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RISK-AVERSE PRICING DECISIONS BASED ON PROSPECT THEORY

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Abstract: *This study examines the risk behaviour of a decision-maker regarding pricing decisions with the aid of the newsvendor model. In this regard, prospect theory and reference point concept are adopted to formulate the value function of the decision-maker. Unlike the traditional reference points (quantity-based), a reference point is deemed a function of the price. It is proved that a convex combination of the maximum-expected profits and expected losses represents the reference point. Closed-form solutions for the optimum price and quantity orders are obtained under uniformly and exponentially distributed demand. Moreover, the risk when the ordering quantity does not match the actual demand is discussed. The results-based numerical experiments reveal that the risk-averse decision-maker manages to increase the price to evade different expected costs, such as shortages and overstocking. Finally, for the same risk aversion level, the maximum reduction percentage of the optimal quantity concerning the price reaches approximately 8% in the exponential distribution, whereas it decreases by approximately 30% under the uniform distribution.*

Key words*: risk behaviour, pricing decisions, newsvendor model, prospect theory, decision-making.*

List of Notations

Q	*Ordered quantity/ number of products*
c	*Unit purchasing /production cost*
p	*Unit selling price*
D	*Expected market demand*
δ	*Random variable of demand error*
z	*Thowsen (1975) quantity transformation*
a, b	*Demand-price regression parameters*
h	*Overstock unit price*
s	*Shortage unit price*
$\pi(p, Q)$	*Profit function*

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$\theta(z)$	*quantity effect factor*
p^*	*Optimal price*
$R(p)$	*Price-based reference point*
α	*Gain loss change index*
$V(\Pi(p,z))$	*Value function*
γ	*Risk aversion level*
${p^*}_{ra}$	*Risk-averse optimal price*
${p^*}_{rn}$	*Risk-neutral optimal price*
β, ω	*Uniform distribution parameters*
${Q_{ra}}^*$	*Optimal ordering quantity for the risk-averse newsvendor*

1. Introduction

The traditional newsvendor (NV) model is a popular operations and inventory management tool for different applications (Khouja, 1999) . Decision-makers adopt the NV model to maximise their expected profit by estimating the optimal advance ordering or stocking quantities (Petruzzi & Dada, 1999). A single-period stochastic demand environment characterises the traditional model. Moreover, it deals with the risk-neutral decision-maker who targets maximising the expected profit. Regardless of the expected encountered risk, profit maximisation in the traditional NV model involves a threefold approach: determining the optimal stocking quantity under a fixed product price (Weng, 2004); determining the optimal product price for different quantity categories (Ye & Sun, 2016) and estimating the joint quantities and price (Raz & Porteus, 2006). The NV model is the recommended tool in many applications and industries, such as advance order quantity of perishable goods and production quantity of restaurants. Moreover, the model significantly affects the air cargo industry. For instance, it is used to manage with cargo capacity allocation (Hellermann, 2006). Wong et al. (2009) used the multi-item of the model to determine the baggage weight limits of passengers for airlines using passenger flights to accommodate cargo. Moreover, the NV model was adopted as a pricing tool to estimate the price of the extra-baggage as a special cargo service (Shaban, et al. 2019a; Shaban, et al., 2019b).

The study of decision-maker's risk behaviour is another growing research stream in the NV problem. In this stream, scholars adopt four risk analysis methods: the expected utility theory (EUT) (Keren & Pliskin, 2006), mean-variance analysis (Rubio-Herrero et al., 2015), conditional value-at-risk (CVaR) criteria (Xinsheng et al., 2015) and prospect theory (PT) and reference point method (R. Wang & Wang, 2018). In our research, we work on the risk behaviour of the decision-maker using PT with reference point analysis.

PT was first introduced to state and overcome the drawbacks of the EUT in risk analysis (Kahneman & Tversky, 1979). In this method, decision-makers use a reference point to state their preference: neutralised, seeking or aversion. This theory has become a widely used method in risk analysis studies. Moreover, a body of research in the risk-based NV approach was developed using PT to study the the decision-maker's risk preference. For example, the seminal work by Schweitzer and Cachon (2000) investigated decision-makers' behaviours and claimed that PT was unable to explain the loss aversion over ordering or stocking quantity; however, Long and Nasiry (2014) used a stochastic reference point to prove the risk aversion behaviour in the NV model. Further, as discussed in the literature review section, studies have been conducted on the relation between PT and risk aversion of the NV.

Apparently, although the risk-based NV model has attracted attention in the past decade, most studies focus only on characterising the reference point concerning the risk-averse NV model, e.g., Wang and Wang (2018). Furthermore, most studies deal with quantity ordering and stocking biases because of decision-makers' risk preferences(Vipin & Amit, 2019). The models are formulated only for one of the expected penalty costs: shortage or overstocking (R. Wang & Wang, 2018). Jammernegg et al. (2022) investigated the asymmetry and heterogeneity of quantity orders. However, by developing a risk aversion model for healthcare systems during the pandemic, Huang et al. (2022) concluded that the pandemic has changed the ordering behaviours of healthcare providers due to the emergency condition. Our paper addresses the price-based NV model to investigate the decision-maker's behaviour using the pricing model. On this subject, we first adopt the setup of Mills (1959) for the demand-based additive price used to formulate the expected profit of the decision-maker. Second, unlike the previous work, we formulate the reference point as a function of the selling price. Third, our model includes both the expected shortage and overstocking values. Fourth, the risk behaviour of the decision-maker during pricing is studied in two aspects: (i) when the overstocked quantity has a positive salvage value and (ii) when it is negative or a penalty cost and how it is applied to the air cargo industry. It is found that the risk-averse decision-maker's product price increases with the increase of risk level. Furthermore, the optimal quantity ordered for the risk-averse decision-maker is derived for both uniform and normal distributions. It is found that they share similar pricing behaviours except that the price difference between each risk aversion level is higher in the exponential distribution than in the uniform distribution.

The rest of the paper is organised as follows. The existing literature of the relevant work is reviewed in Section 2. In Section 3, we formulate the problem in terms of the price-based NV model; moreover, the optimal risk-averse price and quantity are derived in this section. Section 4 contains the numerical analysis for the uniform continuous and exponential distributions. The conclusions and future work are discussed in Section 5.

2. Literature Review

The relevant literature of this research includes three research streams: the studies on the traditional NV model, studies on the risk-averse NV model and related studies on the risk-averse NV models under the PTand reference points.

2.1. Traditional Newsvendor Model

The traditional NV-based price model was first introduced by the economist Whitin (1955), who formulated the model to calculate the advance order quantity and selling price simultaneously. To derive the model, he assumed that the demand probability distribution is a selling price function. Then, he solved the problem in two sequential steps: he first estimated the optimal ordering quantity and then estimated the optimal price considering the optimal quantity equation. Numerous studies have made subsequent improvements and variants to find the optimal price for a fixed class of quantity, e.g., Mills (1959), who developed the most common form of the NV-based price model. He suggested that demand is a random variable that changes with the change in the selling price. A seminal paper by Petruzzi and Dada (1999) summarised most related models in the traditional NV-based pricing model. The traditional NV model was mainly derived and developed to study the risk-neutral decision-maker. For instance, in the aviation industry, it was used to determine air cargo prices, as demonstrated by Shaban et al. (2019). Similarly, Park & Ryu (2023) employed the model in e-commerce retailing to estimate the optimal quantity; however, they found that different decision-makers have different risk behaviours or preferences, which has been included in the traditional NV model. Schweitzer and Cachon (2000) showed this effect through an empirical study, highlighting that managers' decisions differ from the optimal solution resulting from the risk-neutral NV model.

2.2. Risk-averse Newsvendor Model

In this regard, extant research has developed a risk-averse NV model. Researchers in this area use three risk management tools: the mean-variance analysis, value-at-risk (VaR) or CVaR method and EUT. The mean-variance method is used to study the risk-averse NV for optimal ordering with different variable effects, such as in Wu et al., (2009), who studied the impact of stockout cost on optimal ordering quantity for the risk-averse NV. It is also used to determine the optimal pricing decisions for risk-seeking and risk aversion situations (Rubio-Herrero et al., 2015) . However, the mean-variance model is inefficient with asymmetric probability distributions. In our model, we found a closed-form solution for optimal pricing and ordering quantities with an exponential distribution.

Regarding the CVaR (or VaR) criteria, Chen et al. (2009) used this approach to study the risk-averse NV in both additive and multiplicative demand forms. They showed monotonicity characteristics in the optimal ordering and pricing decisions. Unlike most of the research in the traditional and risk-based NV models, Gotoh and Takano (2007) used a convex optimisation function to minimise the CVaR. The authors formulated a linear programming model with two loss functions to demonstrate this objective. Xu (2010) considered a risk-averse seller in the case of emergency purchases, proving that the price of the risk-averse seller is less than the risk-neutral seller in both additive and multiplicative demand functions. Furthermore, NV pricing behaviour changes when the competition factor is involved in pricing and ordering. In the case of emergency orders, the risk-averse NV reduces the price when the level of risk aversion increases (M. Wu et al., 2014). Dai & Meng (2015) studied the marketing effort concerning pricing decisions for the risk-averse NV using the CVaR criterion and showed that the marketing efforts increase the optimal ordering quantity and do not affect the price. Chen Chen (2023) developed machine learning techniques to incorporate decision-makers' risk preferences into the NV models. The author induced that the asymmetric CVaR criterion provides a novel approach to exploring the impact of varying risk preferences for different losses on order decisions.

Additionally, the EUT is used in the literature to study the ordering and pricing behaviours of decision-makers through the NV model. For example, Katariya et al., (2014) made comparisons between the EUT method and mean-variance analysis for the risk-averse NV and then for the EUT and CVaR. They concluded that the results from the EUT and mean-variance analysis were nearly consistent, whereas an inconsistency was found between the EUT and CVaR results. Conversely, when Rabin Rabin (2000) investigated the EUT in the risk-averse NV model, he claimed that the risk-averse NV approach provides unrealistic results in the big stack business compared to results in the small stack business. This claim has been supported by Wang et al. (2009), who stated that the risk-averse decision-maker orders less than the risk-neutral, while pricing higher than the optimal risk-neutral price. Kahneman and Tversky (1979) addressed the limitations of the EUT, showing that the dependence on the probability of the EUT is one of its limitations; hence, they proposed the PT. They replace the standard probability of the event with the event value and weight for this event. The summation of the event's weights may exceed unity, contrary to the probability theory when using the EUT. Furthermore, they stated that the decision-makers decide the stocking quantity based on a certain reference point.

2.3. Newsvendor Models under Prospect Theory

From this standpoint, a fourth stream of research has been established based on PT to study the risk-averse NV model. PT, accompanied by the reference point concept, is used in these studies. For instance, multilocation NV models have been studied by Ho et al. (2010). They applied structural analysis to propose a behavioural theory to examine the decision-maker's preferences at different risk levels. They used the principles of PT and built their model based on the reference point. Consequently, their results are consistent with the PT propositions. Nagarajan and Shechter (2013) conducted an empirical study to investigate the impact of PT on the risk-averse NV model, showing that PT is unable to explain their empirical results. Contrastingly, Long and Nasiry (2014) showed that the PT gives better results in explaining the NV ordering behaviour when reconsidering the formulation of the reference point, which provides a reasonable prediction of the pull-to-centre effect (Shen et al., 2017). Subsequent research was built based on the reference point as a function of the quantity. For example, Wang and Wang (2018) used a probabilistic reference point to study the risk-averse decision-maker ordering behaviour. Additionally, Vipin and Amit (2019) used the reference point based on the quantity to describe the non-linearity ordering of the risk-averse decision-maker when using the NV model. In the industry, the integration between the NV model and PT was used to investigate the optimal purchasing decisions for their commodities using a mismatch cost minimisation criterion and incorporating risk aversion (Xu et al., 2023).

2.4. Research Gaps

The reference point as a function of the ordered quantity gives a reasonable explanation of the risk behaviour of the decision-maker when using the NV model; however, it is not logical to apply the same reference point structure when the decision-maker decides on the optimal selling price. In this regard and unlike previous research, we believe that as risk-averse decision-makers have a reference point regarding the quantity to estimate the optimal ordering or stocking quantity, they also have a reference price when making a pricing decision. Therefore, we model the reference point as a function of the price to study the behaviour of risk-averse decision-makers when they set the optimal selling price. Including the expected losses in the reference point, such as salvage, shortage and opportunity cost, is also necessary. The following section describes the details of the theoretical model and the reference point-based price.

3. Model Formulation and Theoretical Analysis

Suppose a decision-maker orders Q quantity of products at a purchasing cost c per unit. The decision-maker aims to set the unit selling price p of the product to maximise the expected profit. The market demand is uncertain and dependently decreases on the product selling price with an additive uncertainty form, such that $D(p,\delta) = d(p) + \delta$, where δ is the random variable of the demand error. $d(p)$ is the deterministic demand-based price function, where $a > 0$ and $b > 0$. Because of the market demand uncertainty, the decision-maker experiences two scenarios when ordering the quantity Q. First, the quantity Q exceeds the market demand $D(p,\delta)$. Second, the market demand $D(p,\delta)$ is greater than the ordered quantity Q. In the first scenario, the difference between the ordered quantity and the market demand incurs an overstocking cost of $h[Q - D(p,\delta)]$ for the decision-maker, with $h \geq 0$ being the unit overstocking cost. In the second scenario, the shortage due to the high market demand and low ordered quantities causes the decision-maker to incur a loss of $s[D(p,\delta) - Q]$, with s as a unit shortage cost.

Let $f(\cdot)$ be the probability density function of the random variable δ. $F(\cdot)$ is a cumulative distribution function, and the mean and variance of this random variable are μ and σ, respectively. The decision-maker's payoff is described by the profit gained from selling the ordered quantity at any price p. The profit is a function of the ordered quantity and product price and is described as follows:

$$\Pi(p,Q) = \begin{cases} (p-c)Q - s[D(p,Q) - Q], & Q < D(p,Q) \\ pD(p,Q) - cQ - h[Q - D(p,Q)], & Q \geq D(p,Q) \end{cases} \tag{1}$$

This profit is transformed into a price-based profit function by using the Thowsen (1975) formula $Q = d(p) + z$ and the demand-based price function (Petruzzi & Dada, 1999). Then, the profit function changes to

$$\Pi(p,z) = \begin{cases} (p-c)[d(p) + p\delta - cz - h[z-\delta], & \delta < z \\ (p-c)[d(p) + z)] - s[\delta - z)], & \delta \geq z \end{cases} \tag{2}$$

The risk-neutral optimal price, which maximises the profit of the decision-maker from Equation (2) is given by $p^* = \frac{a+bc+\mu-\theta(z)}{2(b+1)}$, where $\theta(z) = \int_0^z (z-x)f(z)dz$ is the quantity effect factor. From the same equation, the *critical fractile* is $(c+h)/(p+s+h) \in [0,1]$ (Whitin, 1955).

In this research, we use PT to analyse the risk behaviour of the decision-maker in pricing decision-making when using the NV model. A reference point-based price is used to characterise the risk behaviour of the decision-maker. The existence of shortage and overstocking penalties is included in this model. We assume that the reference point is based on minimal profit, where the product price characterises the quantity. Let us consider the possible costs in our model, including the opportunity cost plus either shortage or overstocking cost.

$$R(p) = \alpha(p-c)p + (1-\alpha)(p+s+h)p \tag{3}$$

where $0 \leq \alpha \leq 1$ is the gain loss change index. This value indicates the amount of maximum-expected profit relative to the maximum-expected losses. When α increases, the profit increases, whereas the losses decrease. Indeed, the reference point in this model has a broader meaning than in previous models. This reference point is located between the minimum price in the first term and the maximum possible price when the decision-maker adds shortage and/or overstocking penalties, as in the second part. The convexity of the reference point has been adopted in many operations management and marketing studies (Long & Nasiry, 2014; Vipin & Amit, 2019; R. Wang & Wang, 2018).

Considering the zero initial wealth of a risk-averse decision-maker, the value function can be described as follows:

$$V(\Pi(\mathrm{p,z})) = \begin{cases} \Pi(\mathrm{p,z}) - R(p), & R(p) < \Pi(\mathrm{p,z}) \\ -\gamma[R(p) - \Pi(\mathrm{p,z})], & R(p) \geq \Pi(\mathrm{p,z}) \end{cases} \tag{4}$$

where γ is the risk aversion factor. The value of this coefficient captures the decision-maker behaviour. The decision-maker is risk-neutral when $\gamma = 1$ and risk-averse when $\gamma > 1$. Unlike previous studies and models that focused on the quantity-based reference points and the risk behaviour of the decision-maker during the ordering process, Equation (4) provides the value function, the reference to the product price and the quantity factor.

Proposition 1: For a differentiable, decreasing and additive demand function, the risk-averse decision-maker tends to set higher prices than the risk-neutral decision-maker, such that ${p^*}_{ra} \geq {p^*}_{rn}$ when $z = \mu$ and $\alpha c + 1 + (1-\alpha)(-s-h) > \frac{1}{\gamma-(\gamma-1)F(z)}$, where $0 < \alpha < 1$ and $\gamma > 1$.

Proof: By using the value function that characterises the risk behaviour of the decision-maker in Equation (4), the expected wealth or value can be estimated from the following equation:

$$\begin{aligned} E[V(p,z)] = [(p-c)[d(p)+z] - \alpha p(p-c) - p(1-\alpha)(p+s \\ + h)] \int_0^z f(x)dx - (p \\ + h) \int_0^z (z-x) f(x)dx \\ + \gamma[(p-c)[d(p)+z] - \alpha p(p-c) \\ - p(1-\alpha)(p+s+h)] \int_z^\infty f(x)dx \\ - \gamma s \int_z^\infty (x-z) f(x)dx \end{aligned} \quad (5)$$

In the literature, the decreasing linear demand function is $d(p) = a - bp$. By substituting this form in Equation (5), the first derivative of the value function concerning the price can be obtained from Equation (6):

$$\begin{aligned} \frac{\partial E[V(p,z)]}{\partial p} = (1-\gamma)[a - 2bp + bc + z + \alpha c + (1-\alpha)(-s-h) \\ - 2p]F(z) - \theta(z) \\ + \gamma[a - 2bp + bc + z + \alpha c + (1-\alpha)(-s-h) - 2p] \end{aligned} \quad (6)$$

The second partial derivative with $E[V(p,z)]$ regarding the price p yields the following:

$$\frac{\partial^2 E[V(p,z)]}{\partial p^2} = (1-\gamma)[-2(b+1)F(z) - 2\gamma(b+1)$$

This makes the value function strictly concave and the optimum price per unit ${p^*}_{ra}$, satisfying Equation (6), which gives the maximum profit such that

$$(1-\gamma)[a - 2bp + bc + z + \alpha c + (1-\alpha)(-s-h) - 2p]F(z) - \theta(z) + \\ \gamma[a - 2bp + bc + z + \alpha c + (1-\alpha)(-s-h) - 2p] = 0.$$

Thus, the optimal price for the risk-averse decision-maker is as follows:

$$\begin{aligned} {p^*}_{ra} = \frac{(bc + a + z) - \theta(z)}{2(1+b)} + \frac{\theta(z) + \alpha c + (1-\alpha)(-s-h)}{2(b+1)} \\ - \frac{\theta(z)}{2(b+1)[\gamma - \gamma(F(z)]} \end{aligned} \quad (7)$$

For $z = \mu$, the risk-averse price can be written as follows:

$$p^*_{ra} = p^*_{rn} + \frac{\theta(z) + \alpha c + (1-\alpha)(-s-h)}{2(b+1)} - \frac{\theta(z)}{2(b+1)[\gamma - (\gamma-1)F(z)]}$$

For each $\gamma > 1$, $0 \leq F(z) \leq 1$. ■

Proposition 1 describes the relation between the risk-neutral and risk-averse NV models with respect to the pricing decisions. The relation shows that the risk-averse decision-maker tends to increase the price relative to the risk-neutral counterpart to avoid the expected losses. The price increase of the risk-averse decision-maker is a function of the difference between the convex combination of the product cost and negative overstock and/or shortage penalty costs and the ratio of the expected shortage $\theta(z)$ when the decision-maker orders an extremely low quantity and when the ordered quantity $F(z)$ are non-negative values. Furthermore, it is necessary to estimate the optimal quantity compatible with the prices when the decision-maker is risk-averse. Proposition 2 demonstrates the optimal ordering quantity of the risk-averse decision-maker.

Proposition 2: Under uniform distribution $U \sim (\beta, \omega)$, the optimum quantity ordering based on the price-based function for the risk-averse decision-maker is ${Q_{ra}}^* = (\omega - \beta)\left[\frac{\gamma(p+s-c)}{c+h+\gamma(p+s-c)}\right] + d(p) + \beta$.

Proof: The partial derivative of Equation (5) concerning the quantity factor z is demonstrated as follows:

$$\frac{\partial E[V(p,z)]}{\partial z} = -\big(c + h + \gamma(p+s-c)\big)F(z) + \gamma(p+s-c) \tag{8}$$

The second partial derivative of the same equation is $\frac{\partial^2 E[V(p,z)]}{\partial z^2} = -\big(c + h + \gamma(p+s-c)\big)f(z)$.

The first and second derivatives indicate that the value function is strictly concave in the quantity factor z when $\big(c + h + \gamma(p+s-c)\big)F(z) > \gamma(p+s-c)$. A unique z^* value maximises the profit of the risk-averse decision-maker and satisfies Equation (8), such that $-\big(c + h + \gamma(p+s-c)\big)F(z) + \gamma(p+s-c) = 0$; thus,

$$F({z_{ra}}^*) = \frac{\gamma(p+s-c)}{c+h+\gamma(p+s-c)} \tag{9}$$

For a fixed price and referring to the definition by Thowsen (1975), $q^* = z^* + d(p)$. When the demand is uniformly distributed $U \sim (\beta, \omega)$, the cumulative distribution is $F(z^*) = \frac{z^*-\beta}{\omega-\beta}$.

By applying $F(z^*)$ the uniform distribution to Equation (9) and transferring ${z_{ra}}^*$ to ${Q_{ra}}^*$ using Thowsen (1975) formula, we find that ${Q_{ra}}^* = (\omega - \beta)\left[\frac{\gamma(p+s-c)}{c+h+\gamma(p+s-c)}\right] + d(p) + \beta$. ■

According to Petruzzi and Dada (1999), the price-based function gives different variants to the ordering or stocking quantity of the risk-averse decision-maker. The function $d(p)$ can be either linear $d(p) = a - bp$ or an isoelastic $d(p) = ae^{-bp}$ function. Furthermore, the optimal ordering or stocking quantity depends on the demand distribution of each decision-maker, thereby leading to the following lemma.

Lemma 1: When the risk-averse decision-maker demand is exponentially distributed $EXP(1/\lambda)$, the optimum quantity ordering decision is ${Q_{ra}}^{*} = d(p) - \mu \ln\left[1 - \frac{\gamma(p+s-c)}{c+h+\gamma(p+s-c)}\right]$.

Proof: From Equation (9), the optimum quantity order $F({z_{ra}}^{*})$ is a function of the risk aversion factor, and the standard exponential cumulative distribution is $F(z^{*}) = 1 - e^{-\lambda z}$. By equating the standard exponential cumulative distribution to the right-hand side in Equation (8), the optimal ordering/stocking quantity of the risk-averse decision-maker is ${Q_{ra}}^{*} = d(p) - \mu \ln\left[1 - \frac{\gamma(p+s-c)}{c+h+\gamma(p+s-c)}\right]$. ■

Figure 1 provides an overview of the model's reasoning. The model uses three inputs to generate a decision: quantity, anticipated demand and unit costs. The model comprises two scenarios: the NV may order more than the market demand or the order may not meet the actual demand. Both possibilities are considered when calculating the expected profit of the conventional NV model. A reference-based pricing function is devised to address the NV's risk aversion. In this step, the expected profit is replaced by a value function that includes the NV's risk level. This model has the advantage of determining the best prices for a given order and the optimal order quantity for a given unit price.

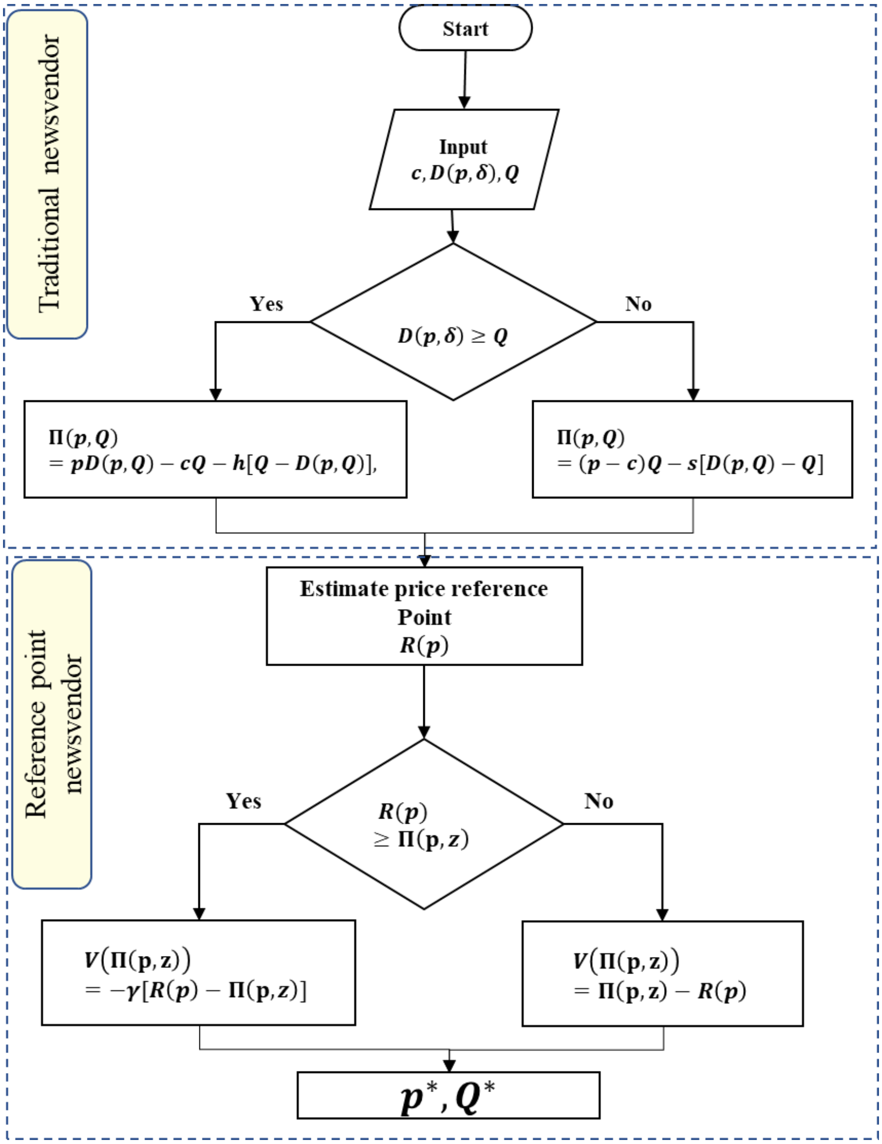


Figure 1. Flowchart of the proposed model.

4. Numerical Analysis and Results

Several numerical analyses were conducted to illustrate the pricing behaviour of a risk-averse decision-maker. The stochastic market demand was estimated based on the linear-decreasing demand-based price function. The demand random error δ was performed based on the uniform continuous distribution and exponential distribution. These experiments were conducted to study the effect of quantity and price on the value function. Furthermore, sensitivity analysis was performed on the relation between the optimal price of the risk-averse decision-maker and the ordered quantity.

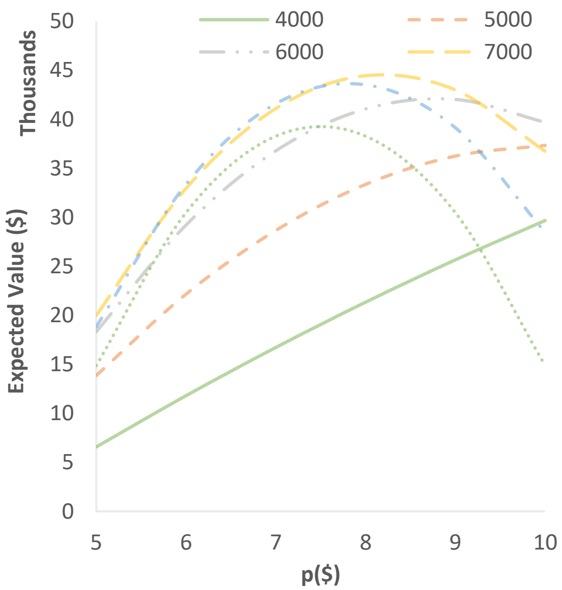
Additionally, the effect of the risk aversion factor on the optimal price and quantity was considered.

The model parameters are summarised as follows. The unit purchasing cost $c = 3$, the unit overstock cost $h = 2$, the unit shortage cost $s = 1.5$ and the profit–loss index is chosen as $\alpha = 0.75$. The experiments are reported for the five values of the risk aversion factor $\gamma = 1, 1.5, 2, 2.5, 3$.To investigate the value function concavity in both price and quantity, we examine the value function for the risk aversion factor $\gamma = 1.5$, and the selected values of the price and the quantity are shown in Table 1.

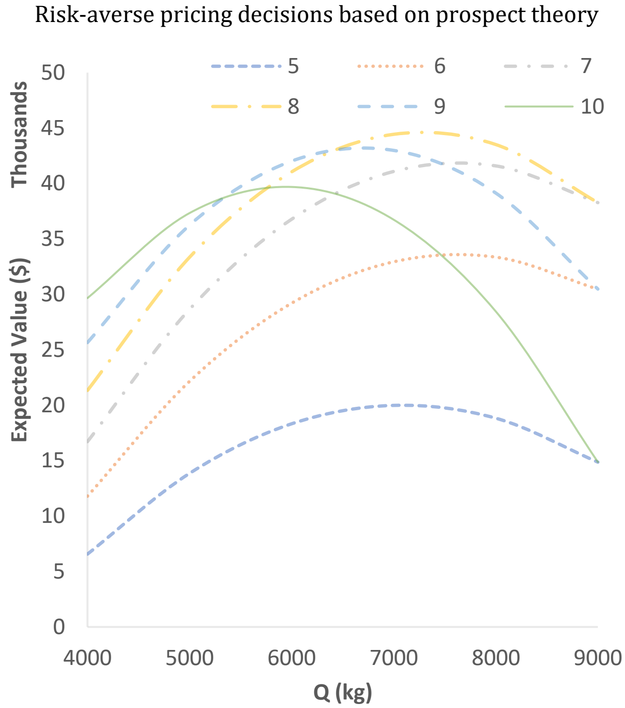
Table 1. Selected values of quantity and price.

p	5	6	7	8	9	10
Q	4000	5000	6000	7000	8000	9000

As stated in the proofs of Propositions 1 and 2, Figure 2(a) reveals that the value function has a unique optimal solution for the price, for a fixed quantity, by obtaining the maxima of the value function concerning the price. Similarly, for different price categories, a unique quantity maximises the value functions for each price, as shown in Figure 2(b).



(a)

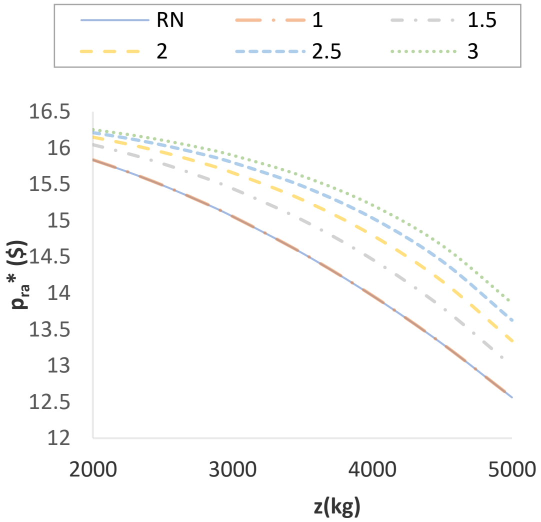


(b)

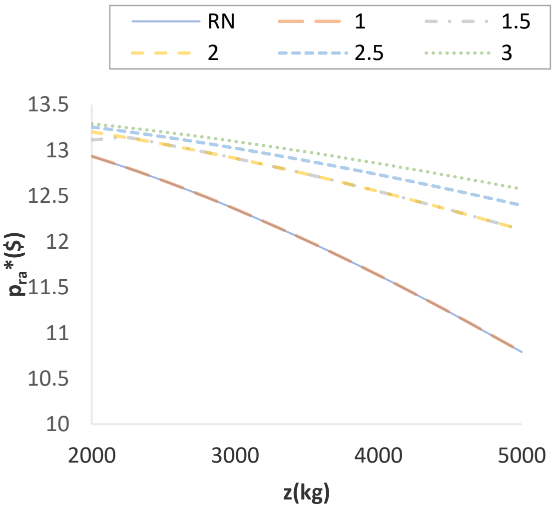
Figure 2. The expected value function for $\gamma = 1.5$: (a) The expected value is a function of price for quantities ranging from 4,000 kg to 9,000 kg; (b) The expected value is a function of quantity for price ranges from 5 to 10 US dollars.

Figure 2(b) shows that the expected value increases systematically for the price values from 5 to 10; then, the expected value curves overlap, and the maximum-expected value is obtained from $p = 7$. This behaviour is compatible with Petruzzi and Dada, (1999), who stated that profit changes by changing both the selling price and ordering/stocking quantity.

We conducted two numerical experiments to investigate the decision-maker's pricing behaviour for different risk aversion levels. Figure 3 shows the pricing behaviour of the risk-averse decision-makers with different risk aversion levels. With a superficial observation, it is easy to see that the demand distribution significantly affects the optimal prices because of the effect of the shortage function on optimum price. In more detail, the shortage function is the uniform distribution, $[\theta(z)]_U = 0.5z^2/(b-a)$, and the shortage function of the exponential distribution is $[\theta(z)]_{Exp} = \mu(e^{-\lambda z} + z\lambda - 1)$. By substituting these two values in the risk-averse optimal price equation, the price behaviour inevitably varies. Furthermore, regardless of the probability distribution of the demand, the price curve is exactly the same as the price line of the risk-neutral decision-maker when $\gamma = 1$. As shown in Proposition 1, the risk-averse decision-maker tends to increase the price to avoid different expected costs, such as shortages and overstocking. The price increases with an increase in the risk aversion level. In the uniform distribution case, the results of the risk-averse decision-maker are relatively scattered from the risk-neutral one. Furthermore, the price decreases with the increase in quantity in both distributions.



(a)

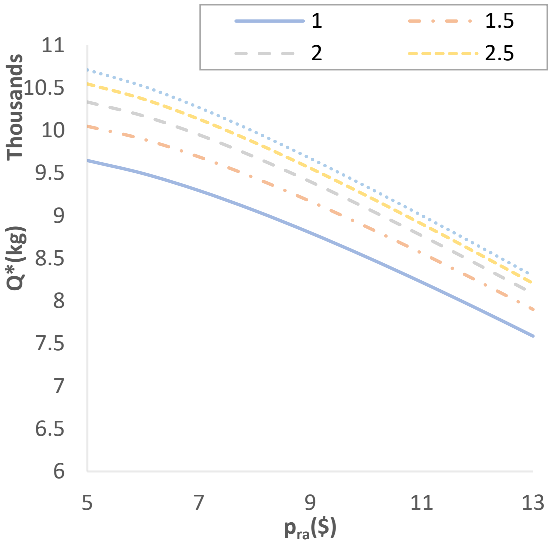


(b)

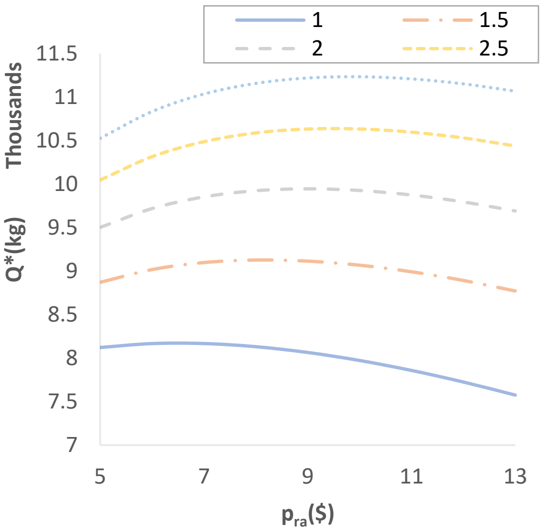
Figure 3. The optimum price for a risk-averse decision-maker,a risk-neutral decision-maker and risk-averse decision-maker with risk levels (1.5,2,2.5,3): (a) Uniform distribution for $U\sim(20000,6000)$; (b) Exponential distribution $Exp\sim(1/_{\lambda} = 40000)$.

An advantage of the NV-based price model is that it can also estimate the optimum ordering quantity. Propositions 2 and Lemma 1 give the optimum ordering quantity when the demand is uniformly distributed and its variation when the demand is exponentially distributed.

Figure 4 shows the decision-maker's behaviour when selecting the optimal quantity concerning the unit price. Similarly, the optimum quantity is affected by the demand distribution.



(a)



(b)

Figure 4. The relation between the optimum risk-averse ordering quantity and product price with risk levels (1.5,2,2.5,3): (a) Uniform distribution $U\sim(20000{,}6000)$; (b) Exponential distribution $Exp\sim({}^{1}/_{\lambda}=4000)$.

Figure 4(a) shows that in the uniformly distributed case, the optimum ordering quantity decreases monotonically with the increase in product unit price. In Figure

4(b), under the exponentially distributed demand case and except for the optimal order when $\gamma = 1$, the optimum quantity moves concavely concerning the unit price of the product, i.e., the decision-maker raises the order when the price increases until a price threshold p_{th} is reached. If the price is beyond this threshold, the decision-maker reduces the order. The optimal ordering curves for $\gamma = 1$ have a similar inclination in both distributions because these two curves represent the risk-neutral NV's ordering behaviour.

For a certain risk aversion level, the maximum reduction percentage of the optimal quantity concerning the price reaches approximately 8% in the exponential distribution, whereas it decreases by approximately 30% under uniform distribution. Moreover, for a specific price, the optimal ordering quantity increases with the increase in risk aversion level; however, the percentage of the increase decreases by raising the risk level. For example, under exponential distribution, at a unit product price of $p = 9$, the percentage of the optimal quantity difference starts at 13% when moving from $\gamma = 1$ to $\gamma = 1.5$, whereas it is 5% between the risk aversion levels $\gamma = 2.5$ and $\gamma = 3$.

5. h-value Variants and the Overbooking Problem

Usually, decision-makers start planning with no information about the actual market demand; therefore, they use different demand forecasting techniques to predict the quantity needed to produce, stock, buy or offer for selling. However, they cannot forecast the exact market demand because it is always random. When the decision-makers' quantity calculations exceed the actual market demand, the difference between the ordering/stocking quantity and market demand is an overestimated quantity Q-D, and the h-value is used. This section discusses the different variations in the h-value. The h-value can be either positive, negative or zero; the variations in the h-value affect the profits of the decision-maker and, in turn, their risk behaviour.

The negative h-value variant is introduced and used as a penalty cost in the above model, meaning that the decision-makers' expenses increase by h for each overestimated unit. The most common application of this negative h-value is the overbooking problem in airlines. For example, airlines offer cargo capacity for booking, and although this capacity is limited, they sell more than their actual capacity because they offer the capacity for a specific flight a year in advance. Again, because the market demand is dynamic and unpredictable, buyers such as logistics companies and freight forwarders cancel some of the order quantity a few days before the flight departure, and others do not show up at all (Shaban et al., 2021). These two reasons lead to an underutilised cargo flight. In this regard, airlines sell the actual flight capacity and overbook to some extent to compensate for the cancellations and no-shows (Weatherford & Bodily, 1992); however, this is not the perfect situation because airlines always have an overbooking risk. The overbooking risk is derived from the overestimated overbooking quantity. As a decision-maker, airlines experience two penalty costs due to the overbooked air cargo quantity: the penalty for delay paid to the consignee and/or freight forwarder and the stacking fee to the airport for the offloaded cargo. The sum of these delay fees represents the h-value.

Many air cargo studies adopted the traditional newsvendor model, especially in capacity allocation and pricing problems. For example, (Shaban et al., 2017) used the price-based NV approach to set the extra-baggage prices concerning the cargo price. The authors considered the negative h-value in terms of the overbooking levels.

Furthermore, Wong et al. (2009) adopted the traditional NV model to estimate the best baggage limits, which gives a better chance to allocate more cargo in the passenger flight and maximise the airline's profit.

The above model shows the possible risk behaviour of airlines when they overestimate the overbooking value and when the overbooking levels are underestimated and the flight departs with an underutilised capacity. The underutilised cargo flight represents a shortage situation.

A negative h-value is used in the air cargo industry when the expected overbooking level exceeds the actual cancellations and no-shows. Conversely, the cargo shortage problem occurs when the expected overbooking level is less than the actual cancellations and no-shows.

Unlike the above model, the positive h-value is the second variant, and it is used in most of the operations and production management applications and is termed as the salvage value (Khouja, 1999; Ye & Sun, 2016). It is less than the selling price and sometimes less than the purchasing cost. In some applications, it is received as a salvage value, such as newspaper returns, whereas in some others, it is a reselling price.

Based on the above description, the risk behaviour of the decision-maker varies with the change in h-value. The change from penalty cost to salvage value affects the risk behaviour of decision-makers as the profit differs due to this change, which starts from the following reference point:

$$R(p) = \alpha(p-c)p + (1-\alpha)(p+s-h)p \tag{10}$$

The new reference point does not change much compared to that in Equation (3), where the h-value changed from positive to negative; however, this needs a detailed interpretation. Because the decision-maker returns some of the purchasing costs when returning or reselling the overstocked quantity, it may appear that the h-value should be added to the profit term. This assumption could be correct if the sold quantity equals the overstocked quantity, but this does not happen in real life. To elaborate, the reference point is a convex combination of the maximum possible profit obtained from selling the entire ordered quantity and possible losses from overstocked or shortage costs and opportunity cost. This means that the second term includes only the unsold quantity in the overstocking scenario or the shortage cost scenario. This leads to the fact that the positive h-value is just a reduction of the possible losses from the overstocking quantity; hence, the correct position of the positive value is in the expected losses, the (1-α) term.

From this perspective, the expected value function changes to

$$\begin{aligned} E[V(p,z)] = [(p-c)[d(p)+z] - \alpha p(p-c) - p(1-\alpha)(p+s \\ - h)]\int_0^z f(x)dx - (p \\ - h))\int_0^z (z-x)f(x)dx \\ + \gamma[(p-c)[d(p)+z] - \alpha p(p-c) \\ - p(1-\alpha)(p+s-h)]\int_z^\infty f(x)dx \\ - \gamma s\int_z^\infty (x-z)f(x)dx \end{aligned} \tag{11}$$

Furthermore, the variant of the h-value affects the pricing decision of risk-averse decision-makers. (Kahneman & Tversky, 1979) referred to this as the reflection effect, which stands for risk-seeking in losses and risk aversion in profits. Similarly, the risk-averse price when the h-value is negative mirrors the price when the h-value is positive. The decision-maker has two opposing behaviour choices concerning the h-value when pricing. In the positive h-value, the salvage value reduces the overstocked losses, and the increase in this amount motivates the decision-maker to increase the price and seek the risk. Conversely, when the h-value turns to a penalty cost, the decision-maker reduces the price to increase the demand such that they sell as much as possible to reduce the unsold quantity and, consequently, the overstock losses. Moreover, the h-value variant is reflected in optimum quantity ordering, where the positive h-value changes the optimum quantity ordering under a uniform distribution to the following:

$$F({z_{ra}}^{*}) = \frac{\gamma(p+s-c)}{c-h+\gamma(p+s-c)} \tag{12}$$

Figure 5 demonstrates the effect of the h-value between salvage and the penalties. It illustrates the change in the decision-maker's behaviour concerning the h-value.

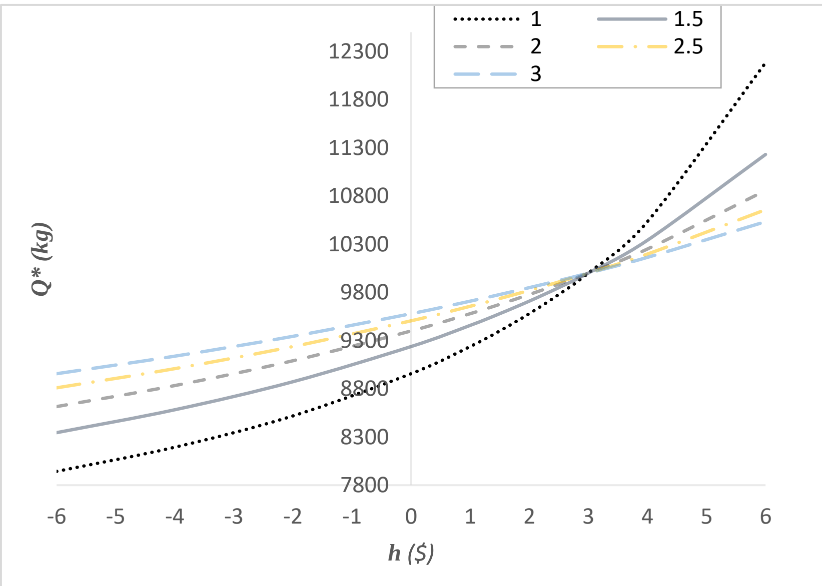


Figure 5. The effect of h-value variants on optimum quantity ordering for different risk aversion levels.

For p = 10 and each risk aversion level, the decision-maker increases the order by decreasing the penalty costs. This behaviour continues even when the decision-maker receives salvage value instead of paying the penalty; however, by increasing the salvage value, decision-makers with different risk aversion levels tend to order a similar quantity. They forget their aversion at the breakeven point, where the unit salvage equals the unit cost, and their aversion becomes unusable. If different decision-makers can resell the unsold quantity at more than its cost but less than its market price, the ordering behaviour changes concerning the risk behaviour, i.e. the less risk-averse decision-maker orders more quantity.

To summarise, although the negative h-value (penalty cost) is a big issue in the air cargo industry, the positive h-value dramatically affects the risk-averse decision-maker. The decision-makers with different risk aversion levels are indifferent when the h-value equals the purchasing cost. Moreover, the salvage value changes the ordering behaviour of risk-averse decision-makers with different risk aversion levels.

6. Conclusions

This research investigates the pricing behaviour of the decision-maker through the price-based NV model. The risk behaviour of a decision-maker is studied based on PT and reference points. Unlike previous studies, the reference point is modelled as a function of the selling price as a convex combination of the maximum possible profit and the expected losses, including shortage, overstocking and opportunity costs. With zero wealth, the model studies the pricing mechanism of different decision-makers with different risk aversion levels. By investigating the model and the effect of the risk aversion factor on optimal price and quantity, it is concluded that the risk-averse decision-maker assigns product prices higher than that of the risk-neutral decision-maker. This price increases with the increase in risk aversion level; however, the same decision-maker prefers to order more quantity than the risk-neutral decision-maker. This result means that the risk-averse decision-maker orders more and sells at higher prices; however, this is not always correct because the risk-averse decision-maker's optimum price depends on the value and type of unsold quantity.

In this regard, the variants of the values of the overstocked/unsold quantity are examined between the positive salvage value and negative penalty costs. The numerical results revealed that the reflection effect dominates the decision-maker's behaviour, reducing the product price to increase the market demand and decreasing the difference between the ordered quantity and actual market demand; thus, the expected losses are minimised. In contrast, when the unsold quantity is returned or resold, the decision-maker increases the price because they are guaranteed to recover some of the expected losses. Consequently, several managerial insights can be regarded from these results; for example, managers are suggested to give up their risk aversion in panic time as they can increase their profits by increasing the prices. This insight is supported by Huang et al. (2022)who concluded that risk-averse managers are unable to relinquish their aversion during catastrophes to save supplies. Another example, as in Figure 5, is the air cargo overbooking problem; sales managers should consider overbooking regardless of their risk preferences as the optimal quantity of overbooking is constant.

Further investigation of this research is required in many different directions. One of these directions is to empirically investigate the decision biases between the NV model and the managers when they set the price. Furthermore, our model refers to the air cargo industry, and a case study is planned to investigate airline pricing behaviour when their risk aversion level changes.

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