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THE M-POLAR FUZZY SET ELECTRE-I WITH REVISED SIMOS' AND AHP WEIGHT CALCULATION METHODS FOR SELECTION OF NON-TRADITIONAL MACHINING PROCESSES

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Abstract: *Using improvements to the recently published m-polar fuzzy set (mFS) elimination and choice translating reality-I (ELECTRE-I) approach for calculating criteria weights, the selection of a Non-Traditional Machining (NTM) process problem from the industry is solved in this research. The criteria weights for the m-polar fuzzy ELECTRE-I method are evaluated using the Analytical Hierarchy Process (AHP) approach and the Revised Simos' method. For the ELECTRE family's criteria weight calculations, the Simos' approach has been revised. Many researchers calculated the weight of the criteria in the selection of the NTM process using the AHP approach. Problems with both physical and intangible properties can be solved using the m-polar fuzzy ELECTRE-I approach. Additionally, it has the ability to solve MCDM issues with more variables. The improved Simos' technique is used in this work because it incorporates user choices for the criteria, or user voting for the criterion. Using expert assistance, the AHP technique prioritizes the criterion based on pair-by-pair comparisons of the criteria. The AHP approach makes compromises between the criteria. The ultimate selection of the process based on the needed aim is affected by both tangible and intangible features in the NTM selection dilemma. The impact of criteria weight techniques on the choice of the NTM process is examined using a single dimensional sensitivity analysis. AHP approach is proven to be less stable for criteria weight variation than the improved Simos' weight calculation method. The updated Simos' method, which takes into account user preferences, performs better for the m-polar fuzzy ELECTRE-I algorithm than the AHP weight calculation method.*

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1. Introduction

The industrial problem of NTM selection, where physical and intangible characteristics affect the ranking of options, is used in this work to identify the research problem. The m-polar fuzzy ELECTRE-I technique is the innovative strategy that addresses this problem. The outranking relationships between two alternatives, which make a clear comparison between two alternatives, are what set the m-polar fuzzy ELECTRE-I approach apart from other methods. Another issue associated with the criteria weight selection method in applying the m-polar fuzzy ELECTRE-I methodology. The Simos updated criterion weight calculation method and the AHP criteria weight calculation are the first two alternatives available for choosing the criteria weights. While different researchers employ the AHP weight calculation approach, Simos' criteria weight calculation method is created for the criteria weight calculation of the ELECTRE family. In this research, a method of calculating the criteria weights suited for the m-polar fuzzy ELECTRE-I for resolving the NTM selection problem is evaluated.

AHP deals with subjective variables or how the objective factors are prioritized. Considering the consistency ratio (CR) to be less than 10% validates the model's acceptance. This makes it more significant to calculate priority values using AHP as opposed to "weighting" for various qualities. The selection process was biased in the prior systems since the decision maker pre-specified the weightage values allocated to certain attributes. The AHP can effectively handle tangible and intangible features in the context of diverse people's subjective evaluations during the decision-making process. A consequence of an unmanageable number of pair-wise comparisons of each attribute's possibilities may occur in some instances. The ELECTRE-I is more effective at handling the quantifiable characteristics and the quantity of choices to be evaluated. Therefore, to benefit from both approaches, a combined strategy is used to choose the best NTM process for a certain work material and shape feature combination.

Careful selection of the best appropriate process for a given application is necessary for effective usage of the capabilities of various NTM processes. The following factors are typically taken into account when choosing an NTM process.

a) Physical characteristics

b) The characteristics of the work material and the shape of the machined feature

c) Process ability

d) Economy

The aforementioned factors make it challenging to compare the machining skills of various NTM methods. A significant obstacle to choosing the best NTM process for a specific case is the shrinking pool of qualified professionals with experience in NTM procedures. Therefore, it is necessary to create a straightforward scientific tool to aid users in choosing the appropriate NTM method in order to satisfy the real-time demands of the machining application.

The main goal of the NTM process selection technique is to discover the factors influencing the choice of NTM process and to find the best possible combination of these factors in relation to the actual needs of the machining application. In order to strengthen the current NTM process selection procedure, additional efforts must be made to identify the attributes that impact the decision to select an NTM process using a straightforward logical approach, to eliminate the unsuitable NTM processes, and to choose the best process.

1.1. NTM selection

The choice of NTM procedures is one of the most urgent problems that industries face. Obtaining elaborate forms over tough materials and executing sophisticated machining with accuracy are aspects of NTM techniques. Several NTM procedures are now in use. The names of the NTM processes that are suitable for machining are listed in the list below. Additionally, NTM processes have a unique set of performance criteria.

- ultrasonic machining (USM) (s1),
- water jet machining (WJM) (s2),
- abrasive jet machining (AJM)(s3),
- electrochemical machining (ECM)(s4),
- chemical machining (CHM)(s5),
- electrical discharge machining (EDM)(s6),
- wire electrical discharge machining (WEDM) (s7),
- electron beam machining (EBM)(s8),
- laser beam machining (LBM) (s9).

We must select the most effective method from the available possibilities in order to produce the surface of revolution on stainless steel and machine precision holes on duralumin. The following factors are taken into account for a more appropriate machining operation.

1. tolerance and surface finish (TSF) (t1),
2. power requirement (PR) (t2),
3. material removal rate (MRR) (t3),
4. cost (C) (t4),
5. efficiency (E) (t5),
6. tooling and fixtures (TF) (t6),
7. tool consumption (TC) (t7),
8. safety (S) (t8),
9. work material (M) (t9), and
10. shape feature (F) (t10).

The bipolar fuzzy set's extension is the mFS (Chen et al., 2014). This method can be used to address issues with criterion subgroups. The mathematical representation of the mFS is $[0,1]^m$, where m stands for a number of established concepts. Multi-criterion group decision-making (MCGDM) issues can be resolved by the mFS algorithm. By combining mFSs with different multi-criterion decision-making (MCDM) strategies, we can address MCDM and MCGDM problems. The ELECTRE-I approach uses outranking linkages to illustrate relationships between alternatives. The investigation in this paper used the mFS ELECTRE-I approach. The mFS hybrid technique is being used by a number of researchers to address selection issues in social and scientific fields. This project was chosen because it would combine the mFS ELECTRE-I algorithm with the updated Simos and AHP criterion weight calculation methods to create a single approach that can be applied to both MCDM and MCGDM situations. In order to implement the mFS ELECTRE-I method, the choice of the NTM process problem is taken into consideration in this study. The mFS, ELECTRE-I, updated Simos', AHP, and NTM selection are introduced in the first half of the article. The critical literature review that discusses the advantages and disadvantages of the existing expert systems in choosing the NTM method is presented in the second half of the paper. Based on literature, it provides a brief overview of how the ELECTRE approach has changed over time and the issues it resolves. The final component of the study provides a step-by-step explanation of the mFS ELECTRE-I integrated updated Simos' and AHP technique. In the fourth section, the mFS ELECTRE-I integrated updated Simos' and AHP algorithm is used to solve NTM selection Example 1a. The second NTM selection case is then resolved using the mFS ELECTRE-I integrated updated Simos' and AHP algorithm in the fifth part. The results validation process is described in section six by contrasting the TOPSIS-AHP method's findings with those from the mFS ELECTRE-I integrated revised Simos' and AHP algorithm. Finally, the work's conclusions are developed in the seventh section.

1.2. Revised Simos' method for Criterion weight calculation

The updated Simos' approach can be used to determine the weight of a criterion in the ELECTRE method family (Figueira & Roy, 2002). In this method, weight is determined by taking into account the ratio between the most important and least important user-suggested criteria. It consists of two steps; the first step allows for the calculation of non-normalized weights, and the second step allows for the calculation of normalized weights for the criterion. Users' preferences for the criterion may be requested without taking the range or the criterion scale into account. The following section of the paper illustrates how Simos' method was put into practice. The paper's goal is to ascertain how the criteria weight calculated by Simos' criterion weight computation affects the rank for the alternatives.

1.3. Analytical Hierarchy Process (AHP)

Another method for determining criterion weights is called AHP, and it mostly involves tradeoffs between the criteria. Similar to the Simos' technique, the AHP method asks users to rank their preferences for criteria without taking range or criterion scale into account. This is the same initial step for both methods (Figueira & Roy, 2002). The ELECTRE method's significance coefficients display the inherent weight, or voting power, which is taken into account for the outranking procedure (Mousseau et al., 2005). The method used to assess the weight of a criterion is the fundamental distinction between the AHP and the updated Simos'. The initial stage in the calculation of AHP weights is the construction of a pairwise comparison matrix using the principle or reflexivity of the criterion. The geometric mean method is used to calculate important degree in the second stage. Consistency values are expressed as vectors in the third phase. The fourth phase assesses the greatest eigenvalue's capacity for judgement. The evaluation of consistency index and consistency ratio, which show if the AHP is reasonable to adopt for a certain case study, is the fifth phase.

1.4. Objectives of the work

- To choose an appropriate NTM method to produce the revolutionized surface on stainless steel.
- To choose an appropriate NTM procedure to drill precise holes in duralumin.
- To discover the relationship of outranking between different NTM processes.

- To use a directed graph to represent the connections between various NTM processes.
- Applying mFS ELECTRE-I to solve selection of NTM process, ELECTRE-I combined the improved Simos' and AHP approach.
- To comprehend how the AHP criterion weight and improved Simos' approach affect the rank
- Performance of the m-polar fuzzy ELECTRE-I algorithm.
- To verify the findings by contrasting them with those of the earlier researcher.
- To find criteria weight stability range, performing single dimensional weight sensitivity analysis.

2. Literature Review

One of the most significant industrial selection issues is NTM process selection, which calls for the creation of scientific and mathematical techniques that take into account all machining applications (N. D. Chakladar & Chakraborty, 2008). To choose the optimum NTM process given a set of input parameters, numerous authors have developed expert systems. A computer-aided NTM selection technique created to benefit decision-makers. The selection of the NTM takes into account the work material and process capabilities such as corner radii taper, hole diameter, hole height to diameter ratio, size tolerance, surface finish, and size tolerance. Unsuitable alternatives are eliminated using a 16-digit classification code that is interactively created, and the remaining options are then sorted accordingly (Cogun, 1993). A two-stage selection process was created, with the first phase being the identification of suitable NTM processes and the second step ranking all of the processes according to the machining operation. It used two multi-attribute decision making (MADM) techniques, such as the approach for ordering preference by resemblance to ideal solution and the AHP, to rank feasible procedures (TOPSIS). For selection purposes, shape features that are needed after machining, process capabilities, and other necessary attributes are taken into account (Yurdakul & Çoğun, 2003). The appropriate NTM procedure is chosen using an AHP-based expert system based on the priority values for various criteria and sub-criteria. It employed a logic table to determine which NTM processes were within the acceptable ranges. It chooses the procedure with the higher acceptance index value (Chakraborty & Dey, 2006). For the purpose of choosing an appropriate NTM process, an expert system based on quality function deployment (QFD) was created. Based on the house of quality (HOQ) matrix, the capabilities of the product and process are compared. To evaluate the score of the NTM process, process characteristic weights are applied. The NTM process is chosen for the specific machining operation based on the process features that carry the highest weight (Chakraborty & Dey, 2007).

The choosing of NTM process required the development of a management information system (MIS). Multidimensionality, isolation, and scalability are three MIS factors that are improved in the first phase of a two-phase expert system. Typical NTM processes have a number of interconnected characteristics; this system determines the relationship between these parameters, demonstrating the system's multidimensionality. The focus of the expert system's second phase changed to a specific machining issue where end users might submit technical data. The software "Machining Expert" was created with the intention of normalising. The expert system uses just normalised data throughout. The created expert system is a unique industrial MIS with a tonne of built-in data (Chakrabarti et al., 2007). A combined

TOPSIS-AHP based expert system was created to choose the appropriate NTM process for a given material and shape feature combination. For the purpose of selection, it took into account the NTM processes' characteristics. The graphic user interface of the TOPSIS-AHP based expert system allows it to separate out the appropriate NTM processes and rank them in descending order of preference. It serves as a user manual for choosing the NTM method (N. D. Chakladar & Chakraborty, 2008). With a graphical user interface and visual aids, an automated expert system built on a pair-wise comparison matrix for qualities was created. The relative importance of the attributes used for NTM process selection is indicated by the pair-wise comparison matrix. Based on qualities and capacities for machining desired shapes on specific materials, this expert system assesses permanent values for NTM processes. Some NTM processes are regarded as acceptable NTM processes because they meet the machining operation's threshold requirement. Additionally, acceptable NTM procedures are ranked by the expert system in decreasing order (N. Das Chakladar et al., 2009). To minimise product costs, improve product quality, and shorten product lead times, a web-based NTM process selection expert system was created. This expert system operated over the internet, allowing designers and engineers with internet access to use it to choose an NTM technique. With the use of the internet, it aided in the exchange of process knowledge and produced wise decision-making. It is possible to implement utilised modules for process selection and expert modules to update the knowledge base in web-based expert systems. It has demonstrated the ability to choose NTM for a variety of industrial items (Edison Chandrasselan et al., 2008). For the purpose of choosing an NTM process, a two-phase decision model was created. In the first phase, the best combination of shape feature and work material for the chosen performance parameters was obtained using the input-minimized Charnes, Cooper, and Rhodes (CCR) model of Data Envelopment Analysis (DEA). The second phase of the ranking process uses a weighted overall efficiency ranking approach to rank the various NTM processes in descending order (Sadhu & Chakraborty, 2011).

The development of an expert system based on the Analytical Network Process (ANP) took into account the interdependence and feedback relationships between various criteria for NTM selection. The selecting process was automated using a graphic user interface (Das & Chakraborty, 2011). Multi-objective decision making techniques were used to choose the NTM process out of the various options with competing criteria. Multi-objective optimization based on ratio analysis (MOORA) is one such technique that is used to address selection issues in the manufacturing setting. The MOORA approach was used to resolve six unique industrial difficulties. Results from the application of the MOORA approach were comparable to those achieved by earlier researchers (Chakraborty, 2011). Data obtained from surveys and discussions with experts may come in crisp or fuzzy forms; either way, they can be managed with the use of various MCDM techniques. A fuzzy based decision model for choosing the NTM method was created to address issues with data input that was both fuzzy and crisp (Temuçin et al., 2014).

A novel approach using the combination of TOPSIS and geometrical analysis of interactive (GAIA) was created to address the issues that arise in the choosing of NTM process for machining hard materials and to provide decision-makers with visual aids. While GAIA provides a graphic user interface to assist decision makers in identifying the optimal NTM process for machining operations, TOPSIS is useful in ranking the choices. With the aid of the proposed strategy, the researchers were able to solve four separate NTM process selection issues. The results found were consistent with those drawn by earlier researchers (Karande & Chakraborty, 2012).

It is necessary to use an appropriate NTM process in order to handle the supplied material's complicated and detailed shape features. For the purpose of automating the selection of the NTM process, a decision-making model based on VISUAL BASICS 6.0 was created. To link product qualities with process characteristics, it is further integrated with QFD. With the aid of four different examples from the business world, the decision model was built, and its use was discovered (Prasad & Chakraborty, 2014). The primary problem in selecting NTM processes is the conflicting nature of the various qualitative and quantitative criteria. A fuzzy axiomatic decision-making method was created to address these kinds of problems. It can be employed to find solutions to issues like micro-drilling operations on hardened tool steel, the creation of tiny holes on titanium, and the manufacture of blind cavities on ceramics. The outcomes are highly pertinent to the choices made by experts in machining (Khandekar & Chakraborty, 2016). The overall score of the NTM process is calculated from the QFD method by taking shape features and work material into consideration. An expert system with a fuzzy analytic hierarchy process is used to evaluate the relative importance of NTM process alternatives with respect to product and process characterization. When integrating QFD to NTM processes, the viewpoint of the customer is taken into account (Roy et al., 2014).

For the purpose of choosing an NTM process, a memory model with case-based reasoning (CBR) is constructed. Information is accessed from the NTM process's previously stored data, and new NTM operations are developed using the indexing that is offered by the NTM process. A software solution is created to carry out this process, and a process is created to retrieve data from the software. The software contains all of the data from NTM processes. This is the most sensible method for choosing the NTM technique (Boral & Chakraborty, 2016). The combination of Factor Relationship (FARE) and Multi-attributive Border Approximation Area Comparison (MABAC) is designed to address the NTM process selection problem with conflicting criteria. In an MCDM setting, the FARE approach deals with criterion weight calculations. NTM processes are ranked according to their technical merits and performance using the MABAC technique. The results acquired using this established model are useful and consistent with those of the earlier researcher (Chatterjee et al., 2017). The challenging issue from the industry is solving complex challenges in the selection of NTM processes. Process engineers needed the right direction while making decisions. With the aid of VISUAL BASICS 6.0, a decision-guiding framework is created that aids in choosing the best NTM method for the desired shape feature and work material (Prasad & Chakraborty, 2018).

The following are reasons for creating scientific and mathematical instruments that can be gleaned from the literature.

- Support in making decisions.
- Based on the machining parameters, determine the NTM process.
- Make decision-making processes automated.
- Make a visual aid for the process engineers' decision-making.
- When cutting titanium, titanium alloys, and hardened tool steels, choose the NTM process.

The ELECTRE method was modified to create ELECTRE-I (Recherche & Roy, 1968). MCDM models employed a number of different techniques, including AHP, TOPSIS, PROMETHEE, and ELECTRE. The literature claims that in terms of performance and decision clarity, the ELECTRE family outperforms all other techniques. In ELECTRE, the "incomparable" preference relation aids in the comparison of alternatives by decision-makers. The Concordance and Discordance indices place a numerical value on the respective benefits and drawbacks (Akram et al., 2019). Numerous researchers have utilised ELECTRE-I in conjunction with fuzzy sets (Zadeh, 1965) to resolve real-world situations with insufficient information. A list of researchers and their published work is shown below in Table 1.

Table 1. Actual life application of fuzzy ELECTRE family

S. No.	method	Problems solved by researchers	Name of researchers
1	fuzzy ELECTRE	selection of academic staff	(Rouyendegh & Erkan, 2013)
2	fuzzy group ELECTRE	evaluation of hazardous waste recycling plants	Hatami-Marbini et al. (2013)
3	fuzzy ELECTRE-I	evaluation of mobile payment models	Asghari et al. (2010)
4	fuzzy ELECTRE-I	evaluating catering firm alternatives	Aytac et al. (2011)
5	fuzzy ELECTRE-I	environmental effect evaluation method	(Kaya & Kahraman, 2011)

The mFS was developed by Chen et al. (2014) as an expansion of the bi-polar fuzzy set. They figured out how to deal with multipolar information, numerous agents, multiple objects, multiple qualities, and/or uncertainty in real-world challenges. The techniques in Table 2, are unable to handle problems in the actual world combining many characteristics and data containing multipolar information. The mFS ELECTRE-I method was created to solve numerous characteristics and data as multipolar information. Real-world issues like finding a diesel plant, finding an airport, and assessing a physical sciences instructor's performance were all resolved using the mFS ELECTRE-I approach (Akram et al., 2019). Salary analysis of company and selection of corrupt country was done with the help of the m-polar fuzzy linguistic method (Adeel et al., 2019). The mFS ELECTRE-I algorithm was implemented for the industrial robot selection problem by Jagtap et al. (2021). The hesitant mFS ELECTRE-I and m-polar hesitant fuzzy ELECTRE-I algorithms were developed by Adeel et al. (2019) to address issues like brick selection for construction and site selection for farming. (Jagtap & Karande, 2021) investigated how normalisation affected the mFS ELECTRE-I algorithm and discovered that vector normalisation is an appropriate method for decision matrix normalisation. Incorporating a suitable weight calculator method into the mFS ELECTRE-I approach is the aim of this work. The parameter weights and rank alternatives are calculated using the mFS ELECTRE-I integrated revised Simos' and AHP approach. The mFS ELECTRE-I integrated updated Simos' and AHP technique is suitable to a wide range of parameter or pole size-related concepts. The manufacturing case studies literature was investigated, and mFS approaches were used to compare results to those of other researchers.

2.1. Research gap

- The expert systems' actual input and normalised input were not investigated.
- There are no expert systems available to handle multipolar data.
- Different approaches were needed to solve the MCDM and MCGDM challenges.
- The researchers did not use the m-polar fuzzy set strategy to choose an NTM process.
- Due to its complexity, the ELECTRE-I approach was not used in the selection of NTM.

2.2. Problems from literature

The difficulty taken into consideration for the selection of NTM was the creation of a surface with a revolution shape on stainless steel and the effective machining of precise holes on duralumin (N. D. Chakladar & Chakraborty, 2008). Parameters are categorised as positive, detrimental, quantitative, and qualitative as shown in Table 2. It also organised the data using cardinal data.

Table 2. Parameter category for NTM selection

Sr. No.	Beneficial	Non- Beneficial	Qualitative	Quantitative
1	material removal rate (MRR)	tolerance and surface finish (TSF)	cost (C)	tolerance and surface finish (TSF)
2	efficiency (E)	power requirement (PR)	efficiency (E)	power requirement(PR)
3	safety (S)	cost (C)	tooling and fixtures (TF)	material removal rate (MRR)
4	work material (M)	tooling and fixtures (TF)	tool consumption (TC)	
5	shape feature (F)	tool consumption (TC)	safety (S)	
6			work material (M)	
7			shape feature (F)	

High values are desired for advantageous parameters in the preceding table, whereas low values are preferred for unfavourable parameters. While qualitative factors are represented by rank value judgement on a range of 1 to 5, quantitative parameters have numerical values (1–low, 3–moderate, 5–high). We need to select the best NTM process to cause a revolution in stainless steel and poke holes in duralumin.

3. Methodology

With the updated Simos and AHP technique of criterion weight, the mFS ELECTRE-I algorithm is implemented in this article. Both exact and inaccurate data can be evaluated using the mFS ELECTRE-I method. Below is a detailed explanation of the mFS ELECTRE-I, the updated Simos', and the AHP approach.

3.1. Revised Simos' Method

The revised Simos' approach involves a step-by-step examination of weight, giving the user's viewpoint weight. By assigning a specific number to the criterion, users can indicate their preference for it (Figueira & Roy, 2002).

STEP I

A series of cards bearing the names of several criteria are given to the user. Users can assign numbers to the cards based on their preferred criteria. The criterion may be numbered by many users. The user has the option to organise the cards in ascending order, from least important to most important. Use the card that is considered to be the least important initially, and the person might number it one, then two, and so on. Users can clip the two criteria together if they believe that their weights are the same. Depending on individual preferences, each criterion can have a different number. If there are n criteria, then the ranks assigned to the cards are n'. If the difference in importance between two weights is regarded as a "u" unit, users are given white cards. A single white card separating two weights indicates a "2u" difference in the weights' relative value. Two white cards indicate a "3u" difference in the weight's relevance. The user will be prompted for the ratio between the least important criterion and most important criterion in accordance with the improved Simos' technique. Suppose z is the ratio.

STEP II

Calculation of non-normalized weights $k\ (r)$: Let e_r represent the quantity of white cards between ranks r and r+1 as shown in Eq. (1).

$$\begin{cases} e'_r = e_r + 1 \\ e = \sum_{r=1}^{n'-1} e_r \\ u = \dfrac{z-1}{e} \end{cases} \tag{1}$$

For all r = 1, 2… n'-1.

The equation below can be used to determine the weight of the criterion.

$k\ (r) = 1 + u\ (e_0 + \ldots + e_{r-1})$ with $e_0 = 0$. All criteria must have the same rank in order for their weights to be equal.

STEP III

Normalized weights (k_i), Let t_i criterion is having rank r and w_i be the weight of the criterion. The non-normalized weight can be written as $k_i' = k(r)$ and shown below in Eq. (2).

$$\begin{cases} K' = \sum_{i=1}^{n} k'_i \\ k_i^* = \dfrac{100}{K'} k'_i \end{cases} \tag{2}$$

Where k_i^* denotes the normalised weight determined by the updated Simos' weight criteria approach. The improved Simos approach is used in the section that follows to compute the weight of the criterion in the NTM selection process.

3.2. Analytical Hierarchy Process (AHP)

The weights of the criterion are determined using the AHP approach.

STEP-I

Let C' be the matrices for pairwise comparisons. A pair wise matrix can be used in the AHP method to estimate the relative importance of attributes. (Saaty, 2002) nine-point scale is used to create a pair wise comparison matrix. As the significance of the criterion rises from left to right, the nine-point scale is split into two groups of numbers (1, 3, 5, 7, 9). The following importance levels are 1(equal), 3(weak), 5(strong), 7(very strong), and 9(absolute). The second set of (2, 4, 6, 8) on the other hand shows a middle preference for the criterion. One measure is compared to another using the reflexivity principle. For the pair-wise comparison matrix, the first set is utilised. The pair-wise comparison matrix's diagonal elements are selfcomparable and share the same value. As they are self-compare, all of the diagonal elements have value one, or $c_{ij} = 1$. (N. D. Chakladar & Chakraborty, 2008).

$$C' = \begin{bmatrix} 1 & c_{12} & \cdots & c_{1n} \\ c_{21} & 1 & \dots & c_{2n} \\ \vdots & \dots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & 1 \end{bmatrix}$$

The strength of the i^{th} attribute's relative value in comparison to the j^{th} attribute is represented by the left and right sides of the diagonal matrix.

STEP-II

The relevance of the attributes is then assessed using a normalised geometric mean. And if w_i represents the significance of its characteristics, w_i can be calculated using Eq. (3).

$$w_i = \frac{\left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} c_{ij}\right)^{\frac{1}{n}}}$$

$$i, j = 1,2,3 \dots n \tag{3}$$

Eq. (4) shows that the total weights for all the criteria will equal one.

$$1 = \sum_{i=1}^{n} w_i \, . \tag{4}$$

STEP-III

To validate the output of the pair wise comparison matrix, a consistency ratio is introduced. The consistency ratio's value demonstrates that it is acceptable. Let's use the column vector E to represent the sum of the weighted values for the importance degree of the characteristics. It has n dimensions. Eq. (5) can be used to mathematically express the matrix *E*.

$$E = CW^T \tag{5}$$

Where $CW^T = \begin{bmatrix} 1 & c_{12} & \cdots & c_{1n} \\ c_{21} & 1 & \dots & c_{2n} \\ \vdots & \dots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & 1 \end{bmatrix} [w_1, \quad w_2, \quad \dots . \quad w_n] = \begin{bmatrix} E_1 \\ E_2 \\ \dots \\ E_n \end{bmatrix}$

STEP-IV

(Saaty, 2002) recommended utilising the Eigen value (λ_{max}), which is used to assess the efficacy of judgement, to prevent inconsistency. The value of λ_{max} closure to n demonstrates a more reliable evaluation. λ_{max} is expressed as shown in Eq. (6).

$$\lambda_{max} = \frac{\sum_{i=1}^{n} E_c}{n} \text{ Where } i=1,2,3 \dots n. \tag{6}$$

STEP-V

With λ_{max} consistency index (*CI*) can be defined as shown in Eq. (7).

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{7}$$

For $CI = 0$ Comparison matrix is perfectly consistent.

Further Consistency Ratio (*CR*) is used for the Consistency check, as shown in Eq. (8).

$$CR = \frac{CI}{RI} \tag{8}$$

RI is a random index that is derived from several pairwise comparison matrices of orders.

For $CR \leq 0.10$, Attribute levels are appropriate.

3.3. The m-polar Fuzzy ELECTRE-I method

1) Let $S=\{s_1,s_2,s_3,.........s_n\}$ set of options (Alternatives) available with $T=\{t_1,t_2,t_3,........t_n\}$ set of criterion.
2) The Decision Matrix can be represented as a value of alternative to criterion with the help of
$U=(u_{ij})=\{u_{ij}^1,u_{ij}^2,u_{ij}^3, ...u_{ij}^m\}$
3) Data input without normalization

OR

Eq. (9) shows the vector normalising technique to create a normalised decision matrix. In the m-polar fuzzy ELECTRE-I algorithm, there is an optional step; for the rank evaluation of alternatives, we can simply utilise real data. The use of normalised and non-normalized data has an impact on rank evaluation. In this work, further findings from both normalised and non-normalized data sets are reported.

$$N=\frac{u_{ij}}{\sqrt{\sum_{i=1}^{m} u_{ij}^2}} \tag{9}$$

4) $F=(f_{ij})$ is created by giving the m-polar decision matrix weights. As illustrated in Eq. (10), the weighted normalised matrix F is created by multiplying each column of the u_{ij} by w_j obtained from revised Simos' and AHP weight calculation method.
$F=(f_{ij})=(f_{ij}^1,f_{ij}^2,f_{ij}^3,........f_{ij}^m)$ here $f_{ij}=w_j u_{ij}$ (10)
5) The concordance sets are obtained under the next condition. With the value obtained y_{ij}, a weighted normalised matrix is further evaluated to compare the elements and ascertain the column-wise superiority of one component over the other component. The concordance set includes numbers for each of these dominating elements. Eq. (11) can be used to mathematically demonstrate the concordance set.
$$K_{pq}= \{1\leq j \leq t: y_{pj} \geq y_{qj}, p\neq q; p, q =1,2, ...n\}$$
Here $y_{ij}=f_{ij}^1+ f_{ij}^2+f_{ij}^3........+ f_{ij}^m$, (11)
6) The next circumstance results in discordance sets. In order to determine which element (criterion) has the most insignificant column-wise advantage over the other elements for the value determined by y_{ij}, a weighted normalised matrix is further examined for the element comparison. Then, a discordance set is numbered to include all of these least value criteria. Eq. (12) can be used to mathematically demonstrate the discordance set.
$$V_{pq}= \{1 \leq j \leq t: y_{pj} \leq y_{qj}, p\neq q; p, q=1,2, ...n\}$$
Here $y_{ij}=f_{ij}^1+ f_{ij}^2+f_{ij}^3........+ f_{ij}^m$, (12)
7) The weights of all such elements (criterion) from the concordance set of each element are added to generate the index's concordance value, which is mathematically represented as indicated in Eq. (13).
$k_{pq}=\sum_{j\in k_{pq}} w_j$, for all p,q. (13)
8) All of the concordance indices created by Eq. (13) can be used to construct the concordance matrix K.

$$K=\begin{pmatrix} - & k_{12} & k_{13} & k_{1n} \\ k_{21} & - & .. & k_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ k_{n1} & k_{n2} & \cdots & - \end{pmatrix}$$

9) The following Eq. (14) is used to evaluate the values of the discordance indices.

$$v_{\mathrm{pq}}=\frac{max_{j\in V_{pq}}\sqrt{\frac{1}{m}\left[\left(f^{1}{}_{pj}-f^{1}{}_{qj}\right)^{2}+\left(f^{2}{}_{pj}-f^{2}{}_{qj}\right)^{2}+\cdots+\left(f^{m}{}_{pj}-f^{m}{}_{qj}\right)^{2}\right]}}{max_{j}\sqrt{\frac{1}{m}\left[\left(f^{1}{}_{pj}-f^{1}{}_{qj}\right)^{2}+\left(f^{2}{}_{pj}-f^{2}{}_{qj}\right)^{2}+\cdots+\left(f^{m}{}_{pj}-f^{m}{}_{qj}\right)^{2}\right]}} \text{ for all } p,q \quad (14)$$

10) With all of the discordance indices created by Eq. (14), discordance matrix V can be generated.

$$V=\begin{pmatrix} - & v_{12} & v_{13} & v_{1n} \\ v_{21} & - & .. & v_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ v_{n1} & v_{n2} & \cdots & - \end{pmatrix}$$

11) Eq. (15) and Eq. (16) illustrate how levels of concordance (k) and discordance (v) might be defined.

$$\bar{k}=\frac{1}{n(n-1)}\sum_{\substack{p=1\\p\neq q}}^{n}\sum_{\substack{q=1\\q\neq p}}^{n}k_{pq} \quad (15)$$

$$\bar{v}=\frac{1}{n(n-1)}\sum_{\substack{p=1\\p\neq q}}^{n}\sum_{\substack{q=1\\q\neq p}}^{n}v_{pq} \quad (16)$$

12) The Concordance dominance matrix and the Discordance dominance matrix can be created from the Concordance and Discordance levels. The if and then statements for obtaining the concordance matrix A and discordance matrix B elements are shown in Eq. (17) and Eq. (18).

$$A=\begin{pmatrix} - & a_{12} & a_{13} & a_{1n} \\ a_{21} & - & .. & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & - \end{pmatrix}$$

Here

$$a_{pq}=\begin{cases}1, k_{pq}\geq \bar{k}\\ 0, k_{pq}\leq \bar{k}\end{cases} \quad (17)$$

$$B=\begin{pmatrix} - & b_{12} & b_{13} & b_{1n} \\ b_{21} & - & .. & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & - \end{pmatrix}$$

Here

$$b_{pq}=\begin{cases}1, v_{pq}\geq \bar{v}\\ 0, v_{pq}\leq \bar{v}\end{cases} \quad (18)$$

13) Point-to-point multiply the values of matrices A and B to generate the aggregate dominance matrix D.

$$D=\begin{pmatrix} - & d_{12} & d_{13} & d_{1n} \\ d_{21} & - & .. & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \cdots & - \end{pmatrix}$$

To solve the selection problem using the mFS ELECTRE-I integrated AHP approach, the aforementioned fourteen states must be followed. In order to rate relationships between alternatives, matrices A, B, and D are used. Figure 1, shows research methodology followed in this research paper.

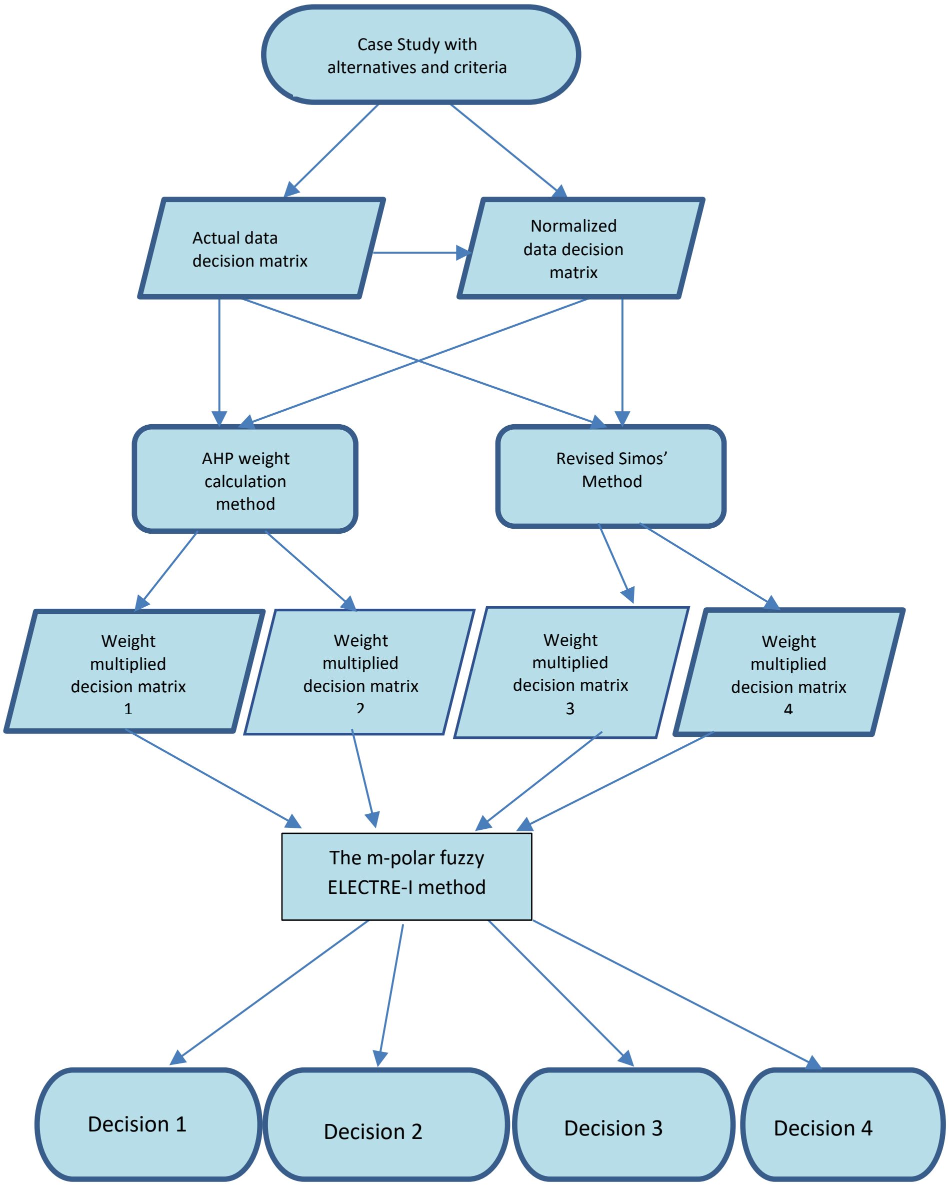


Figure 1. Research Methodology

3.4. Sensitivity Analysis

Sensitivity analysis is a methodology to determine how changes in the input affect the results of the MCDM process. It demonstrates the power of the MCDM method. According to (Ustinovichius & Simanaviciene, 2010) sensitivity analysis examines the relationship between sources of input uncertainty and uncertainty in a model's output. Its tool for evaluating the uncertainty in the MCDM model. As a result, sensitivity analysis can be performed to verify the findings and identify the input model's stable output range. Local weight stability interval indicates the range of weights within which rank of best alternatives remains unchanged and Global weight stability interval shows range of weights for which rank of overall alternatives remains unchanged (Karande et al., 2016).In MCDM approaches, input performance data and criteria weight have an impact on outputs. Calculating criteria weights can be done using a variety of methods. Weight obtained using various methods varies from one another. Due to the methods used in their computations and the inherent uncertainty involved, criteria weights in MCDM are frequently contested. Similar to AHP, the method for calculating weight is based on the decision makers-viewpoint. Manufacturer’s Data on input performance cannot be subjected to sensitivity analysis because it came from reputable sources. As a result, sensitivity analysis is carried out to investigate the impact of changing the weight of the criterion on the alternatives' final rankings. The stability of the specific designed model is demonstrated by the sensitivity analysis of the criteria weight calculation approach. In this study, sensitivity analysis is used to compare and contrast two alternative criteria weighting approaches. It leads to the discovery of a more reliable criteria weighting technique appropriate for the m-polar fuzzy ELECTRE-I approach.

3.4.1. Single Dimensional weight sensitivity Analysis (SDWSA)

The greatest criteria weight in the SDWSA is variable for evaluating the range of solutions that are practical. All other criteria weights are modified while modifying the highest criteria weight in a way that satisfies the additive principle of criteria weight. The weights of the criterion will not remain proportional to variation in this manner. The greatest criterion weights are taken into account in this procedure since they affect the rankings of the alternatives. The highest criteria weight is constrained in this method because it becomes impracticable if other criteria's weights are negative during weight variation. It is important to determine the highest and lowest weights in order to find a consistent range of the criteria weight. The most influential criteria's minimum weight is zero, and their maximum weight is shown in Eq. (19).

$$W_{\max} = [\, W_{\text{highest}} + (n\text{-}1) \times W_{\text{Lowest}}] \tag{19}$$

Where, $W_{\max}$ is the maximum varied weight of highest criteria, and the W_{highest} is the weight of influential criteria. W_{Lowest} is the least influential criteria. After the implementation of the sensitivity analysis to MCDM method, we can identify the local stability range and global stability range. In the local stability range rank of the best alternative remains unaltered. In the global stability range ranks of all the alternatives remains unaltered.

4. Example 1: Selection of NTM process for surface of revolution on stainless steel

In order to choose the best NTM process for producing a surface of revolution feature on stainless steel, it is first necessary to identify a number of significant factors that will influence the selection process. These characteristics include power requirement (PR), material removal rate (MRR), cost (C), efficiency (E), tooling and fixtures (TF), tool consumption (TC), safety (S), work material (M), and shape feature (F) . TSF (m), PR (kW), and MRR (mm3/min) are three of these properties that are quantitative in nature and have absolute numerical values. As opposed to C, E, TF, TC, S, M, and F, which have qualitative measures and call for a ranking value judgement on a scale of 1–5 (1 being the lowest, 3 the middle, and 5 the highest). Benefiting qualities where high values are desirable are MRR, E, S, M, and F. On the other side, low values are recommended for the non-beneficial qualities TSF, PR, C, TF, and TC. The relatedness is ensured using a five-point scale because the data for the criterion TSF, PR, and MRR are cardinal in nature whereas the data for the other criteria, such as C, E, etc., may change over time.

They chose nine different NTM procedures, each with ten parameters, to solve the stainless-steel problem (N. D. Chakladar & Chakraborty, 2008). Table 3, provides the decision matrix for the same scenario. While the remaining parameters are given qualitative values using a five-point scale, the TSF, PR, and MRR parameters are given real values (quantitative).

Table 3. Decision Matrix: performance of various NTM processes to different attributes (N. D. Chakladar & Chakraborty, 2008)

Sr.No	Parameters									
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
1	1.0	10.00	500.0	2	4	2	3	1	4	1
2	2.5	0.22	0.8	1	4	2	2	3	4	1
3	2.5	0.24	0.5	1	4	2	2	3	4	1
4	3.0	100.00	400.0	5	2	3	1	3	5	4
5	3.0	0.40	15.0	3	3	2	1	3	5	1
6	3.5	2.70	800.0	3	4	4	4	3	5	1
7	3.5	2.50	600.0	3	4	4	4	3	5	1
8	2.5	0.20	1.6	4	5	2	1	3	4	1
9	2.0	1.40	0.1	3	5	2	1	1	4	1

The normalised decision matrix generated using the vector normalisation method from Table 3, is displayed in Table 4.

Table 4. Normalized Decision Matrix

Sr. No.	Parameters									
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
s_1	0.122	0.099	0.421	0.219	0.334	0.248	0.412	0.124	0.298	0.204
s_2	0.307	0.002	0.0007	0.109	0.334	0.248	0.274	0.372	0.298	0.204
s_3	0.307	0.002	0.0004	0.109	0.334	0.248	0.274	0.372	0.298	0.204
s_4	0.368	0.994	0.336	0.548	0.167	0.372	0.137	0.372	0.372	0.816
s_5	0.368	0.004	0.012	0.329	0.250	0.248	0.137	0.372	0.372	0.204
s_6	0.430	0.026	0.673	0.329	0.334	0.496	0.549	0.372	0.372	0.204
s_7	0.430	0.024	0.505	0.329	0.334	0.496	0.549	0.372	0.372	0.204
s_8	0.307	0.002	0.001	0.439	0.418	0.248	0.137	0.372	0.298	0.204
s_9	0.245	0.013	0.0001	0.329	0.418	0.248	0.137	0.124	0.298	0.204

4.1. Criterion weight calculation by Revised Simos' Method

STEP-I

The criteria for NTM selection are arranged in ascending order in Table 5, in accordance with the preferences of the users.

Table 5. Criterion preference for NTM selection as per user suggestion.

Rank	Subset	Number of cards according to the rank
1	$\{t_8, t_7, t_6\}$	3
2	$\{t_5\}$	1
3	$\{t_2, t_1, t_4\}$	3
4	White Card	1
5	$\{t_3\}$	1
6	$\{t_{10}\}$	1
7	$\{t_9\}$	1

STEP-II

The user-provided criteria preferences are used to create non-normalized weights as shown in Table 6.

Table 6. Non-Normalized weight for z= 5.5.

Rank r	Criterion in the rank r	Number of white cards according to rank r, e_r	e_r	Non-normalized weights $k(r)$	Total
1	$\{t_8, t_7, t_6\}$	0	1	1.00	3 × 1 = 3.00
2	$\{t_5\}$	0	1	1.75	1×1.75= 1.75
3	$\{t_2, t_1, t_4\}$	0	1	2.5	3×2.5=7.5
4	$\{t_3\}$	1	2	3.25	1×3.25=3.25
5	$\{t_{10}\}$	0	1	4.75	1×4.75=4.75
6	$\{t_9\}$	...	...	5.50	1×5.50=5.50
Sum	10	1	6		25.75

STEP-III

Normalized weights are obtained with the help of steps explained in section 3.1, and shown in Table 7.

Table 7. Normalized weight of each criterion for w = 1 and z = 5.5.

Rank	Criteria	N	Normalized Weight k^*	Normalized weight k''	Normalized weight k'
1	t_8	8	3.88	3.8	3.9
1	t_7	7	3.88	3.8	3.9
1	t_6	6	3.88	3.8	3.9
2	t_5	5	6.79	6.7	6.8
3	t_2	2	9.708	9.7	9.7
3	t_1	1	9.708	9.7	9.7
3	t_4	4	9.708	9.7	9.7
4	t_3	3	12.621	12.6	12.6
5	t_{10}	10	18.446	18.4	18.4
6	t_9	9	21.3592	21.3	21.4
				99.5	100

4.2. Criterion weight calculation by AHP method

Finally, a pair-wise comparison for the parameters in Table 8, is provided using a nine-point scale. The significance of the scale's values is further broken down into the following categories: (1- equally important, 3- weakly important, 5- strongly important, 7- very strongly important, and 9- absolutely crucial), while (2, 4, 6, and 8) demonstrate a middle-ground preference value.

STEP-I

Table 8. Pair wise Comparison Matrix C.

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
t_1	1	0.5	3	2	0.33	0.25	0.2	0.16	5	4
t_2	2	1	3	2	0.5	0.33	0.25	0.2	5	4
t_3	0.33	0.33	1	0.5	0.2	0.16	0.14	0.12	2	3
t_4	0.5	0.5	2	1	0.25	0.2	0.14	0.13	4	3
t_5	3	2	5	4	1	0.5	0.33	0.25	7	6
t_6	4	3	6	5	2	1	0.5	0.33	8	7
t_7	5	4	7	6	3	2	1	0.5	9	8
t_8	6	5	8	7	4	3	2	1	9	9
t_9	0.2	0.2	0.5	0.25	0.13	0.12	0.11	0.11	1	0.5
t_{10}	0.25	0.25	0.33	0.33	0.16	0.13	0.12	0.11	2	1
Total	22.28	16.78	35.83	28.08	11.57	7.69	4.79	2.91	52	45.5

Finding matrix Normalized matrix C'.

$$C'=\begin{bmatrix} 0.04 & 0.03 & 0.08 & 0.07 & 0.03 & 0.03 & 0.04 & 0.05 & 0.09 & 0.08 \\ 0.09 & 0.06 & 0.08 & 0.07 & 0.04 & 0.04 & 0.05 & 0.07 & 0.09 & 0.08 \\ 0.01 & 0.02 & 0.02 & 0.017 & 0.017 & 0.02 & 0.03 & 0.04 & 0.04 & 0.06 \\ 0.02 & 0.03 & 0.05 & 0.03 & 0.02 & 0.02 & 0.03 & 0.04 & 0.07 & 0.06 \\ 0.13 & 0.12 & 0.14 & 0.14 & 0.08 & 0.06 & 0.06 & 0.08 & 0.13 & 0.13 \\ 0.18 & 0.18 & 0.16 & 0.17 & 0.17 & 0.13 & 0.1 & 0.11 & 0.15 & 0.15 \\ 0.22 & 0.23 & 0.19 & 0.21 & 0.26 & 0.26 & 0.2 & 0.17 & 0.17 & 0.17 \\ 0.27 & 0.3 & 0.22 & 0.25 & 0.34 & 0.39 & 0.41 & 0.34 & 0.17 & 0.2 \\ 0.009 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 & 0.02 & 0.04 & 0.02 & 0.01 \\ 0.01 & 0.014 & 0.01 & 0.01 & 0.01 & 0.01 & 0.02 & 0.04 & 0.04 & 0.02 \end{bmatrix}$$

STEP-II

To determine the weights of the criteria, apply Eq. (3). We obtained the following criterion weight values for example-1,

$$\boldsymbol{W=[0.0571\ 0.0695\ 0.0293\ 0.040\ 0.1108\ 0.1532\ 0.2120\ 0.2907\ 0.0161\ 0.0201]^T}$$

W_{TSF}= 0.0571, W_{PR} = 0.0695, W_{MRR} = 0.0293, W_C = 0.040, W_E = 0.1108, W_{TF} = 0.1532, W_{TC} = 0.212, W_S = 0.290, W_M = 0.016, W_F = 0.0201.

STEP-III

$$\boldsymbol{CW=[0.586\ \ 0.835\ \ 0.304\ \ 0.673\ \ 2.88\ \ 3.114\ \ 2.457\ \ 1.98\ \ 0.007\ \ 0.005]^T}$$

STEP-IV

$$\lambda_{max}= {}^{1}/_{10}\,(\frac{0.586}{0.0571}+\frac{0.835}{0.069}+\frac{0.304}{0.0293}+\frac{0.673}{0.04}+\frac{2.88}{0.1108}+\frac{3.114}{0.1532}+\frac{2.457}{0.2120}$$

$$+\frac{1.98}{0.2907}+\frac{0.007}{0.0161}+\frac{0.005}{0.0201})$$

λ_{max} = 11.46

STEP-V

$CI = \frac{11.46-10}{10-1} = 0.162$

Saaty et al. (2002) provided random indexes for different sizes of matrices as below in Table 9.

Table 9. Random indexes for different sizes of matrices

Num. of Criteria	1	2	3	4	5	6	7	8	9	10	11	12
Random Index	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48

For Ten criterions random index (RI) is 1.49.

$CR = \frac{CI}{RI} = \frac{0.162}{1.49} = 0.10$

Eq. (6) and Eq. (8) are used for the evaluation of the maximum Eigen value (λ_{max}) and consistency ratio (CR). For the pair wise comparison matrix shown in Table 8, λ_{max} = 11.46, and CR = 0.10 It shows consistency, as this value is less than 0.10. We can consider AHP priorities given by the expert.

The evaluation of the maximal Eigen value (λ_{max}) and CR is performed using Eq. (6) and Eq. (8). λ_{max} = 11.46, and CR = 0.10 for the pair-wise comparison matrix in Table 8. Given that this figure is smaller than 0.10, consistency can be seen. We can take into account the expert's suggested AHP priorities.

Table 10. Weighted Normalized Matrix

Sr. No.	Parameters									
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
s_1	0.007	0.007	0.012	0.008	0.037	0.038	0.087	0.036	0.004	0.004
s_2	0.017	0.0001	0.0002	0.004	0.037	0.038	0.058	0.108	0.004	0.004
s_3	0.017	0.0001	0.0001	0.004	0.037	0.038	0.058	0.108	0.004	0.004
s_4	0.021	0.0691	0.009	0.021	0.018	0.057	0.029	0.108	0.006	0.016
s_5	0.021	0.0002	0.0003	0.013	0.027	0.038	0.029	0.108	0.006	0.004
s_6	0.024	0.0018	0.019	0.013	0.037	0.076	0.116	0.108	0.006	0.004
s_7	0.024	0.0017	0.014	0.013	0.037	0.076	0.116	0.108	0.006	0.004
s_8	0.017	0.0001	0.0003	0.017	0.046	0.038	0.029	0.108	0.004	0.004
s_9	0.014	0.0009	0.0003	0.013	0.046	0.038	0.029	0.036	0.004	0.004

Table 10, shows the weighted normalised matrix. A weighted normalised decision matrix is used to perform Eq. (9) and Eq. (10) from the procedure. The results are displayed in Table 11, as a concordance set and Table 12, as discordance set.

Table 11. Concordance Set

j	1	2	3	4	5	6	7	8	9
K_{1j}	-	{2,3,4,5,6,7, [illegible]	{2,3,4,5,6,7, [illegible]	{3,5,7}	{2,3,5,6,7,10 [illegible]	{2,5,10}	{2,5,10}	{2,3,6,7,9,10 [illegible]	{2,3,6,7,8,9, [illegible]
K_{2j}	{1,5,6,8,9,10 [illegible]	-	{1,3,4,5,6,7, [illegible]	{5,7,8}	{5,6,7,8,10}	{5,8,10}	{5,8,10}	{1,2,6,7,8,9, [illegible]	{1,3,6,7,8,9, [illegible]
K_{3j}	{1,5,6,8,9,10 [illegible]	{1,2,4,5,6,7, [illegible]	-	{5,7,8}	{5,6,7,8,10}	{5,8,10}	{5,8,10}	{1,2,6,7,8,9, [illegible]	{1,3,6,7,8,9, [illegible]
K_{4j}	{1,2,4,6,8,9, [illegible]	{1,2,3,4,6,8, [illegible]	{1,2,3,4,6,8, [illegible]	-	{1,2,3,4,6,7, [illegible]	{2,4,8,9,10}	{2,4,8,9,10}	{1,2,3,4,6,7, [illegible]	{1,2,3,4,6,7, [illegible]
K_{5j}	{1,4,6,8,9,10 [illegible]	{1,2,3,4,6,8, [illegible]	{1,2,3,4,6,8, [illegible]	{1,5,7,8,9}	-	{4,8,9,10}	{4,8,9,10}	{1,2,3,6,7,8, [illegible]	{1,3,4,6,7,8, [illegible]
K_{6j}	{1,3,4,5,6,7, [illegible]	{1,2,3,4,5,6, [illegible]	{1,2,3,4,5,6, [illegible]	{1,3,5,6,7,8, [illegible]	{1,2,3,4,5,6, [illegible]	-	{1,2,3,4,5,6, [illegible]	{1,2,3,6,7,8, [illegible]	{1,2,3,4,6,7, [illegible]
K_{7j}	{1,3,4,5,6,7, [illegible]	{1,2,3,4,5,6, [illegible]	{1,2,3,4,5,6, [illegible]	{1,3,5,6,7,8, [illegible]	{1,2,3,4,5,6, [illegible]	{1,4,5,6,7,8, [illegible]	-	{1,2,3,6,7,8, [illegible]	{1,2,3,4,6,7, [illegible]
K_{8j}	{1,4,5,6,8,9, [illegible]	{1,3,4,5,6,8, [illegible]	{1,3,4,5,6,8, [illegible]	{5,7,8}	{4,5,6,7,8,10 [illegible]	{4,5,8,10}	{4,5,8,10}	-	{1,3,4,5,6,7, [illegible]
K_{9j}	{1,4,5,6,8,9, [illegible]	{2,4,5,6,9,10 [illegible]	{2,4,5,6,9,10 [illegible]	{5,7}	{2,4,5,6,7,10 [illegible]	{4,5,10}	{4,5,10}	{2,5,6,7,9,10 [illegible]	-

Table 12. Discordance Set

j	1	2	3	4	5	6	7	8	9
V_{1j}	-	{1,8}	{1,8}	[illegible] 10}	{1,4,8,9}	[illegible] 9}	[illegible] 9}	{1,4,5,8}	{1,4,5}
V_{2j}	{2,3,4,7}	-	{2}	[illegible] 10}	{1,2,3,4,9}	[illegible] 9}	[illegible] 9}	{3,4,5}	{2,4,5}
V_{3j}	{2,3,4,7}	{3}	-	[illegible] 10}	{1,2,3,4,9}	[illegible] 9}	[illegible] 9}	{3,4,5}	{2,4,5}
V_{4j}	{3,5,7}	{5,7}	{5,7}	-	{5}	{1,3,5,6,7}	{1,3,5,6,7}	{4,5}	{2,5}
V_{5j}	{2,3,5,7}	{5,7}	{5,7}	{2,3,4,6,10}	-	{1,2,3,5,6,7}	{1,2,3,5,6,7}	{4,5}	{2,5}
V_{6j}	{2}	{}	{}	{2,4,10}	{}	-	{}	{4,5}	{5}
V_{7j}	{2}	{}	{}	{2,4,10}	{}	{2,3}	-	{4,5}	{5}
V_{8j}	{2,3,7}	{2,7}	{2,7}	[illegible] 10}	{1,2,3,9}	{1,2,3,6,7,9}	{1,2,3,6,7,9}	-	{2}
V_{9j}	{2,3,7}	{1,3,7,8}	{1,3,7,8}	[illegible] 9,10}	{1,3,8,9}	[illegible] 9}	[illegible] 9}	{1,3,4,8}	-

Concordance and discordance indices, concordance matrix K, and discordance matrix V are calculated using Eq. (13) and Eq. (14), respectively, from the algorithm.

$$K = \begin{bmatrix} - & 0.651 & 0.651 & 0.3521 & 0.5949 & 0.2004 & 0.2004 & 0.5002 & 0.7909 \\ 0.648 & - & 0.9293 & 0.6135 & 0.7868 & 0.4216 & 0.4216 & 0.8187 & 0.7785 \\ 0.648 & 0.9695 & - & 0.6135 & 0.7868 & 0.4216 & 0.4216 & 0.8187 & 0.7785 \\ 0.6467 & 0.676 & 0.676 & - & 0.888 & 0.4364 & 0.4364 & 0.888 & 0.888 \\ 0.5772 & 0.676 & 0.676 & 0.6867 & - & 0.3669 & 0.3669 & 0.848 & 0.8185 \\ 0.9293 & 0.9988 & 0.9988 & 0.8692 & 0.9988 & - & 0.9988 & 0.848 & 0.888 \\ 0.9293 & 0.9988 & 0.9988 & 0.8692 & 0.9988 & 0.9 & - & 0.848 & 0.888 \\ 0.688 & 0.7173 & 0.7173 & 0.6135 & 0.8268 & 0.4616 & 0.4616 & - & 0.9293 \\ 0.688 & 0.4097 & 0.4097 & 0.3228 & 0.6056 & 0.1709 & 0.1709 & 0.5817 & - \end{bmatrix}$$

$$V = \begin{bmatrix} - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.1590 \\ 0.4038 & - & 1 & 1 & 0.3016 & 1 & 1 & 0.4525 & 0.1284 \\ 0.4038 & 0.6323 & - & 1 & 0.3016 & 1 & 1 & 0.4525 & 0.1284 \\ 0.8074 & 0.4421 & 0.4222 & - & 0.1347 & 1 & 1 & 0.4031 & 0.3854 \\ 0.8074 & 1 & 1 & 1 & - & 1 & 1 & 1 & 0.2568 \\ 0.0699 & 0 & 0 & 0.7698 & 0 & - & 0 & 0.1060 & 0.1060 \\ 0.0717 & 0 & 0 & 0.7713 & 0 & 1 & - & 0.1060 & 0.1060 \\ 0.8074 & 1 & 1 & 1 & 0.1895 & 1 & 1 & - & 0.0114 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & - \end{bmatrix}$$

Matrix A and Matrix B are evaluated using Eq. (17) and Eq. (18). It appears as a comparison of each matrix K and V element with respect to levels of concordance and discordance. For instance, in example-1, Eq. (15) yields a concordance level of 0.6820, whereas Eq. (16) yields a discordance level of 0.640. Values above 0.6820, satisfy condition 1 for concordance matrix K, whereas values below 0.6820 satisfy condition 0. Values below 0.640, are one and above 0.640, are 0 when the discordance matrix is compared to the discordance level of 0.640.

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The ultimate ranking of the alternatives is shown in Matrix D, which displays the total matrix. The ELECTRE-I approach provides a precise comparison of the available options. Electre-I has a special quality that provides outranking relationships for other alternatives.

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

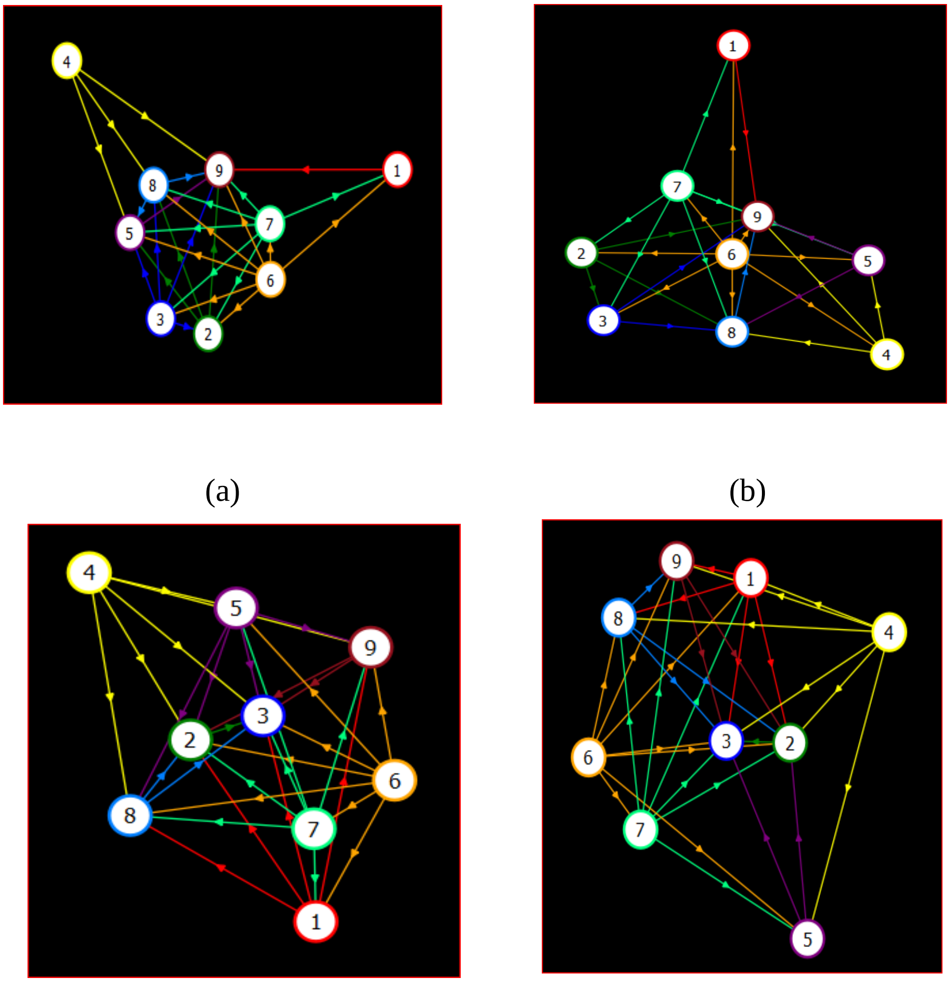
An overlap matrix for matrix A, and matrix B is provided by matrix D. A vibrant directed graph is displayed in Figure 1. A preference for an alternative is represented by a number of coloured lines connected to nodes. Table 13, provides a clear comparison of the available options. The values of elements from the A, B, and D matrices are used to illustrate the outranking relationships between NTM processes in Table 13. The comparison between two options is shown by the notation a=1, b=1, and d=1. The relationships between the choices in all other combinations are "Incomparable." As an illustration, the possibilities are ranked as follows: 6-7-3-2-4-8-1-5-9. It is possible to list the NTM process in the following order: EDM, WEDM, AJM, WJM, ECM, EBM, USM, CHM, LBM.

Table 13. Outranking relations between NTM processes

Comparison of NTMP's	K_{pq}	V_{pq}	k_{pq}	v_{pq}	a	b	d	Ranking
(1,2)	{2,3,4,5,6,7,9,10}	{1,8}	0.651	1	0	0	0	IC
(1,3)	{2,3,4,5,6,7,9,10}	{1,8}	0.651	1	0	0	0	IC
(1,4)	{3,5,7}	{1,2,4,6,8,9,10}	0.352	1	0	0	0	IC
(1,5)	{2,3,5,6,7,10}	{1,4,8,9}	0.594	1	0	0	0	IC
(1,6)	{2,5,10}	{1,3,4,6,7,8,9}	0.200	1	0	0	0	IC
(1,7)	{2,5,10}	{1,3,4,6,7,8,9}	0.200	1	0	0	0	IC
(1,8)	{2,3,6,7,9,10}	{1,4,5,8}	0.500	1	0	0	0	IC
(1,9)	{2,3,6,7,8,9,10}	{1,4,5}	0.790	0.159	1	1	1	1 -- 9
(2,1)	{1,5,6,8,9,10}	{2,3,4,7}	0.648	0.403	0	1	0	IC
(2,3)	{1,3,4,5,6,7,8,9,10	{2}	0.929	1	1	0	0	IC
(2,4)	{5,7,8}	{1,2,3,4,6,9,10}	0.613	1	0	0	0	IC
(2,5)	{5,6,7,8,10}	{1,2,3,4,9}	0.786	0.301	1	1	1	2--5
(2,6)	{5,8,10}	{1,2,3,4,6,7,9}	0.421	1	0	0	0	IC
(2,7)	{5,8,10}	{1,2,3,4,6,7,9}	0.421	1	0	0	0	IC
(2,8)	{1,2,6,7,8,9,10}	{3,4,5}	0.818	0.452	1	1	1	2--8
(2,9)	{1,3,6,7,8,9,10}	{2,4,5}	0.778	0.128	1	1	1	2--9
(3,1)	{1,5,6,8,9,10}	{2,3,4,7}	0.648	0.403	0	1	0	IC
(3,2)	{1,2,4,5,6,7,8,9,10	{3}	0.969	0.632	1	1	1	3--2
(3,4)	{5,7,8}	{1,2,3,4,6,9,10	0.613	1	0	0	0	IC
(3,5)	{5,6,7,8,10}	{1,2,3,4,9}	0.786	0.301	1	1	1	3--5
(3,6)	{5,8,10}	{1,2,3,4,6,7,9}	0.421	1	0	0	0	IC
(3,7)	{5,8,10}	{1,2,3,4,6,7,9}	0.421	1	0	0	0	IC
(3,8)	{1,2,6,7,8,9,10}	{3,4,5}	0.818	0.452	1	1	1	3--8
(3,9)	{1,3,6,7,8,9,10}	{2,4,5}	0.778	0.128	1	1	1	3--9
(4,1)	{1,2,4,6,8,9,10}	{3,5,7}	0.646	0.807	0	0	0	IC
(4,2)	{1,2,3,4,6,8,9,10}	{5,7}	0.676	0.442	0	1	0	IC
(4,3)	{1,2,3,4,6,8,9,10}	{5,7}	0.676	0.422	0	1	0	IC
(4,5)	{1,2,3,4,6,7,8,9,10	{5}	0.888	0.134	1	1	1	4 -- 5

Comparison of NTMP's	K_{pq}	V_{pq}	k_{pq}	v_{pq}	a	b	d	Ranking
(4,6)	{2,4,8,9,10}	{1,3,5,6,7}	0.436	1	0	0	0	IC
(4,7)	{2,4,8,9,10}	{1,3,5,6,7}	0.436	1	0	0	0	IC
(4,8)	{1,2,3,4,6,7,8,9,10	{4,5}	0.888	0.403	1	1	1	4 -- 8
(4,9)	{1,2,3,4,6,7,8,9,10	{2,5}	0.888	0.385	1	1	1	4 -- 9
(5,1)	{1,4,6,8,9,10}	{2,3,5,7}	0.577	0.807	0	0	0	IC
(5,2)	{1,2,3,4,6,8,9,10}	{5,7}	0.676	1	0	0	0	IC
(5,3)	{1,2,3,4,6,8,9,10}	{5,7}	0.676	1	0	0	0	IC
(5,4)	{1,5,7,8,9}	{2,3,4,6,10}	0.686	1	1	0	0	IC
(5,6)	{4,8,9,10}	{1,2,3,5,6,7}	0.366	1	0	0	0	IC
(5,7)	{4,8,9,10}	{1,2,3,5,6,7}	0.366	1	0	0	0	IC
(5,8)	{1,2,3,6,7,8,9,10}	{4,5}	0.848	1	1	0	0	IC
(5,9)	{1,3,4,6,7,8,9,10}	{2,5}	0.818	0.256	1	1	1	5 -- 9
(6,1)	{1,3,4,5,6,7,8,9,10	{2}	0.929	0.069	1	1	1	6 -- 1
(6,2)	1,2,3,4,5,6,7,8,9,10	{}	0.998	0	1	1	1	6 -- 2
(6,3)	1,2,3,4,5,6,7,8,9,10	{}	0.998	0	1	1	1	6 -- 3
(6,4)	{1,3,5,6,7,8,9}	{2,4,10}	0.869	0.769	1	0	0	IC
(6,5)	{1,2,3,4,5,6,7,8,9,1	{}	0.998	0	1	1	1	6 -- 5
(6,7)	{1,2,3,4,5,6,7,8,9,1	{}	0.998	0	1	1	1	6 -- 7
(6,8)	{1,2,3,6,7,8,9,10}	{4,5}	0.848	0.106	1	1	1	6 -- 8
(6,9)	{1,2,3,4,6,7,8,9,10	{5}	0.888	0.106	1	1	1	6 -- 9
(7,1)	{1,3,4,5,6,7,8,9,10	{2}	0.929	0.071	1	1	1	7 -- 1
(7,2)	{1,2,3,4,5,6,7,8,9,1	{}	0.998	0	1	1	1	7 -- 2
(7,3)	{1,2,3,4,5,6,7,8,9,1	{}	0.998	0	1	1	1	7 -- 3
(7,4)	{1,3,5,6,7,8,9}	{2,4,10}	0.869	0.771	1	0	0	IC
(7,5)	{1,2,3,4,5,6,7,8,9,1	{}	0.998	0	1	1	1	7 -- 5
(7,6)	{1,4,5,6,7,8,9,10}	{2,3}	0.9	1	1	0	0	IC
(7,8)	{1,2,3,6,7,8,9,10}	{4,5}	0.848	0.106	1	1	1	7 -- 8
(7,9)	{1,2,3,4,6,7,8,9,10}	{5}	0.888	0.106	1	1	1	7 -- 9
(8,1)	{1,4,5,6,8,9,10}	{2,3,7}	0.688	0.807	1	0	0	IC
(8,2)	{1,3,4,5,6,8,9,10}	{2,7}	0.717	1	1	0	0	IC
(8,3)	{1,3,4,5,6,8,9,10}	{2,7}	0.717	1	1	0	0	IC
(8,4)	{5,7,8}	{1,2,3,4,6,9,10}	0.613	1	0	0	0	IC
(8,5)	{4,5,6,7,8,10}	{1,2,3,9}	0.826	0.189	1	1	1	8--5
(8,6)	{4,5,8,10}	{1,2,3,6,7,9}	0.461	1	0	0	0	IC
(8,7)	{4,5,8,10}	{1,2,3,6,7,9}	0.461	1	0	0	0	IC
(8,9)	{1,3,4,5,6,7,8,9,10}	{2}	0.929	0.011	1	1	1	8 -- 9
(9,1)	{1,4,5,6,8,9,10}	{2,3,7}	0.688	1	1	0	0	IC
(9,2)	{2,4,5,6,9,10}	{1,3,7,8}	0.409	1	0	0	0	IC
(9,3)	{2,4,5,6,9,10}	{1,3,7,8}	0.409	1	0	0	0	IC
(9,4)	{5,7}	{1,2,3,4,6,8,9,10}	0.322	1	0	0	0	IC
(9,5)	{2,4,5,6,7,10}	{1,3,8,9}	0.605	1	0	0	0	IC
(9,6)	{4,5,10}	{1,2,3,6,7,8,9}	0.170	1	0	0	0	IC
(9,7)	{4,5,10}	{1,2,3,6,7,8,9}	0.170	1	0	0	0	IC
(9,8)	{2,5,6,7,9,10}	{1,3,4,8}	0.581	1	0	0	0	IC

IC* - Incomparable



(a) (b)

(c) (d)

Figure 2. Color directed graph, for example-1 : a) Normalized data input with AHP weight; b) Actual data input with AHP weight; c) Actual data input with revised Simos' method; d) Normalized data input with revised Simos' method.

Results 1: Figure 2 (a), shows the rank order of NTM process alternatives is given by the normalised data input with AHP weight calculations to the m-polar fuzzy ELECTRE-I algorithm and is given as EDM-WEDM-AJM-WJM-ECM-EBM-USM-CHM-LBM.

Result 2: Figure 2 (b), shows the rank order of the NTM process alternatives, as determined by actual data input with AHP weight calculations to the m-polar fuzzy ELECTRE-I algorithm, is given as 6-7-2-4-3-5-1-8-9 and EDM-WEDM-WJM-ECM-AJM-CHM-USM-EBM-LBM.

Result 3: Figure 2 (c), shows actual data input using a modified version of Simos' weight calculation to create an m-polar fuzzy the rank order of the NTM process alternatives according to the ELECTRE-I algorithm is given as 6-7-4-1-5-8-9-2-3 and EDM-WEDM-ECM-USM-CHM-EBM-LBM-WJM-AJM.

Result 4: Figure 2 (d), shows normalized data input using the m-polar fuzzy version of Simos' weight calculation. According to the ELECTRE-I algorithm, the NTM process options are ranked 6-4-7-1-8-5-9-2-3 and EDM-ECM-WEDM-USM-EBM-CHM-LBM-WJM-AJM (Table 14).

Table 14. Validation of results by comparing results with those with reference. (N. D. Chakladar & Chakraborty, 2008)

Example 1					
Ranks	TOPSIS-AHP method (N. D. Chakladar & Chakraborty, 2008)	m-polar fuzzy ELECTRE-I with data normalization and integrated AHP	m-polar fuzzy ELECTRE-I with actual data and integrated AHP	m-polar fuzzy ELECTRE-I with actual data and integrated revised Simos' method	m-polar fuzzy ELECTRE-I with data normalization and integrated revised Simos' method
1	ECM	EDM	EDM	EDM	EDM
2	EDM	WEDM	WEDM	WEDM	ECM
3	WEDM	AJM	WJM	ECM	WEDM
4	USM	WJM	ECM	USM	USM
5	EBM	ECM	AJM	CHM	EBM
6	CHM	EBM	CHM	EBM	CHM
7	LBM	USM	USM	LBM	LBM
8	WJM	CHM	EBM	WJM	WJM
9	AJM	LBM	LBM	AJM	AJM

4.3. SDWSA for Simos' criteria weight method for example-1

In the section 3.4, sensitivity analysis detailed, example-1 employs a single dimensional weight sensitivity analysis (SDWSA). According to Table 15, below "work material" is regarded as the criteria with the greatest influence on rank performance according to Simos' criteria weight approach because it has the highest value in comparison to other criteria. To determine rank variations for various criteria weights, the weight of the "work material" was adjusted from least to maximum.

Table 15. Criteria weight variation with weight additive constraint for Simos' criteria weight method

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Criteria weight calculated using Simos' weight calculation method									
9.7	9.7	12.6	9.7	6.8	3.9	3.9	3.9	21.4	18.4
Criteria weight after implementation of weight additive constraint									
10.967	10.967	13.867	10.967	8.067	5.167	5.167	5.167	10	19.667
9.855	9.855	12.755	9.855	6.955	4.055	4.055	4.055	20	18.555
8.745	8.745	11.645	8.745	5.845	2.945	2.945	2.945	30	17.445
7.633	7.633	10.533	7.633	4.733	1.833	1.833	1.833	40	16.333
6.522	6.522	9.422	6.522	3.622	0.722	0.722	0.722	50	15.222
5.8	5.8	8.7	5.8	2.9	0	0	0	56.5	14.5

The ranks of alternatives from the m-polar fuzzy ELECTRE-I technique are calculated using six different criterion weight combinations from table 15, in the procedure. Table 16, displays the rank determined using Simos' criteria weight approach and the m-polar fuzzy ELECTRE-I for six different combinations of criteria weights.

Table 16. Alternatives with their ranks for criteria weight variation

alternatives	1	2	3	4	5	6
s_1	4	4	4	4	4	4
s_2	7	8	8	8	8	8
s_3	8	9	9	9	9	9
s_4	2	2	1	1	1	1
s_5	6	6	5	5	5	5
s_6	1	1	2	2	2	2
s_7	3	3	3	3	3	3
s_8	5	5	6	6	6	6
s_9	9	7	7	7	7	7

The X-axis in Figure 3, reflects changes in criteria weight, and the Y-axis shows changes in alternative rankings. The global stability range is the same as the local stability range for the variation in the criteria weight, which is from 30 to 56.5. It demonstrates how stable rank performance for alternatives across a wide range is given by variation in criteria weight.

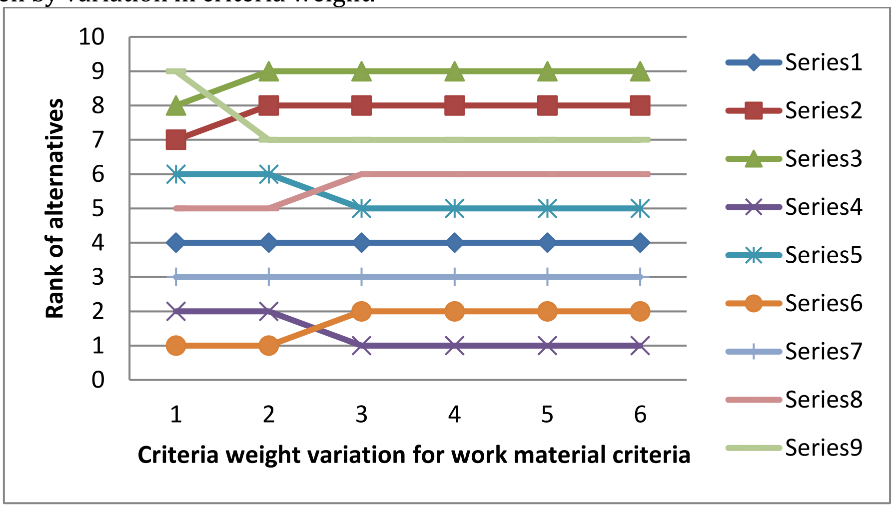


Figure 3. Single dimensional weight sensitivity analysis for example-1 considering Simos' method

4.4. SDWSA for AHP criteria weight method

In the section 3.4, sensitivity analysis detailed, example-1 employs a single dimensional weight sensitivity analysis (SDWSA). According to Table 17, below, "Safety" is regarded as the criteria with the greatest influence on rank performance according to AHP criteria weight approach because it has the highest value in comparison to other criteria. To determine rank variations for various criteria weights, the weight of the "Safety" was adjusted from least to maximum.

Table 17. Criteria weight variation with weight additive constraint for AHP criteria weight method

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Criteria weight calculated using AHP weight calculation method									
0.057	0.065	0.023	0.04	0.118	0.152	0.212	0.29	0.016	0.020
Criteria weight after implementation of weight additive constraint									
0.0893	0.1017	0.0615	0.0722	0.143	0.1854	0.2442	0.0001	0.0482	0.0523
0.0782	0.0906	0.0504	0.0611	0.1319	0.1743	0.2331	0.1	0.0371	0.0412
0.0671	0.0795	0.0393	0.05	0.1208	0.1632	0.222	0.2	0.026	0.0301
0.056	0.0585	0.0183	0.029	0.1097	0.1521	0.2109	0.3	0.0149	0.019
0.0449	0.0573	0.0171	0.0278	0.0986	0.141	0.1998	0.4	0.0038	0.0079
0.0404	0.0528	0.0126	0.0233	0.0941	0.1365	0.1953	0.44	0	0.0034

The ranks of alternatives from the m-polar fuzzy ELECTRE-I technique are calculated using six different criterion weight combinations from Table 17, in the procedure. Table 18, displays the rank determined using AHP criteria weight approach and the m-polar fuzzy ELECTRE-I for six different combinations of criteria weights.

Table 18. Alternatives with their ranks for criteria weight variation

alternatives	1	2	3	4	5	6
s_1	3	6	7	7	7	7
s_2	6	3	3	4	5	5
s_3	7	4	4	3	4	3
s_4	4	5	5	5	3	4
s_5	9	8	8	8	8	8
s_6	1	1	1	1	1	1
s_7	2	2	2	2	2	2
s_8	5	7	6	6	6	6
s_9	8	9	9	9	9	9

The X-axis in Figure 4, reflects changes in criteria weight, and the Y-axis shows changes in alternative rankings. The local stability range is not same as the global stability range for the variation in the criteria weight, which is from 0 to 0.44. It demonstrates how unstable rank performance for alternatives across a wide range is given by variation in criteria weight as there is no global stability range.

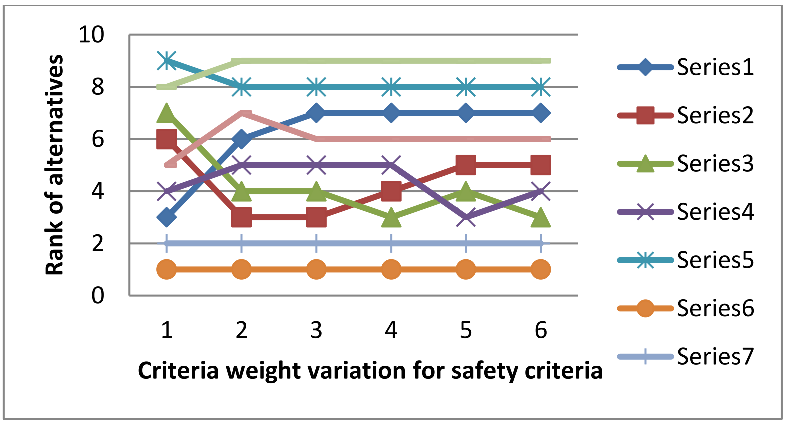


Figure 4. Single dimensional weight sensitivity analysis for example-1 considering AHP method

5. Example 2: Selection of NTM process to efficiently machine precision holes on duralumin

In order to effectively cut precision holes on duralumin, (N. D. Chakladar & Chakraborty, 2008) have chosen nine alternative NTM methods while taking ten parameters into consideration. Table 19, contains the decision matrix for the identical case. While the remaining parameters are given qualitative values using a five-point scale, the TSF, PR, and MRR parameters are given real values (quantitative).

Table 19. Decision Matrix: performance of various NTM processes to different attributes. (N. D. Chakladar & Chakraborty, 2008)

Sr.	Process	Parameters									
		t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
1	s_1	1.0	10.00	500.0	2	4	2	3	1	4	1
2	s_2	2.5	0.22	0.8	1	4	2	2	3	3	1
3	s_3	2.5	0.24	0.5	1	4	2	2	3	3	1
4	s_4	3.0	100.00	400.0	5	2	3	1	3	5	4
5	s_5	3.0	0.40	15.0	3	3	2	1	3	5	4
6	s_6	3.5	2.70	800.0	3	4	4	4	3	4	5
7	s_7	3.5	2.50	600.0	3	4	4	4	3	4	5
8	s_8	2.5	0.20	1.6	4	5	2	1	3	4	1
9	s_9	2.0	1.40	0.1	3	5	2	1	1	4	1

Table 20, shows the normalised decision matrix.

Table 20. Normalized Decision Matrix

Sr. No.	Parameters									
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
s_1	0.122	0.099	0.421	0.219	0.334	0.248	0.412	0.124	0.328	0.107
s_2	0.307	0.002	0.007	0.109	0.334	0.248	0.274	0.372	0.246	0.107
s_3	0.307	0.002	0.004	0.109	0.334	0.248	0.274	0.372	0.246	0.107
s_4	0.368	0.994	0.336	0.548	0.167	0.372	0.137	0.372	0.411	0.428
s_5	0.368	0.004	0.012	0.329	0.250	0.248	0.137	0.372	0.411	0.428
s_6	0.430	0.026	0.673	0.329	0.334	0.496	0.549	0.372	0.328	0.536
s_7	0.430	0.024	0.505	0.329	0.334	0.496	0.549	0.372	0.328	0.536
s_8	0.307	0.002	0.001	0.439	0.418	0.248	0.137	0.372	0.328	0.107
s_9	0.245	0.013	0.001	0.329	0.418	0.248	0.137	0.124	0.328	0.107

5.1. Criterion weight calculation by Revised Simos' Method

Weight for the improved Simos' technique can be determined from section 4.1, for the NTM Process selection Criteria and shown in Table 21.

Table 21. Criteria weight obtained from revised Simos' method

Criteria	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Criteria Weight	9.7	9.7	12.6	9.7	6.8	3.9	3.9	3.9	21.4	18.4

5.2. Criterion weight calculation by AHP method

Weight for the improved Simos' technique can be determined from section 4.2, for the NTM Process selection Criteria and shown in Table 22.

Table 22. Criteria weight obtained from AHP method

Criteria	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Criteria Weight	0.057	0.069	0.029	0.040	0.110	0.153	0.212	0.290	0.016	0.020

Table 23. Weighted Normalized Matrix for AHP weight calculations

Sr. No.	Parameters									
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
1	0.007	0.007	0.0123	0.008	0.037	0.038	0.087	0.035	0.005	0.002
2	0.017	0.001	0.0002	0.004	0.037	0.038	0.058	0.107	0.003	0.002
3	0.017	0.001	0.0001	0.004	0.037	0.038	0.058	0.107	0.003	0.002
4	0.021	0.069	0.009	0.021	0.018	0.057	0.029	0.107	0.006	0.008
5	0.021	0.002	0.003	0.013	0.027	0.038	0.029	0.107	0.006	0.008
6	0.024	0.001	0.019	0.013	0.037	0.076	0.116	0.107	0.005	0.010
7	0.024	0.001	0.014	0.013	0.037	0.076	0.116	0.107	0.005	0.010
8	0.017	0.003	0.001	0.017	0.046	0.038	0.029	0.107	0.005	0.002
9	0.014	0.009	0.002	0.013	0.046	0.038	0.029	0.035	0.005	0.002

In Table 23, the algorithm's Eq. (9) and Eq. (10) are applied to a weighted normalized decision matrix. Results are presented as a concordance set in Table 24, and a discordance set in Table 25.

Concordance and discordance indices, concordance matrix K, and discordance matrix V are calculated using Eq. (15) and Eq. (16), respectively, from the algorithm.

$$K = \begin{bmatrix} - & 0.6509 & 0.6509 & 0.3521 & 0.5748 & 0.1963 & 0.1963 & 0.5001 & 0.7901 \\ 0.6312 & - & 0.9285 & 0.6128 & 0.766 & 0.4008 & 0.4008 & 0.8019 & 0.7617 \\ 0.6312 & 0.9687 & - & 0.6128 & 0.766 & 0.4008 & 0.4008 & 0.8019 & 0.7617 \\ 0.6459 & 0.6752 & 0.6752 & - & 0.8872 & 0.4155 & 0.4155 & 0.8872 & 0.8872 \\ 0.5764 & 0.6752 & 0.6752 & 0.706 & - & 0.346 & 0.346 & 0.8472 & 0.8177 \\ 0.9285 & 0.998 & 0.998 & 0.8725 & 0.982 & - & 0.998 & 0.8472 & 0.8872 \\ 0.9285 & 0.998 & 0.998 & 0.8725 & 0.982 & 0.8992 & - & 0.8472 & 0.8872 \\ 0.6872 & 0.7165 & 0.7165 & 0.6128 & 0.806 & 0.4568 & 0.4568 & - & 0.9285 \\ 0.6872 & 0.4096 & 0.4096 & 0.3228 & 0.5855 & 0.1668 & 0.1668 & 0.5816 & - \end{bmatrix}$$

$$V = \begin{bmatrix} - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.1590 \\ 0.4048 & - & 1 & 1 & 0.3016 & 1 & 1 & 0.4525 & 0.1287 \\ 0.4048 & 0.6323 & - & 1 & 0.3016 & 1 & 1 & 0.4525 & 0.1287 \\ 0.8094 & 0.4221 & 0.4222 & - & 0.1347 & 1 & 1 & 0.4031 & 0.3863 \\ 0.8094 & 1 & 1 & 1 & - & 1 & 1 & 1 & 0.2574 \\ 0.0701 & 0 & 0 & 0.7698 & 0.0150 & - & 0 & 0.1060 & 0.1060 \\ 0.07196 & 0 & 0 & 0.7713 & 0.0150 & 1 & - & 0.1060 & 0.1060 \\ 0.8094 & 1 & 1 & 1 & 0.3489 & 1 & 1 & - & 0.0114 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & - \end{bmatrix}$$

Matrix A and Matrix B are evaluated using Eq. (17) and Eq. (18). The comparison of all of the matrix's elements yields matrices A and B, with a Concordance level of 0.617754 and a Discordance level of 0.583391, respectively.

Table 24. Concordance Set

j	1	2	3	4	5	6	7	8	9
K_{1j}	-	{2,3,4,5,6,7,9,10}	{2,3,4,5,6,7,9,10}	{3,5,7}	{2,3,5,6,7}	{2,5,9}	{2,5,9}	{2,3,6,7,9,10}	{2,3,6,7,8,9,10}
K_{2j}	{1,5,6,8,10}	-	{1,3,4,5,6,7,8,9,10}	{5,7,8}	{5,6,7,8}	{5,8}	{5,8}	{1,2,6,7,8,10}	{1,3,6,7,8,10}
K_{3j}	{1,5,6,8,10}	{1,2,4,5,6,7,8,9,10}	-	{5,7,8}	{5,6,7,8}	{5,8}	{5,8 }	{1,2,6,7,8,10}	{1,3,6,7,8,10}
K_{4j}	{1,2,4,6,8,9,10}	{1,2,3,4,6,8,9,10}	{1,2,3,4,6,8,9,10}	-	{1,2,3,4,6,7,8,9,10}	{2,4,8,9}	{2,4,8,9}	{1,2,3,4,6,7,8,9,10}	{1,2,3,4,6,7,8,9,10}
K_{5j}	{1,4,6,8,9,10}	{1,2,3,4,6,8,9,10}	{1,2,3,4,6,8,9,10}	{1,5,7,8,9,10}	-	{4,8,9}	{4,8,9}	{1,2,3,6,7,8,9,10}	{1,3,4,6,7,8,9,10}
K_{6j}	{1,3,4,5,6,7,8,9,10}	{1,2,3,4,5,6,7,8,9,10}	{1,2,3,4,5,6,7,8,9,10}	{1,3,5,6,7,8,10}	{1,2,3,4,5,6,7,8,10}	-	{1,2,3,4,5,6,7,8,9,10}	{1,2,3,6,7,8,9,10}	{1,2,3,4,6,7,8,9,10}
K_{7j}	{1,3,4,5,6,7,8,9,10}	{1,2,3,4,5,6,7,8,9,10}	{1,2,3,4,5,6,7,8,9,10}	{1,3,5,6,7,8,10}	{1,2,3,4,5,6,7,8,10}	{1,4,5,6,7,8,9,10}	-	{1,2,3,6,7,8,9,10}	{1,2,3,4,6,7,8,9,10}
K_{8j}	{1,4,5,6,8,9,10}	{1,3,4,5,6,8,9,10}	{1,3,4,5,6,8,9,10}	{5,7,8}	{4,5,6,7,8}	{4,5,8,9}	{4,5,8,9}	-	{1,3,4,5,6,7,8,9,10}
K_{9j}	{1,4,5,6,8,9,10}	{2,4,5,6,9,10}	{2,4,5,6,9,10}	{5,7}	{2,4,5,6,7}	{4,5,9}	{4,5,9}	{2,5,6,7,9,10}	-

Table 25. Discordance Set.

j	1	2	3	4	5	6	7	8	9
V_{1j}	-	{1,8}	{1,8}	{1,2,4,6,8,9,10}	{1,4,8,9,10}	{1,3,4,6,7,8,10}	{1,3,4,6,7,8,10}	{1,4,5,8}	{1,4,5}
V_{2j}	{2,3,4,7,9}	-	{2}	{1,2,3,4,6,9,10}	{1,2,3,4,9,10}	{1,2,3,4,6,7,9,10}	{1,2,3,4,6,7,9,10}	{3,4,5,9}	{2,4,5,9}
V_{3j}	{2,3,4,7,9}	{3}	-	{1,2,3,4,6,9,10}	{1,2,3,4,9,10}	{1,2,3,4,6,7,9,10}	{1,2,3,4,6,7,9,10}	{3,4,5,9}	{2,4,5,9}
V_{4j}	{3,5,7}	{5,7}	{5,7}	-	{5}	{1,3,5,6,7,10}	{1,3,5,6,7,10}	{5}	{5}
V_{5j}	{2,3,5,7}	{5,7}	{5,7}	{2,3,4,6}	-	{1,2,3,5,6,7,10}	{1,2,3,5,6,7,10}	{4,5}	{2,5}
V_{6j}	{2}	{}	{}	{2,4,9}	{9}	-	{}	{4,5}	{5}
V_{7j}	{2}	{}	{}	{2,4,9}	{9}	{2,3}	-	{4,5}	{5}
V_{8j}	{2,3,7}	{2,7}	{2,7}	{1,2,3,4,6,9,10}	{1,2,3,9,10}	{1,2,3,6,7,10}	{1,2,3,6,7,10}	-	{2}
V_{9j}	{2,3,7}	{1,3,7,8}	{1,3,7,8}	{1,2,3,4,6,8,9,10}	{1,3,8,9,10}	{1,2,3,6,7,8,10}	{1,2,3,6,7,8,10}	{1,3,4,8}	-

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

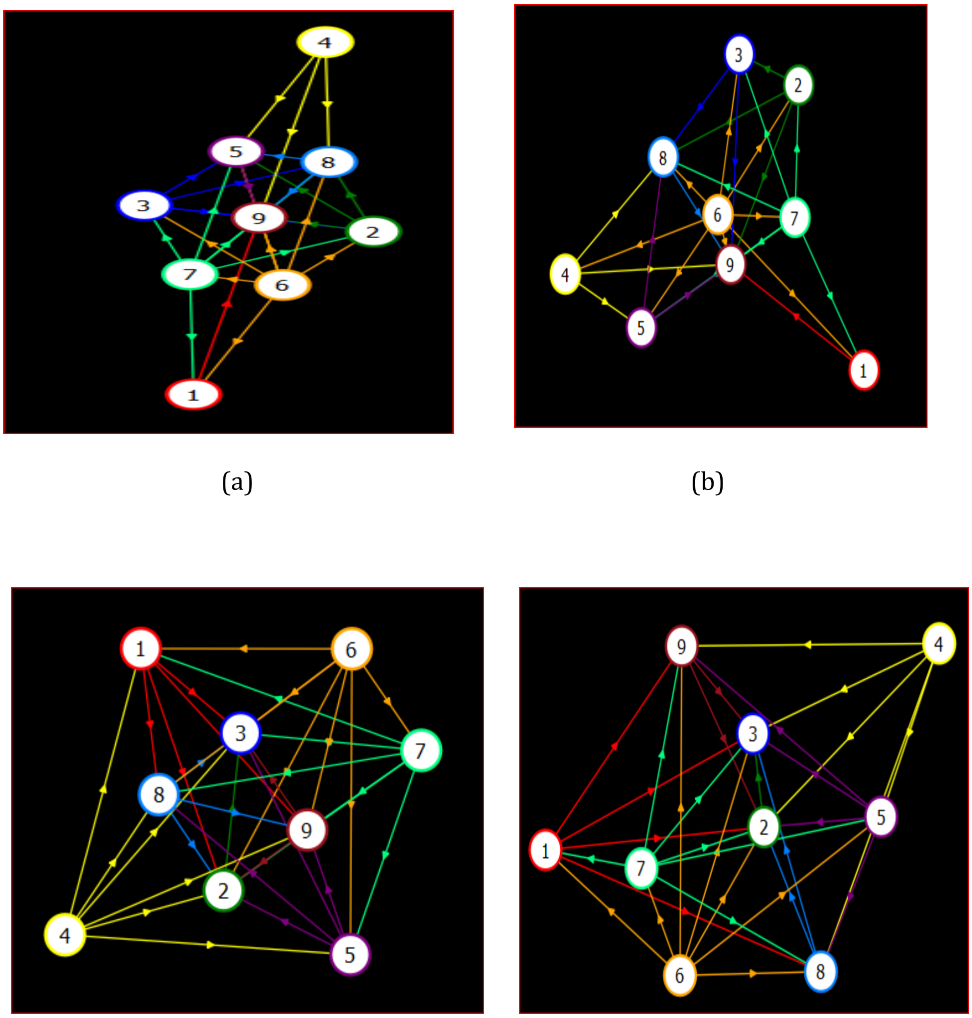
An overlap matrix for matrix A and matrix B is provided by matrix D. A vibrant directed graph is shown in Figure 5. A preference for an alternative is represented by a number of coloured lines connected to nodes. Table 26, presents a clear comparison of the options. Table 26, displays outranking correlations between NTM processes using data from the A, B, and D matrices. The formula a = 1, b = 1, and d = 1 compares two options. When we assessed the number of comparable alternatives, the rank order changed from "incomparable" relationships between alternatives in all other combinations to 6-7-3-2-4-8-1-5-9. The NTM process can be summarised as EDM-WEDM-AJM-WJM-ECM-EBM-USM-CHM-LBM, in that order.

Table 26. Outranking relations between NTM processes

Comparison of NTMP's	K_{pq}	V_{pq}	k_{pq}	v_{pq}	a	b	d	Ranking
(1,2)	{2,3,4,5,6,7,9,10}	{1,8}	0.65	1	0	0	0	IC
(1,3)	{2,3,4,5,6,7,9,10}	{1,8}	0.65	1	0	0	0	IC
(1,4)	{3,5,7}	{1,2,4,6,8,9,10}	0.35	1	0	0	0	IC
(1,5)	{2,3,5,6,7}	{1,4,8,9,10}	0.57	1	0	0	0	IC
(1,6)	{2,5,9}	{1,3,4,6,7,8,10}	0.19	1	0	0	0	IC
(1,7)	{2,5,9}	{1,3,4,6,7,8,10}	0.19	1	0	0	0	IC
(1,8)	{2,3,6,7,9,10}	{1,4,5,8}	0.50	1	0	0	0	IC
(1,9)	{2,3,6,7,8,9,10}	{1,4,5}	0.79	0.159	1	1	1	1 -- 9
(2,1)	{1,5,6,8,10}	{2,3,4,7,9}	0.63	0.4048	0	1	0	IC
(2,3)	{1,3,4,5,6,7,8,9,10}	{2}	0.92	1	1	0	0	IC
(2,4)	{5,7,8}	{1,2,3,4,6,9,10}	0.61	1	0	0	0	IC
(2,5)	{5,6,7,8}	{1,2,3,4,9,10}	0.76	0.3016	1	1	1	2--5
(2,6)	{5,8}	{1,2,3,4,6,7,9,10}	0.40	1	0	0	0	IC
(2,7)	{5,8}	{1,2,3,4,6,7,9,10}	0.40	1	0	0	0	IC
(2,8)	{1,2,6,7,8,10}	{3,4,5,9}	0.80	0.4525	1	1	1	2--8
(2,9)	{1,3,6,7,8,10}	{2,4,5,9}	0.76	0.1287	1	1	1	2--9
(3,1)	{1,5,6,8,10}	{2,3,4,7,9}	0.63	0.4048	0	1	0	IC
(3,2)	{1,2,4,5,6,7,8,9,10}	{3}	0.96	0.6323	1	1	1	3--2
(3,4)	{5,7,8}	{1,2,3,4,6,9,10}	0.61	1	0	0	0	IC
(3,5)	{5,6,7,8}	{1,2,3,4,9,10}	0.76	0.3016	1	1	1	3--5
(3,6)	{5,8}	{1,2,3,4,6,7,9,10}	0.40	1	0	0	0	IC
(3,7)	{5,8}	{1,2,3,4,6,7,9,10}	0.40	1	0	0	0	IC
(3,8)	{1,2,6,7,8,10}	{3,4,5,9}	0.80	0.4525	1	1	1	3--8
(3,9)	{1,3,6,7,8,10}	{2,4,5,9}	0.76	0.1287	1	1	1	3--9
(4,1)	{1,2,4,6,8,9,10}	{3,5,7}	0.64	0.809	0	0	0	IC
(4,2)	{1,2,3,4,6,8,9,10}	{5,7}	0.67	0.4221	0	1	0	IC
(4,3)	{1,2,3,4,6,8,9,10}	{5,7}	0.67	0.4222	0	1	0	IC
(4,5)	{1,2,3,4,6,7,8,9,10}	{5}	0.88	0.1347	1	1	1	4 -- 5
(4,6)	{2,4,8,9}	{1,3,5,6,7,10}	0.41	1	0	0	0	IC
(4,7)	{2,4,8,9}	{1,3,5,6,7,10}	0.41	1	0	0	0	IC
(4,8)	{1,2,3,4,6,7,8,9,10}	{5}	0.88	0.4031	1	1	1	4 -- 8
(4,9)	{1,2,3,4,6,7,8,9,10}	{5}	0.88	0.3863	1	1	1	4 -- 9
(5,1)	{1,4,6,8,9,10}	{2,3,5,7}	0.57	0.8094	0	0	0	IC
(5,2)	{1,2,3,4,6,8,9,10}	{5,7}	0.67	1	0	0	0	IC
(5,3)	{1,2,3,4,6,8,9,10}	{5,7}	0.67	1	0	0	0	IC
(5,4)	{1,5,7,8,9,10}	{2,3,4,6}	0.70	1	1	0	0	IC
(5,6)	{4,8,9}	{1,2,3,5,6,7,10}	0.34	1	0	0	0	IC
(5,7)	{4,8,9}	{1,2,3,5,6,7,10}	0.34	1	0	0	0	IC
(5,8)	{1,2,3,6,7,8,9,10}	{4,5}	0.84	1	1	0	0	IC
(5,9)	{1,3,4,6,7,8,9,10}	{2,5}	0.81	0.2574	1	1	1	5 -- 9
(6,1)	{1,3,4,5,6,7,8,9,10}	{2}	0.92	0.0701	1	1	1	6 -- 1
(6,2)	{1,2,3,4,5,6,7,8,9,10}	{}	0.99	0	1	1	1	6 -- 2
(6,3)	{1,2,3,4,5,6,7,8,9,10}	{}	0.99	0	1	1	1	6 -- 3
(6,4)	{1,3,5,6,7,8,10}	{2,4,9}	0.87	0.7698	1	0	0	IC
(6,5)	{1,2,3,4,5,6,7,8,10}	{9}	0.98	0.015	1	1	1	6 -- 5

Comparison of NTMP's	K_{pq}	V_{pq}	k_{pq}	v_{pq}	a	b	d	Ranking
(6,7)	{1,2,3,4,5,6,7,8,9,10}	{}	0.99	0	1	1	1	6 -- 7
(6,8)	{1,2,3,6,7,8,9,10}	{4,5}	0.84	0.106	1	1	1	6 -- 8
(6,9)	{1,2,3,4,6,7,8,9,10}	{5}	0.88	0.106	1	1	1	6 -- 9
(7,1)	{1,3,4,5,6,7,8,9,10}	{2}	0.92	0.0719	1	1	1	7 -- 1
(7,2)	{1,2,3,4,5,6,7,8,9,10}	{}	0.99	0	1	1	1	7 -- 2
(7,3)	{1,2,3,4,5,6,7,8,9,10}	{}	0.99	0	1	1	1	7 -- 3
(7,4)	{1,3,5,6,7,8,10}	{2,4,9}	0.87	0.771	1	0	0	IC
(7,5)	{1,2,3,4,5,6,7,8,10}	{9}	0.98	0.015	1	1	1	7 -- 5
(7,6)	{1,4,5,6,7,8,9,10}	{2,3}	0.89	1	1	0	0	IC
(7,8)	{1,2,3,6,7,8,9,10}	{4,5}	0.84	0.106	1	1	1	7 -- 8
(7,9)	{1,2,3,4,6,7,8,9,10}	{5}	0.88	0.106	1	1	1	7 -- 9
(8,1)	{1,4,5,6,8,9,10}	{2,3,7}	0.68	0.809	1	0	0	IC
(8,2)	{1,3,4,5,6,8,9,10}	{2,7}	0.71	1	1	0	0	IC
(8,3)	{1,3,4,5,6,8,9,10}	{2,7}	0.71	1	1	0	0	IC
(8,4)	{5,7,8}	{1,2,3,4,6,9,10}	0.61	1	0	0	0	IC
(8,5)	{4,5,6,7,8}	{1,2,3,9,10}	0.80	0.348	1	1	1	8--5
(8,6)	{4,5,8,9}	{1,2,3,6,7,10}	0.45	1	0	0	0	IC
(8,7)	{4,5,8,9}	{1,2,3,6,7,10}	0.45	1	0	0	0	IC
(8,9)	{1,3,4,5,6,7,8,9,10}	{2}	0.92	0.011	1	1	1	8 -- 9
(9,1)	{1,4,5,6,8,9,10}	{2,3,7}	0.68	1	1	0	0	IC
(9,2)	{2,4,5,6,9,10}	{1,3,7,8}	0.40	1	0	0	0	IC
(9,3)	{2,4,5,6,9,10}	{1,3,7,8}	0.40	1	0	0	0	IC
(9,4)	{5,7}	{1,2,3,4,6,8,9,10}	0.32	1	0	0	0	IC
(9,5)	{2,4,5,6,7}	{1,3,8,9,10}	0.58	1	0	0	0	IC
(9,6)	{4,5,9}	{1,2,3,6,7,8,10}	0.16	1	0	0	0	IC
(9,7)	{4,5,9}	{1,2,3,6,7,8,10}	0.16	1	0	0	0	IC
(9,8)	{2,5,6,7,9,10}	{1,3,4,8}	0.58	1	0	0	0	IC

IC* - Incomparable



(a) (b) (c) (d)

Figure 5. Color directed graph, for example-II: a) Directed graph for NTM selection with normalization and AHP method; b) Directed graph for NTM selection without normalization with AHP method; c) Actual data with revised Simos' method; d) Normalized data with revised Simos' method

Result 1: Figure 5 (a), shows normalized data input with AHP weight calculations to the m-polar fuzzy ELECTRE-I algorithm produces the rank order of NTM process alternatives in decreasing order of 6-7-3-2-4-8-1-5-9 and EDM-WEDM-AJM-WJM-ECM-EBM-USM-CHM-LBM.
Result 2: Figure 5 (b), shows the rank order of the NTM process alternatives, as determined by actual data input with AHP weight calculations to the m-polar fuzzy ELECTRE-I algorithm, is given as 6-7-2-4-3-5-1-8-9 and EDM-WEDM-WJM-ECM-AJM-CHM-USM-EBM-LBM.

Result 3: Figure 5 (c), shows actual data input using a modified version of Simos' weight calculation to create an m-polar fuzzy the rank order of the NTM process alternatives according to the ELECTRE-I algorithm is given as 6-7-4-1-5-8-9-2-3 and EDM-WEDM-ECM-USM-CHM-EBM-LBM-WJM-AJM.

Result 4: Figure 5 (d), shows normalized data input using the m-polar fuzzy version of Simos' weight calculation The rank order of the NTM process alternatives according to the ELECTRE-I algorithm is given as 6-7-4-1-5-8-9-2-3 and EDM-WEDM-ECM-USM-CHM-EBM-LBM-WJM-AJM.

In table 27, results from examples-2, using the mFS ELECTRE-I integrated AHP methodology are contrasted with those from the study using the TOPSIS-AHP method (N. D. Chakladar & Chakraborty, 2008). The rankings attained are seen to be uniform with the m-polar fuzzy ELECTRE-I with revised Simos' method.

Table 27. Result validation for the m-polar fuzzy ELECTRE-I method

Example-2					
Ranks	TOPSIS-AHP method (N. D. Chakladar & Chakraborty, 2008)	m-polar ELECTRE-I with data normalization integrated AHP	m-polar ELECTRE-I with actual data and integrated AHP	m-polar fuzzy ELECTRE-I with actual data and integrated revised Simos' method	m-polar fuzzy ELECTRE-I with data normalization and integrated revised Simos' method
1	EDM	EDM	EDM	EDM	EDM
2	ECM	WEDM	WEDM	WEDM	WEDM
3	WEDM	AJM	WJM	ECM	ECM
4	CHM	WJM	ECM	USM	USM
5	USM	ECM	AJM	CHM	CHM
6	EBM	EBM	CHM	EBM	EBM
7	LBM	USM	USM	LBM	LBM
8	WJM	CHM	EBM	WJM	WJM
9	AJM	LBM	LBM	AJM	AJM

5.3. SDWSA for Simos' criteria weight calculations for example-2

In the section 3.4, sensitivity analysis detailed, example-2 employs a single dimensional weight sensitivity analysis (SDWSA). According to Table 28, below, "work material" is regarded as the criteria with the greatest influence on rank performance according to Simos' criteria weight approach because it has the highest value in comparison to other criteria. To determine rank variations for various criteria weights, the weight of the "work material" was adjusted from least to maximum.

Table 28. Criteria weight variation with weight additive constraint for Simos' criteria weight method

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Criteria weight calculated using Simos' weight calculation method									
9.7	9.7	12.6	9.7	6.8	3.9	3.9	3.9	21.4	18.4
Criteria weight after implementation of weight additive constraint									
10.967	10.967	13.867	10.967	8.067	5.167	5.167	5.167	10	19.667
9.855	9.855	12.755	9.855	6.955	4.055	4.055	4.055	20	18.555
8.745	8.745	11.645	8.745	5.845	2.945	2.945	2.945	30	17.445
7.633	7.633	10.533	7.633	4.733	1.833	1.833	1.833	40	16.333
6.522	6.522	9.422	6.522	3.622	0.722	0.722	0.722	50	15.222
5.8	5.8	8.7	5.8	2.9	0	0	0	56.5	14.5

The ranks of alternatives from the m-polar fuzzy ELECTRE-I technique are calculated using six different criterion weight combinations from table 28, in the procedure. Table 29, displays the rank determined using Simos' criteria weight approach and the m-polar fuzzy ELECTRE-I for six different combinations of criteria weights.

Table 29. Alternatives with their ranks for criteria weight variation

Alternatives	1	2	3	4	5	6
s_1	4	4	4	4	4	4
s_2	8	8	8	8	8	8
s_3	9	9	9	9	9	9
s_4	2	2	2	1	1	1
s_5	5	5	5	5	5	5
s_6	1	1	1	2	2	2
s_7	3	3	3	3	3	3
s_8	6	6	6	6	6	6
s_9	7	7	7	7	7	7

The X-axis in Figure 6, reflects changes in criteria weight, and the Y-axis shows changes in alternative rankings. The global stability range is the same as the local stability range for the variation in the criteria weight, which is from 0 to 56.5. It demonstrates how stable rank performance for alternatives across a wide range is given by variation in criteria weight.

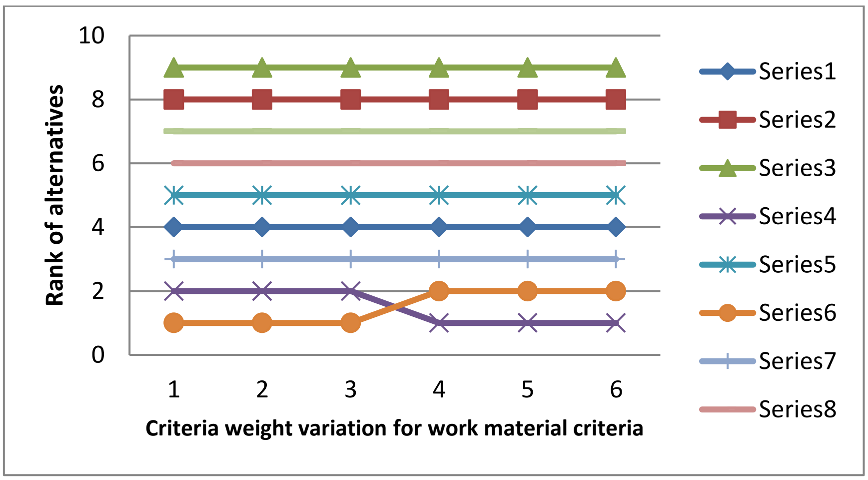


Figure 6. Single dimensional weight sensitivity analysis for example-2 considering Simos' method

5.4. SDWSA for AHP criteria weight calculations

In the section 3.4, sensitivity analysis detailed, example-2 employs a single dimensional weight sensitivity analysis (SDWSA). According to Table 30, below, "Safety" is regarded as the criteria with the greatest influence on rank performance according to AHP criteria weight approach because it has the highest value in comparison to other criteria. To determine rank variations for various criteria weights, the weight of the "Safety" was adjusted from least to maximum.

Table 30. Criteria weight variation with weight additive constraint for AHP criteria weight method

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Criteria weight calculated using AHP weight calculation method									
0.057	0.069	0.029	0.04	0.110	0.153	0.212	0.29	0.016	0.020
Criteria weight after implementation of weight additive constraint									
0.089	0.101	0.061	0.072	0.143	0.185	0.244	0.001	0.048	0.052
0.078	0.090	0.050	0.061	0.131	0.174	0.233	0.1	0.037	0.041
0.067	0.079	0.039	0.05	0.120	0.163	0.222	0.2	0.026	0.030
0.056	0.058	0.018	0.029	0.109	0.152	0.210	0.3	0.014	0.019
0.044	0.057	0.017	0.027	0.098	0.141	0.199	0.4	0.003	0.007
0.040	0.052	0.012	0.023	0.094	0.136	0.195	0.44	0	0.003

The ranks of alternatives from the m-polar fuzzy ELECTRE-I technique are calculated using six different criterion weight combinations from table 30, in the procedure. Table 31, displays the rank determined using AHP criteria weight approach and the m-polar fuzzy ELECTRE-I for six different combinations of criteria weights.

Table 31. Alternatives with their ranks for criteria weight variation

Alternatives	1	2	3	4	5	6
s_1	3	6	7	7	7	7
s_2	5	3	3	4	5	5
s_3	6	4	4	3	4	3
s_4	4	5	5	5	3	4
s_5	8	8	8	8	8	8
s_6	1	1	1	1	1	1
s_7	2	2	2	2	2	2
s_8	7	7	6	6	6	6
s_9	9	9	9	9	9	9

The X-axis in Figure 7, reflects changes in criteria weight, and the Y-axis shows changes in alternative rankings. The local stability range is not same as the global stability range for the variation in the criteria weight, which is from 0 to 0.44. It demonstrates how unstable rank performance for alternatives across a wide range is given by variation in criteria weight as there is no global stability range.

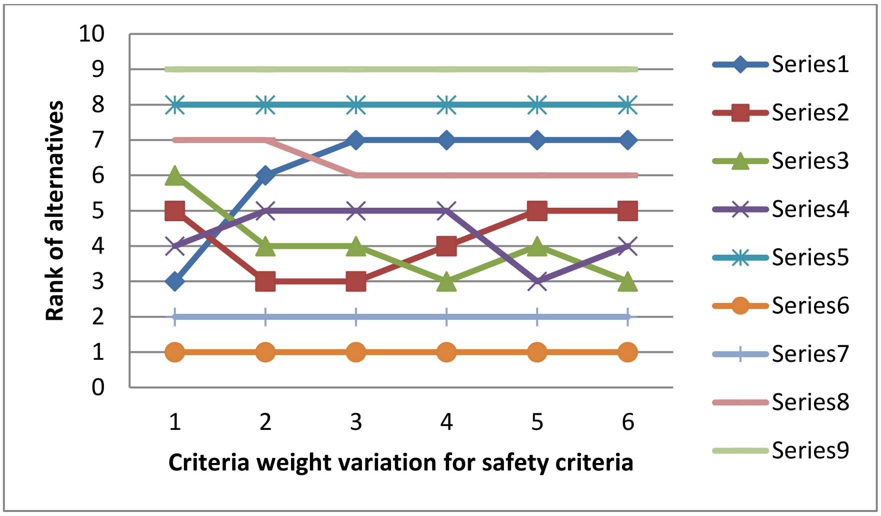


Figure 7. Single dimensional weight sensitivity analysis for example-1 considering AHP method

6. Conclusions

The rank performance obtained by the mFS ELECTRE-I with improved Simos' and AHP weight calculation approach shows variation in the ordering of alternatives when compared to the TOPSIS-AHP strategy. The improved Simos' and AHP weight calculations in the mFS ELECTRE-I have led to a higher ranking for the NTM procedure. For instance, it can be concluded from the results that EDM is the best-suited procedure for duralumin precision hole machining and stainless-steel surface revolution. The use of the mFS ELECTRE-I with updated Simos' and AHP weight computation enables, as shown in figures 1 and 5, a clear comparison between two choices using a coloured directed graph. The revised Simos' and AHP technique paired with the m-polar ELECTRE-I algorithm is used to assign weights to parameters for the input of combined (measured and imprecise) information. Based on the examples in this work and the difficulties resolved in the literature, we can conclude that the mFS ELECTRE-I with revised Simos' weight computation can be used for MCDM and MCGDM. The algorithm has credibility because the findings are supported by past research (N. D. Chakladar & Chakraborty, 2008).

With Simos' weight calculation method and AHP weight calculation method, additional single dimensional weight sensitivity analyses (SDWSA) are carried out for examples 1 and 2. The SDWSA results demonstrate that the Simos weight calculation method is stable for changes in the criteria weight. Calculating SDWSA to AHP weights reveals unstable rank performance with changing criteria weights. As seen in example 1, the Simos weight calculation approach yields a global stability range for the "work material" requirement of between 30 and 56.5. While the global stability range for the "safety" criteria is not included in the AHP weight calculation. For the "work material" criteria in Example 2, the Simos weight calculation method yields a worldwide stability range from 0 to 56.5. For example-2, the AHP weight calculation approach lacks a worldwide stability range. The aforementioned observation for SDWSA demonstrates that Simos' criterion weight calculation method, when used with mFS ELECTRE-I, is more stable for criteria weight variation than AHP weight calculation method with mFS ELECTRE-I approach. The study has some limitations, including the need for additional iterations to arrive at final weights with a low consistency ratio when using the AHP weight calculation method. AHP can be used with additional iterations in the future. The mFS ELECTRE-I method can be used with Simos' weight calculating approach. To achieve results in the NTM selection process, it is advised that Simos' weight calculation approach be combined with the mFS ELECTRE-I algorithm. To determine the stability range of each criterion weight calculation method, a single dimensional weight sensitivity analysis is used. Because the mFS ELECTRE-I methodology does not allow for the computation of performance scores, high dimensional weight sensitivity analysis cannot be used. In the future, the Simos' criteria weight calculation technique and the AHP weight calculation method can both use the high dimensional weight sensitivity analysis for the mFS ELECTRE-I method.

This is the general algorithm for many industrial selection problems. For applications of the mFS ELECTRE-I with improved Simos' algorithm, robot selection, flexible production system selection, fast prototyping selection, and other industrial challenges can be taken into consideration.

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References

Adeel, A., Akram, M., Ahmed, I., & Nazar, K. (2019). Novel m-polar fuzzy linguistic ELECTRE-I method for group decision-making. Symmetry, 11(4), 1–26.

Adeel, A., Akram, M., & Koam, A. N. A. (2019). Multi-Criteria Decision-Making under mHF ELECTRE-I and HmF ELECTRE-I. Energies, 12(9), 1–30.

Akram, M., Waseem, N., & Liu, P. (2019). Novel Approach in Decision Making with m–Polar Fuzzy ELECTRE-I. International Journal of Fuzzy Systems, 21(4), 1117–1129.

Asghari, F., Amidian, A. A., Muhammadi, J., & Rabiee, H. R. (2010). A fuzzy ELECTRE approach for evaluating mobile payment business models. Proceedings - 2010 International Conference on Management of e-Commerce and e-Government, ICMeCG 2010, 351–355.

Aytac, E., Tus, I. A., & Kundakci, N. (2011). Fuzzy ELECTRE I Method for Evaluating Catering Firm Alternatives. Ege Academic Review, 11(2011), 125–134.

Boral, S., & Chakraborty, S. (2016). A case-based reasoning approach for non-traditional machining processes selection. Advances in Production Engineering And Management, 11(4), 311–323.

Chakladar, N. Das, Das, R., & Chakraborty, S. (2009). A digraph-based expert system for non-traditional machining processes selection. International Journal of Advanced Manufacturing Technology, 43(3–4), 226–237.

Chakladar, N. D., & Chakraborty, S. (2008). A combined TOPSIS-AHP-method-based approach for non-traditional machining processes selection. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 222(12), 1613–1623.

Chakrabarti, S., Mitra, S., & Bhattacharyya, B. (2007). Development of an management information system as knowledge base model for machining process characterisation. International Journal of Advanced Manufacturing Technology, 34(11–12), 1088–1097.

Chakraborty, S. (2011). Applications of the MOORA method for decision making in manufacturing environment. International Journal of Advanced Manufacturing Technology, 54(9–12), 1155–1166.

Chakraborty, S., & Dey, S. (2006). Design of an analytic-hierarchy-process-based expert system for non-traditional machining process selection. International Journal of Advanced Manufacturing Technology, 31(5–6), 490–500.

Chakraborty, S., & Dey, S. (2007). QFD-based expert system for non-traditional machining processes selection. Expert Systems with Applications, 32(4), 1208–1217.

Chatterjee, P., Mondal, S., Boral, S., Banerjee, A., & Chakraborty, S. (2017). A novel hybrid method for non-traditional machining process selection using factor relationship and multi-attribute border approximation method. Facta Universitatis, Series: Mechanical Engineering, 15(3), 439–456.

Chen, J., Li, S., Ma, S., & Wang, X. (2014). M-Polar fuzzy sets: An extension of bipolar fuzzy sets. Scientific World Journal, 2014.

Cogun, C. (1993). Computer-aided system for selection of nontraditional machining operations. Computers in Industry, 22(2), 169–179.

Das, S., & Chakraborty, S. (2011). Selection of non-traditional machining processes using analytic network process. Journal of Manufacturing Systems, 30(1), 41–53.

Edison Chandrasselan, R., Jehadeesan, R., & Raajenthiren, M. (2008). Web-based knowledge base system for selection of non-traditional machining processes. Malaysian Journal of Computer Science, 21(1), 45–56.

Figueira, J., & Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. European Journal of Operational Research, 139(2), 317–326.

Hatami-Marbini, A., Tavana, M., Moradi, M., & Kangi, F. (2013). A fuzzy group Electre method for safety and health assessment in hazardous waste recycling facilities. Safety Science, 51(1), 414–426.

Jagtap, M., & Karande, P. (2021). Effect of normalization methods on rank performance in single valued m-polar fuzzy ELECTRE-I algorithm. Materials Today:

Proceedings.

Jagtap, M., Karande, P., & Athawale, V. M. (2021). Rank Assessment of Robots Using m-Polar Fuzzy ELECTRE-I Algorithm. Proceedings of the International Conference on Industrial Engineering and Operations Management, 246–255.

Karande, P., & Chakraborty, S. (2012). Application of PROMETHEE-GAIA method for non-traditional machining processes selection. Management Science Letters, 2(6), 2049–2060.

Karande, P., Zavadskas, E. K., & Chakraborty, S. (2016). A study on the ranking performance of some MCDM methods for industrial robot selection problems. International Journal of Industrial Engineering Computations, 7(3), 399–422.

Kaya, T., & Kahraman, C. (2011). An integrated fuzzy AHP-ELECTRE methodology for environmental impact assessment. Expert Systems with Applications, 38(7), 8553–8562.

Khandekar, A. V., & Chakraborty, S. (2016). Application of fuzzy axiomatic design principles for selection of non-traditional machining processes. International Journal of Advanced Manufacturing Technology, 83(1–4), 529–543.

Mousseau, V., Roy, B., & Paris-dauphine, U. (2005). Electre Methods. In: Multiple Criteria Decision Analysis: State of the Art Surveys. International Series in Operations Research & Management Science, vol 78. (133-162), Springer, New York.

Prasad, K., & Chakraborty, S. (2014). A decision-making model for non-traditional machining processes selection. Decision Science Letters, 3(4), 467–478.

Prasad, K., & Chakraborty, S. (2018). A decision guidance framework for non-traditional machining processes selection. Ain Shams Engineering Journal, 9(2), 203–214.

Recherche, R. O., & Roy, O. B. (1968). Classement et choix en présence de points de vue multiples. Revue Française D'Automatique, D'Informatique Et De recherche opérationnelle. Recherche opérationnelle, 2, 57-75.

Rouyendegh, B. D., & Erkan, T. E. (2013). An application of the Fuzzy ELECTRE method for academic staff selection. Human Factors and Ergonomics In Manufacturing, 23(2), 107–115.

Roy, M. K., Ray, A., & Pradhan, B. B. (2014). Non-traditional machining process selection using integrated fuzzy AHP and QFD techniques: a customer perspective. Production and Manufacturing Research, 2(1), 530–549.

Saaty, T. L. (2002). Decision making with the Analytic Hierarchy Process. Scientia Iranica, 9(3), 215–229.

Sadhu, A., & Chakraborty, S. (2011). Non-traditional machining processes selection using data envelopment analysis (DEA). Expert Systems with Applications, 38(7), 8770–8781.

Temuçin, T., Tozan, H., Vayvay, Ö., Harničárová, M., & Valíček, J. (2014). A fuzzy based decision model for nontraditional machining process selection. International Journal of Advanced Manufacturing Technology, 70(9–12), 2275–2282.

Ustinovichius, L., & Simanaviciene, R. (2010). The sensitivity analysis for cooperative decision by TOPSIS method. Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 6240 LNCS, 89–96.

Yurdakul, M., & Çoğun, C. (2003). Development of a multi-attribute selection procedure for non-traditional machining processes. Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 217(7), 993–1009.

Zadeh, L. . (1965). Fuzzy Set Theory. Information and Control, 8, 338–353.