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DECISION MAKING ON CONSISTENT CUSTOMER CENTRIC INVENTORY MODEL WITH QUALITY SUSTENANCE AND SMART WAREHOUSE MANAGEMENT COST PARAMETERS

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Abstract: *Customer acquaintance and quality of the products decide the survival span of all the business sectors. Every production firm attempts to manufacture novel products with attractive and compatible features to satisfy the demands of different types of customers, especially with special preference to the consistent kind of customers. A decision-making inventory model compassing of the initiatives taken by the production firm for the consistent customers is developed with the inclusion of costs associated with quality conservation and warehouse management. The reliability of the model is validated with numerical examples and inferences on parametric variations are made on comparing the results employing MATLAB software. The proposed model shall be further discussed in a fuzzy environment.*

Key words: *Consistent customer, quality, warehouse management, inventory.*

1. Introduction

A manufacturing firm is bounded with competing challenges from its initial to terminal points of production process. The growing competition at all market levels makes these manufacturing firms imbibe new production technology and customer centric strategies. Many companies are shielding themselves from their business opponents using the weapon of customer centricity. Business is made customer centric, as customers are the buyers and users of the products, and it is they who decide the destiny of the success rate of all business. The production sectors make attempts to fulfil the expectancies of the customers on products ranging from designing to marketing. As customers are one of the pivotal stakeholders, the industrial sectors of this present age circumscribe customer-centric approaches in all

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their business entities. Omur (2020) has discussed the benefits of customer centric strategies in supply chain management. The attributes of customer centeredness and the positive outcomes are outlined with illustrations. Implementation of customer centric orientation results in inventory challenges and different demand patterns.

The business sectors that adopt customer-centered approaches in their production activities must also be watchful over the existence of different types of customers as customer's purchasing behavior highly influences the business profit. In general academicians categorize customers based on their purchasing behavior. The sellers always concentrate on increasing the customers with consistent buying capacity and these customers are referred as consistent customers in this research work. It is quite natural for every production firm to take extra efforts in satisfying the demands of such type of consistent customers. The consequences of such additional initiatives result in inventory management without shortages and warehouse management for fulfilling the varied demands of the customers.

Inventory models are formulated to determine the optimal order quantity and optimal order time. The conventional economic order quantity and economic production quantity models are extended based on the needs of the decision makers and at many a times these inventory models provide solutions to various production conflicts. These models are also associated with other aspects of inventory management and one such is warehouses which are owned or rented by the production sectors for stocking their products to tackle fluctuating demand patterns. Researchers have framed inventory models integrating warehouses but not in the context of customer centricity with separate demand. Also, another factor that constraints the decision makers is the existence of uncertainty. To resolve such crisis, an inventory model is put forwarded in this paper.

This model considers smart manufacturing systems also. The cost of implementing it is also included in this model. SMP has replaced the traditional production technique with technological innovations. The Internet of Things (IoT) integrated technology has now surpassed human interaction, resulting in a smart manufacturing system (SMP).

2. Literature Review

The EPQ model developed by E.W. Taft is extended by researchers to include practical issues related to inventory. One such issue is varying demand. These inventory models with warehouses mentioned in Table 1 have used the concepts of stocking items in owned or rented warehouses to tackle fluctuating demand and have included the concepts such as Deterioration, Shortages, Credit policies and the cost factors associated with the inventory.

Table 1. Chronological development of Inventory models related warehouses and associated Cost factors

Authors	Number of warehouses	Owned/Rented	Manual /Automated Management System	Cost parameters
Yang (2004)	2	1 rented and 1 owned	Manual	Ordering cost, Holding cost, Deterioration cost, Shortage cost
Lee (2006)	2	1 owned and 1 rented	Manual	Holding cost held in RW/OW, Deterioration rates in RW/OW
(Singh & Malik, 2010)	2	1 owned and 1 rented	Manual	Replenishment cost, Holding cost, Deterioration cost, Shortage cost
Singh et al. (2011)	2	1 rented and 1 owned	Manual	Ordering cost, Holding cost RW, Holding cost OW, Deteriorating cost, Shortage cost, Opportunity cost due to lost sales
Kumar Sett et al. (2012)	2	1 owned and 1 rented	Manual	Setup cost, Carrying cost and Deteriorating cost
Yadav & Swami (2013)	2	1 owned and 1 rented	Manual	Ordering cost, Holding cost, Backlogging cost, Opportunity cost, Purchase cost
Singh & Rathore (2016)	2	1 owned and 1 rented	Manual	Ordering cost, Purchase cost, Shortage cost, Lost sale cost
Rastogi et al. (2017)	2	1 rented and 1 owned	Manual	Holding cost, Deterioration cost, Shortage cost, Lost sale cost
A K et al., (2017)	2	1 owned and 1 rented	Manual	Ordering cost, Holding cost RW, Holding cost OW, Deteriorating cost RW, Deteriorating cost OW
Kaliraman et al. (2017)	2	1 owned and 1 rented	Manual	Ordering cost, Stock holding cost in RW, Stock holding cost in OW, Deterioration cost, Opportunity cost with interest, Interest

Authors	Number of warehouses	Owned/Rented	Manual /Automated Management System	Cost parameters
				earned
Panda et al. (2019)	2	1 owned and 1 rented	Manual	Storage cost of OW/RW, Deterioration cost at OW/RW, Holding cost, Interest earned/Charged by supplier
Kumar et al. (2020)	2	1 owned and 1 rented	Manual	Ordering cost, Stock holding cost, Deterioration cost, Interest payable, Interest earned
Chandra (2021)	2	1 rented and 1 owned	Manual	Ordering cost, Holding cost RW, Holding cost OW, Deteriorating cost RW, Deteriorating cost OW, backorder cost
Bhavani et al. (2022)	1	For Both Cases	Manual	Deterioration cost, Carbon emission cost, Green investment cost
Duary et al. (2022)	2	1 owned and 1 rented	Manual	Deterioration rate, Inventory level, Holding cost, Rate of interest earned/ charged.
Proposed Model	1	1 owned	Automated	Total quality inspection cost, Precaution cost of defective items before production/times of production/at time of production distribution, Mending cost, Shipping Cost from production to warehouse, Shipping cost of the products from warehouse to the special customers, Fixed maintenance cost, Variable maintenance cost,

Authors	Number of warehouses	Owned/Rented	Manual /Automated Management System	Cost parameters
				Software cost, Hardware cost, Personnel cost, Equipment maintenance cost

As shown in Table 1, Demand pattern is a significant parameter for inventory management. Nevertheless, the growing competitions make the manufacturing firms imbibe new production technology and customer centric strategies to survive. Thus, it is essential to include those aspects in the inventory model.

There are few research questions put forward.

1. How will the smart production system enhance business by being customer centric and dealing with consistent customers?
2. What happens if the demands of a general customer and a consistent customer vary and what will be the Average cost of Production in such situations?
3. Do loyal consumers of a firm receive any benefits?

2.1 Motivation, Research Gaps and Contributions

The following research gaps have been highlighted because of the present findings on inventory models with warehouses.

1. Inventory models dealing with conventional production systems have been researched more than smart production systems.
2. Models of inventory in warehouses have dealt only with concepts such as demand, turnover, and assignment and have neglected consistent customers and the demand fluctuations associated with them.

To close this research gap the concept of consistent customers is introduced and the EPQ model dealing with demand fluctuations between the normal customers and consistent customers are discussed. The concept used in this model is that a particular portion of items produced are separately transferred to the warehouse exclusively to satisfy the demands of consistent customers. This will help in demand fulfilment of consistent customers continuously and effectively.

3. Mathematical Formulation of the Inventory Model

In this part, a production inventory model is constructed that incorporates actions undertaken by the production business for consistent clients, as well as expenditures related with quality conservation and warehouse management.

3.1 Notations

This section makes extensive use of the notations listed below

P is the Production rate per cycle

D is the Demand rate per cycle

C_s is the Set-up cost

C_H is the Holding cost

I_Q is the Quality inspection cost

P_B is the Precaution cost of defective items before production

P_P is the Precaution cost at times of production

P_D is the Precaution cost at times of product distribution

M_β is the Mending cost

$S_w\alpha$ is the Shipping cost of the products from production to warehouse

$S_S\alpha$ is the Shipping cost of the products from warehouse to special customers

F_M is the Fixed maintenance cost

V_M is the Variable maintenance cost

S is the Software cost

H is the Hardware cost

P_S is the Personnel cost

E_M is the Equipment maintenance cost

3.2 Assumptions of the Model

The model's assumptions are as follows.

[1] Shortages are not permitted in this model.
[2] The time horizon is unbounded.
[3] No deterioration.

3.3 Problem Description

Let us consider a production system that produces products incorporating customer centric strategies during the time period [0, *T*]. The production system adopts the policy of satisfying two demand patterns pertaining to general and special categories of customers. Consistent customers are referred to as special type of customers and others are referred as general customers. The inventory gets accumulated in the rate of *P-D* during the time period [0, t_1] and a portion α of Q(t) is stored as special inventory and it is stocked in warehouses to meet the demands of consistent customers (*Ds*). The demands of general customers (*D*) and the demand of the consistent customers are simultaneously satisfied during the time period (t_1, *T*], where $t_1 < T$. The above production problem is modelled with the inclusion of associated cost parameters as follows to find the optimal order quantity.

3.4 Model Development

Case 1: D and Ds are constants

$$\frac{dQ}{dt} = P - D \tag{1}$$

$$\frac{dQ}{dt} = -\left[D + Ds + \alpha Q(t)\right] \quad t_1 < t \le T$$

$$\frac{dQ}{dt} + \alpha Q(t) = -\left[D + Ds\right] \tag{2}$$

$$Q(0) = 0 \tag{3}$$

$$Q(T) = 0 \tag{4}$$

Let $Q(t_1) = I_{max}$ (5)

Solving (1) and (3) with initial conditions we get

$$Q(t) = (P - D)t \text{ or } 0 < t \le t_1 \tag{6}$$

Solving (2) with (4) we get

$$Q(t) = \frac{(D + Ds)}{\alpha}\left[e^{\alpha T - \alpha t} - 1\right]$$

$$Q(t) = (D + D_S)[T - t] \text{or } t_1 < t \le T \tag{7}$$

Using the equations (5) in (6) and (7) we get

$$Imax = (P - D)t_1$$

$$= (D + D_s)[T - t_1]$$

$$\therefore t_1 = \frac{Imax}{P - D} \tag{8}$$

and

$$T - t_1 = \frac{Imax}{D + D_S} \tag{9}$$

By adding equations, (8) and (9) we get,

$$T = I_{max}\left(\frac{(P - Ds)}{(P - D)(D + D_S)}\right) \tag{10}$$

$$I_{max} = \frac{T(P - D)(D + D_{S)}}{(P - D_S)}$$

Holding cost

$$= C_H\left[\int_0^{t_1} Q(t)\,dt + \int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{C_H}{2}\left[(P - D)t_1^2 + (D + D_S)(T - t_1)^2\right]$$

$$= \frac{C_H}{2}\left[\frac{(P - D)(D + D_S)}{(P - D_S)}\right]T^2$$

Total quality inspection cost

$$= I_Q\int_0^{t_1} Q(t)\,dt$$

$$= \frac{I_Q}{2}\left[(P-D)t_1^{\,2}\right]$$

$$= \frac{I_Q}{2}\left[\frac{(P-D)(D+D_S)^2}{(P-D_S)^2}\right]T^2$$

Precaution cost at times of production

$$= P_P\int_0^{t_1} Q(t)\,dt$$

$$= \frac{P_P}{2}\left[\frac{(P-D)(D+D_S)^2}{(P-D_S)^2}\right]T^2$$

Precaution cost at times of product distribution

$$= P_D\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{P_D}{2}\left[(D+D_S)(T-t_1)^2\right]$$

$$= \frac{P_D}{2}\left[\frac{(P-D)^2(D+D_S)}{(P-D_S)^2}\right]T^2$$

Mending cost

$$= M_\beta\int_0^{t_1} Q(t)\,dt$$

$$= \frac{M_\beta}{2}\left[\frac{(P-D)(D+D_S)^2}{(P-D_S)^2}\right]T^2$$

Shipping cost of products from production place to warehouse

$$= S_w\alpha\int_0^{t_1} Q(t)\,dt$$

$$= \frac{S_w\alpha}{2}\left[(P-D)t_1^{\,2}\right]$$

$$= \frac{S_w\alpha}{2}\left[\frac{(P-D)(D+D_S)^2}{(P-D_S)^2}\right]T^2$$

Shipping cost of products from warehouse to special customers

$$= S_S\alpha\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$=\frac{S_S\alpha}{2}\left[\left(D+D_S\right)\left(T-t_1\right)^2)\right]$$

$$=\frac{S_S\alpha}{2}\left[\frac{\left(P-D\right)^2\left(D+D_S\right)}{\left(P-D_S\right)^2}\right]T^2$$

Total cost = Set up cost + Holding cost + Total quality inspection cost + Precaution cost of defective items before production + Precaution cost at times of production + Production cost at time of production distribution + Mending cost + Shipping cost from production to warehouse + Shipping cost of the products from warehouse to the special customers + Fixed maintenance cost + Variable maintenance cost + Software cost + Hardware cost + Personnel cost + Equipment maintenance cost

$$=C_S+\frac{C_H}{2}\left[\frac{\left(P-D\right)\left(D+D_S\right)}{\left(P-D_S\right)}\right]T^2+\frac{I_Q}{2}\left[\frac{\left(P-D\right)\left(D+D_S\right)^2}{\left(P-D_S\right)^2}\right]T^2$$

$$+P_B+\frac{P_P}{2}\left[\frac{\left(P-D\right)\left(D+D_S\right)^2}{\left(P-D_S\right)^2}\right]T^2+\frac{P_D}{2}\left[\frac{\left(P-D\right)^2\left(D+D_S\right)}{\left(P-D_S\right)^2}\right]T^2$$

$$+\frac{M_\beta}{2}\left[\frac{\left(P-D\right)\left(D+D_S\right)^2}{\left(P-D_S\right)^2}\right]T^2+\frac{S_W\alpha}{2}\left[\frac{\left(P-D\right)\left(D+D_S\right)^2}{\left(P-D_S\right)^2}\right]T^2$$

$$+\frac{S_S\alpha}{2}\left[\frac{\left(P-D\right)^2\left(D+D_S\right)}{\left(P-D_S\right)^2}\right]T^2+F_M+V_M+S+H+P_S+E_M$$

Total average cost Tac(T)

$$=\frac{C_S+P_B+F_M+V_M+S+H+P_S+E_M}{T}+\frac{C_H}{2}\left[\frac{\left(P-D\right)\left(D+D_S\right)}{\left(P-D_S\right)}\right]T$$

$$+\left(\frac{I_Q+P_P+M_\beta+S_W\alpha}{2}\right)\left[\frac{\left(P-D\right)\left(D+D_S\right)^2}{\left(P-D_S\right)^2}\right]T$$

$$+\frac{(P_D+S_S\alpha)}{2}\left[\frac{\left(P-D\right)^2\left(D+D_S\right)}{\left(P-D_S\right)^2}\right]T$$

So, the classical EPQ is

MinTac(T)= $\dfrac{C_S+P_B+F_M+V_M+S+H+P_S+E_M}{T}+\dfrac{C_H}{2}\left[\dfrac{\left(P-D\right)\left(D+D_S\right)}{\left(P-D_S\right)}\right]T$

$$+\left(\frac{I_Q+P_P+M_\beta+S_W\alpha}{2}\right)\left[\frac{\left(P-D\right)\left(D+D_S\right)^2}{\left(P-D_S\right)^2}\right]T+$$

$$\frac{(P_D+S_S\alpha)}{2}\left[\frac{(P-D)^2(D+D_S)}{(P-D_S)^2}\right]T \text{ such that } T>0 \tag{11}$$

Solving (11) we can show that Tac(T) will be minimum for

$$T^*=\sqrt{\frac{2(C_S+P_B+F_M+V_M+S+H+P_S+E_M)(P-D_S)^2}{\begin{bmatrix}C_H(P-D_S)+(I_Q+P_P+M_\beta+S_W\alpha)(D+D_S)+\\(P_D+S_S\alpha)(P-D)\end{bmatrix}(P-D)(D+D_S)}}$$

$$\mathrm{Tac}^*(T^*)=\sqrt{\frac{2(C_S+P_B+F_M+V_M+S+H+P_S+E_M)\begin{bmatrix}C_H(P-D_S)+(I_Q+P_P+M_\beta+S_W\alpha)\\(D+D_S)+(P_D+S_S\alpha)(P-D)\end{bmatrix}(P-D)(D+D_S)}{(P-D_S)^2}}$$

Case 2: D is linear, and Ds is constant

$$\frac{dQ}{dt}=P-D \quad 0<t\le t_1 \tag{12}$$

$$\frac{dQ}{dt}=-\left[D+Ds+\alpha Q(t)\right] \; t_1<t\le T$$

$$\frac{dQ}{dt}+\alpha Q(t)=-\left[(a+bt)+Ds\right] \tag{13}$$

$$Q(0)=0 \tag{14}$$

$$Q(T)=0 \tag{15}$$

Let $Q(t_1)=I_{max}$ (16)

Solving (12) and (14) with initial conditions we get

$$Q(t)=(P-a)t \text{ for } 0<t\le t_1 \tag{17}$$

Solving (13) with (15) we get

$$Q(t)=\left(\frac{b}{\alpha}\right)[T-t] \text{ for } t_1<t\le T \tag{18}$$

Using the equations (16) in (17) and (18) we get

$$Imax=(P-a)t_1$$

$$=\left(\frac{b}{\alpha}\right)\left[T-t_1\right]$$

$$t_1=\frac{Imax}{P-a} \tag{19}$$

and

$$T-t_1=\frac{\alpha Imax}{b} \tag{20}$$

By adding equations, (18) and (20) we get,

$$T=I_{max}\left(\frac{\alpha\left(P-a\right)+b}{b\left(P-a\right)}\right) \tag{21}$$

$$I_{max}=\frac{Tb\left(P-a\right)}{\alpha\left(P-a\right)+b}$$

Holding cost

$$=C_H\left[\int_0^{t_1}Q\left(t\right)dt+\int_{t_1}^{T}Q\left(t\right)dt\right]$$

$$=\frac{C_H}{2}\left[\left(P-a\right)t_1^{\,2}+\left(\frac{b}{\alpha}\right)\left(T-t_1\right)^2\right]$$

$$=\frac{C_H}{2}\left[\frac{b\left(P-a\right)}{\alpha\left(P-a\right)+b}\right]T^2$$

Total quality inspection cost

$$=I_Q\int_0^{t_1}Q\left(t\right)dt$$

$$=\frac{I_Q}{2}\left[\left(P-a\right)t_1^{\,2}\right]$$

$$=\frac{I_Q}{2}\left[\frac{b^2\left(P-a\right)}{\left(\alpha\left(P-a\right)+b\right)^2}\right]T^2$$

Precaution cost at times of production

$$=P_P\int_0^{t_1}Q\left(t\right)dt$$

$$=\frac{P_P}{2}\left[\frac{b^2\left(P-a\right)}{\left(\alpha\left(P-a\right)+b\right)^2}\right]T^2$$

Precaution cost at times of product distribution

$$= P_D\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{P_D}{2}\left[\frac{b}{\alpha}\left(T-t_1\right)^2\right]$$

$$= \frac{P_D}{2}\left[\frac{\alpha b\left(P-a\right)^2}{\left(\alpha\left(P-a\right)+b\right)^2}\right]T^2$$

Mending cost

$$= M_\beta\int_0^{t_1} Q(t)\,dt$$

$$= \frac{M_\beta}{2}\left[\frac{b^2\left(P-a\right)}{\left(\alpha\left(P-a\right)+b\right)^2}\right]T^2$$

Shipping cost of products from production place to warehouse

$$= S_w\alpha\int_0^{t_1} Q(t)\,dt$$

$$= \frac{S_W\alpha}{2}\left[\left(P-a\right)t_1^{\,2}\right]$$

$$= \frac{S_W\alpha}{2}\left[\frac{b^2\left(P-a\right)}{\left(\alpha\left(P-a\right)+b\right)^2}\right]T^2$$

Shipping cost of products from warehouse to special customers

$$= S_S\alpha\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{S_S\alpha}{2}\left[\frac{b}{\alpha}\left(T-t_1^{\,2}\right)\right]$$

$$= \frac{S_S\alpha}{2}\left[\frac{\alpha b\left(P-a\right)^2}{\left(\alpha\left(P-a\right)+b\right)^2}\right]T^2$$

Total cost = Set up cost + Holding cost + Total quality inspection cost + Precaution cost of defective items before production + Precaution cost at times of production + Production cost at time of production distribution + Mending cost + Shipping cost from production to warehouse + Shipping cost of the products from warehouse to the special customers + Fixed maintenance cost + Variable maintenance cost + Software cost + Hardware cost + Personnel cost + Equipment maintenance cost

$$= C_S + \frac{C_H}{2}\left[\frac{b(P-a)}{\alpha(P-a)+b}\right]T^2 + \frac{I_Q}{2}\left[\frac{b^2(P-a)}{(\alpha(P-a)+b)^2}\right]T^2 + P_B + \frac{P_P}{2}\left[\frac{b^2(P-a)}{(\alpha(P-a)+b)^2}\right]$$

$$T^2 + \frac{P_D}{2}\left[\frac{\alpha b(P-a)^2}{(\alpha(P-a)+b)^2}\right]T^2 + \frac{M_\beta}{2}\left[\frac{b^2(P-a)}{(\alpha(P-a)+b)^2}\right]T^2 + \frac{S_W\alpha}{2}\left[\frac{b^2(P-a)}{(\alpha(P-a)+b)^2}\right]$$

$$T^2 + \frac{S_S\alpha}{2}\left[\frac{\alpha b(P-a)^2}{(\alpha(P-a)+b)^2}\right]T^2 + F_M + V_M + S + H + P_S + E_M$$

Total average cost Tac(T)

$$= \frac{C_S + P_B + F_M + V_M + S + H + P_S + E_M}{T} + \frac{C_H}{2}\left[\frac{b(P-a)}{\alpha(P-a)+b}\right]T +$$

$$\left(\frac{I_Q + P_P + M_\beta + S_W\alpha}{2}\right)\left[\frac{b^2(P-a)}{(\alpha(P-a)+b)^2}\right]T + \frac{(P_D + S_S\alpha)}{2}\left[\frac{\alpha b(P-a)^2}{(\alpha(P-a)+b)^2}\right]T$$

So, the classical EPQ is

MinTac(T)= $= \frac{C_S + P_B + F_M + V_M + S + H + P_S + E_M}{T} + \frac{C_H}{2}\left[\frac{b(P-a)}{\alpha(P-a)+b}\right]T +$

$$\left(\frac{I_Q + P_P + M_\beta + S_W\alpha}{2}\right)\left[\frac{b^2(P-a)}{(\alpha(P-a)+b)^2}\right]T$$

$$+ \frac{(P_D + S_S\alpha)}{2}\left[\frac{\alpha b(P-a)^2}{(\alpha(P-a)+b)^2}\right]T \text{ such that } T > 0 \tag{22}$$

Solving (2.11) we can show that Tac(T) will be minimum for

$$T^* = \sqrt{\frac{2(C_S + P_B + F_M + V_M + S + H + P_S + E_M)(\alpha(P-a)+b)^2}{C_H b(P-a)\left[\alpha(P-a)+b\right] + (I_Q + P_P + M_\beta + S_W\alpha)(b^2(P-a)) + (P_D + S_S\alpha)(\alpha b(P-a)^2)}}$$

$$\text{Tac}^*(T^*) = \sqrt{\frac{2b(P-a)(C_S + P_B + F_M + V_M + S + H + P_S + E_M)\left[C_H(\alpha(P-a)+b) + (I + P_P + M_\beta + S_W\alpha)b + (P_D + S_S\alpha)\alpha(P-a)\right]}{(\alpha(P-a)+b)^2}}$$

Case 3: D and Ds are linear

$$\frac{dQ}{dt} = P - D \quad 0 < t \leq t_1 \tag{23}$$

$$\frac{dQ}{dt} = -\left[D + Ds + \alpha Q(t)\right] \; t_1 < t \leq T$$

$$\frac{dQ}{dt} + \alpha Q(t) = -\left[(a + bt) + (c + et)\right] \tag{24}$$

$$Q(0) = 0 \tag{25}$$

$$Q(T) = 0 \tag{26}$$

Let $Q(t_1) = I_{max}$ (27)

Solving (23) and (25) with initial conditions we get

$$Q(t) = (P - a)t \text{ for } \; 0 < t \leq t_1 \tag{28}$$

Solving (24) with (26) we get

$$Q(t) = \left(\frac{b + e}{\alpha}\right)[T - t] \text{for } \; t_1 < t \leq T \tag{29}$$

Using the equations (27) in (28) and (29) we get

$$Imax = (P - a)t_1$$

$$= \left(\frac{b + e}{\alpha}\right)[T - t_1]$$

$$t_1 = \frac{Imax}{P - a} \tag{30}$$

and

$$T - t_1 = \frac{\alpha Imax}{b + e} \tag{31}$$

By adding equations, (30) and (31) we get,

$$T = I_{max}\left(\frac{\alpha(P - a) + (b + e)}{(b + e)(P - a)}\right) \tag{32}$$

$$I_{max} = \frac{Tb(P - a)}{\alpha(P - a) + (b + e)}$$

Holding cost

$$= C_H\left[\int_0^{t_1} Q(t)\,dt + \int_{t_1}^{T} Q(t)\,dt\right]$$

$$=\frac{C_H}{2}\left[(P-a)t_1^{\,2}+\left(\frac{b}{\alpha}\right)(T-t_1)^2\right]$$

$$=\frac{C_H}{2}\left[\frac{(b+e)(P-a)}{\alpha(P-a)+(b+e)}\right]T^2$$

Total quality inspection cost

$$=I_Q\int_0^{t_1}Q(t)\,dt$$

$$=\frac{I_Q}{2}\left[(P-a)t_1^{\,2}\right]$$

$$=\frac{I_Q}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2$$

Precaution cost at times of production

$$=P_P\int_0^{t_1}Q(t)\,dt$$

$$=\frac{P_P}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2$$

Precaution cost at times of product distribution

$$=P_D\left[\int_{t_1}^{T}Q(t)\,dt\right]$$

$$=\frac{P_D}{2}\left[\frac{b}{\alpha}(T-t_1)^2\right]$$

$$=\frac{P_D}{2}\left[\frac{\alpha(b+e)(P-a)^2}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2$$

Mending cost

$$=M_\beta\int_0^{t_1}Q(t)\,dt$$

$$=\frac{M_\beta}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2$$

Shipping cost of products from production place to warehouse

$$=S_w\alpha\int_0^{t_1}Q(t)\,dt$$

$$=\frac{S_W\alpha}{2}\left[(P-a)t_1^{\,2}\right]$$

$$=\frac{S_W\alpha}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2$$

Shipping cost of products from warehouse to special customers

$$=S_S\alpha\left[\int_{t_1}^{T}Q(t)\,dt\right]$$

$$=\frac{S_S\alpha}{2}\left[\frac{(b+e)}{\alpha}\left(T-t_1^{\,2}\right)\right]$$

$$=\frac{S_S\alpha}{2}\left[\frac{\alpha(b+e)(P-a)^2}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2$$

Total cost = Set up cost + Holding cost + Total quality inspection cost + Precaution cost of defective items before production + Precaution cost at times of production + Production cost at time of production distribution + Mending cost + Shipping cost from production to warehouse + Shipping cost of the products from warehouse to the special customers + Fixed maintenance cost + Variable maintenance cost + Software cost + Hardware cost + Personnel cost + Equipment maintenance cost

$$=C_S+\frac{C_H}{2}\left[\frac{(b+e)(P-a)}{\alpha(P-a)+(b+e)}\right]T^2+\frac{I_Q}{2}\left[\frac{(b+e)b^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2+P_B+$$
$$\frac{P_P}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2+\frac{P_D}{2}\left[\frac{\alpha(b+e)(P-a)^2}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2+$$
$$\frac{M_\beta}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2+\frac{S_W\alpha}{2}\left[\frac{(b+e)^2(P-a)}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2+$$
$$\frac{S_S\alpha}{2}\left[\frac{\alpha(b+e)(P-a)^2}{\left(\alpha(P-a)+(b+e)\right)^2}\right]T^2+F_M+V_M+S+H+P_S+E_M$$

Total average cost Tac(T)

$$= \frac{C_S + P_B + F_M + V_M + S + H + P_S + E_M}{T} + \frac{C_H}{2}\left[\frac{(b+e)(P-a)}{\alpha(P-a)+(b+e)}\right]T +$$

$$\left(\frac{I_Q + P_P + M_\beta + S_W\alpha}{2}\right)\left[\frac{(b+e)^2(P-a)}{(\alpha(P-a)+(b+e))^2}\right]T + \frac{(P_D + S_S\alpha)}{2}$$

$$\left[\frac{\alpha(b+e)(P-a)^2}{(\alpha(P-a)+(b+e))^2}\right]T$$

So the classical EPQ is

MinTac(T)=

$$= \frac{C_S + P_B + F_M + V_M + S + H + P_S + E_M}{T} + \frac{C_H}{2}\left[\frac{(b+e)(P-a)}{\alpha(P-a)+(b+e)}\right]T +$$

$$\left(\frac{I_Q + P_P + M_\beta + S_W\alpha}{2}\right)\left[\frac{(b+e)^2(P-a)}{(\alpha(P-a)+(b+e))^2}\right]T +$$

$$+\frac{(P_D + S_S\alpha)}{2}\left[\frac{\alpha(b+e)(P-a)^2}{(\alpha(P-a)+(b+e))^2}\right]T \text{ such that } T > 0 \tag{33}$$

Solving (33) we can show that Tac(T) will be minimum for

$$T^* = \sqrt{\frac{2(C_S + P_B + F_M + V_M + S + H + P_S + E_M)(\alpha(P-a)+(b+e))^2}{\begin{gathered}C_H(b+e)(P-a)\left[\alpha(P-a)+(b+e)\right]+ \\ (I_Q + P_P + M_\beta + S_W\alpha)\left((b+e)^2(P-a)\right)+ \\ (P_D + S_S\alpha)\left(\alpha(b+e)(P-a)^2\right)\end{gathered}}}$$

$$\mathrm{Tac}^*(T^*) = \sqrt{\frac{\begin{gathered}2(b+e)(P-a)(C_S + P_B + F_M + V_M + S + H + P_S + E_M) \\ \left[\begin{gathered}C_H(\alpha(P-a)+(b+e)) + (I + P_P + M_\beta + S_W\alpha)(b+e) + \\ (P_D + S_S\alpha)\alpha(P-a)\end{gathered}\right]\end{gathered}}{(\alpha(P-a)+(b+e))^2}}$$

Case 4: D is constant and Ds is linear

$$\frac{dQ}{dt} = P - D \qquad 0 < t \le t_1 \tag{34}$$

$$\frac{dQ}{dt} = -\left[D + Ds + \alpha Q(t)\right] \; t_1 < t \le T$$

$$\frac{dQ}{dt} + \alpha Q(t) = -\left[D + (a + bt)\right] \tag{35}$$

$$Q(0) = 0 \tag{36}$$

$$Q(T) = 0 \tag{37}$$

Let $Q(t_1) = I_{max}$ (38)

Solving (34) and (36) with initial conditions we get

$$Q(t) = (P - D)t \text{ for } 0 < t \le t_1 \tag{39}$$

Solving (35) with (37) we get

$$Q(t) = \left(\frac{b}{\alpha}\right)[T - t] \text{ for } t_1 < t \le T \tag{40}$$

Using the equations (38) in (39) and (40) we get

$$Imax = (P - D)t_1$$

$$= \left(\frac{b}{\alpha}\right)[T - t_1]$$

$$t_1 = \frac{Imax}{P - D} \tag{41}$$

and

$$T - t_1 = \frac{\alpha Imax}{b} \tag{42}$$

By adding equations, (41) and (42) we get,

$$T = I_{max}\left(\frac{\alpha(P - D) + b}{b(P - D)}\right) \tag{43}$$

$$I_{max} = \frac{Tb(P - D)}{\alpha(P - D) + b}$$

Holding cost

$$= C_H\left[\int_0^{t_1} Q(t)\,dt + \int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{C_H}{2}\left[(P - D)t_1^2 + \left(\frac{b}{\alpha}\right)(T - t_1)^2\right]$$

$$= \frac{C_H}{2}\left[\frac{b(P-D)}{\alpha(P-D)+b}\right]T^2$$

Total quality inspection cost

$$= I_Q \int_0^{t_1} Q(t)\,dt$$

$$= \frac{I_Q}{2}\left[(P-D)t_1^{\,2}\right]$$

$$= \frac{I_Q}{2}\left[\frac{b^2(P-D)}{\left(\alpha(P-D)+b\right)^2}\right]T^2$$

Precaution cost at times of production

$$= P_P \int_0^{t_1} Q(t)\,dt$$

$$= \frac{P_P}{2}\left[\frac{b^2(P-D)}{\left(\alpha(P-D)+b\right)^2}\right]T^2$$

Precaution cost at times of product distribution

$$= P_D\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{P_D}{2}\left[\frac{b}{\alpha}(T-t_1)^2\right]$$

$$= \frac{P_D}{2}\left[\frac{\alpha b(P-D)^2}{\left(\alpha(P-D)+b\right)^2}\right]T^2$$

Mending cost

$$= M_\beta \int_0^{t_1} Q(t)\,dt$$

$$= \frac{M_\beta}{2}\left[\frac{b^2(P-D)}{\left(\alpha(P-D)+b\right)^2}\right]T^2$$

Shipping cost of products from production place to warehouse

$$= S_w \alpha \int_0^{t_1} Q(t)\,dt$$

$$= \frac{S_W \alpha}{2}\left[(P-D)t_1^{\,2}\right]$$

$$= \frac{S_W \alpha}{2}\left[\frac{b^2 (P-D)}{\left(\alpha (P-D)+b\right)^2}\right] T^2$$

Shipping cost of products from warehouse to special customers

$$= S_S \alpha \left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= \frac{S_S \alpha}{2}\left[\frac{b}{\alpha}\left(T - t_1^{\,2}\right)\right]$$

$$= \frac{S_S \alpha}{2}\left[\frac{\alpha b (P-D)^2}{\left(\alpha (P-D)+b\right)^2}\right] T^2$$

Total cost = Set up cost + Holding cost + Total quality inspection cost + Precaution cost of defective items before production + Precaution cost at times of production + Production cost at time of production distribution + Mending cost + Shipping cost from production to warehouse + Shipping cost of the products from warehouse to the special customers + Fixed maintenance cost + Variable maintenance cost + Software cost + Hardware cost + Personnel cost + Equipment maintenance cost

$$= C_S + \frac{C_H}{2}\left[\frac{b(P-D)}{\alpha (P-D)+b}\right] T^2 + \frac{I_Q}{2}\left[\frac{b^2 (P-D)}{\left(\alpha (P-D)+b\right)^2}\right] T^2 + P_B +$$

$$\frac{P_P}{2}\left[\frac{b^2 (P-D)}{\left(\alpha (P-D)+b\right)^2}\right] T^2 + \frac{P_D}{2}\left[\frac{\alpha b (P-D)^2}{\left(\alpha (P-D)+b\right)^2}\right] T^2 +$$

$$\frac{M_\beta}{2}\left[\frac{b^2 (P-D)}{\left(\alpha (P-D)+b\right)^2}\right] T^2 + \frac{S_W \alpha}{2}\left[\frac{b^2 (P-D)}{\left(\alpha (P-D)+b\right)^2}\right] T^2$$

$$+ \frac{S_S \alpha}{2}\left[\frac{\alpha b (P-D)^2}{\left(\alpha (P-D)+b\right)^2}\right] T^2 + F_M + V_M + S + H + P_S + E_M$$

Total average cost Tac(T)

$$= \frac{C_S + P_B + F_M + V_M + S + H + P_S + E_M}{T} + \frac{C_H}{2}\left[\frac{b(P-D)}{\alpha (P-a)+b}\right] T +$$

$$\left(\frac{I_Q + P_P + M_\beta + S_W \alpha}{2}\right)\left[\frac{b^2 (P-D)}{\left(\alpha (P-D)+b\right)^2}\right] T + \frac{(P_D + S_S \alpha)}{2}\left[\frac{\alpha b (P-D)^2}{\left(\alpha (P-D)+b\right)^2}\right] T$$

So the classical EPQ is

MinTac(T)= $= \dfrac{C_S + P_B + F_M + V_M + S + H + P_S + E_M}{T} + \dfrac{C_H}{2}\left[\dfrac{b(P-D)}{\alpha(P-a)+b}\right]T +$

$$\left(\frac{I_Q + P_P + M_\beta + S_W\alpha}{2}\right)\left[\frac{b^2(P-D)}{\left(\alpha(P-D)+b\right)^2}\right]T +$$

$$+\frac{(P_D + S_S\alpha)}{2}\left[\frac{\alpha b(P-D)^2}{\left(\alpha(P-D)+b\right)^2}\right]T \text{ such that } T > 0 \tag{44}$$

Solving (44) we can show that Tac(T) will be minimum for

$$T^* = \sqrt{\frac{2\left(C_S + P_B + F_M + V_M + S + H + P_S + E_M\right)\left(\alpha(P-D)+b\right)^2}{C_H b(P-D)\left[\alpha(P-D)+b\right] + \left(I_Q + P_P + M_\beta + S_W\alpha\right) \left(b^2(P-D)\right) + (P_D + S_S\alpha)\left(\alpha b(P-D)^2\right)}}$$

$$\text{Tac}^*\left(T^*\right) = \sqrt{\frac{2b(P-D)\left(C_S + P_B + F_M + V_M + S + H + P_S + E_M\right)\begin{bmatrix} C_H\left(\alpha(P-D)+b\right) + \left(I + P_P + M_\beta + S_W\alpha\right)b \\ +(P_D + S_S\alpha)\alpha(P-D) \end{bmatrix}}{\left(\alpha(P-D)+b\right)^2}}$$

4. Numerical Example

This model is an extension of the Classical EPQ model as represented in (Kumar Das & Kumar Roy, 2017)

The Inventory system where D and Ds are constants is analyzed with the following input parameters provided in Table 2.

Table 2. Model Values of Parameters for Case 1

C_S	P_B	F_M	V_M	S	H	P_S	E_M
10	20	20	20	13	10	5	5

P	D_S	C_H	I_Q	P_P	M_β	S_W	α
10000	500	10	5	5	10	12	0.5

D	P_D	S_S
1000	10	5

The Inventory system where D is linear and Ds is constant is analysed with the following input parameters provided in Table 3.

Table 3. Model Values of Parameters for Case 2

Cs	P_B	F_M	V_M	S	H	P_S	E_M
10	20	20	20	13	10	5	5

α	P	a	b	C_H	I_Q	Pp	M_β
0.5	10000	150	20	10	5	5	10

S_W	P_D	S_S
12	10	5

The Inventory system where D and D_S are linear is analysed with the following input parameters provided in Table 4.

Table 4. Model Values of Parameters for Case 3

Cs	P_B	F_M	V_M	S	H	P_S
10	20	20	20	13	10	5

E_M	α	P	a	b	C_H	I_Q
5	0.5	10000	150	20	10	5

Pp	M_β	S_W	P_D	S_S	c	e
5	10	12	10	5	1.7	3

The Inventory system where D is constant, and Ds is linear is analyzed with the following input parameters provided in Table 5.

Table 5. Model Values of Parameters for Case 4

Cs	P_B	F_M	V_M	I	H	P_S	E_M
10	20	20	20	13	10	5	5

α	P	a	b	C_H	I_Q	Pp	M_β
0.5	10000	105	20	10	5	5	10

S_W	P_D	S_S	D
12	10	5	500

5. Sensitivity Analysis

The sensitivity analysis is performed for each of the different cases using the model parameters.

In the production model where D and Ds are constants, the parameters P is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 1.

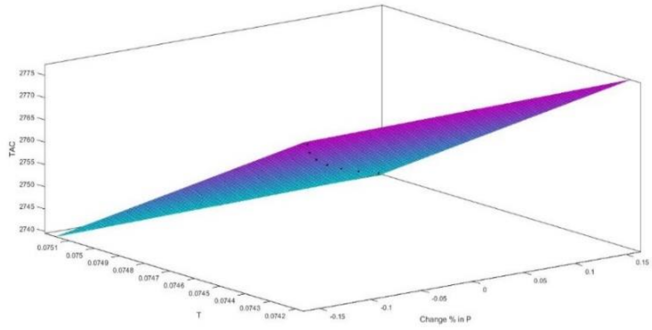

Figure 1. Pictorial Representation of TAC & T with respect to Change in P

In the production model where D and Ds are constants, the parameter α is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 2.

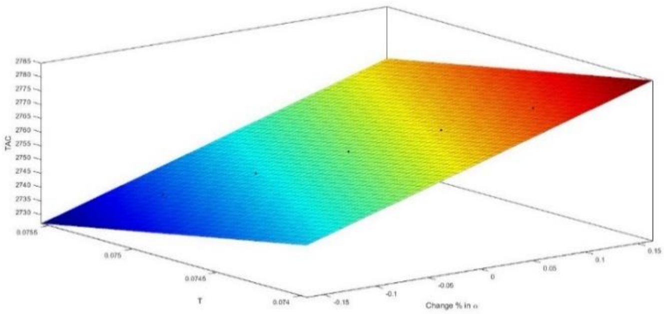

Figure 2. Pictorial Representation of TAC & T with respect to Change in α

As noted in the above tables,

1. The value of T increases as P increases and decreases when T decreases.
2. The value of Total average cost increases with respect to decrease in P and Total average cost decreases with respect to increase in P.
3. The value of T decreases as α increases and increases when T decreases.
4. The value of Total average cost increases with respect to increase in α and Total average cost decreases with respect to decrease in P.

In the production model where D is linear and Ds are constants, the parameter a is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 3.

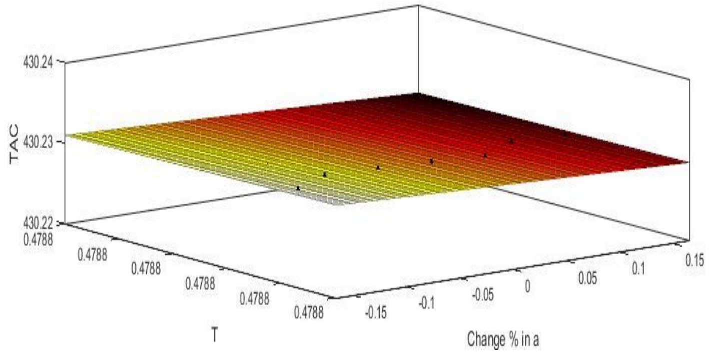

Figure 3. Pictorial Representation of TAC & T with respect to Change in a

In the production model where D is linear and Ds is constants, the parameters α is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 4.

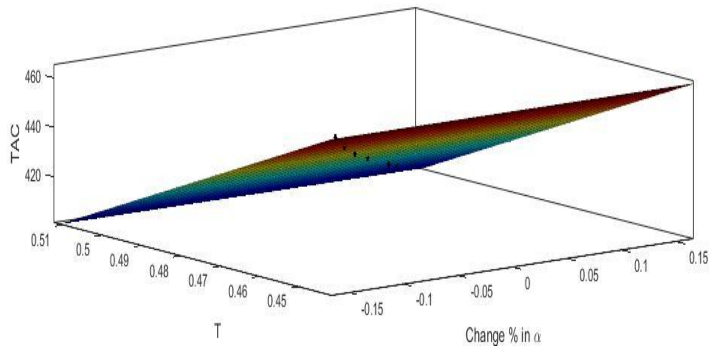

Figure 4. Pictorial Representation of TAC & T with respect to Change in α

In the production model where D is linear and Ds is constants, the parameters b is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 5.

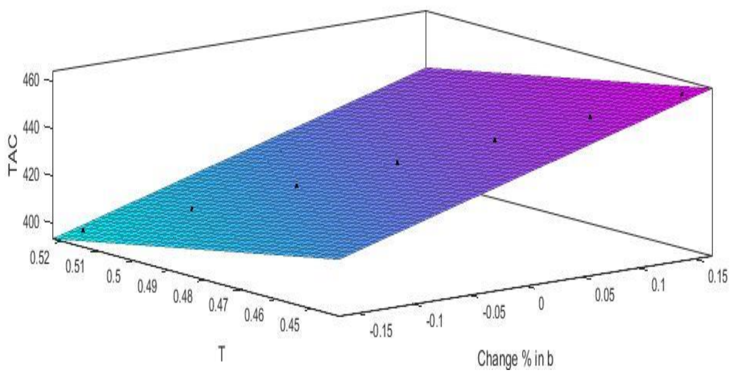

Figure 5. Pictorial Representation of TAC & T with respect to Change in b

In the production model where D is linear and Ds is constants, the parameter P are varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 6.

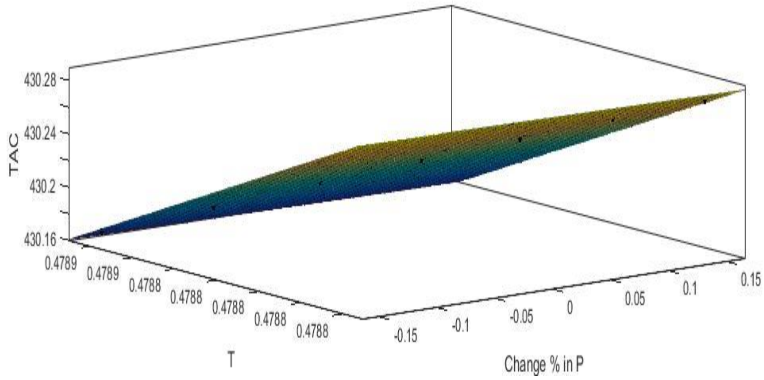

Figure 6. Pictorial Representation of TAC & T with respect to Change in P

As noted in the above tables,

1. The value of T increases as parameter a increases and decreases when a decreases.
2. The value of Total average cost increases with respect to decrease in a and Total average cost decreases with respect to increase in a.
3. The value of T increases as α increases and decreases when α decreases.
4. The value of Total average cost increases with respect to increase in α and Total average cost decreases with respect to decrease in P.
5. The value of T decreases as parameter b increases and increases when b decreases.
6. The value of Total average cost increases with respect to increase in b and Total average cost decreases with respect to decrease in b.
7. The value of T decreases as parameter P increases and increases when P decreases.
8. The value of Total average cost increases with respect to increase in b and Total average cost decreases with respect to decrease in b.

In the production model where D and Ds are linear, the parameters a are varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 7.

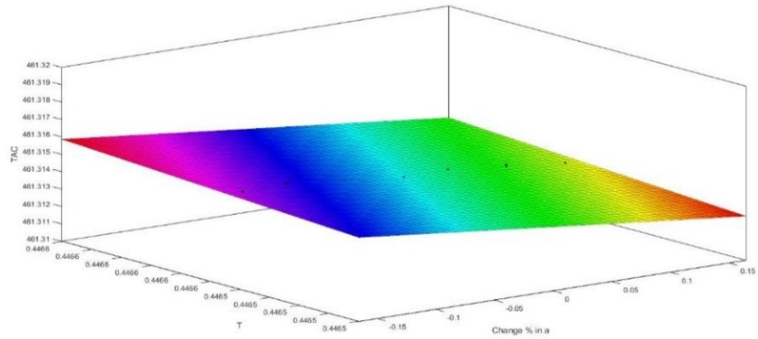

Figure 7. Pictorial Representation of TAC & T with respect to Change in a

In the production model where D and Ds are linear, the parameter α is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 8.

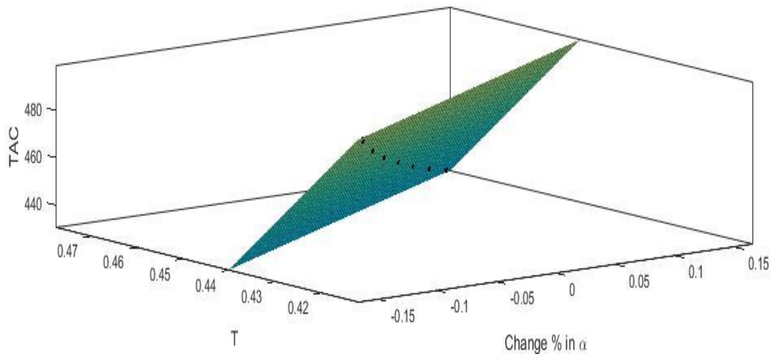

Figure 8. Pictorial Representation of TAC & T with respect to Change in α

In the production model where D and Ds are linear, the parameters b is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 9.

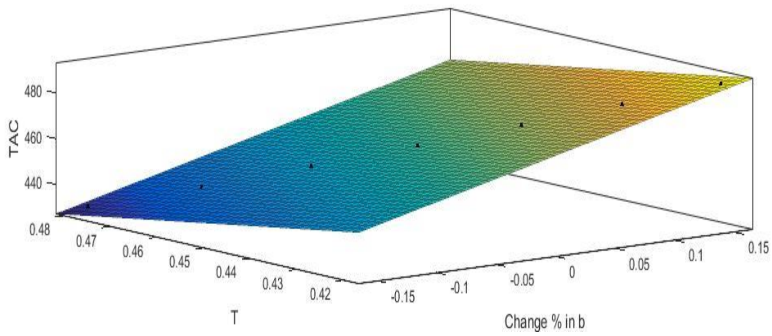

Figure 9. Pictorial Representation of TAC & T with respect to Change in b

In the production model where D and Ds are linear, the parameter c is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 10.

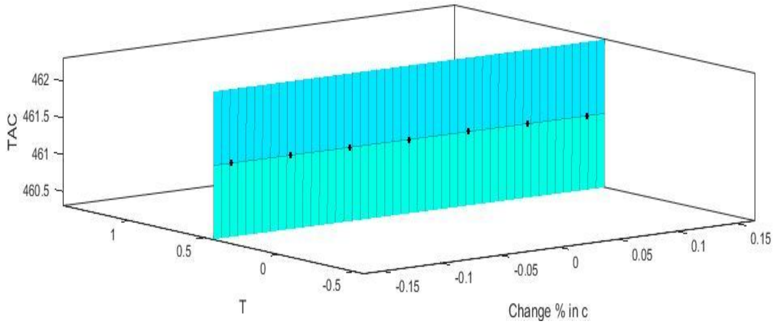

Figure 10. Pictorial Representation of TAC & T with respect to Change in c

In the production model where D and Ds are linear, the parameters e is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 11.

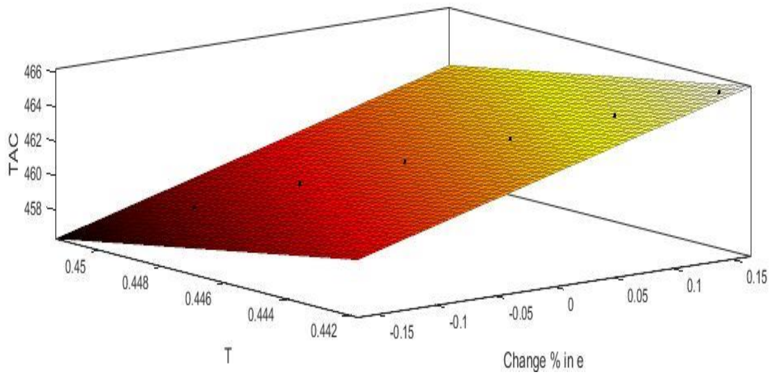

Figure 11. Pictorial Representation of TAC & T with respect to Change in e

As noted in the above tables,

1. The value of T increases as parameter a increases and decreases when a decreases.
2. The value of Total average cost decreases with respect to increase in a and Total average cost increases with respect to increase in a.
3. The value of T increases as α increases and decreases when α decreases.
4. The value of Total average cost decreases with respect to increase in α and Total average cost increases with respect to decrease in α.
5. The value of T decreases as parameter b increases and increases when b decreases.
6. The value of Total average cost increases with respect to increase in b and Total average cost decreases with respect to decrease in b.

7. The value of T remains constant with respect to change in c.
8. The value of Total average cost remains constant with respect to change in parameter c.
9. The value of T decreases as parameter e increases and increases when e decreases.
10. The value of Total average cost increases with respect to increase in e and Total average cost decreases with respect to decrease in e.

In the production model where D is constant and Ds is linear, the parameters α is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 12.

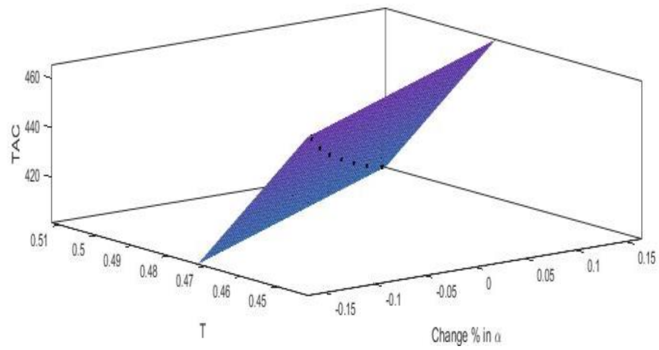

Figure 12. Pictorial Representation of TAC & T with respect to Change in α

In the production model where D is constant and Ds is linear, the parameters b and is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 13.

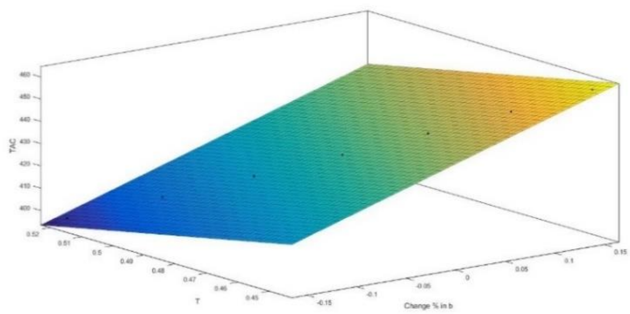

Figure13. Pictorial Representation of TAC & T with respect to Change in b

In the production model where D is constant and Ds is linear, the parameter P is varied from -15% to 15% with intervals of 5% and the variation in T and TAC are noted. It is represented in Figure 14.

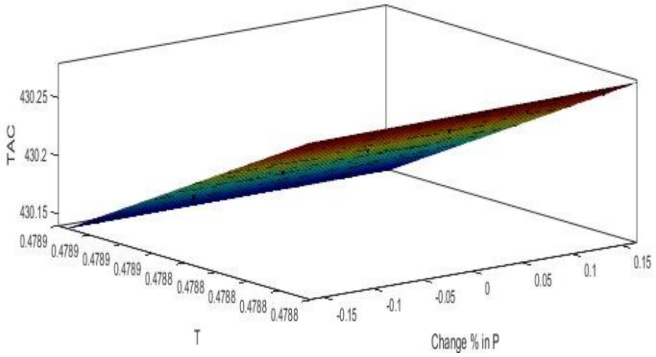

Figure 14. Pictorial Representation of TAC & T with respect to Change in b

As noted in the above tables,

1. The value of T decreases as P increases and increases when P decreases.
2. The value of Total average cost increases with respect to increases in P and Total average cost decreases with respect to decreases in P.
3. The value of T increases as α increases and decreases when α decreases.
4. The value of Total average cost decreases with respect to increase in α and Total average cost increases with respect to decrease in α.
5. The value of T decreases as b increases and increases when b decreases.
6. The value of Total average cost increases with respect to increase in b and Total average cost decreases with respect to decrease in b.

6. Conclusions

To build the business utilizing the smart production setup, this article suggests an inventory model centered on a customer-centric approach. This method will relocate a portion of the manufactured inventory to a specialized warehouse to meet consistent customers' needs. Consumers and manufacturers benefit equally from this setup since the consistent customers' demands will be met entirely and without delay, and the business will function smoothly with clients who are loyal to them. Secondary numerical data are used to illustrate the established model in this research.

The sensitivity analysis is performed to identify changes in the optimal production quantity and ANR due to changing parameters with fluctuating demand between general customer and consistent customer. Under fuzzy, intuitionistic, and neutrosophic contexts, the model's deterministic character can be expanded and addressed. This model will surely assist decision-makers in considering this customer-centric approach.

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