

FLEXIBLE FUZZY-ROBUST OPTIMIZATION METHOD IN CLOSED-LOOP SUPPLY CHAIN NETWORK PROBLEM MODELING FOR THE ENGINE OIL INDUSTRY

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Abstract: *This study models a closed loop supply chain network for the Iranian engine oil market. The primary goal of the created model is to summarize tactical choices like choosing the best degree of discount and allocating the best flow of products across facilities as well as strategic decisions like selecting a supplier and finding new facilities. The three aim functions of reducing overall expenses, optimizing employment rate, and limiting unrealized demand are considered. The novel flexible fuzzy robust optimization approach also controls the uncertainty parameters and the meta-heuristics algorithm for solving the model. This investigation showed that the network's overall transportation and operational expenses have risen as the rate of uncertainty and dependability has grown. MOGWO was chosen as an effective algorithm and employed in solving numerical examples of more significant size after the final examination of comparison indices between solution techniques (case study). According to the findings of a case study, the four oil businesses, Behran, Sepahan, Iranol, and Pars, were chosen as the best production hubs since they can generate 514 million liters of engine oil annually. As a consequence, building the network cost a total of 434321010 million Rials, required the employment of more than 37 thousand individuals, and left 90 million liters of fuel short.*

Key words: *Closed-loop supply chain network, reliability, flexible fuzzy robust optimization, discount, metaheuristic algorithms.*

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1. Introduction

Economic and industrial transformation is occurring extremely quickly nowadays. The notion of the supply chain (SC) was developed due to the rapid development of technology and the constrained resources available to enterprises. Governments, organizations, and environmental corporations have given the creation of mathematical models of SC networks much consideration due to the state of the global economy and the significance of the environment. To do this, firms aim to establish an integrated environment across the SC network to fulfill all their objectives concurrently. SC is one of the most significant strategic challenges that has sparked market competitiveness between organizations and enterprises (Pishvae & Razmi, 2012). Organizations have been forced to optimize their SCs to remain competitive and increase their share of product sales in international marketplaces. This has enabled them to react quickly to client requests while maintaining the highest quality and cost-efficiency standards. As a result, the whole SC—from raw material suppliers through the delivery of products to customers—needs to be carefully watched, managed, and regulated. As a result, SC management may be described as a process that involves planning, carrying out, and overseeing all activities linked to the supply of goods, their manufacture, storage, and delivery to consumers (Ghahremani Nahr et al., 2020; Zahiri et al., 2014). The SC consists of a collection of businesses and institutions that exchange two different kinds of goods and information flows. Companies and organizations provide the raw ingredients for this SC, which is then managed to create a variety of goods. On the other side, it oversees the manufactured goods' storage, shipping, and sales. The fundamental objective of all SC challenges is to satisfy consumer demands (Pop et al., 2015). Rapid technological advancements, the creation of new industrial goods, and the shortening of product life cycles have contributed to a rise in product waste and, as a result, environmental problems. Organizations should not focus just on economic factors when determining their company's sustainability. Benefits of sustainable development such as improving human health, promoting chances for education and empowerment, assisting with social and environmental development, reducing pollution, creating more green space, etc., should also be considered (Pahlevan et al., 2021). As a result, in modern human cultures, people increasingly seek an excellent existence in a healthy environment with favorable social circumstances rather than just the price of the commodity they are purchasing (Abad & Pasandideh, 2022; Ghasempoor Anaraki et al., 2021).

As a result, governments' worries about this problem and their rules and regulations on environmental problems have prompted businesses to collect waste items, which has given rise to a novel idea known as reverse SC (Cricelli et al., 2021). The process of organizing, carrying out, and effectively managing the influx and storage of relevant products and information to increase the value or ensure correct disposal, which incorporates two significant commercial and environmental problems, is known as the reverse SC. In the commercial factor, direct and indirect profits through the reuse and recycling of waste products and returned economic capital can be used for the organization. Environmental factors like market and customer pressures and ethical incentives to improve environmental conditions are raised (Chen et al., 2021; Seyedrezaei et al., 2012; Soleimani et al., 2013). Consequently, the expenses of raw material supply, transportation, and manufacturing are decreased, which lowers the cost of the final product and, inevitably, increases profits (Amin et al., 2017).

The closed-loop supply chain network (CLSCN) is a supplement to the two kinds of chains, and it has the following features (Ghahremani Nahr et al., 2020):

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- The recycling of waste materials via the acquisition of items from the reverse SC
- Promoting and dispersing things that have been recycled and remade
- Reproducing abandoned goods for sale in the first and second-hand markets by transporting them from clients to collecting centers to manufacturing centers.

Due to government norms and regulations and environmental problems, integrating two forward and reverse SCs has introduced new layers to the network's SC, including the ability to retrieve and separate certainty (Zahedi et al., 2021).

The modeling and solution of the green CLSCN problem concerning numerous economic, social, and environmental elements for the engine oil industry are covered in this article due to the significance of designing CLSCN in many industries. In this approach, several facets of tactical and strategic decisions—such as choosing the best flow allocation and figuring out the right degree of discount—as well as strategic decisions—such as choosing a supplier and finding suitable facilities—are analyzed. As a consequence, the article's significant characteristics can be confused for the following:

- Creating a complete CLSCN model that takes into account problem stability factors.
- Controlling the uncertain problem parameters with the flexible fuzzy robust optimization (FFRO) method
- By identifying an appropriate initial solution, use meta-heuristic algorithms to solve the problem.
- Consider choices relating to determining the best discount amount from suppliers and the dependability of product delivery.

The structure of the article is as follows. In the second part, the literature review has been reviewed. The third part presents the mathematical model of the CLSCN for the engine oil industry. In the fourth part, the methods of solving the problem of chromosome encoding and decoding are introduced. In the fifth section, the analysis of different examples is discussed. In the sixth, conclusions and future solutions are discussed.

2. Literature Review

Garg et al. (2015) provided a CLSCN model that included the effects of the surrounding environmental circumstances. They did it in the context of a real case study by using the proposed interactive multi-objective programming approach algorithm. According to the data, there is a possibility that the number of people using vehicles may decline along with the increase in demand. Zohal and Soleimani, 2016 looked at the solution of a multi-objective logistics model and how to develop it using a green technique based on CO₂ emissions. They did this by using a heuristic algorithm in the gold business. The methodology was used in the gold industry. They employed a linear integer programming model and other methods to save costs and decrease pollution.

They developed an algorithm based on optimizing ant colonies to solve the problem. Alshamsi and Diabat (2018) developed a model to estimate the best placement and node capacity for complex integer linear programming. They tried to solve the problem using the conventional Banders analysis method, but it was computationally inefficient for them. After developing and refining the technique using several different accelerator approaches, an ideal answer was finally discovered. In research for an integrated SC forward and reverse network, Fathollahi-Fard et al. (2018) developed a three-tier location-allocation model. Within this model, they

considered a skeleton game involving clients, distribution centers, and recycling. At each level of the SC, the costs related to allocation and placement are reduced to the absolute minimum. They overcame the issue using meta-heuristic methods such as particle swarm optimization, forbidden search, slaughter, and water wave optimization. The experiment results show that Water wave optimization (WWO) is much more effective than other meta-heuristic algorithms. Ghahremani Nahr et al. (2018) developed a CLSCN that considers the three tiers of production centers, end users, collection centers, and destruction centers to reduce the total costs of placement and allocation. They used the League championship algorithm (LCA) to provide a modified chromosome based on priority to solve the recommended model and evaluate its efficacy compared to other chromosomes. The results showed that the method effectively solves problems that include small and large samples by providing priority-based chromosomes.

In their attempt to create a multi-objective CLSCN, Mardan et al. (2019) used an accelerated port decomposition strategy as part of their methodology. They provided a model to select the appropriate facility location, transit volumes, and inventory balance in cost to reduce total costs and environmental emissions. The model's objective was to determine the proper facility location, transit volumes, and inventory balance in cost. The given issue was handled using the Banders decomposition approach, and the computational outputs were then analyzed for several numerical examples that were compiled. Ahmadi and Amin (2019) proposed the mixed-integer linear programming model to maximize total profit in the CLSC network while maximizing the number of eligible providers via a fuzzy approach for supplier selection. In addition, random demand and return are accounted for by the model. It was suggested that orders be sent to an efficient supplier. Yadegari et al. (2019) integrated the Memetic algorithm with an innovative local search strategy to solve a CLSCN issue. This problem features a unique integer programming model for a single-product and multi-period SC problem. In the Iranian polymer industry, they made use of the model that was provided. The findings showed that utilizing a memetic algorithm to tackle problems with a large sample size provides favorable results, demonstrating the importance of adopting such an approach. Mohtashami et al. (2020) conducted research to minimize the environmental impact and the amount of energy used. This was accomplished by taking into consideration the queuing system that was present in a green CLSCN. To achieve this goal, they devised a forward. They reversed green SC and a queuing system to reduce the time spent waiting and increase the time spent traveling throughout the transportation network. They also used a meta-heuristic strategy to handle complex problems. Sadeghi et al. (2020) used a complex integer linear programming model to develop an environmentally friendly CLSCN that gives product reuse the highest priority while also considering the challenge of location routing. The model that has been proposed is a multi-period, multi-product model that takes into consideration not only the location of the facility but also the various routes that the fleet travels.

Yolmeh and Saif (2021) proposed a nonlinear mixed-integer model to determine the most accurate estimate of the optimal output for the manufacturing line. In addition, they suggested a CLSCN model operating under uncertainty, which took into account assembly and disassembly options in the context of the production line balance. They increased the amount of assembly and the number of return items in the line where the product was being disassembled. In order to investigate the model, they used Banders' analytical technique. According to the issue sensitivity analysis results, the costs connected with the production system become more expensive whenever

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the rate of return or the demand for the product increases. Zahedi et al. (2021) developed a model for multimodal mobility in a CLSCN to increase the current value to its maximum potential. They determined the SC network architecture's benefits, drawbacks, and overall current value throughout the anticipated horizon. Metaheuristic algorithms such as Simulated Annealing (SA), Genetic Algorithm (GA), KA, and KAGA were used to solve the model successfully. The model was used in a real-world case study focused on the aluminum industry. According to the data, when compared to other algorithms, KAGA was able to generate high outcomes in a shorter amount of time.

Zhang et al. (2019) regulated the uncertain parameters of a CLSCN issue by using the fuzzy programming technique. This was done while considering the uncertain aspects of demand, maintenance expenditures, transportation, and product price. They established a network consisting of SC members. In order to find a solution to the issue, they combined a genetic algorithm with Monte Carlo simulations. Maximizing supply capability, minimizing total network design costs, and minimizing total network design costs were the three objective functions included in the integrated supplier selection model and transportation decisions presented by Govindan et al. (2017) in a multi-objective under the uncertainty of some parameters. This model was presented in a multi-objective under uncertainty of some parameters. In addition to contributing to the provision of raw materials, the supplier plays a role in lowering the overall amount of greenhouse gas emissions caused by transportation. They used the Maxmin strategy to obtain the optimal value for all three objective functions simultaneously. They also used the fuzzy programming method to control the uncertain demand and transportation cost components. Both of these approaches were successful. The findings indicated that implementing this strategy in India might potentially positively affect both the economy and the environment there.

Jabbarzadeh et al. (2018) constructed a CLSCN under destructive hazards to save costs. They considered tactical choices such as where to locate production centers and how to gather best and distribute flow. Due to the system's environmental unpredictability, they employed a potentially reliable approach to regulating the parameters before applying the suggested model to the glass sector in Iran's Hamadan area. By considering various transportation choices while considering demand uncertainty and the product return rate, Haddadsisakht and Ryan (2018) proposed a model for developing a CLSCN. They employed unstable/probabilistic approaches to handle uncertain parameters. By imposing a price on carbon emissions, their primary objective was to reduce the expenses of the whole SC network. Under the unpredictability of demand characteristics, operational expenses, and transportation costs, Ghahremani-Nahr et al. (2019) created a CLSCN model. The model was controlled using robust fuzzy programming. The findings demonstrated that when the level of uncertainty rises, the system as a whole incurs higher costs. The Whale Optimization Algorithm (WOA) was utilized by creating a priority-based chromosome to find a problem with the issue. Darestani and Hemmati (2019) looked at a CLSCN and distributed goods using a queuing mechanism when uncertain demand and transportation costs were high. The robust optimization approach was used. The solution model's utility function and Torabi-Hasini (TH) were solved using three comprehensive criteria approaches. The findings show that the TH method effectively solves the two-objective model. In the context of demand unpredictability and transportation costs, Ghahremani Nahr (2020) constructed a CLSCN and examined how it affected the objective functions of reducing network design costs and greenhouse gas emissions. The uncertain parameters were controlled using a reliable manner. The findings point to a consistent rise in expenses against an increase in

uncertain demand. In research on the water supply system and wastewater collection under uncertainty, Fathollahi-Fard et al. (2020) provided an integrated model of a sustainable CLSCN. They added a case study to the multi-objective stochastic optimization model. By considering cost management and CO₂ emissions in the design of a CLSCN, Huang et al. (2020) created a two-objective accurate mixed-programming model to strike a balance between managing environmental effects and cutting operational expenses. To solve the randomly built model, they employed the scenario-based technique and the Epsilon constraint method under demand uncertainty. Abad and Pasandideh (2022) proposed a model and applied a novel Pareto accelerator bending decomposition technique to solve it in research to create a green CLSCN with unpredictable demand. To enhance the rate of returned items being returned for recycling and destruction, Gholizadeh et al. (2020) did research to employ a CLSCN. They provided a model for implementing location-allocation and routing decisions in this respect. They used a reliable technique and a genetic algorithm based on problems to manage uncertain factors.

To create a sustainable CLSCN, Khalili Nasr et al. (2021) developed a model in which supplier allocation and selection choices were considered concurrently. They took into account in the model the objective functions of reducing overall costs, eliminating negative environmental consequences, increasing employment by opening additional centers, minimizing unmet demand, and optimizing supplier sustainability. The results demonstrate the remarkable effectiveness of the TH technique in quickly addressing the five-objective model. In a fuzzy optimization technique, research, Liu et al. (2021) modeled a green CLSCN under demand uncertainty. They put the Coca-Cola concept into practice while considering the objective functions of reducing greenhouse gas emissions and the overall cost of network construction. The results demonstrate how system expenses are managed under uncertain circumstances. A multi-objective model for the green CLSCN under uncertain circumstances was put up by Boronoos et al. (2021). Both greenhouse gas emissions and SC network costs were concurrently reduced in this model for the forward and reverse SCs. The transportation and operation expenses, regarded as uncertain and triangular fuzzy values, are controlled using the robust fuzzy technique. The outcomes of the TH approach demonstrate that when prices rise, so do greenhouse gas emissions under uncertain situations.

Ziari and Sajadieh (2022) presented a mixed integer linear programming model for designing a closed-loop supply chain network and optimizing pricing policies under stochastic disturbance. The reuse of returned products as a flexibility strategy to deal with energy waste and improve supply efficiency is applied in this paper. In addition, in this article, the determination of the optimal price for the final and returned products has been done. The goal of the model is to maximize the profit of the supply chain network in the glass industry. Soon et al. (2022) proposed a sustainable closed-loop supply chain model that balances economic, environmental, and social responsibilities. They considered costs and customer demands different types of products at different quality levels under conditions of uncertainty using a robust probabilistic planning method. The results show that the robust probabilistic planning method is more effective in dealing with uncertainties than the probabilistic planning method. Kalantari et al. (2022) designed a closed-loop supply chain problem under uncertainty to maximize net present value and minimize carbon dioxide emissions. They made various decisions, including choosing suitable suppliers, choosing the type of transportation, and optimal flow between facilities to solve the decision problem efficiently. Their model decision-making method was Neutrosophic. The findings show

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Fuzzy programming has been used to control uncertainty parameters through fuzzy triangular numbers. Guo et al. (2022) proposed a multi-objective mixed integer programming model with the objectives of minimum total cost, minimization of environmental damage, and maximum social responsibility. They used a robust fuzzy programming approach to deal with the uncertainty caused by the dynamic business environment. They also proposed an efficiency-based optimization method, combining meta-heuristics and efficiency evaluation, to solve the developed multi-objective model. Aliahmadi et al. (2023) modeled and solved a closed loop supply chain network considering uncertainty. The goal of this paper was to maximize net present value. For this, they used the flexible fuzzy robust optimization method. Also, to solve the problem, they proposed four algorithms: Grey wolf optimizer (GWO), Antlion optimizer (ALO), Particle Swarm Optimization (PSO), and Genetic-Harris Hawks Optimization (G-HHO). The results showed the high efficiency of the solution methods provided by them.

The above literature studies show the existence of gaps in the field of applying reliability relationships in the closed loop supply network for the engine oil industry. Also, the use of effective counter methods for uncertainty is another research gap that has been addressed in this research. On the other hand, the use of efficient methods to solve the problem, which has not received much attention in research, has been used in this article.

3. Problem Definition

The green CLSCN for engine oil products is mathematically modeled in this section for situations where the problem's parameters are uncertain, such as (demand, transmission, and operating costs). The SC network model in this section was built using a variety of tactical and strategic decisions. The best location for potential facilities, including the choice of suppliers, the positioning of production and distribution hubs, the location of collection and recycling facilities, and the location of distribution and distribution hubs, can be determined by evaluating the capacity level. After selecting the suppliers, this section specifies the best level of discount that the provider should provide for selling engine oil raw materials. Figures 1 and 2 show that a general discount is frequently intended for the delivery of raw materials by chosen suppliers.

Due to its high demand and non-decomposability, engine oil requires a detailed study to provide a mathematical model to analyze its costs and other objective functions. Therefore, this product has been investigated in this study.

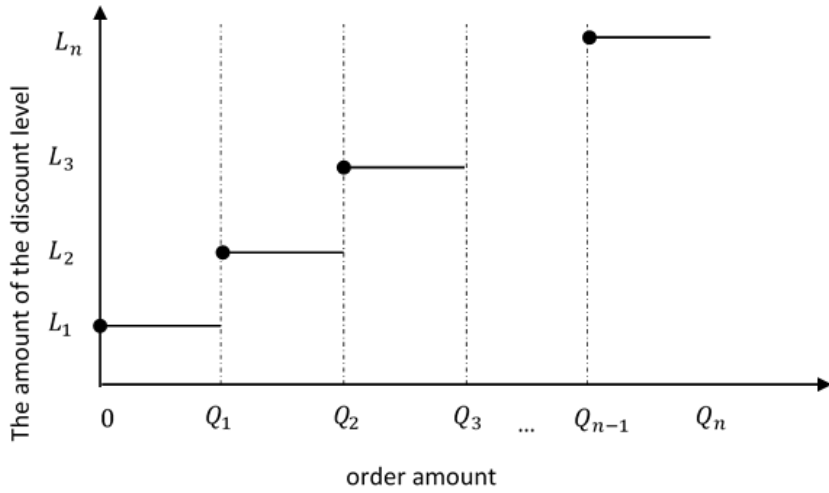


Figure 1. The order amount at different levels of discount

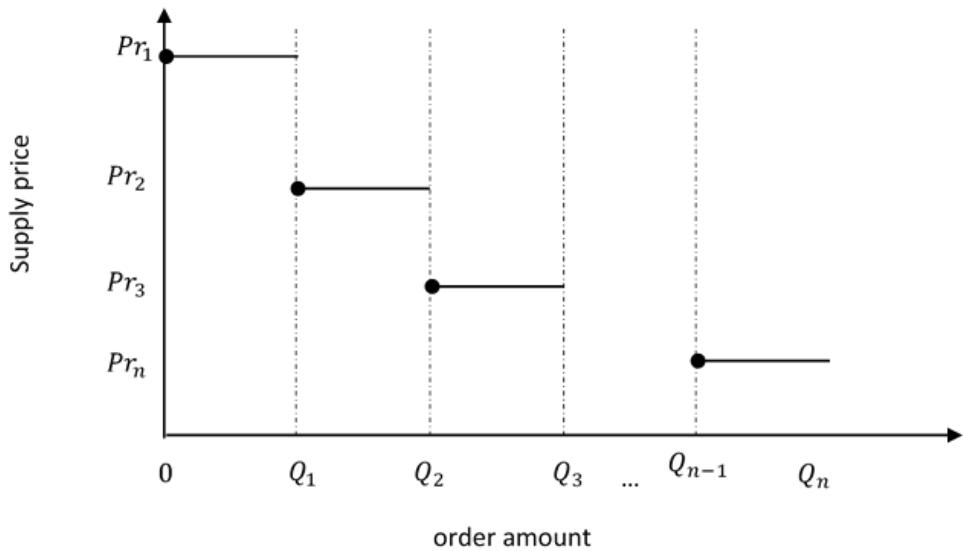
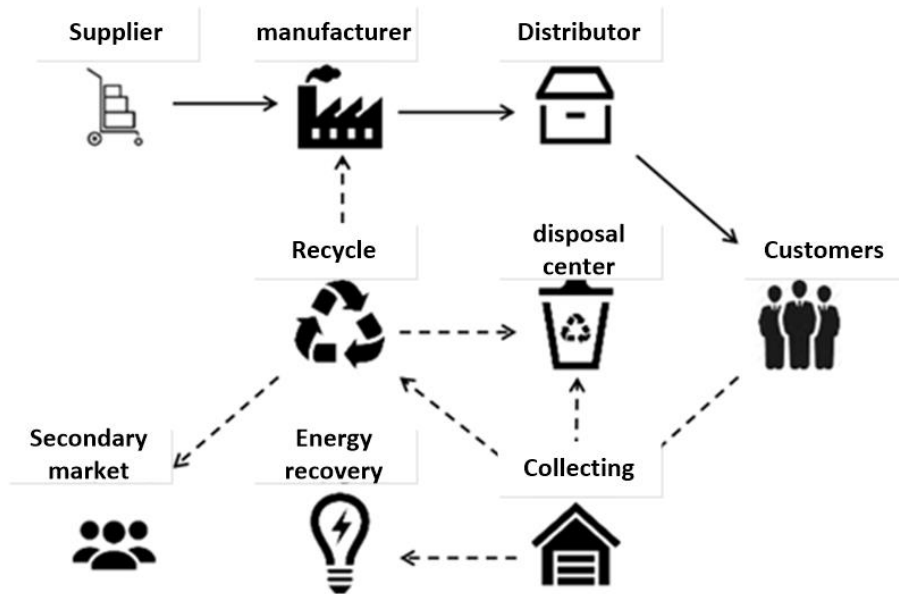


Figure 2. Supply price by the supplier at different levels of discount

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Strategically positioned production facilities buy raw materials from the selected supplier and make payments based on the size of the discount. The second type of SC network option employs the proper flow of raw materials and engine oil products between various levels of the SC network. Figure 3 illustrates the optimal distribution of commodities among different levels, including suppliers, production and distribution, customers, collection, disposal and recycling, secondary markets, and



energy recovery.

Figure 3. Green CLSCN for engine oil products

The fundamental objective of the suggested model, as shown in Figure 3, is to satisfy the greatest cost of uncertain customer demand for a range of engine oil products while incurring the least customers and the most incredible social res. Therefore, the model aims to maximize the number of jobs created by building more potential centers and minimizing unmet customer demand. It also seeks to minimize all costs associated with network design, including facility location, transfer costs, operating costs, and the cost of purchasing raw materials at a discount. The FFRO approach regulates the indefinite parameters since some of the problem's most crucial factors are considered endlessly. Consequently, the following hypotheses may be used to describe the CLSCN model for engine oil products:

- Indefinite consideration is given to demand factors, transportation costs, operational expenses, and trapezoidal fuzzy numbers.
- A shortage is permitted.
- The transportation fleet is seen as diverse.
- The cost of facilities varies depending on their capacity.
- The suppliers are providing an all-inclusive discount.

3.1. Sets

$a \in A$ customers

$k \in K$	distribution centers
$j \in J$	production centers
$i \in I$	suppliers
$l \in L$	disposal centers
$n \in N$	recycling centers
$m \in M$	collection centers
$e \in E$	energy recovery centers
$b \in B$	secondary market
$g \in G$	capacity levels
$p \in P$	products (engine oil products)
$h \in H$	raw material
$v \in V$	vehicle
$c \in C$	discount levels

3.2. Parameters

\overline{Dem}_{ap}	The demand for product $p \in P$ Customer $a \in A$
O_{hp}	The number of raw materials $h \in H$ needed to produce a product $p \in P$
α_{ap}	A fraction of returned product $p \in P$ by customer $a \in A$
β_{mp}	A fraction of shipped product $p \in P$ of collection center $m \in M$ to the energy recovery center
γ_{mp}	A fraction of shipped product $p \in P$ of collection center $m \in M$ to the recycling center $\beta_{mp} + \gamma_{mp} < 1, \forall m, p$
δ_{np}	A fraction of shipped product $p \in P$ of recycling center $n \in N$ to the disposal center
σ_{np}	A fraction of shipped product $p \in P$ of recycling center $n \in N$ to the production center $\delta_{np} + \sigma_{np} < 1, \forall n, p$
$CapJ_{jpg}$	The capacity of production center $j \in J$ for product $p \in P$ at capacity level $g \in G$
$CapK_{kpg}$	The capacity of distribution center $k \in K$ for product $p \in P$ at capacity level $g \in G$
$CapM_{mpg}$	The capacity of collection center $m \in M$ for product $p \in P$ at capacity level $g \in G$
$CapN_{npg}$	The capacity of recycling center $n \in N$ for product $p \in P$ at capacity level $g \in G$
\overline{CapI}_{ih}	The capacity of supplier $i \in I$ for raw material $h \in H$
$CapL_{lp}$	The capacity of disposal center $l \in L$ for product $p \in P$
$Capw_v$	Weight capacity of vehicle $v \in V$
$Capv_v$	Volume capacity of vehicle $v \in V$
w_h	Weight of raw material $h \in H$
w_p	Weight of product $p \in P$
v_h	The volume of raw material $h \in H$
v_p	The volume of product $p \in P$
F_i	Cost of selecting a supplier $i \in I$
$F_{\nabla g}$	Cost of establishing node $\nabla \in \{J, K, M, N\}$ with capacity level $g \in G$
FC_v	Cost of selecting the vehicle $v \in V$
\overline{FV}_v	Cost of shipment vehicle $v \in V$ between two centers

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$D_{\nabla, \nabla'}$	Distance between node ∇ and node ∇' (∇, ∇') = $(K, A), (J, K), (I, J), (N, J), (A, M), (M, E), (M, N), (M, L), (N, L), (N, B)$
$\widetilde{P}C_{jp}$	The production cost of product $p \in P$ in the production center $j \in J$
$\widetilde{S}C_{ih}$	Cost of raw material $h \in H$ from supplier $i \in I$
$\widetilde{D}C_{kp}$	Cost of product $p \in P$ in distribution center $k \in K$
$\widetilde{C}C_{mp}$	Cost of product $p \in P$ in the collection center $m \in M$
$\widetilde{R}C_{np}$	Cost of product $p \in P$ in the recycling center $n \in N$
$\widetilde{L}C_{lp}$	Cost of product $p \in P$ at the disposal center $l \in L$
$\widetilde{R}\widetilde{P}C_{jp}$	Cost of product $p \in P$ in the production center $j \in J$
$Co2_v$	Co2 emissions of vehicle $v \in V$
$E_{\nabla g}$	Co2 emission of node $\nabla \in \{J, K, M, N\}$ with capacity level $g \in G$
Pe_{jp}	Co2 emission of product $p \in P$ in the production center $j \in J$
Ce_{mp}	Co2 emission of product $p \in P$ in the collection center $m \in M$
Re_{np}	Co2 emission of product $p \in P$ in the recycling center $n \in N$
Le_{lp}	Co2 emission of product $p \in P$ in the disposal center $l \in L$
ϑ	Cost of overplus Co2 emissions
$JOB_{\nabla g}$	The number of jobs created in node $\nabla \in \{J, K, M, N\}$ with capacity level $g \in G$
θ_{job}	The importance coefficient of the created job
Re_{ijh}	Reliability between supplier $i \in I$ and production center $j \in J$ for raw material $h \in H$
Re_{jkp}	Reliability between node $j \in J$ and distribution center $k \in K$ for product $p \in P$
Re_{kap}	Reliability between distribution center $k \in K$ and customer $a \in A$ for product $p \in P$
Va_{ihc}	The lower limit of the discount period of raw material h at the discount level c by the supplier i
Pr_{ihc}	Purchase price of raw material h at discount level c by the supplier i

3.3. Decision Variables

$Q_{\nabla \nabla' p}$	Quantity of product $p \in P$ transferred between node ∇ and node ∇' (∇, ∇') = $(K, A), (J, K), (I, J), (N, J), (A, M), (M, E), (M, N), (M, L), (N, L), (N, B)$
S_{ap}	Shortage of product $p \in P$ for customer $a \in A$
$U_{\nabla g}$	1; If the node $\nabla \in \{J, K, M, N\}$ is established with a capacity level $g \in G$ 0; Otherwise
U_i	1; If the supplier $i \in I$ is selected 0; Otherwise
$Y_{\nabla \nabla' v}$	1; If vehicle $v \in V$ is assigned between node ∇ and node ∇' (∇, ∇') = $(K, A), (J, K), (I, J), (N, J), (A, M), (M, E), (M, N), (M, L), (N, L), (N, B)$ 0; otherwise
Z_{ihc}	1; If supplier $i \in I$ offers discount level $c \in C$ for raw material $h \in H$ 0; Otherwise

3.4. Green CLSCN Model for Engin Oil Industry

$$\begin{aligned}
 \min Z_1 = & \sum_{j \in J} \sum_{g \in G} F_{jg} U_{jg} + \sum_{k \in K} \sum_{g \in G} F_{kg} U_{kg} + \sum_{m \in M} \sum_{g \in G} F_{mg} U_{mg} \\
 & + \sum_{n \in N} \sum_{g \in G} F_{ng} U_{ng} + \sum_{i \in I} F_i U_i \\
 & + \left[\sum_{v \in V} (FC_v + \widetilde{FV}_v) \left(\sum_{i \in I} \sum_{j \in J} Y_{ijv} + \sum_{j \in J} \sum_{k \in K} Y_{jkv} + \sum_{k \in K} \sum_{a \in A} Y_{kav} \right. \right. \\
 & + \sum_{a \in A} \sum_{m \in M} Y_{amv} + \sum_{m \in M} \sum_{l \in L} Y_{mlv} + \sum_{m \in M} \sum_{e \in E} Y_{mev} \\
 & + \sum_{m \in M} \sum_{n \in N} Y_{mnv} + \sum_{n \in N} \sum_{l \in L} Y_{nlv} + \sum_{n \in N} \sum_{b \in B} Y_{nbv} \\
 & \left. \left. + \sum_{n \in N} \sum_{j \in J} Y_{njv} \right) \right] + \sum_{i \in I} \sum_{j \in J} \sum_{h \in H} \widetilde{SC}_{ih} Q_{ijh} \\
 & + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \widetilde{PC}_{jp} Q_{jkp} + \\
 & \sum_{k \in K} \sum_{a \in A} \sum_{p \in P} \widetilde{DC}_{kp} Q_{kap} + \sum_{a \in A} \sum_{m \in M} \sum_{p \in P} \widetilde{CC}_{mp} Q_{amp} + \sum_{m \in M} \sum_{n \in N} \sum_{p \in P} \widetilde{RC}_{np} Q_{mnp} + \\
 & \sum_{m \in M} \sum_{l \in L} \sum_{p \in P} \widetilde{LC}_{lp} Q_{mlp} + \sum_{n \in N} \sum_{l \in L} \sum_{p \in P} \widetilde{LC}_{lp} Q_{nlp} + \sum_{n \in N} \sum_{j \in J} \sum_{p \in P} \widetilde{PRC}_{jp} Q_{njp} + \\
 & \left[\sum_{v \in V} \vartheta Co2_v \left(\sum_{i \in I} \sum_{j \in J} D_{ij} Y_{ijv} + \sum_{j \in J} \sum_{k \in K} D_{jk} Y_{jkv} + \sum_{k \in K} \sum_{a \in A} D_{ka} Y_{kav} \right. \right. \\
 & + \sum_{a \in A} \sum_{m \in M} D_{am} Y_{amv} + \sum_{m \in M} \sum_{l \in L} D_{ml} Y_{mlv} \\
 & + \sum_{m \in M} \sum_{e \in E} D_{me} Y_{mev} + \sum_{n \in N} \sum_{b \in B} D_{nb} Y_{nbv} + \sum_{n \in N} \sum_{l \in L} D_{nl} Y_{nlv} \\
 & \left. \left. + \sum_{m \in M} \sum_{n \in N} D_{mn} Y_{mnv} + \sum_{n \in N} \sum_{j \in J} D_{nj} Y_{njv} \right) \right] +
 \end{aligned} \tag{1}$$

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$$\left[\vartheta \left(\sum_{j \in J} \sum_{g \in G} E_{jg} U_{jg} + \sum_{k \in K} \sum_{g \in G} E_{kg} U_{kg} + \sum_{m \in M} \sum_{g \in G} E_{mg} U_{mg} + \sum_{n \in N} \sum_{g \in G} E_{ng} U_{ng} \right. \right. \\ \left. \left. + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} P e_{jp} Q_{jkp} + \sum_{a \in A} \sum_{m \in M} \sum_{p \in P} C e_{mp} Q_{amp} \right. \right. \\ \left. \left. + \sum_{m \in M} \sum_{n \in N} \sum_{p \in P} R e_{np} Q_{mnp} + \sum_{m \in M} \sum_{l \in L} \sum_{p \in P} L e_{lp} Q_{mlp} \right. \right. \\ \left. \left. + \sum_{n \in N} \sum_{l \in L} \sum_{p \in P} L e_{lp} Q_{nlp} + \sum_{n \in N} \sum_{j \in J} \sum_{p \in P} R P e_{jp} Q_{njp} \right) \right] \\ + \sum_{i \in I} \sum_{j \in J} \sum_{h \in H} \sum_{c \in C} P r_{ihc} \cdot Z_{ihc} \cdot Q_{ijh}$$

$$\max Z_2 = \theta_{job} \left\{ \begin{array}{l} \sum_{j \in J} \sum_{g \in G} J O B_{jg} U_{jg} + \sum_{k \in K} \sum_{g \in G} J O B_{kg} U_{kg} + \\ \sum_{m \in M} \sum_{g \in G} J O B_{mg} U_{mg} + \sum_{n \in N} \sum_{g \in G} J O B_{ng} U_{ng} \end{array} \right\} \quad (2)$$

$$\min Z_3 = \sum_{a \in A} \sum_{p \in P} S_{ap} \quad (3)$$

s. t.:

$$\sum_k Q_{kap} + S_{ap} = \widetilde{Dem}_{ap}, \forall a \in A, p \in P \quad (4)$$

$$1 - \prod_{i \in I} \prod_{a \in A} \left(1 - \left(1 - \prod_{j \in J} \prod_{k \in K} \prod_{h \in H} \prod_{p \in P} \prod_{v \in V} \left(1 - \left(\begin{array}{l} R e_{ijh} Y_{ijv} * \\ R e_{jkp} Y_{jkv} * \\ R e_{kap} Y_{kav} \end{array} \right) \right) \right) \right) \\ \geq 0.95 \quad (5)$$

$$\sum_{a \in A} Q_{kap} = \sum_{j \in J} Q_{jkp}, \forall k \in K, p \in P \quad (6)$$

$$\sum_{i \in I} \sum_{h \in H} O_{hp} Q_{ijh} + \sum_{n \in N} Q_{njp} = \sum_{k \in K} Q_{jkp}, \forall j \in J, p \in P \quad (7)$$

$$\alpha_{ap} \sum_{k \in K} Q_{kap} = \sum_{m \in M} Q_{amp}, \forall a \in A, p \in P \quad (8)$$

$$\beta_{mp} \sum_{a \in A} Q_{amp} = \sum_{e \in E} Q_{mep}, \forall m \in M, p \in P \quad (9)$$

$$\gamma_{mp} \sum_{a \in A} Q_{amp} = \sum_{n \in N} Q_{mnp}, \forall m \in M, p \in P \quad (10)$$

$$\sum_{a \in A} Q_{amp} = \sum_{l \in L} Q_{mlp} + \sum_{n \in N} Q_{mnp} + \sum_{e \in E} Q_{mep}, \forall m \in M, p \in P \quad (11)$$

$$\delta_{np} \sum_{m \in M} Q_{mnp} = \sum_{l \in L} Q_{nlp}, \forall n \in N, p \in P \quad (12)$$

$$\sigma_{np} \sum_{m \in M} Q_{mnp} = \sum_{j \in J} Q_{njp}, \forall n \in N, p \in P \quad (13)$$

$$\sum_{m \in M} Q_{mnp} = \sum_{l \in L} Q_{nlp} + \sum_{j \in J} Q_{njp} + \sum_{b \in B} Q_{nbp}, \forall n \in N, p \in P \quad (14)$$

$$\sum_{k \in K} Q_{jkp} \leq \sum_{g \in G} CapJ_{jpg} U_{jg}, \forall j \in J, p \in P \quad (15)$$

$$\sum_{a \in A} Q_{k ap} \leq \sum_{g \in G} CapK_{kpg} U_{kg}, \forall k \in K, p \in P \quad (16)$$

$$\sum_{a \in A} Q_{amp} \leq \sum_{g \in G} CapM_{mpg} U_{mg}, \forall m \in M, p \in P \quad (17)$$

$$\sum_{m \in M} Q_{mnp} \leq \sum_{g \in G} CapN_{npg} U_{ng}, \forall n \in N, p \in P \quad (18)$$

$$\sum_{j \in J} Q_{ijh} \leq CapI_{ih} U_i, \forall i \in I, h \in H \quad (19)$$

$$\sum_{m \in M} Q_{mlp} + \sum_{n \in N} Q_{nlp} \leq CapL_{lp}, \forall l \in L, p \in P \quad (20)$$

$$\sum_{g \in G} U_{ng} \leq 1, \forall n \in N \quad (21)$$

$$\sum_{g \in G} U_{mg} \leq 1, \forall m \in M \quad (22)$$

$$\sum_{g \in G} U_{jg} \leq 1, \forall j \in J \quad (23)$$

$$\sum_{g \in G} U_{kg} \leq 1, \forall k \in K \quad (24)$$

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$$\sum_{h \in H} Q_{ijh} w_h \leq \sum_{v \in V} Capw_v Y_{ijv}, \forall i \in I, j \in J, v \in V \quad (25)$$

$$\sum_{p \in P} Q_{jkp} w_p \leq \sum_{v \in V} Capw_v Y_{jkv}, \forall j \in J, k \in K, v \in V \quad (26)$$

$$\sum_{p \in P} Q_{kap} w_p \leq \sum_{v \in V} Capw_v Y_{kav}, \forall k \in K, a \in A, v \in V \quad (27)$$

$$\sum_{p \in P} Q_{amp} w_p \leq \sum_{v \in V} Capw_v Y_{amv}, \forall a \in A, m \in M, v \in V \quad (28)$$

$$\sum_{p \in P} Q_{mlp} w_p \leq \sum_{v \in V} Capw_v Y_{mlv}, \forall m \in M, l \in L, v \in V \quad (29)$$

$$\sum_{p \in P} Q_{mnp} w_p \leq \sum_{v \in V} Capw_v Y_{mnv}, \forall m \in M, n \in N, v \in V \quad (30)$$

$$\sum_{p \in P} Q_{mep} w_p \leq \sum_{v \in V} Capw_v Y_{mev}, \forall m \in M, e \in E, v \in V \quad (31)$$

$$\sum_{p \in P} Q_{nlp} w_p \leq \sum_{v \in V} Capw_v Y_{nlv}, \forall n \in N, l \in L, v \in V \quad (32)$$

$$\sum_{p \in P} Q_{njp} w_p \leq \sum_{v \in V} Capw_v Y_{njv}, \forall n \in N, j \in J, v \in V \quad (33)$$

$$\sum_{p \in P} Q_{nbp} w_p \leq \sum_{v \in V} Capw_v Y_{nbv}, \forall n \in N, b \in B, v \in V \quad (34)$$

$$\sum_{h \in H} Q_{ijh} v_h \leq \sum_{v \in V} Capv_v Y_{ijv}, \forall i \in I, j \in J, v \in V \quad (35)$$

$$\sum_{p \in P} Q_{jkp} v_p \leq \sum_{v \in V} Capv_v Y_{jkv}, \forall j \in J, k \in K, v \in V \quad (36)$$

$$\sum_{p \in P} Q_{kap} v_p \leq \sum_{v \in V} Capv_v Y_{kav}, \forall k \in K, a \in A, v \in V \quad (37)$$

$$\sum_{p \in P} Q_{amp} v_p \leq \sum_{v \in V} Capv_v Y_{amv}, \forall a \in A, m \in M, v \in V \quad (38)$$

$$\sum_{p \in P} Q_{mlp} v_p \leq \sum_{v \in V} Capv_v Y_{mlv}, \forall m \in M, l \in L, v \in V \quad (39)$$

$$\sum_{p \in P} Q_{mnp} v_p \leq \sum_{v \in V} Capv_v Y_{mnv}, \forall m \in M, n \in N, v \in V \tag{40}$$

$$\sum_{p \in P} Q_{mep} v_p \leq \sum_{v \in V} Capv_v Y_{mev}, \forall m \in M, e \in E, v \in V \tag{41}$$

$$\sum_{p \in P} Q_{nlp} v_p \leq \sum_{v \in V} Capv_v Y_{nlv}, \forall n \in N, l \in L, v \in V \tag{42}$$

$$\sum_{p \in P} Q_{njp} v_p \leq \sum_{v \in V} Capv_v Y_{njv}, \forall n \in N, j \in J, v \in V \tag{43}$$

$$\sum_{p \in P} Q_{nbp} v_p \leq \sum_{v \in V} Capv_v Y_{nbv}, \forall n \in N, b \in B, v \in V \tag{44}$$

$$\sum_{c \in C} Z_{ihc} \leq 1, \forall i \in I, h \in H \tag{45}$$

$$\sum_{c \in C} Z_{ihc} = U_i, \forall i \in I, h \in H \tag{46}$$

$$\sum_{j \in J} Q_{ijh} \geq \sum_{c \in C} Va_{ihc} \cdot Z_{ihc}, \forall i \in I, h \in H \tag{47}$$

$$Q_{kap}, Q_{jkp}, Q_{ijh}, Q_{njp}, Q_{amp}, Q_{mep}, Q_{mnp}, Q_{mlp}, Q_{nlp}, Q_{nbp}, S_{ap} \geq 0 \tag{48}$$

$$U_{jg}, U_{kg}, U_{mg}, U_{ng}, Y_{kav}, Y_{jkv}, Y_{ijv}, Y_{njv}, Y_{amv}, Y_{mev}, Y_{mnv}, Y_{mlv}, Y_{nlv}, Y_{nbv}, Z_{ihc} \in \{0,1\} \tag{49}$$

Eq. (1) shows the first goal function, and one of its components is the maximum profits derived from the design of the SC network. The network's earnings are affected in this manner because of the sales of products on the main market, substandard goods on the secondary market, and the generation and sale of energy produced from mediocre return items. The operational costs for manufacturing commodities, the fixed costs of buildings, the variable and fixed costs of running vehicles, and the costs associated with generating more carbon dioxide than is permitted are all included in the costs of the SC network. The second objective function, represented by Eq. (2), has as its primary purpose the promotion of a rise in the total number of employment opportunities made available as a direct consequence of establishing new potential centers. This considers the normal amount of time off work that must be taken because of an accident. Eq. (3), the third objective function, emphasizes minimizing unsatisfied customer demand or, more explicitly, revenue leakage. In the main market, satisfying client demand while also considering the presence of scarcity is shown by Eq. (4). According to Eq. (5), there should be a dependability of at least 95% in the transfer of components that are related to engine oil. The equilibrium connection between the distribution of completed items from the production center to the ultimate customers

Flexible fuzzy-robust optimization method in closed-loop supply chain network problem... is depicted by Eq. (6), which may be found below. The process of constructing completed items from their respective raw components is represented by Eq. (7). The number of low-quality items that are sent back to the store by consumers is represented by Eq. (8). The value shown in Eq. (9) represents the percentage of resources of poor quality that have the potential to be transformed into energy and money. The answer to this question may be found in Eq. (10), which shows the percentage of poor returned items that might be copied or sold. Eq. (11) illustrates the equilibrium between collecting customer-returned items at the collection center and the equation. Equation displays the percentage of substandard products that must either be buried or destroyed after being returned Eq. (12). The value shown in Eq. (13) represents the proportion of substandard items returned to the production site and have the potential to be resold and manufactured again. The equilibrium relationship in the recycling plant is shown by Eq. (14), which may be found below. If a prospective center with a certain capacity level is selected, Eqs. (15) to (20), which also describe the relationships between potential and fixed facility capacity, show that the potential capacity level must be met for the choice to be valid. Eqs. (20) to (24) guarantee that a potential center may be used to the maximum extent of its capabilities. The formulae for choosing a heterogeneous vehicle depending on the weight of raw materials and completed products are provided from equation number 25 to Eq. (34). Eq. (35) and (44), which are shown below, demonstrate the equations for choosing a heterogeneous vehicle. These equations are dependent on the number of raw materials and final commodities. Eq. (45) and (46) guarantee that if a source is chosen to furnish raw materials, there can only be one discount level applied for that supplier. The equation below shows the maximum amount of raw materials that may be bought at each of the discount levels that are specified in Eq. (47). Different kinds of choice variables are shown in Eq. (48) and, respectively, Eq. (49).

3.5. FFRO Method

As a consequence, the model for the CLSCN problem for engine oil products regulated by the FFRO approach will be as follows:

$$\begin{aligned}
 \min Z_1 = & \sum_{j \in J} \sum_{g \in G} F_{jg} U_{jg} + \sum_{k \in K} \sum_{g \in G} F_{kg} U_{kg} + \sum_{m \in M} \sum_{g \in G} F_{mg} U_{mg} \\
 & + \sum_{n \in N} \sum_{g \in G} F_{ng} U_{ng} + \sum_{i \in I} F_i U_i + \\
 & \left[\sum_{v \in V} \left(FC_v + \left[\frac{FV_v^1 + FV_v^2 + FV_v^3 + FV_v^4}{4} \right] \right) \left(\sum_{i \in I} \sum_{j \in J} Y_{ijv} + \sum_{j \in J} \sum_{k \in K} Y_{jkv} \right. \right. \\
 & + \sum_{k \in K} \sum_{a \in A} Y_{kav} + \sum_{a \in A} \sum_{m \in M} Y_{amv} + \sum_{m \in M} \sum_{l \in L} Y_{mlv} \\
 & + \sum_{m \in M} \sum_{e \in E} Y_{mev} + \sum_{m \in M} \sum_{n \in N} Y_{mnv} + \sum_{n \in N} \sum_{l \in L} Y_{nlv} \\
 & \left. \left. + \sum_{n \in N} \sum_{b \in B} Y_{nbv} + \sum_{n \in N} \sum_{j \in J} Y_{njv} \right) \right] + \tag{50}
 \end{aligned}$$

$$\begin{aligned}
 & \sum_{i \in I} \sum_{j \in J} \sum_{h \in H} \left[\frac{Sc_{ih}^1 + Sc_{ih}^2 + Sc_{ih}^3 + Sc_{ih}^4}{4} \right] Q_{ijh} \\
 & \quad + \sum_{a \in A} \sum_{m \in M} \sum_{p \in P} \left[\frac{Cc_{mp}^1 + Cc_{mp}^2 + Cc_{mp}^3 + Cc_{mp}^4}{4} \right] Q_{amp} + \\
 & \sum_{k \in K} \sum_{a \in A} \sum_{p \in P} \left[\frac{Dc_{kp}^1 + Dc_{kp}^2 + Dc_{kp}^3 + Dc_{kp}^4}{4} \right] Q_{kap} \\
 & \quad + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} \left[\frac{Pc_{jp}^1 + Pc_{jp}^2 + Pc_{jp}^3 + Pc_{jp}^4}{4} \right] Q_{jkp} + \\
 & \sum_{m \in M} \sum_{n \in N} \sum_{p \in P} \left[\frac{Rc_{np}^1 + Rc_{np}^2 + Rc_{np}^3 + Rc_{np}^4}{4} \right] Q_{mnp} \\
 & \quad + \sum_{m \in M} \sum_{l \in L} \sum_{p \in P} \left[\frac{Lc_{lp}^1 + Lc_{lp}^2 + Lc_{lp}^3 + Lc_{lp}^4}{4} \right] Q_{mlp} + \\
 & \sum_{n \in N} \sum_{l \in L} \sum_{p \in P} \left[\frac{Lc_{lp}^1 + Lc_{lp}^2 + Lc_{lp}^3 + Lc_{lp}^4}{4} \right] Q_{nlp} + \\
 & \sum_{n \in N} \sum_{j \in J} \sum_{p \in P} \left[\frac{PRc_{jp}^1 + PRc_{jp}^2 + PRc_{jp}^3 + PRc_{jp}^4}{4} \right] Q_{njp} + \\
 & \left[\sum_{v \in V} \vartheta Co2_v \left(\sum_{i \in I} \sum_{j \in J} D_{ij} Y_{ijv} + \sum_{j \in J} \sum_{k \in K} D_{jk} Y_{jkv} + \sum_{k \in K} \sum_{a \in A} D_{ka} Y_{kav} \right. \right. \\
 & \quad + \sum_{a \in A} \sum_{m \in M} D_{am} Y_{amv} + \sum_{m \in M} \sum_{l \in L} D_{ml} Y_{mlv} \\
 & \quad + \sum_{m \in M} \sum_{e \in E} D_{me} Y_{mev} + \sum_{n \in N} \sum_{b \in B} D_{nb} Y_{nbv} + \sum_{n \in N} \sum_{l \in L} D_{nl} Y_{nlv} \\
 & \quad \left. \left. + \sum_{m \in M} \sum_{n \in N} D_{mn} Y_{mnv} + \sum_{n \in N} \sum_{j \in J} D_{nj} Y_{njv} \right) \right] +
 \end{aligned}$$

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$$\begin{aligned}
 & \left[\vartheta \left(\sum_{j \in J} \sum_{g \in G} E_{jg} U_{jg} + \sum_{k \in K} \sum_{g \in G} E_{kg} U_{kg} + \sum_{m \in M} \sum_{g \in G} E_{mg} U_{mg} + \sum_{n \in N} \sum_{g \in G} E_{ng} U_{ng} \right. \right. \\
 & \quad + \sum_{j \in J} \sum_{k \in K} \sum_{p \in P} P e_{jp} Q_{jkp} + \sum_{a \in A} \sum_{m \in M} \sum_{p \in P} C e_{mp} Q_{amp} \\
 & \quad + \sum_{m \in M} \sum_{n \in N} \sum_{p \in P} R e_{np} Q_{mnp} + \sum_{m \in M} \sum_{l \in L} \sum_{p \in P} L e_{lp} Q_{mlp} \\
 & \quad + \sum_{n \in N} \sum_{l \in L} \sum_{p \in P} L e_{lp} Q_{nlp} + \sum_{n \in N} \sum_{j \in J} \sum_{p \in P} R P e_{jp} Q_{njp} \left. \right) \\
 & \quad + \sum_{i \in I} \sum_{j \in J} \sum_{h \in H} \sum_{c \in C} P r_{ihc} \cdot Z_{ihc} \cdot Q_{ijh} \\
 & \quad + \eta \sum_a \sum_p [Dem_{ap}^4 - (1 - \alpha) Dem_{ap}^3 - \alpha Dem_{ap}^4] \\
 & \quad + \varrho \sum_{a \in A} \sum_{p \in P} \left[\left(\left(\frac{Dem_{ap}^1 + Dem_{ap}^2}{2} \right) \right. \right. \\
 & \quad \left. \left. + \left(\frac{Dem_{ap}^4 - Dem_{ap}^3 + Dem_{ap}^2 - Dem_{ap}^1}{4} \right) \right) (1 - \varepsilon) \right] \\
 & \max Z_2 = \theta_{job} \left\{ \begin{aligned} & \sum_{j \in J} \sum_{g \in G} JOB_{jg} U_{jg} + \sum_{k \in K} \sum_{g \in G} JOB_{kg} U_{kg} + \\ & \sum_{m \in M} \sum_{g \in G} JOB_{mg} U_{mg} + \sum_{n \in N} \sum_{g \in G} JOB_{ng} U_{ng} \end{aligned} \right\} \tag{51}
 \end{aligned}$$

$$\min Z_3 = \sum_{a \in A} \sum_{p \in P} S_{ap} \tag{52}$$

s. t.:

$$\begin{aligned}
 \sum_{k \in K} Q_{kap} + S_{ap} &= [\alpha Dem_{ap}^4 + (1 - \alpha) Dem_{ap}^3] + \\
 & \left[\left(\left(\frac{Dem_{ap}^1 + Dem_{ap}^2}{2} \right) + \left(\frac{Dem_{ap}^4 - Dem_{ap}^3 + Dem_{ap}^2 - Dem_{ap}^1}{4} \right) \right) (1 \right. \\
 & \quad \left. - \varepsilon) \right], \forall a \in A, p \in P \tag{53}
 \end{aligned}$$

$$Eqs. (5) - (49) \tag{54}$$

4. Solving Methods

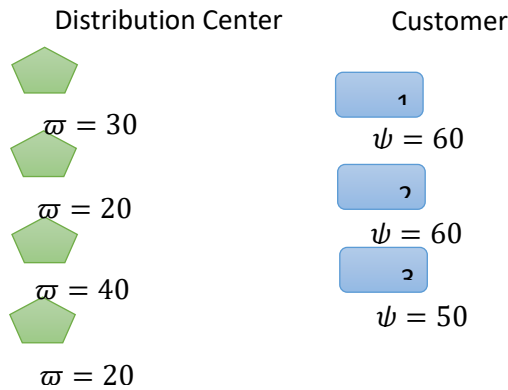
Meta-heuristic algorithms have been used to tackle the issue because of the proposed model's Mixed Integer Nonlinear Programming (MINLP) and the requirement to make decisions on location allocation and routing after presenting the multi-objective CLSCN model.

4.1. Initial Solution and Its Decoding

When attempting to solve a problem using meta-heuristic algorithms, it is essential to understand how to decode and display the original result to be successful. The challenge with the SC network requires a variety of approaches to decision-making from a variety of perspectives. As shown in Figure 4, one of the most important things to consider when designing an SC network is where to put the supply centers and how to distribute flow between the two facilities appropriately. Figure 4 depicts the initial solution for three different customers, four different distribution facilities, and three different types of vehicles. The fake information utilized in the chromosomal decoding of Figure 4 is presented in Table 1.

Table 1. Required data for decoding the initial solution

ξ	i_1	i_2	i_3	i_4	ψ
j_1	15	7	12	10	60
j_2	9	13	10	14	60
j_3	10	12	18	20	50
ϖ	30	20	40	20	



Node	$ I + J $						
	i_1	i_2	i_3	i_4	j_1	j_2	j_3
$Rand()$	6	1	3	4	2	5	7
V	3	2	1	3	-	-	-

Figure 4. Initial solution

The initial solution is shown as a $2 * (|I| + |J|)$ the matrix in Figure 4. We shall carry out the procedures in each part to decrypt:

- 1- As the initial portion of the allocation, the changed chromosome's greatest priority is chosen.
- 2- The customer or distribution facility with the lowest transportation costs is chosen using the information about the consumer or distribution center from step (1).
- 3- The appropriate vehicle is chosen to transport items between the two floors.
- 4- Based on the minimal value, optimal flow allocation between the chosen levels is accomplished (distribution center supply, customer demand, and vehicle capacity).
- 5- Updates will be made to the distribution center's supply and consumer demand.
- 6- The priority for that center is dropped to 0 if there is no supply for the demand center or no consumer demand.
- 7- Steps 1 through 6 are repeated until all distribution centers have the same priority, which is 0.
- 8- A consumer will experience a shortage if their priorities are not zero.
- 9- Distribution centers are chosen as the best SC network centers and are used to the fullest extent possible.

The initial solution and decoding technique were given above for a two-echelon SC network. The final design chromosome is depicted in Figure 5, given the 10-echelon CLSCN. Since several products are involved in the problem.

		Product (P) →									
Nodes		I	J	J	K	K	A	A	M	M	L
Priority		$v(I + J)$	$v(J + K)$	$v(K + A)$	$v(A + M)$	$v(M + L)$					
Nodes		M	N	M	E	N	L	N	J	N	B
Priority		$v(M + N)$	$v(M + E)$	$v(N + L)$	$v(N + J)$	$v(N + B)$					

Figure 5. The final solution designed for a CLSCN

The problem of nonlinear complex integer programming is solved by employing NSGA II, MOPSO, and MOGWO, all based on the initial solution provided in this section. The following is a list of the several methods that can be used to solve the problem.

4.2. NSGA II

The operational complexity of this algorithm is lower than that of other algorithms, making it one of the quickest and most powerful optimization algorithms. It ensures a desirable range for changing objective functions and allows the designer to choose the optimum design. The maintenance of elitism and dispersion is considered concurrently in NSGA II. This approach bases determining a new population in each stage on the dominance principle. At each phase of the solution, chooses the best-undefeated solutions before moving on to the next. In the case when there are two maximizing objective functions, g_1 and g_2 , solution x outperforms solution y . ($x < y$) if $g_1(x) \geq g_1(y)$ and $g_2(x) \geq g_2(y)$, or if $g_1(x) > g_1(y)$ and $g_2(x) > g_2(y)$. In addition, the concept is known as congestion distance is utilized in this method to ensure that the solutions' density is distributed appropriately.

To sort a population of size n based on the levels of non-defeat, each solution is frequently compared to every other solution in the population to determine whether or not it is defeated. These solutions are added to the set designated as F1. The answers found in the first boundary are ignored for the time being during the just detailed procedure. The only difference this time is that the solutions are relocated to the F2 set and given second place. This process is repeated for each issue the population is still struggling with. One of the prerequisites for the evolutionary algorithm to approach the ideal Pareto boundary is that the collection of solutions that have been identified must, at all times, preserve the diversity and breadth of the solutions that have been found. Organizing non-defeats is a way that may be used to arrive at better answers, and the process of variety also attempts to maintain these solutions in a manner that is diverse and all-encompassing. Swarming at a distance in this manner is the means through which this objective can be attained using this strategy. As the swarming distance between two solutions reduces, the density of solutions close to each other increases. Choose the alternatives for the following phase that are further away from crowded areas and located in places with a lower population density. As a direct consequence of this, the solutions that are developed contain a greater variety and dispersion. Within NSGA II, congestion spacing is essential for diversifying population responses and demonstrating the response density in close proximity to a specific response. Figure 6 illustrates the NSGA II pseudocode.

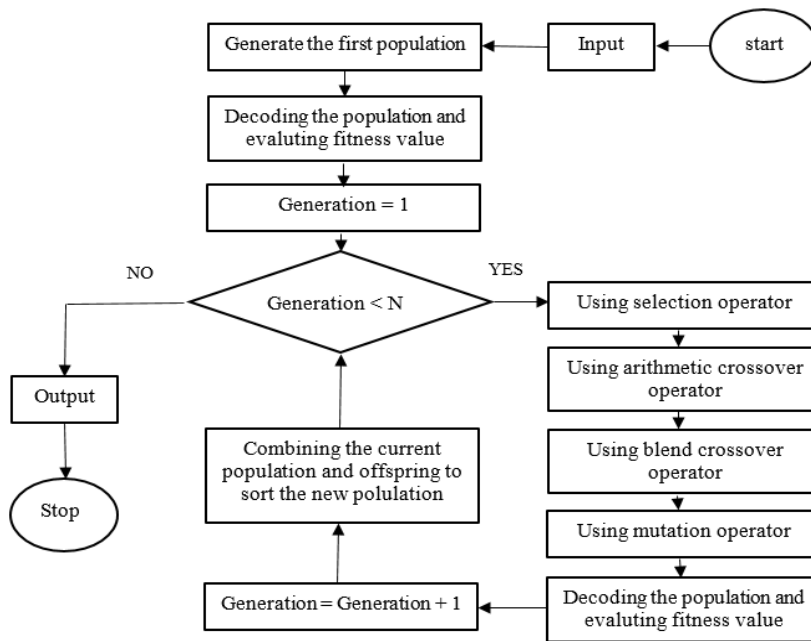


Figure 6. NSGA II pseudocode

4.3. MOGWO

Canis lupus, sometimes known as the gray wolf, is a member of the *Canidae* family. Gray wolves are the top predators in the food pyramid and chain. They are also at the top of the food pyramid. Most gray wolves choose to make their home in groups known as packs. The average wolf pack contains anywhere from five to twelve members. The male and female leadership team members are referred to as alphas. Alpha is the primary decision-maker for various issues, including hunting, sleeping arrangements, wake-up times, and other topics. Choices made by the Alpha are communicated to the group, although there have been examples of democratic behavior in which an Alpha follows the other wolves' lead in the pack. When there is a community, everyone in the herd rallies behind Alpha. The dominant wolf is another name for the alpha wolf. This is because the entire pack must carry out the orders the alpha wolf gave. The alpha wolves are not allowed to mate outside the herd at any time.

It is essential to remember that the Alpha is not always the member of the herd who possesses the greatest physical prowess but rather the one who is the most skilled in herd management. The rank that comes after Alpha in the social structure of gray wolves is called Beta. Beta wolves offer advice and assistance to Alpha when making decisions for the herd. If the Alpha wolf were to perish or get old for whatever reason, the best possible replacement would be the Beta wolf, which may be male or female, depending on the circumstances. Beta is responsible for carrying out Alpha's directives throughout the herd and providing feedback to Alpha. The Omega wolf is the lowest-ranking member of the gray wolf pack, according to its social structure. Wolf Omega plays the role of the unfortunate victim. It is common practice for the more strong and dominant wolves to demand obedience from Omega wolves. They are the only wolves left that can consume food. If a wolf does not fit an Alpha or an Omega

profile, we call it a Delta. Alpha and Beta wolves must maintain control over Delta wolves. However, they are the ones who rule Omega. In the mathematical modeling of the wolf social hierarchy, alpha is considered the best option since it represents the best possible course of action. Therefore, Beta and Delta are the two responses that are the second and third most appropriate. The final possible response, Omega, is selected from those that remain. To successfully hunt, gray wolves must identify and encircle their prey. Figure 7 shows the algorithm for the Pseudocode of MOGWO.

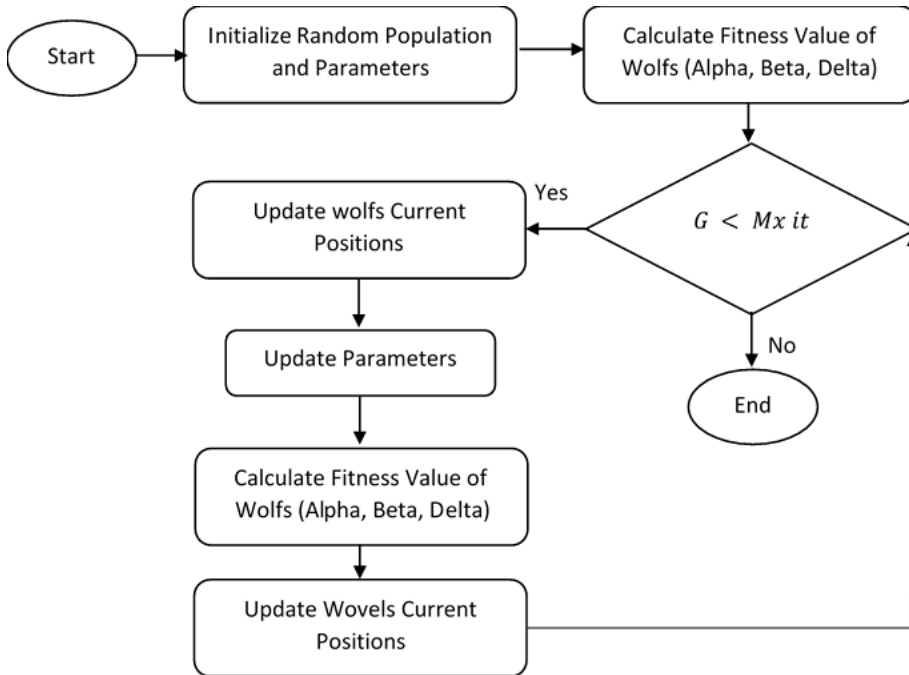


Figure 7. Pseudocode of MOGWO

4.4. MOPSO

In general, the PSO has many characteristics with other algorithms, like ants or genetics, but there are also significant distinctions that set this algorithm apart and make it easy. For instance, this method is more straightforward than genetic algorithms since it does not involve operators like intersection and leap; therefore, it does not call for using number strings or decryption stages. The motion of a particle group can be broken down into two categories: the definite and the probabilistic. Each particle is motivated to go toward the greatest solution ever discovered, g^* or the best current solution, x^* . The following describes the MOPSO's overall procedure.

- Create the initial population in step 1
- Organize the population's undefeated members into a separate archive or repository.
- Calculating the target space found
- Out of all the archive members, each particle chooses a leader.
- Update particle position and velocity

Each particle possesses not only the X_t position but also the highest value that has ever been measured for it. These findings result from contrasting each particle's

Flexible fuzzy-robust optimization method in closed-loop supply chain network problem... efforts to locate the most desirable conclusion possible. In addition, every particle in the group compares its ideal values to those of the other particles to determine which outcomes have been the most successful across the board for the entire group. Each particle alters its position based on the following information to achieve the best possible result. Therefore, Eqs. (68) and (69) can be used to express each particle's velocity and any subsequent changes in location.

$$V_i^{t+1} = wV_i^t + c_1rand(pbest_i - X_i^t) + c_2rand(gbest_i - X_i^t) \quad (68)$$

$$X_i^{t+1} = X_i^t + V_i^{t+1} \quad (69)$$

- Each particle updates its greatest personal recollection.
- Recessive members of the archive are deleted, and new undefeated members are added.
- The process terminates if the stop criteria are satisfied, and the best particle in the swarm is used to solve the issue. Otherwise, go on to step 4.

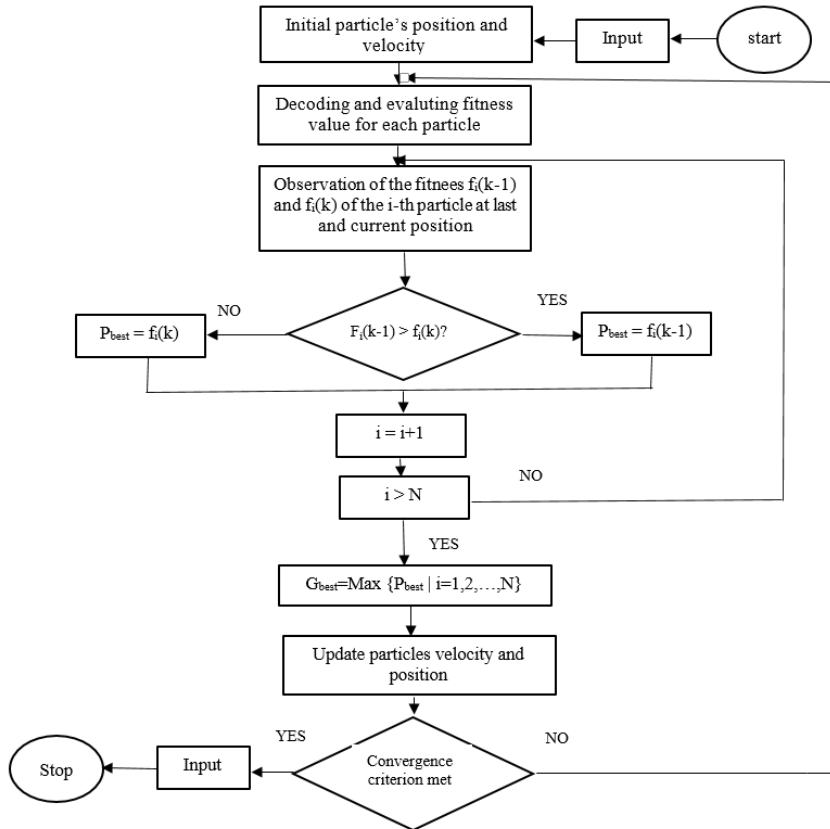


Figure 8. Pseudocode of MOPSO

Figure 8 shows the algorithm for the Pseudocode of MOPSO.

5. Results

5.1. Small Size Instance

In this section, an instance with meta-heuristic algorithms and the Epsilon constraint is analyzed, and finally using, the comparison indicators of solution methods such as (Number of Pareto Front (NPF), Maximum Spread Index (MSI), Space Metric (SM), Mean of Ideal Deviations (MID)and CPU-Time) The best solution method for a case study in Iran has been selected. Hence the numerical example is considered small, as in Table 2, and the random data used to solve the problem in Table 3.

Table 2. Small-size sample problem

Set	A	K	J	M	E	B	H	V	P	L	N	I	G	C
Number	5	4	4	3	3	4	2	6	2	4	3	4	3	3

Table 3. parameters of the problem

Exact Parameter	Approximate Interval	Exact Parameter	Approximate Interval
O_{hp}	$(1, 2) \sim U$	w_h	$(0.03, 0.05) \sim U$
α_{ap}	$(0.1, 0.2) \sim U$	w_p	$\sim U (0.08, 0.1)$
β_{mp}, γ_{mp}	$(0.3, 0.4) \sim U$	v_h	$\sim U (0.3, 0.5)$
δ_{np}, σ_{np}	$(0.2, 0.3) \sim U$	v_p	$\sim U (0.5, 0.8)$
$Pr_{ap}, Pr_{ep}, Pr_{bp}, CapL_{lp}$	$(1000 \sim U, 1200)$	$F_i, F_{jg}, F_{kg}, F_{mg}, F_{ng}$	$(10000 \sim U, 120000)$
$CapI_{ih}, CapJ_{jpg}, CapK_{kpg}$	$(5000 \sim U, 6000)$	FC_v	$(1000, 1200) \sim U$
$CapM_{mpg}, CapN_{npg}$	$(2000 \sim U, 3000)$	$D_{ka}, D_{jk}, D_{ij}, D_{nj}, D_{am}$	$\sim U (10, 100)$
$Capw_v, Capv_v$	$(500, 700) \sim U$	$D_{me}, D_{mn}, D_{ml}, D_{nl}, D_{nb}$	$\sim U (10, 100)$
Va_{ihc}	$(0, 3000) \sim U$	$Co2_v$	$\sim U (5, 8)$
Pr_{ihc}	$(5, 10) \sim U$	$E_{jg}, E_{kg}, E_{mg}, E_{ng}$	$\sim U (50, 100)$
$Pe_{jp}, Ce_{mp}, Re_{np}$	$(1, 3) \sim U$	ϑ	0.5
Le_{lp}, RPe_{jp}	$(1, 3) \sim U$	θ_{job}	1
$JOB_{jg}, JOB_{kg}, JOB_{mg}, JOB_{ng}$	$(500, 1000) \sim U$	$Re_{ijh}, Re_{jkp}, Re_{kap}$	$\sim U (0.1, 0.4)$

Uncertain Parameter	L1	L2	L3	L4
\widetilde{Dem}_{ap}	(1250) $\sim U(1000)$	(1750) $\sim U(1250)$	$(1750, 2250)$ $\sim U$	(3000) $\sim U(2250)$
\widetilde{FV}_v	$(5, 7)$ $\sim U$	$(7, 8)$ $\sim U$	$(8, 10)$ $\sim U$	$(10, 12)$ $\sim U$
$\widetilde{RC}_{np}, \widetilde{LC}_{lp}, \widetilde{RPC}_{jp}$	$(1, 2)$ $\sim U$	$(2, 3)$ $\sim U$	$(3, 4)$ $\sim U$	$(4, 5)$ $\sim U$
$\widetilde{PC}_{jp}, \widetilde{SC}_{ih}, \widetilde{DC}_{kp}, \widetilde{CC}_{mp}$	$(1, 2)$ $\sim U$	$(2, 3)$ $\sim U$	$(3, 4)$ $\sim U$	$(4, 5)$ $\sim U$

Therefore, these algorithms' parameters were changed using the Taguchi approach before attempting to solve the sample problem in tiny sizes using meta-heuristic algorithms. This strategy is designed to make algorithms more effective in generating efficient answers in a shorter time while avoiding local searches. Therefore, as a result of the three goals of the established model, comparison indices of the solution approach and following Eqs. (70) and (71) have been utilized to determine the RPD of each Taguchi experiment.

$$R_i = \frac{|NPF + MSI + SM + MID + CPU_{Time}|}{5} \tag{70}$$

$$RPD_i = \frac{R_i - R_i^*}{R_i^*} \tag{71}$$

In the above relation, R_i is the solution to each Taguchi test, R_i^* is the best solution to each algorithm test, and RPD_i is the scaled value of the results, which will be a number between 0 and 1. Accordingly, the parameters set by the Taguchi method for methods are obtained as described in Table 4.

Table 4. Optimal parameters of meta-heuristic algorithms by Taguchi method

Method	Factor	Optimal L	The Optimal Amount
NSGA II	N pop	3	200
	Pc	2	0.8
	Pm	2	0.05
MOPSO	N particle	3	200
	C1	1	1.5
	C2	2	1
	W	3	0.5
MOGWO	N Wolf	3	200
	A	3	3
	C	1	1.5

After parameter tuning, the designed model is solved using the mentioned methods, and the Pareto front obtained from solving the problem is obtained in Figure 9.

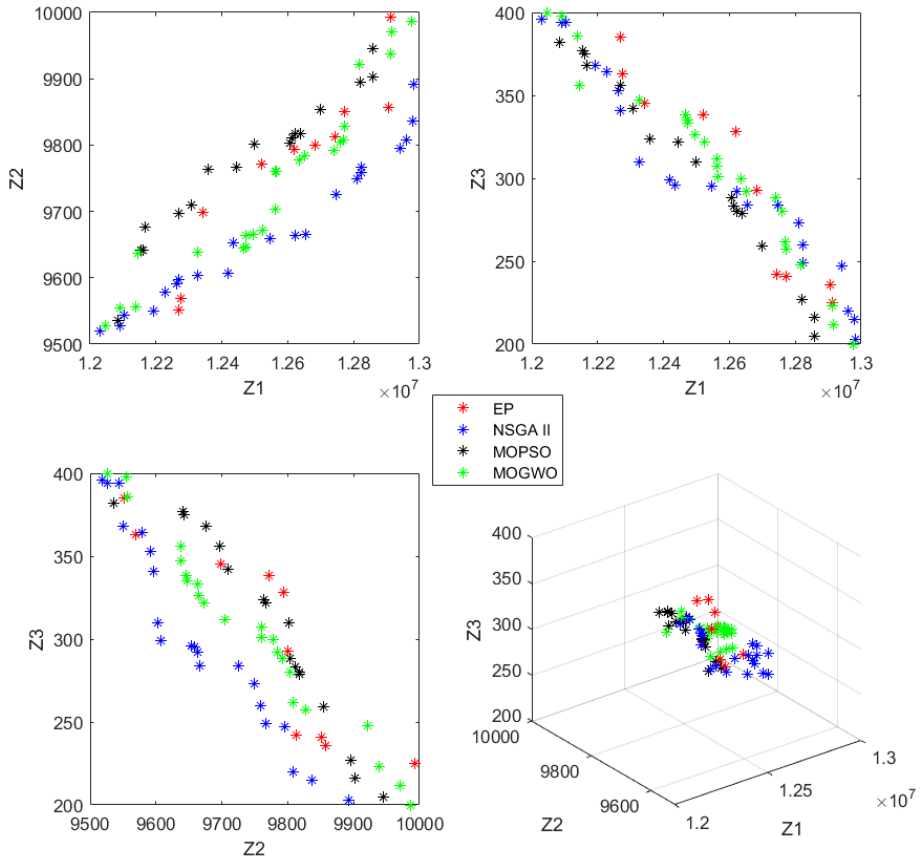


Figure 9. Pareto front obtained by solving a small-size instance

As seen in Figure 9, the overall network design cost will decrease as scarcity increases since fewer raw materials must be manufactured, distributed, or purchased. This is because there will be less demand for these activities. In addition, as the scarcity has decreased, there has been a decline in the number of production, distribution, and other potential centers, which has resulted in a decrease in employment. When looking at the pattern of changes in the first and second objective functions, on the other hand, it is clear that as the number of facilities has increased, so have the costs that are connected with maintaining the network as a whole. This can be seen when studying the pattern of changes. Table 5 also includes the descriptive statistics produced by employing various approaches to tackle the problem with the small sample size. The Epsilon method has utilized the best mean of the first objective function, which can be seen in Table 5 of the MOPSO, to restrict the best means of the second and third objective functions.

Table 5. The results obtained by solving small size sample problem

Solving Method	Descriptive Statistics	Z1	Z2	Z3
Epsilon constraint	The lowest amount	12268056.59	9551.00	225.00
	The maximum amount	12911832.58	9993.00	385.00
	Average	12603990.73	9769.60	299.60
	Standard deviation	231738.96	126.76	56.62
NSGA II	The lowest amount	12029387.81	9519.00	203.00
	The maximum amount	12981848.72	9892.00	396.00
	Average	12535210.69	9670.95	301.76
	Standard deviation	317690.48	107.08	58.36
MOPSO	The lowest amount	12084001.35	9536.00	205.00
	The maximum amount	12859728.19	9945.00	382.00
	Average	12480482.46	9769.47	305.47
	Standard deviation	252570.65	104.24	55.74
MOGWO	The lowest amount	12046940.50	9527.00	200.00
	The maximum amount	12976085.12	9986.00	400.00
	Average	12557205.59	9740.91	305.35
	Standard deviation	262298.50	128.34	54.00

The Epsilon constraint method has the worst computational time, despite obtaining the best mean of the Z2 and Z3. Therefore, solving the numerical example in a small size in 1624.39 seconds by the Epsilon method has been limited. Meta-heuristic algorithms have solved a small numerical example with a computational error of less than 1% in less than 60 seconds. To better evaluate the results, the variables related to supplier selection and the location of potential facilities are shown in Table 6.

Table 6. Results obtained from strategic decisions of the model by different solution methods

Solving Method	Epsilon Constraint	NSGA II	MOPSO	MOGWO
Supplier	2-3	2-3	1-4	2-4
Production center (capacity level)	:(2)2 :(2)1 (1)3	:(3)2 :(2)1 (2)4	:(1)3 :(2)1 (1)4	:(2)2 :(2)1 (2)4
Distribution center (capacity level)	(1)3 :(3)2	(2)2 :(2)1	(3)3 :(2)1	(3)3 :(3)2
Collection center (capacity level)	(1)3 :(2)2	(2)3 :(2)2	(3)2 :(2)1	(1)3 :(3)1
Recycling center (capacity level)	(1)1	(2)1	(1)1	(2)2

According to Table 6, it can be seen that all methods have worked the same in determining the number of potential centers; only the facility's location has been different due to the impact of objective functions on location. The following two sensitivity analyses have been performed to investigate the effect of problem parameters such as uncertainty rate and reliability on objective functions.

The generated Pareto front was evaluated in the prior section regarding an uncertainty rate of 0.5 and a reliability of 0.95. These figures were presented in the preceding section. First, the influence of the uncertainty rate on the objective functions in the small-size sample problem is studied by varying it within a range of 0.1 to 0.9, and the findings are displayed in Table 7.

Table 7. Value of objective functions at different uncertainty rates

Uncertainty Rate	Z1	Z2	Z3
0.1	12495416.25	9991.26	201.46
0.2	12514952.75	9934.74	224.36
0.3	12556784.66	9892.10	248.67
0.4	12581164.21	9817.64	284.67
0.5	12603990.73	9769.60	299.60
0.6	12674651.67	9418.46	326.67
0.7	12741253.49	8765.25	348.77
0.8	12794631.74	8419.46	369.04
0.9	12846652.92	8245.90	394.15

According to Table 7, it has been observed that as the rate of uncertainty has increased, the total amount of demand has increased due to its direct impact on the demand for engine oil. The probability of shortage or non-satisfaction with demand has increased (Z3). The number of potential centers for supply, production, distribution, recycling, and collection has dropped, as has the number of workers in the SC network. This is a direct result of the rising shortages that have been occurring. On the other hand, due to increased demand, the model is attempting to fulfill the greatest possible quantity of demand, which has led to a rise in the costs connected with production, transportation, and distribution. As a result, with the increase in the rate of uncertainty, it is reasonable to assert that the overall network's costs have climbed, the amount of labor utilized within the network has dropped, and the amount of items that are in short supply has also increased. Figure 10 also depicts the general

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 pattern of shifts that occur in the mean values of objective functions as a function of
 the rate of uncertainty.

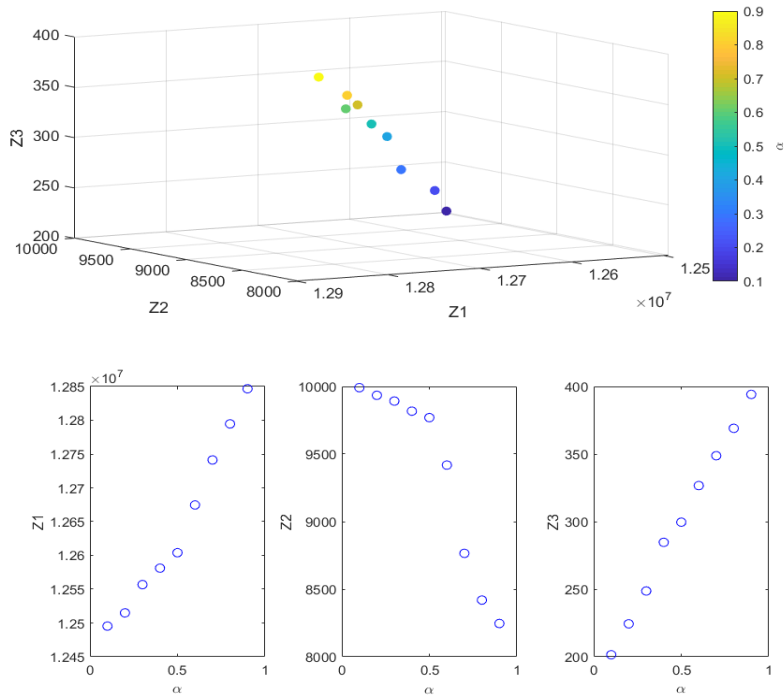


Figure 10. The changes in the objective functions at different uncertainty rates

In a separate piece of research, the confidence level of 95% in the movement of engine oil products through the forward SC network was investigated. In another study, this figure goes from 91% to 99%, and the objective functions derived from that study are listed in Table 8.

Table 8. Averages of objective functions in different reliability

Reliability	Z1	Z2	Z3
0.91	12448644.67	9620.34	295.25
0.92	12457985.14	9643.97	295.25

0.93	12517482.47	9643.97	299.60
0.94	12584654.67	9643.97	299.60
0.95	12603990.73	9769.60	299.60
0.96	12648727.42	9769.60	299.60
0.97	12694653.18	9769.60	299.60
0.98	12721646.82	9810.26	303.48
0.99	12741543.64	9810.26	303.48

Table 8 shows that with the increase of reliability in the SC network, the total costs of CLSCN have increased due to the use of transportation options with high reliability. Accordingly, increasing reliability does not have much effect on the second and third target functions and only in a few cases increases the workforce by relocating production and distribution centers and increasing the deficit by reducing the production capacity of new locations. Figure 11 also shows the changes in objective functions in different reliability capabilities.

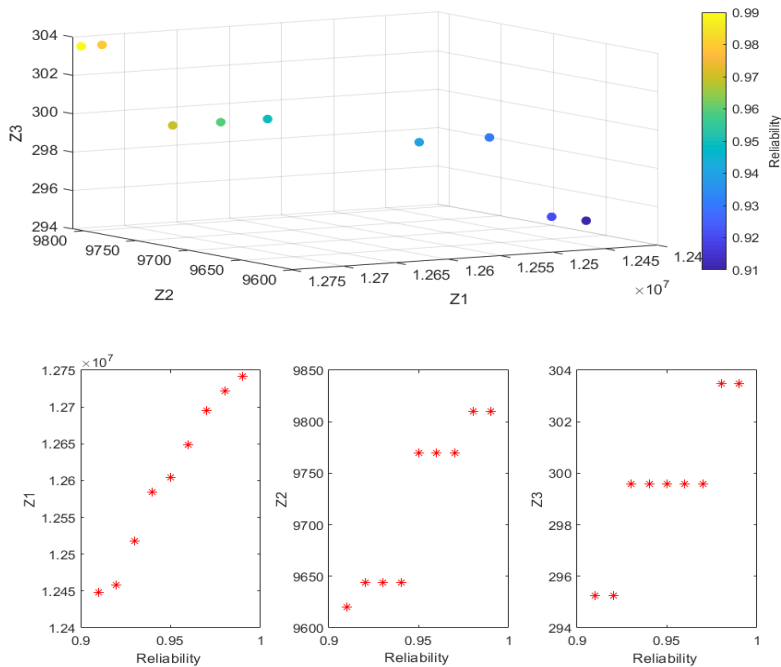


Figure 11. The changes in the means of objective functions in different reliability

Comparative indicators have been studied to select the most efficient method for solving numerical examples of larger sizes. This was done after considering the various effective solutions obtained from utilizing multiple approaches to finding a solution to a problem involving a sample of a smaller size. Table 9 compares the

Flexible fuzzy-robust optimization method in closed-loop supply chain network problem... different approaches to problem-solving when applied to the resolution of a small-size sample problem.

Table 9. Comparisons of different solution methods in small size sample problem

Indicator	NPF	MSI	SM	MID	CPU_time
Epsilon constraint	10	643776.16	0.728	335950.35	1624.39
NSGA II	21	952461.00	0.705	505832.13	56.82
MOPSO	17	775726.96	0.777	396491.64	42.39
MOGWO	23	929144.75	0.963	510273.87	40.77

According to the analysis, MOGWO has been selected as an efficient algorithm for solving numerical examples in larger sizes (a case study of engine oil in Iran) due to the higher number of efficient solutions and shorter solution time than other solutions.

5.2. Case Study of Engine Oil

The distribution of engine oil products in Iran has been considered to solve the problem in a larger size. For this purpose, 13 provincial centers (East Azerbaijan, West Azerbaijan, Isfahan, Alborz, Tehran, Khorasan Razavi, Khuzestan, Fars, Qom, Kerman, Kermanshah, Golestan, and Hamedan) have been considered as potential production centers. There are also three other potential centers in each center of the province. Hence 36 potential producer points are considered for the SC problem. Also, all the centers of the country's region are considered distribution and collection centers.

Considering the rate of the uncertainty of the amount of demand in the fuzzy trapezoidal set as $\widehat{Dem}_{ap} = (0.9Dem_{ap}^1, 0.95Dem_{ap}^2, 1.05Dem_{ap}^3, 1.1Dem_{ap}^4)$, is included. The amount of engine oil demand in each province of the country in terms of million liters in 1399 is specified in Table 10.

Table 10. Demand for engine oil in different provinces of Iran (million liters)

State	Million liters	State	Million liters	State	Million liters
East Azarbaijan	40	Southern Khorasan	15	Qom	16

Western Azerbaijan	11	Khorasan Razavi	79	Kurdistan	14
Ardabil	11	North Khorasan	11	Kerman	13
Esfahan	41	Khuzestan	20	Kermanshah	16
Alborz	13	Zanjan	13	Hamedan	17
Ilam	11	Semnan	15	Yazd	17
Bushehr	17	Golestan	26	Guilan	12
Tehran	52	Fars	17	Lorestan	20
Chahar Mahal Bakhtiari	11	Qazvin	11	Mazandaran	20
Kohgiluyeh and Boyerahmad	12	Hormozgan	2	Central	13
Sistan and Baluchestan	13				

Based on the results of comparing the indicators, MOGWO has been selected as a suitable solution method that can solve the problem in the shortest time and with the slightest error. Therefore, the algorithm has solved the green CLSCN model for the oil industry, and nine efficient solutions have been obtained in Table 11.

Table 11. Efficient solutions obtained from solving the case study in Iran

Efficient Solution	Z1 (million Rials)	Z2 (n)	Z3 (million liters)
1	3655687	19846	112
2	3719383	22570	108
3	3924140	22746	105
4	3926944	28388	104
5	3991300	29195	103
6	4096580	31148	97
7	4160196	31767	96
8	4303770	36240	94
9	4343210	37392	90

Also, the Pareto front is obtained from solving a case study in Iran, as shown in Figure 12.

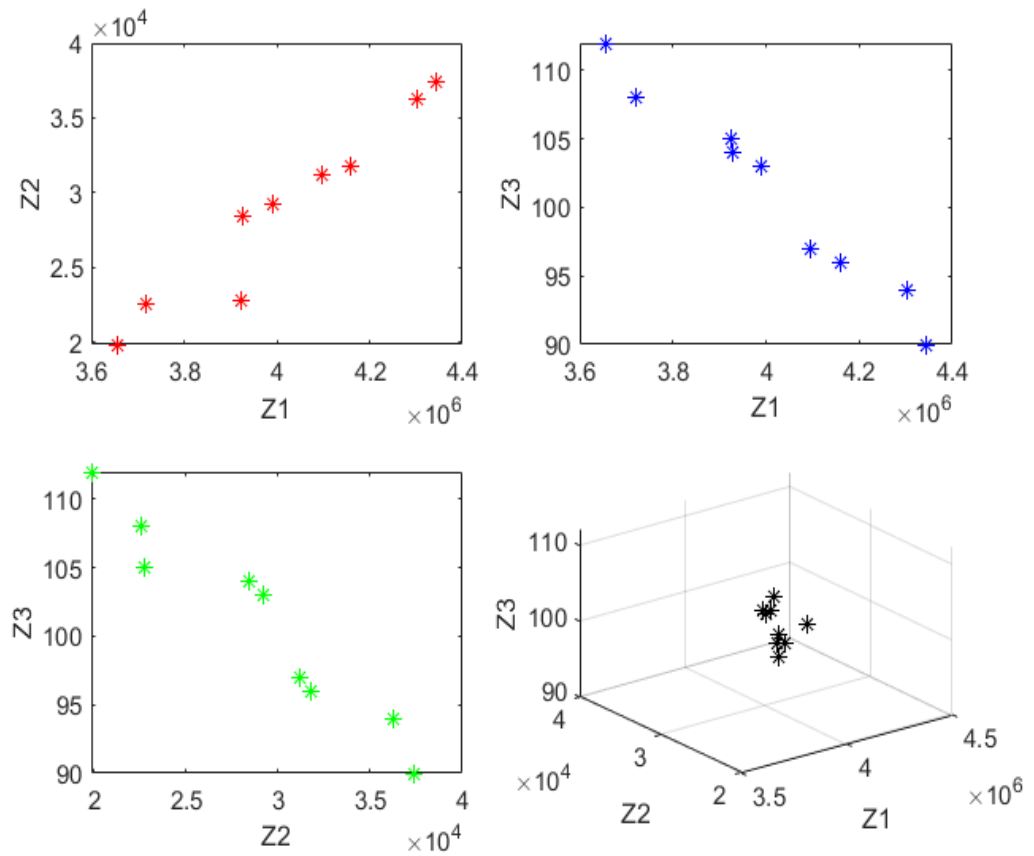


Figure 12. Pareto front obtained from solving a case study in Iran

According to the results obtained in the last efficient solution, it was found that four oil companies, Behran, Sepahan, Iranol, and Pars, in Isfahan, Tehran, and Karaj provinces can produce 514 million liters of engine oils and distribute 503 million liters per year. Therefore, the best optimal location of production and distribution facilities and the optimal allocation of the flow of engine oil products in the country is obtained, as shown in Figures 13 and 14. As a result, the total cost of establishing the network is 434321010 million Rials, the workforce employed is 37392 people, and the shortage is 90 million liters.



Figure 13. Optimal location of producer and distributor facilities and optimal flow allocation



Figure 14. Optimal location of distributor facilities and centers of demand and optimal flow allocation

5.3. Discussion

In this article, a model was designed for the engine oil industry, and by implementing it in Iran, various solutions were presented for the decision maker, which is suitable for each of the solutions, the location of production centers, distribution, and also how to transfer products between Different levels of the supply chain network was obtained. Based on this, the decision maker can make the best decision for implementing this network in Iran according to the cost and other goals. Nine proposed solutions are obtained for this article's case study.

6. Conclusion

This paper discusses modeling and solving a CLSCN problem for the engine oil industry under uncertain conditions. For this purpose, a three-objective model was designed to minimize the total costs of network design, maximize the workforce, and minimize the lack of demand. Four accurate and meta-heuristic methods were used to solve the problem. Due to the uncertainty in the parameters of demand, transfer costs, and operating costs, a new FFRO method was used. The results of numerical calculations in small-size examples showed that the Epsilon method obtained efficient solution constraint, NSGA II 21 efficient solutions, MOPSO 17 efficient solutions, and MOGWO 23 efficient solutions. The computational results also showed a very high computational time of the Epsilon constraint method compared to other meta-heuristic algorithms. Meta-heuristics algorithms were designed to solve the three-objective model in less than 60 seconds, with a difference of less than 1%.

On the other hand, by examining the uncertainty rates and reliability, it was observed that with the increase of the uncertainty rate, the costs of the whole network increased due to the rise in demand. At the same time, the number of hired workforce decreased. Also, due to the increase in the rate of uncertainty and as a result of demand, the shortage in the network has increased. On the other hand, to improve the network's reliability to send products from the supplier to the end customers securely, the costs related to the transfer method should be increased. With the final review of comparison indices between solution methods, MOGWO was selected as an efficient algorithm due to the number of more efficient solutions in a shorter time and was used to solve numerical examples in larger sizes (case study). This analysis showed that four oil companies, Behran, Sepahan, Iranol, and Pars in Isfahan, Tehran, and Karaj provinces, could produce 514 million liters of engine oils and distribute 503 million liters annually. As a result, the total cost of establishing the network was 434321010 million Rials, the workforce employed was 37392 people, and the shortage was 90 million liters.

6.1. Limitation

Failure to consider the objective environmental function in the supply chain network, along with another aim of sustainability functions, is one of the limitations of the research. Also, the complexity of the model, the lack of accurate methods to solve the problem in much larger and more realistic sizes, and the lack of a queue system in the production center of engine oil products are among the other research limitations.

6.2. Future Research

After modeling and problem-solving, suggestions for improvement are provided, including considering the inventory maintenance system not addressed in this model and other optimal discount levels in delivering raw materials for engine oil production. Due to the MINLP of the model, the use of heuristic methods is also recommended. Also, considering the limitations of the research, it is suggested to develop the model in three objective sustainability functions (economic, social, and ecological), and considering the queuing theory system in the production center of engine oil products can also add to the richness of the research. Along with innovative methods, the development of accurate methods such as branch-boundary is also suggested.

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