by the decision maker corresponding to its importance. The presented model is constructed under the assumption of variable returns to scale (VRS) and eliminating constraint (26) converts it to the constant returns-to-scale (CRS). The objective function (16) evaluates the efficiency score in the input-oriented case. Constraints (17) and (18) are the constraints related to the i-th input and the r-th output of the k-th subunit (*Divk*) of the p-th decision-making unit (*DMUp*), respectively. The set of Constraints (19)-(25) are related to intermediate products. Optimal value for binary variable $g_{dp}^{(k,h)}$in these constraints, determines the role of the intermediate product $\tilde{z}_{dj}^{(k,h)}$.

Intermediate measure $\tilde{z}_{dj}^{(k,h)}$belongs to set *input-type* if the model considers $\tilde{z}_{dj}^{(k,h)}$as an input to *Divh*. This type of intermediate measures decreases in the optimal solution of the proposed model and their nonzero slacks enter directly into the efficiency scores to which the objective is oriented. Intermediate measure $\tilde{z}_{dj}^{(k,h)}$belongs to set *output-type* if it is treated as an output from *Divk*. This type of intermediate measures increases in the optimal solution of the proposed model and their nonzero slacks enter directly into the efficiency scores to which the objective is oriented.

Suppose $L^{(k,h)}$ denotes the set of all intermediate products between *Divk* and *Divh* and $L_{out}^{(k,h)}$ and $L_{in}^{(k,h)}$ denote the sets of *input-type* and *output-type* intermediate measures, respectively. Let $g_{dp}^{*(k,h)}$ denote the optimal value of decision variable $g_{dp}^{(k,h)}$,then it can be easily concluded that:

- If $g_{dp}^{*(k,h)} = 0$ then $z_d^{(k,h)} \in L_{out}^{(k,h)}$, $\forall d$, $\forall(k,h)$
- If $g_{dp}^{*(k,h)} = 1$ then $z_d^{(k,h)} \in L_{in}^{(k,h)}$, $(\forall d$), $\forall(k,h)$
- $L_{in}^{(k,h)} \cap L_{out}^{(k,h)} = \{\}$, $\forall(k,h)$
- $L_{in}^{(k,h)} \cup L_{out}^{(k,h)} = L^{(k,h)}$. $\forall(k,h)$

For further explanation, if $g_d^{*(k,h)} = 0$, then constraints (19) to (25) that are related to the intermediate products are converted to the constraints (28) and (29):

$$\sum_{j=1}^{n} \lambda_j^k \tilde{z}_{dj}^{(k,h)} - s_{dp}^{(k,h)+} = \tilde{z}_d^{(k,h)} = \tilde{z}_{dp}^{(k,h)}, \quad \forall d\ , \forall(k,h) \tag{28}$$

$$\sum_{j=1}^{n} \lambda_j^k \tilde{z}_{dj}^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h \tilde{z}_{dj}^{(k,h)} \quad \forall d \;, \forall (k,h) \tag{29}$$

Also, if $g_{dp}^{*(k,h)} = 1$, then constraints (15) to (21) are converted to the constraints (30) and (31):

$$\sum_{j=1}^{n} \lambda_j^k \tilde{z}_{dj}^{(k,h)} + s_{dp}^{(k,h)-} = \tilde{z}_d^{(k,h)} = \tilde{z}_{dp}^{(k,h)}, \qquad \forall d \;, \forall (k,h) \tag{30}$$

$$\sum_{j=1}^{n} \lambda_j^k \tilde{z}_{dj}^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h \tilde{z}_{dj}^{(k,h)} \quad \forall d \;, \forall (k,h) \tag{31}$$

In proposed model, due to the fact that all inputs and outputs and intermediate products are of the type of neutrosophic data, so the efficiency obtained from the model is also a neutrosophic number. Next, we intend to convert the model into an interval model. The neutrosophic values of the model are represented by the convex neutrosophic numbers $\tilde{X}_{ij}^k$, $\tilde{Z}_{dj}^{(k,h)}$, and $\tilde{Y}_{rj}^k$, with the truth, indeterminacy, and falsity membership functions presented in Eqs. (32)-(34):

$$\tilde{X}_{ij}^k = \left\{\tilde{x}_{ij}^k, T_{\tilde{x}_{ij}^k}, I_{\tilde{x}_{ij}^k}, F_{\tilde{x}_{ij}^k} | \tilde{x}_{ij}^k \in S(\tilde{x}_{ij}^k)\right\} \tag{32}$$

$$\tilde{Z}_{dj}^{(k,h)} = \left\{\tilde{z}_{dj}^{(k,h)}, T_{\tilde{z}_{dj}^{(k,h)}}, I_{\tilde{z}_{dj}^{(k,h)}}, F_{\tilde{z}_{dj}^{(k,h)}} | \tilde{z}_{dj}^{(k,h)} \in S(\tilde{z}_{dj}^{(k,h)})\right\} \tag{33}$$

$$\tilde{Y}_{rj}^k = \left\{\tilde{y}_{rj}^k, T_{\tilde{y}_{rj}^k}, I_{\tilde{y}_{rj}^k}, F_{\tilde{y}_{rj}^k} | \tilde{y}_{rj}^k \in S(\tilde{y}_{rj}^k)\right\} \tag{34}$$

where, $S(.)$is the support of the set. So, it is sufficient to determine the left shape and the right shape function of the aggregate coefficient of Definition 4. So, $\forall i, j, r, d, (k,h)$, the upper and lower limits for each neutrosophic variable in an φ variation degree are presented in Eq. (35):

$$\begin{cases} (X_{ij}^k)_\varphi^L \le x_{ij}^k \le (X_{ij}^k)_\varphi^U \\ (Y_{rj}^k)_\varphi^L \le y_{rj}^k \le (Y_{rj}^k)_\varphi^U \\ (Z_{dj}^{(k,h)})_\varphi^L \le z_{dj}^{(k,h)} \le (Z_{dj}^{(k,h)})_\varphi^U \end{cases} \tag{35}$$

Using φ variation degree of neutrosophic numbers, the neutrosophic NDEA-SBM easily can be converted into an interval problem. To do this, we created two-step mathematical programming that models the upper and lower efficiencies of the *DMUp* as presented in Eqs. (36) and (37), respectively.

$$(\rho_p)^U_\varphi = \begin{cases} max \\ (X^k_{ij})^L_\varphi \le x^k_{ij} \le (X^k_{ij})^U_\varphi \\ (Y^k_{rj})^L_\varphi \le y^k_{rj} \le (Y^k_{rj})^U_\varphi \\ (Z^{(k,h)}_{dj})^L_\varphi \le z^{(k,h)}_{dj} \le (Z^{(k,h)}_{dj})^U_\varphi \\ \forall i,j,r,d,(k,h) \\ min \sum_{k=1}^{K} w_k[\\ 1 - \frac{1}{m_k + \sum_{d=1}^{l_{(f,k)}} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{s^{k-}_{ip}}{x_{ip}} + \sum_{d=1}^{l_{(f,k)}} \frac{s^{(f,k)-}_{dp}}{z^{(f,k)}_{dp}})] \end{cases} \tag{36}$$

$$s.t.$$

$$\left\{ \begin{array}{l} \sum_{j=1}^{n} \lambda^k_j x^k_{ij} + s^{k-}_{ip} = x^k_{ip} \\ \sum_{j=1}^{n} \lambda^k_j y^k_{rj} - s^{k+}_{rp} = y^k_{rp} \\ \sum_{j=1}^{n} \lambda^h_j z^{(k,h)}_{dj} + s^{(k,h)-}_{dp} = z^{(k,h)}_d \\ s^{(k,h)-}_{dp} \le M g^{(k,h)}_d \\ z^{(k,h)}_{dp} - M(1 - g^{(k,h)}_d) \le z^{(k,h)}_d \le z^{(k,h)}_{dp} \\ \qquad +M(1 - g^{(k,h)}_d) \\ \sum_{j=1}^{n} \lambda^k_j z^{(k,h)}_{dj} - s^{(k,h)+}_{dp} = z'^{(k,h)}_d \\ s^{(k,h)+}_{dp} \le M(1 - g^{(k,h)}_d) \\ z^{(k,h)}_{dp} - M g^{(k,h)}_{dp} \le z'^{(k,h)}_d \le z^{(k,h)}_{dp} + M g^{(k,h)}_{dp} \\ \sum_{j=1}^{n} \lambda^k_j z^{(k,h)}_{dj} = \sum_{j=1}^{n} \lambda^h_j z^{(k,h)}_{dj} \\ \sum_{j=1}^{n} \lambda^k_j = 1 \\ g^{(k,h)}_d = \{0,1\}; z^{(k,h)}_d, z'^{(k,h)}_d : free, \lambda^k_j \ge 0, \\ \quad s^{k+}_{rp} \ge 0, s^{k-}_{ip} \ge 0, s^{(k,h)-}_{dp} \ge 0, s^{(k,h)+}_{dp} \ge 0 \end{array} \right\}$$

$$(\rho_p)^L_\varphi = \tag{37}$$

$$\begin{cases} min \\ (X_{ij}^{k})_{\varphi}^{L} \le x_{ij}^{k} \le (X_{ij}^{k})_{\varphi}^{U} \\ (Y_{rj}^{k})_{\varphi}^{L} \le y_{rj}^{k} \le (Y_{rj}^{k})_{\varphi}^{U} \\ (Z_{dj}^{(k,h)})_{\varphi}^{L} \le z_{dj}^{(k,h)} \le (Z_{dj}^{(k,h)})_{\varphi}^{U} \\ \forall i,j,r,d,(k,h) \\ min \sum_{k=1}^{K} w_k \,[\\ 1-\frac{1}{m_k+\sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{s_{ip}^{k-}}{x_{ip}} + \sum_{d=1}^{l(f,k)} \frac{s_{dp}^{(f,k)-}}{z_{dp}^{(f,k)}})] \end{cases}$$

$$s.t.$$

$$\left\{\begin{array}{l} \sum_{j=1}^{n} \lambda_j^k x_{ij}^k + s_{ip}^{k-} = x_{ip}^k \\ \sum_{j=1}^{n} \lambda_j^k y_{rj}^k - s_{rp}^{k+} = y_{rp}^k \\ \sum_{j=1}^{n} \lambda_j^h z_{dj}^{(k,h)} + s_{dp}^{(k,h)-} = z_d^{(k,h)} \\ s_{dp}^{(k,h)-} \le M g_d^{(k,h)} \\ z_{dp}^{(k,h)} - M(1-g_d^{(k,h)}) \le z_d^{(k,h)} \le z_{dp}^{(k,h)} \\ \qquad +M(1-g_d^{(k,h)}) \\ \sum_{j=1}^{n} \lambda_j^k z_{dj}^{(k,h)} - s_{dp}^{(k,h)+} = z'^{(k,h)}_d \\ s_{dp}^{(k,h)+} \le M(1-g_d^{(k,h)}) \\ z_{dp}^{(k,h)} - M g_{dp}^{(k,h)} \le z'^{(k,h)}_d \le z_{dp}^{(k,h)} + M g_{dp}^{(k,h)} \\ \sum_{j=1}^{n} \lambda_j^k z_{dj}^{(k,h)} = \sum_{j=1}^{n} \lambda_j^h z_{dj}^{(k,h)} \\ \sum_{j=1}^{n} \lambda_j^k = 1 \\ g_d^{(k,h)} = \{0,1\}; z_d^{(k,h)}, z'^{(k,h)}_d : free, \lambda_j^k \ge 0, \\ \quad s_{rp}^{k+} \ge 0, s_{ip}^{k-} \ge 0, s_{dp}^{(k,h)-} \ge 0, s_{dp}^{(k,h)+} \ge 0 \end{array}\right\}$$

By comparing models (36) and (37), it can be easily concluded that $(\rho_p^*)_{\varphi}^{L} \le (\rho_p^*)_{\varphi}^{U}$ and therefore based on this result, the interval $\left[(\rho_p^*)_{\varphi}^{L}, (\rho_p^*)_{\varphi}^{U}\right]$ was considered as the φ variation degree for *DMUp* performance. To solve the two-step models (36) and

(37), we will turn it into a one-step model. According to the Pareto efficiency concept, for *DMUp* with a specific aggregate coefficient $\varphi_{(\alpha,\beta,\gamma)} = \varphi$, maximum efficiency will occur when the inputs are at their lowest and the outputs are at their highest. For other units, the inputs should be at their maximum and the outputs should be at their minimum.

Similarly, for *DMUp* the minimum performance will occur when the inputs have the highest value and the outputs have the lowest value. Also, for other units, the inputs should have the lowest value and the outputs should have the highest value. Relationships (38) to (49) calculate the upper limit of efficiency.

$$Min(\rho_p)_\varphi^U = \sum_{k=1}^{K} w_k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} \left(\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_\varphi^L}{(x_{ip})_\varphi^L} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_\varphi^L}{(z_{dp}^{(f,k)})_\varphi^L}\right)\right] \tag{38}$$

s.t.

$$(x_{ip}^k)_\varphi^L = \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j^k (x_{ij}^k)_\varphi^U + \lambda_p^k (x_{ip}^k)_\varphi^L + (s_{ip}^{k-})_\varphi^L \tag{39}$$

$$(y_{rp}^k)_\varphi^U = \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j^k (y_{rj}^k)_\varphi^L + \lambda_p^k (y_{rp}^k)_\varphi^U - (s_{rp}^{k+})_\varphi^U \tag{40}$$

$$z_d^{(k,h)} = \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j^h (z_{dj}^{(k,h)})_\varphi^U + \lambda_p^h (z_{dp}^{(k,h)})_\varphi^L + (s_{dp}^{(k,h)-})_\varphi^L \tag{41}$$

$$(s_{dp}^{(k,h)-})_\varphi^L \le M g_d^{(k,h)} \tag{42}$$

$$(z_{dp}^{(k,h)})_\varphi^L - M(1 - g_d^{(k,h)}) \le z_d^{(k,h)} \le (z_{dp}^{(k,h)})_\varphi^L + M(1 - g_d^{(k,h)}) \tag{43}$$

$$z'^{(k,h)}_d = \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j^k (z_{dj}^{(k,h)})_\varphi^L + \lambda_p^k (z_{dp}^{(k,h)})_\varphi^U - (s_{dp}^{(k,h)+})_\varphi^U \tag{44}$$

$$(s_{dp}^{(k,h)+})_\varphi^U \le M\left(1 - g_d^{(k,h)}\right) \quad \forall (k,h) \tag{45}$$

$$(z_{dp}^{(k,h)})_\varphi^U - M g_{dp}^{(k,h)} \le z'^{(k,h)}_d \le (z_{dp}^{(k,h)})_\varphi^U + M g_{dp}^{(k,h)} \tag{46}$$

$$\sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j^k (z_{dj}^{(k,h)})_\varphi^L + \lambda_p^k (z_{dp}^{(k,h)})_\varphi^U = \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j^h (z_{dj}^{(k,h)})_\varphi^L + \lambda_p^h (z_{dp}^{(k,h)})_\varphi^U \tag{47}$$

$$\sum_{j=1}^{n} \lambda_j^k = 1 \tag{48}$$

$$g_d^{(k,h)} = \{0,1\}; z_d^{(k,h)}, {z'}_d^{(k,h)}: free, \lambda_j^k \geq 0, (s_{rp}^{k+})_\varphi^L \geq 0, (s_{ip}^{k-})_\varphi^U \geq 0, (s_{dp}^{(k,h)-})_\varphi^U \geq 0, (s_{dp}^{(k,h)+})_\varphi^L \geq 0 \tag{49}$$

Relationships (50) to (61) calculate the lower limit of efficiency.

$$\left(\rho_p\right)_\varphi^L = Min \sum_{k=1}^{K} w_k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} \left(\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_\varphi^U}{(x_{ip})_\varphi^U} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_\varphi^U}{(z_{dp}^{(f,k)})_\varphi^U}\right)\right] \tag{50}$$

$$(x_{ip}^k)_\varphi^U = \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j^k (x_{ij}^k)_\varphi^L + \lambda_p^k (x_{ip}^k)_\varphi^U + (s_{ip}^{k-})_\varphi^U \tag{51}$$

$$(y_{rp}^k)_\varphi^L = \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j^k (y_{rj}^k)_\varphi^U + \lambda_p^k (y_{rp}^k)_\varphi^L - (s_{rp}^{k+})_\varphi^L \tag{52}$$

$$z_d^{(k,h)} = \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j^h (z_{dj}^{(k,h)})_\varphi^L + \lambda_p^h (z_{dp}^{(k,h)})_\varphi^U + (s_{dp}^{(k,h)-})_\varphi^U \tag{53}$$

$$(s_{dp}^{(k,h)-})_\varphi^U \leq M g_\varphi^{(k,h)} \tag{54}$$

$$(z_{dp}^{(k,h)})_\varphi^U - M(1 - g_d^{(k,h)}) \leq z_d^{(k,h)} \leq (z_{dp}^{(k,h)})_\varphi^U + M(1 - g_d^{(k,h)}) \tag{55}$$

$${z'}_d^{(k,h)} = \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j^k (z_{dj}^{(k,h)})_\varphi^U + \lambda_p^k (z_{dp}^{(k,h)})_\varphi^L - (s_{dp}^{(k,h)+})_\varphi^L \tag{56}$$

$$(s_{dp}^{(k,h)+})_\varphi^L \leq M(1 - g_d^{(k,h)}) \tag{57}$$

$$(z_{dp}^{(k,h)})_\varphi^L - M g_{dp}^{(k,h)} \leq {z'}_d^{(k,h)} \leq (z_{dp}^{(k,h)})_\varphi^L + M g_{dp}^{(k,h)} \tag{58}$$

$$\sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j^k (z_{dj}^{(k,h)})_\varphi^U + \lambda_p^k (z_{dp}^{(k,h)})_\varphi^L = \sum_{\substack{j=1 \\ j\neq p}}^{n} \lambda_j^h (z_{dj}^{(k,h)})_\varphi^L + \lambda_p^h (z_{dp}^{(k,h)})_\varphi^U \tag{59}$$

$$\sum_{j=1}^{n} \lambda_j^k = 1 \tag{60}$$

$$g_d^{(k,h)} = \{0,1\}; z_d^{(k,h)}, {z'}_d^{(k,h)}: free, \lambda_j^k \geq 0, (s_{rp}^{k+})_\varphi^L \geq 0, (s_{ip}^{k-})_\varphi^U \geq 0, (s_{dp}^{(k,h)-})_\varphi^U \geq 0, (s_{dp}^{(k,h)+})_\varphi^L \geq 0. \tag{61}$$

It should be noted that the presented models in Eqs. (38)-(49) and Eqs. (50)-(61) are input-oriented VRS-based models. In the following, we introduce the concepts of output-oriented and overall efficiency, as well as the divisional efficiency.

Definition 4 (overall efficiency of neutrosophic NDEA-SBM). To evaluate the upper and lower bounds of the overall efficiency score of *DMUp* we can replace the objective functions (38) and (50), respectively with Eqs. (62) and (63).

$$(\eta_p)_{\varphi}^{U} = min \frac{\sum_{k=1}^{K} w_k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_{\varphi}^{L}}{(x_{ip})_{\varphi}^{L}} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_{\varphi}^{L}}{(z_{dp}^{(f,k)})_{\varphi}^{L}}) \right]}{\sum_{k=1}^{K} w_k \left[1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{*(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_{\varphi}^{U}}{(y_{rp})_{\varphi}^{U}} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{*(k,h)+})_{\varphi}^{U}}{(z_{dp}^{(k,h)})_{\varphi}^{U}}) \right]} \tag{62}$$

$$(\eta_p)_{\varphi}^{L} = min \frac{\sum_{k=1}^{K} w_k \left[1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_{\varphi}^{U}}{(x_{ip})_{\varphi}^{U}} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_{\varphi}^{U}}{(z_{dp}^{(f,k)})_{\varphi}^{U}}) \right]}{\sum_{k=1}^{K} w_k \left[1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{*(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_{\varphi}^{L}}{(y_{rp})_{\varphi}^{L}} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{*(k,h)+})_{\varphi}^{L}}{(z_{dp}^{(k,h)})_{\varphi}^{L}}) \right]} \tag{63}$$

Definition 5 (non-oriented divisional efficiency of neutrosophic NDEA-SBM). After solving the non-oriented neutrosophic NDEA-SBM model by replacing the optimal values in the formulas (64) and (65), the boundaries for efficiency score of *Divk* is obtained:

$$({\eta_p}^{k})_{\varphi}^{U} = \frac{1 - -\frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_{\varphi}^{L}}{(x_{ip})_{\varphi}^{L}} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_{\varphi}^{L}}{(z_{dp}^{(f,k)})_{\varphi}^{L}})}{1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_{\varphi}^{U}}{(y_{rp})_{\varphi}^{U}} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{(k,h)+})_{\varphi}^{U}}{(z_{dp}^{(k,h)})_{\varphi}^{U}})} \tag{64}$$

$$({\eta_p}^{k})_{\varphi}^{L} = \frac{1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_{\varphi}^{U}}{(x_{ip})_{\varphi}^{U}} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_{\varphi}^{U}}{(z_{dp}^{(f,k)})_{\varphi}^{U}})}{1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_{\varphi}^{L}}{(y_{rp})_{\varphi}^{L}} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{(k,h)+})_{\varphi}^{L}}{(z_{dp}^{(k,h)})_{\varphi}^{L}})} \tag{65}$$

Definition 6 (Input-oriented divisional efficiency of neutrosophic NDEA-SBM). To calculate the bounds of the input-oriented divisional efficiency of *Divk* of *DMUp*, it is sufficient to substitute the optimal values obtained from the calculation of the upper and lower bounds of the input-oriented efficiency in formulas (66) and (67):

$$(\rho_p^{k*})_{\varphi}^{U} = 1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_{\varphi}^{L}}{(x_{ip})_{\varphi}^{L}} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_{\varphi}^{L}}{(z_{dp}^{(f,k)})_{\varphi}^{L}}) \tag{66}$$

$$(\rho_p^{k*})_{\varphi}^{L} = 1 - \frac{1}{m_k + \sum_{d=1}^{l(f,k)} g_d^{(f,k)}} (\sum_{i=1}^{m_k} \frac{(s_{ip}^{k-})_{\varphi}^{U}}{(x_{ip})_{\varphi}^{U}} + \sum_{d=1}^{l(f,k)} \frac{(s_{dp}^{(f,k)-})_{\varphi}^{U}}{(z_{dp}^{(f,k)})_{\varphi}^{U}}) \tag{67}$$

Definition 7 (Output-oriented overall efficiency of neutrosophic NDEA-SBM). To obtain the output-oriented overall efficiency of DMU_p, taking into account the direct effect of the inefficiency of the intermediate products in the calculations, we replace the objective functions (38) and (50) with Eqs. (68) and (69), respectively in neutrosophic NDEA-SBM.

$$(\psi_p^*)_{\varphi}^{U} = \max \sum_{k=1}^{K} w_k [1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{*(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_{\varphi}^{U}}{(y_{rp})_{\varphi}^{U}} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{*(k,h)+})_{\varphi}^{U}}{(z_{dp}^{(k,h)})_{\varphi}^{U}})] \tag{68}$$

$$(\psi_p^*)_\varphi^L = \max \sum_{k=1}^{K} w_k [1+\frac{1}{r_k+l_{(k,h)}-\sum_{d=1}^{l(k,h)} g_d^{(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_\varphi^L}{(y_{rp})_\varphi^L} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{*(k,h)+})_\varphi^L}{(z_{dp}^{(k,h)})_\varphi^L})] \tag{69}$$

Definition 8 (Output-oriented divisional efficiency of neutrosophic NDEA-SBM). Replacing the optimal values obtained from the calculation of the lower and upper bounds of input-oriented efficiency in the formulas (70) and (71), we can measure the divisional efficiency for *Divk* of the *DMUp*.

$$(\psi_p^k)_\varphi^L = 1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_\varphi^L}{(y_{rp})_\varphi^L} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{(k,h)+})_\varphi^L}{(z_{dp}^{(k,h)})_\varphi^L}) \tag{70}$$

$$(\psi_p^{k*})_\varphi^U = 1 + \frac{1}{r_k + l_{(k,h)} - \sum_{d=1}^{l(k,h)} g_d^{(k,h)}} (\sum_{r=1}^{r_k} \frac{(s_{rp}^{k+})_\varphi^U}{(y_{rp})_\varphi^U} + \sum_{d=1}^{l(k,h)} \frac{(s_{dp}^{(k,h)+})_\varphi^U}{(z_{dp}^{(k,h)})_\varphi^U}) \tag{71}$$

In the next section a case study is presented to verify our model.

4. Airlines Efficiency: A Case of Iran

In this section, with the help of the proposed model, we evaluate the efficiency of airlines in Iran. We will first examine the airlines and their various divisions that are interconnected as a network with a series structure. After that, we will introduce and implement the model under study. Then we will present and analyze the results.

4.1. Airlines in Iran

Iran's aviation industry is a dynamic and influential industry due to its infrastructural role and its great relationship with all factors affecting economic growth. Due to the widespread use of transportation, it can be considered as one of the main factors underlying the economic development of the country. Therefore, competition in aviation has always been increasing and these changes have led to the recognition of the efficiency of airlines and awareness of their status in relation to competitors is a very important issue.

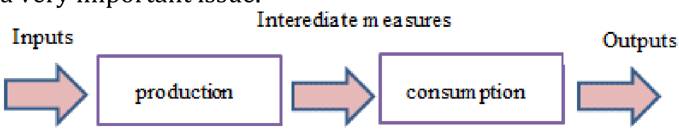

Figure 1. Airline network structure.

The aviation industry can be viewed from a perspective with two parts: production and consumption. Which in turn are of fundamental importance. The two departments, which are interconnected, are jointly responsible for providing air services.

Figure 1 is a structure of an airline whose various units are connected in series.

4.2. The Data Studied in This Research in Airlines

Figure 2 exhibits the network structure of airlines consisting of production and consumption divisions.

The production division (Division 1) uses the inputs number of employees and number of aircraft and produces Tons-kilometers supplied per year and seat-kilometers supplied per year. Then it becomes an intermediate input for the consumption division. In the consumption division (Division2), airlines utilize Airport coverage on domestic flights and Airport coverage on foreign flights inputs as well as the intermediate inputs from production division to produce Passenger share of foreign flights, Passenger share of domestic flights, Ton-km performed and Passenger-km performed.

In this study, 13 companies of domestic airlines have been studied whose names are presented as follows:

Qeshm Air (DMU1), Caspian (DMU2), Kish Air (DMU3), Mahan Air (DMU4), Iran Airtour (DMU5), Iran Air (DMU6), Ata (DMU7), Aseman (DMU8), Zagros (DMU9), Taban (DMU10), Pouya (DMU11), Karun (DMU12), and Meraj (DMU13).

The indicators of the two-stage model are described as follows:

Number of employees (X_{11}): The total number of employees related to the airline includes flight attendant, pilot, co-pilot, security, repairs, administrative and financial and commercial, airport services, repairs and engineering and other personnel, which is considered as the entrance to the production department.

Number of aircraft (X_{12}): The number of air fleet used in the airline, which is considered as the input of the production sector.

Seat- Kilometer supplied per year $\left(Z_1^{(1,2)}\right)$: The sum of multiplying the number of airline seats by the distance traveled in each flight stage.

Tons-kilometers supplied per year($Z_2^{(1,2)}$): is the product of the total amount of portable weight in terms of tons at each origin and destination of the flight in the distance between the same origin and destination of the flight.

Passenger kilometers carried per year (Y_1): The sum obtained by multiplying the number of income-generating passengers carried by the distance traveled in each flight stage.

Tons-Kilometers of cargo carried per year (Y_2): The product of the total amount of weight carried in terms of tons at each origin and destination of the flight in the distance between the same origin and destination of the flight.

Share of passengers transferred in domestic flights (Y_3): The ratio of passengers transferred during the year in domestic flights of the airline to the total number of passengers transferred during the year in domestic flights. This index is considered as the output of the second stage or the consumption stage.

Share of passengers transferred on international flights (Y_4): The ratio of passengers transferred during the year on international flights of the airline under study to the total number of passengers transferred during the year on international flights.

Airport coverage on international flights (X_{21}): The equipment and facilities of airlines are in accordance with the policies and standards of how many foreign airports play a key role in the production process of airline services.

Airport coverage on domestic flights (X_{22}): The equipment and facilities of airlines are in accordance with the policies and standards of how many domestic airports play a key role in the production process of airline services.

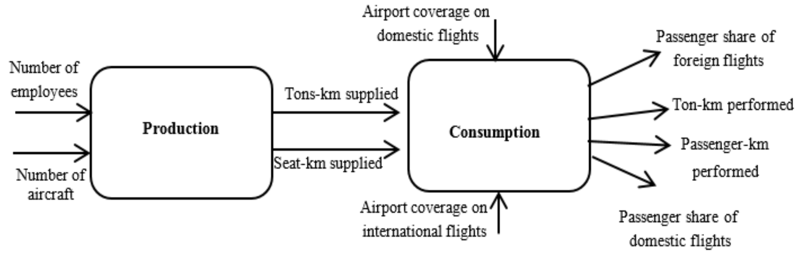

Figure 2. A structure of airlines that communicate in series

4.3. Calculation of Airline Efficiency

In this section we apply our proposed approach to measure the relative efficiency of Iranian airlines. To solve the proposed model and measure the bounds of efficiency for airlines and detecting the role of intermediate measures we employed GAMS (Generalized Algebraic Modeling System) and the results were obtained at different levels. Tables 1 and 2 report the performance limits of 13 airlines at $\varphi_{(0,1.0,1.0)}$.

It should be noted that the limits of efficiency are evaluated with the input-oriented VRS models.

According to Tables 1 and 2, Caspian, Mahan, Iran Airtour, Pouya Karun and Meraj airlines have been recognized as efficient at $\varphi_{(0,1.0,1.0)}$.

Among the airlines, Iran Air has the lowest efficiency level at $\varphi_{(0,1.0,1.0)}$. Both intermediate products, i.e., Ton-Kilometer supplied and seat-Kilometer supplied at $\varphi_{(0,1.0,1.0)}$, are considered as the output of the production unit in the calculation of Iran Air's efficiency.

At $\varphi_{(0,1.0,1.0)}$, Qeshm Air has inefficiencies in the production sector and the consumption sector is quite efficient.

Kish Air, Iran Air, Ata and Aseman are also inefficient in both production and consumption sectors, and inefficiency due to intermediate products is not seen in both parts of these companies.

Most of Taban's inefficiency is due to the consumption sector. Considering that both intermediate products in this part of the company are detected in the model as input-type, so the role of intermediate products in the inefficiency of the consumption sector of this company is quite prominent and managers should pay attention to the excesses of these intermediate products.

The high inefficiency in Iran Air is due to inefficiency in both sectors, which is more inefficient in the production sector than in the consumption sector. This inefficiency is more due to separate inputs to each division and not to intermediate products.

Zagros company has significant inefficiency in the consumption sector. Part of this inefficiency is due to the inefficiency of the intermediate seat-kilometer product, which is considered as the input of the consumption sector by the model.

The inefficiency of Ata Airlines is also due to intermediate products.

Figure 3 reports the lower and upper bounds of performance for these 13 airlines at $\varphi_{(0,1.0,1.0)}$.

According to Tables 3 and 4, Caspian, Iran Airtour and Meraj, Mahan and Pouya airlines have been recognized as efficient at $\varphi_{(0.2,0.7,0.9)}$. Among the companies, Iran Air has the lowest efficiency level at $\varphi_{(0.2,0.7,0.9)}$. Both intermediate products, i.e., Ton-Kilometer supplied and Seat-Kilometer supplied at $\varphi_{(0.2,0.7,0.9)}$, are considered as the output of the production unit in the calculation of Iran Air's efficiency.

Kish Air is extremely inefficient in both production and consumption at this level. Part of the inefficiency of the consumption sector is related to the inefficiency caused by the intermediate product Ton-Kilometers.

Most of Taban's inefficiency is due to the consumption sector. Considering that both intermediate products in this part of the company are detected as *input-type*, so the role of intermediate products in the inefficiency of the consumption sector of this company is quite prominent. As a result, managers must pay more attention to the surplus of these intermediate products.

Also, the high inefficiency in Iran Air is due to inefficiency in both divisions, which is more inefficient in the production sector than in the consumption sector. This inefficiency is more due to separate inputs to each sector and not to intermediate products.

Zagros Company has significant inefficiency in the consumption sector. Part of this inefficiency is due to the inefficiency of the intermediate product seat-kilometer, which is considered as the input of the consumption sector by the model.

Ata Airlines also has inefficiencies in both sectors, which is not due to intermediate products.

Table 1. Lower bound of efficiency at $\varphi_{(0,1.0,1.0)}$

DMU	Overall eff	Eff of Div1	Eff of Div2	$\mathbf{g}_1^{*(1,2)}$	$\mathbf{g}_2^{*(1,2)}$
DMU01	0.86	0.73	1.00	0.00	1.00
DMU02	1.00	1.00	1.00	1.00	0.00
DMU03	0.54	0.66	0.43	0.00	0.00
DMU04	1.00	1.00	1.00	1.00	1.00
DMU05	1.00	1.00	1.00	1.00	1.00
DMU06	0.22	0.25	0.19	0.00	0.00
DMU07	0.47	0.53	0.40	0.00	0.00
DMU08	0.69	0.48	0.90	0.00	0.00
DMU09	0.34	0.64	0.04	0.00	1.00
DMU10	0.64	0.74	0.55	1.00	1.00
DMU11	1.00	1.00	1.00	1.00	1.00
DMU12	1.00	1.00	1.00	1.00	1.00
DMU13	1.00	1.00	1.00	1.00	1.00

According to Tables 5 and 6, Caspian, Mahan, Iran Airtour, Pouya, Aseman, Zagros, Karun and Meraj airlines have been recognized as efficient at the level of $\varphi_{(0.5,0.8,0.9)}$.

Among these companies, Iran Air has the lowest efficiency level at $\varphi_{(0.5,0.8,0.9)}$. Both intermediate products, i.e., Ton-Kilometer supplied and seat-Kilometer supplied at the level of $\varphi_{(0.5,0.8,0.9)}$, are not considered as *input-type* in the calculation of Iran Air's efficiency. It should be noted that the inefficiency caused by these two intermediate products in comparison with the inefficiency of other inputs is negligible.

Table 2. Upper bound of efficiency at $\varphi_{(0,1.0,1.0)}$

DMU	Overall eff	Eff of Div1	Eff of Div2	$\mathbf{g}_1^{*(1,2)}$	$\mathbf{g}_2^{*(1,2)}$
DMU01	1.00	1.00	1.00	1.00	1.00
DMU02	1.00	1.00	1.00	1.00	1.00
DMU03	1.00	1.00	1.00	1.00	0.00
DMU04	1.00	1.00	1.00	0.00	0.00
DMU05	1.00	1.00	1.00	1.00	1.00
DMU06	0.63	0.63	0.63	0.00	0.00
DMU07	0.81	0.88	0.74	0.00	0.00
DMU08	1.00	1.00	1.00	1.00	1.00
DMU09	1.00	1.00	0.99	0.00	1.00
DMU10	1.00	1.00	1.00	1.00	1.00
DMU11	1.00	1.00	1.00	1.00	1.00
DMU12	1.00	1.00	1.00	1.00	1.00
DMU13	1.00	1.00	1.00	1.00	1.00

Table 3. Lower bound of efficiency at $\varphi_{(0,1.0,1.0)}$

DMU	Overall eff	Eff of Div1	Eff of Div2	$\mathbf{g}_1^{*(1,2)}$	$\mathbf{g}_2^{*(1,2)}$
DMU01	0.90	0.81	1.00	1.00	1.00
DMU02	1.00	1.00	1.00	1.00	1.00
DMU03	0.57	0.69	0.45	1.00	0.00
DMU04	1.00	1.00	1.00	0.00	0.00
DMU05	1.00	1.00	1.00	0.00	1.00
DMU06	0.36	0.25	0.47	0.00	0.00
DMU07	0.50	0.57	0.43	0.00	0.00
DMU08	0.97	1.00	0.94	0.00	0.00
DMU09	0.62	1.00	0.24	0.00	1.00
DMU10	0.64	0.75	0.52	1.00	1.00
DMU11	1.00	1.00	1.00	1.00	1.00
DMU12	0.83	1.00	0.67	0.00	0.00
DMU13	1.00	1.00	1.00	1.00	1.00

Most of the inefficiency of Taban Company is due to the Consumption sector. Due to the fact that the intermediate products in this section of the company are not detected by the model as input-type, so the role of intermediate products in the inefficiency of the Consumption division of this company is not very prominent and in order to increase the level of efficiency, external inputs should be improved. Also, the high value of inefficiency in Iran Air is due to inefficiency in both sectors. Inefficiency in consumption sector is more than that of production sector. This inefficiency is more due to separate inputs to each sector and not to intermediate products.

Table 4. Upper bound of efficiency at $\varphi_{(0.2,0.7,0.9)}$

DMU	Overall eff	Eff of Div1	Eff of Div2	$\mathbf{g}_1^{*(1,2)}$	$\mathbf{g}_2^{*(1,2)}$
DMU01	1.00	1.00	1.00	0.00	0.00
DMU02	1.00	1.00	1.00	0.00	0.00
DMU03	1.00	1.00	1.00	0.00	0.00
DMU04	1.00	1.00	1000	0.00	0.00
DMU05	1.00	1.00	1.00	0.00	0.00
DMU06	1.00	1.00	1.00	0.00	0.00
DMU07	0.77	0.83	0.71	0.00	0.00
DMU08	1.00	1.00	1.00	0.00	0.00
DMU09	1.00	1.00	1.00	0.00	1.00
DMU10	0.66	1.00	0.32	1.00	1.00
DMU11	1.00	1.00	1.00	1.00	1.00
DMU12	1.00	1.00	1.00	0.00	0.00
DMU13	1.00	1.00	1.00	1.00	1.00

Table 5. Lower bound of efficiency at $\varphi_{(0.5,0.8,0.9)}$

DMU	Overall eff	Eff of Div1	Eff of Div2	$\mathbf{g}_1^{*(1,2)}$	$\mathbf{g}_2^{*(1,2)}$
DMU01	0.89	0.78	1.00	0.00	1.00
DMU02	1.00	1.00	1.00	1.00	1.00
DMU03	0.68	0.82	0.54	0.00	0.00
DMU04	1.00	1.00	1.00	1.00	1.00
DMU05	1.00	1.00	1.00	1.00	1.00
DMU06	0.42	0.42	0.42	0.00	0.00
DMU07	0.61	0.68	0.54	0.00	0.00
DMU08	1.00	1.00	1.00	1.00	0.00
DMU09	1.00	1.00	1.00	1.00	1.00
DMU10	0.79	0.83	0.74	0.00	0.00
DMU11	1.00	1.00	1.00	0.00	0.00
DMU12	1.00	1.00	1.00	0.00	0.00
DMU13	1.00	1.00	1.00	0.00	0.00

Qeshm Air company is also inefficient in the production sector and in the consumption sector, which is part of the inefficiency of the consumption sector due to the seat-kilometer's surplus. Ata and Taban companies are also inefficient in both sectors, both of which are not due to intermediate products.

Table 6. Upper bound of efficiency at $\varphi_{(0.5,0.8,0.9)}$

DMU	Overall eff	Eff of Div1	Eff of Div2	$\mathbf{g}_1^{*(1,2)}$	$\mathbf{g}_2^{*(1,2)}$
DMU01	0.91	0.96	0.85	0.00	1.00
DMU02	1.00	1.00	1.00	0.00	0.00
DMU03	0.76	0.87	0.66	0.00	0.00
DMU04	1.00	1.00	1.00	0.00	0.00
DMU05	1.00	1.00	1.00	0.00	0.00
DMU06	0.48	0.53	0.43	0.00	0.00
DMU07	0.72	0.75	0.69	0.00	0.00
DMU08	1.0\0	1.00	1.00	0.00	1.00
DMU09	1.00	1.00	1.00	0.00	0.00
DMU10	0.88	1.00	0.76	0.00	0.00
DMU11	1.00	1.00	1.00	0.00	1.00
DMU12	1.00	1.00	1.00	0.00	1.00
DMU13	1.00	1.00	1.00	0.00	1.00

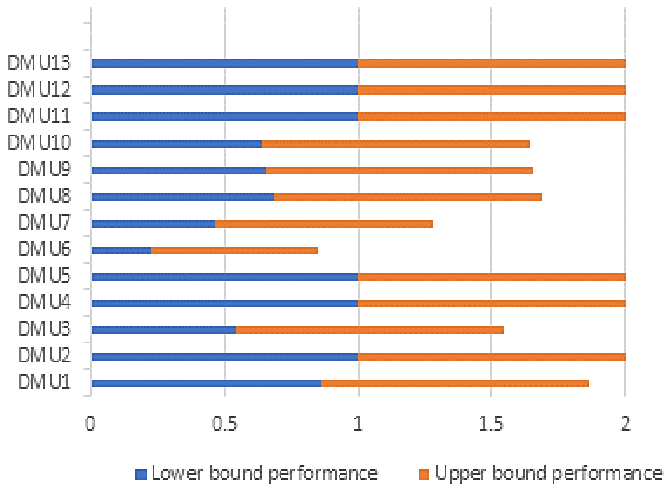

Figure 3. The lower and upper bounds of performance at $\varphi_{(0,1.0,1.0)}$

5. Conclusions

Data envelopment analysis (DEA) is one of the most powerful tools for evaluating the performance of decision-making units (DMUs). In real world problems, to provide a more detailed performance evaluation, there are some issues that need to be addressed. DMUs have often multi division structures and some divisions that are connected together with some intermediate products. In efficiency measurement it is very essential to detect the sources of inefficiencies caused by these intermediate measures. On the other hand, in many situations the data available are not necessarily crisp, and the information collected are often uncertain and indeterminate. Therefore, this paper focuses on providing a model to address aforementioned issues.

The proposed network DEA model computes the overall and divisional efficiencies of DMUs with network structure by taking the inefficiencies due to linking activities in to account in the presence of fuzzy neutrosophic data. The proposed model categorizes the intermediate measures into two groups of *input-typ*e and *output-type* and enters their excesses and shortfalls directly into the objective function which is

Oriented. The model also keeps the continuity of flow of linking activities between divisions. To help explain the idea of the proposed model and represent its capability, in this paper an illustrative example of Iranian airline companies presented and the efficiencies of 13 airlines were computed. Decomposition the overall efficiency of an airline into divisional efficiencies provides meaningful results. The results obtained from the model reveal the inefficient divisions and identify their sources of inefficiencies.

The general structure of airlines is a network structure consisting of production and consumption divisions and data available in transportation industry are sometimes uncertain. considering all these aspects, we applied our model to compute the overall and divisional efficiency scores of 13 Iranian airlines. According to the results Iran Air has the lowest efficiency score among the airlines in each level. The results also show that the high inefficiency in Iran Air is due to inefficiency in both divisions and the linking activities are not the sources of inefficiency.

Suggestions for future research that may be helpful:

(i). Airlines are each made up of two parts that are interconnected, and each part has several sub-sections, all of which are related in different ways. Therefore, a model can be presented that examines the sections along with the subdivisions as a nested network.

(ii) In line with the privatization policies of Iranian government organizations, part of the body of production and consumption of the country's aviation are being transferred to private companies. Therefore, in order to make a better decision to transfer these government organizations to the private sector, their performance can be examined in comparison with each other and the obtained results can be given to the country's aviation organization, which is in charge of aviation affairs in the country.

(iii) The proposed model involves some binary variables to detect the role of intermediate measures endogenously. The existence of binary variables makes the computation complex. Converting the model to a linear programming removes the barriers of this research is recommended for future researches.

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Ddatasets generated during the current study are available from the corresponding author on reasonable request.

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