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MODIFICATION OF THE CRITIC METHOD USING FUZZY ROUGH NUMBERS

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Abstract: *This paper presents a new approach in the modification of the CRiteria Importance Through Intercriteria Correlation (CRITIC) method using fuzzy rough numbers. In the modified CRITIC method (CRITIC-M), the normalization procedure of the home matrix elements was improved and the aggregation function for information processing in the normalized home matrix was improved. By introducing a new way of normalization, smaller deviations between normalized elements are obtained, which affects smaller values of standard deviation. Thus, the relationships between the data in the initial decision matrix are presented in a more objective way. The introduction of a new way of aggregating the values of weights in the CRITIC-M method enables a more comprehensive view of information in the initial decision matrix, which leads to obtaining more objective values of weights. A new concept of fuzzy rough numbers was used to address uncertainties in the CRITIC-M methodology.*

Key words: *MCDM, fuzzy sets, rough sets, fuzzy rough numbers, CRITIC-M.*

1. Introduction

Determining criterion weights is one of the key problems that arises in multi-criteria optimization models. In order to develop effective methods for determining the weight of the criteria, researchers around the world in recent years in the literature pay considerable attention to this problem. Most authors suggest dividing the model for determining the weights of criteria into subjective and objective (Zhu et al., 2015).

Subjective approaches reflect the subjective opinion and intuition of the decision maker. In this approach, the weight of the criteria are determined based on the preferences of the decision maker. Traditional methods of determining weights of criteria include tradeoff method (Keeney & Raiffa, 1976), proportional (ratio) method, Swing method (Weber et al., 1988) and Conjoint method (Green &

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Srinivasan, 1990), Analytic Hierarchy Process (AHP) model (Saaty, 1980), SMART (the Simple Multi Attribute Rating Technique) method (Keeney & Raiffa, 1976), MACBETH (Measuring Attractiveness by Categorical Based Evaluation Technique) method (Bana e Costa & Vansnick, 1994), Direct point allocation method (Poyhonen & Hamalainen, 2001), Ratio or direct significance weighting method (Weber & Borcherding, 1993), Resistance to change method (Rogers & Bruen, 1998), WLS (Weighted Lest Square) method (Graham, 1987) and FPP (the Fuzzy Preference Programming method) method (Mikhailov, 2000). Recent subjective methods include multipurpose linear programming (Costa & Climaco, 1999), linear programming (Mousseau et al., 2000), SWARA (Step-wise Weight Assessment Ratio Analysis) method (Valipour et al., 2017), BWM (Best Worst Method) (Rezaei, 2015) and FUCOM (FUll COnsistency Method) (Pamučar et al., 2018).

Among the most known objective methods are the following: Entropy method (Shannon & Weaver, 1947), CRITIC method (CRiteria significance Through Intercriteria Correlation) (Srđević et al., 2003) and FANMA method whose name was derived from the names of the authors of the method (Žižović et al., 2020).

The CRITIC method is one of the most well-known and most frequently used objective methods. The CRITIC method belongs to the group of correlation methods, which uses standard deviations of the standardized criterion values of variants to determine the contrast of criteria, as well as the correlation coefficients of all pairs of columns. In this study, certain limitations were identified when applying the classical CRITIC method and a modification of the CRITIC method (CRITIC-M) in a fuzzy rough environment was proposed.

The rest of the work is organized as follows. The following section shows the preliminary settings for fuzzy rough numbers. Section 3 presents the mathematical foundations of the classical CRITIC method. While section 4 shows a modification of the CTIRIC method in a fuzzy rough environment. The fifth section of the paper presents the application of the fuzzy rough CRITIC-M method through an example from the literature. Concluding remarks and directions for future research are given in Section 6.

2. Preliminaries on fuzzy rough numbers

In the fuzzy rough concept, fuzzy theory was used to represent uncertainty in information, while rough theory was used to create flexible boundary intervals of fuzzy numbers. The use of hybrid fuzzy rough numbers eliminates the limitation of classic fuzzy type 2 numbers that have a predefined imprint of uncertainty.

We assume that *U* universe contains all of the objects and let *Y* be an arbitrary object from *U*. We assume there is a set of k classes which represent the preferences of the DM, $G^* = (A_1, A_2, ..., A_k)$, with the condition that they belong to a series which satisfies the condition $A_1 < A_2 <, ..., < A_k$. All objects are defined in the universe and connected with the preferences of the DM. Each element A_i ($1 \le i \le k$) represents a fuzzy number that is defined as $A_q = (a_{1q}, a_{2q}, a_{3q})$. Since element A_i from the class of objects G^* is represented as fuzzy number $A_q = (a_{1q}, a_{2q}, a_{3q})$, for each value a_{1q}, a_{2q} and a_{3q} we obtain one class of objects that is represented in the interval $I(a_1)_q = \{I(a_1)_{lq}, I(a_1)_{uq}\}$, $I(a_2)_q = \{I(a_2)_{lq}, I(a_2)_{uq}\}$ and $I(a_3)_q = \{I(a_3)_{lq}, I(a_3)_{uq}\}$ where the condition is fulfilled that $I(a_j)_{lq} \le I(a_j)_{uq}$ ($j = 1,2,3;\ 1 \le q \le k$), as well as the condition $I(a_1)_q, I(a_2)_q, I(a_3)_q \in G^*$. Then $I(a_j)_{lq}$ and $I(a_j)_{uq}$ ($j = 1,2,3;\ 1 \le q \le k$) respectively represent the lower and upper border of the intervals of the *q*-th class of objects. If both limits of the classes of objects (upper and lower limits) respectively are compared so that $I^*(a_j)_{l1} < I^*(a_j)_{l2} <,...,< I^*(a_j)_{ls}; I^*(a_j)_{u1} < I^*(a_j)_{u2} <,...,< I^*(a_j)_{um}$ ($j=1,2,3$; $1 \le s,m \le k$), then for any of the classes of objects $I^*(a_j)_{lq} \in G^*$ and $I^*(a_j)_{uq} \in G^*$ ($j=1,2,3; 1 \le q \le k$) we can define the lower approximation $I^*(a_j)_{lq}$ using the following equations

$$\underline{Apr}\left(I^*(a_1)_{lq}\right) = \cup\left\{Y \in U / G^*(Y) \le I^*(a_1)_{lq}\right\};\ \left(1 \le q \le k\right) \tag{1}$$

$$\underline{Apr}\left(I^*(a_2)_{lq}\right) = \cup\left\{Y \in U / G^*(Y) \le I^*(a_2)_{lq}\right\};\ \left(1 \le q \le k\right) \tag{2}$$

$$\underline{Apr}\left(I^*(a_3)_{lq}\right) = \cup\left\{Y \in U / G^*(Y) \le I^*(a_3)_{lq}\right\};\ \left(1 \le q \le k\right) \tag{3}$$

And the upper approximation of $I^*(a_j)_{uq}$ using the following equations

$$\overline{Apr}\left(I^*(a_1)_{uq}\right) = \cup\left\{Y \in U / G^*(Y) \ge I^*(a_1)_{uq}\right\};\ \left(1 \le q \le k\right) \tag{4}$$

$$\overline{Apr}\left(I^*(a_2)_{uq}\right) = \cup\left\{Y \in U / G^*(Y) \ge I^*(a_2)_{uq}\right\};\ \left(1 \le q \le k\right) \tag{5}$$

$$\overline{Apr}\left(I^*(a_3)_{uq}\right) = \cup\left\{Y \in U / G^*(Y) \ge I^*(a_3)_{uq}\right\};\ \left(1 \le q \le k\right) \tag{6}$$

Both classes of objects (object classes $I^*(a_j)_{lq}$ and $I^*(a_j)_{uq}$) are defined by their lower limits $\underline{Lim}\left(I^*(a_j)_{lq}\right);\ j=1,2,3$, and upper limits $\overline{Lim}\left(I^*(a_j)_{uq}\right);\ j=1,2,3$. The lower limits are defined by the following equations

$$\underline{Lim}\left(I^*(a_1)_{lq}\right) = \frac{1}{M_{L(a_1)}} \sum G^*(Y) \middle| Y \in \underline{Apr}\left(I^*(a_1)_{lq}\right);\ \left(1 \le q \le k\right) \tag{7}$$

$$\underline{Lim}\left(I^*(a_2)_{lq}\right) = \frac{1}{M_{L(a_2)}} \sum G^*(Y) \middle| Y \in \underline{Apr}\left(I^*(a_2)_{lq}\right);\ \left(1 \le q \le k\right) \tag{8}$$

$$\underline{Lim}\left(I^*(a_3)_{lq}\right) = \frac{1}{M_{L(a_3)}} \sum G^*(Y) \middle| Y \in \underline{Apr}\left(I^*(a_3)_{lq}\right);\ \left(1 \le q \le k\right) \tag{9}$$

where $M_{L(a_1)}$, $M_{L(a_2)}$ and $M_{L(a_3)}$ respectively represent the number of objects included in the lower approximation of the classes of objects $I^*(a_1)_{lq}$, $I^*(a_2)_{lq}$ and $I^*(a_3)_{lq}$. The upper limits $\overline{Lim}\left(I^*(a_j)_{uq}\right);\ j=1,2,3$ are defined by equations (10)-(12)

$$\overline{Lim}\left(I^*(a_1)_{uq}\right) = \frac{1}{M_{U(a_1)}} \sum G^*(Y) \middle| Y \in \overline{Apr}\left(I^*(a_1)_{uq}\right);\ \left(1 \le q \le k\right) \tag{10}$$

$$\overline{Lim}\left(I^*(a_2)_{uq}\right) = \frac{1}{M_{U(a_2)}} \sum G^*(Y) \middle| Y \in \overline{Apr}\left(I^*(a_2)_{uq}\right);\ \left(1 \le q \le k\right) \tag{11}$$

$$\overline{Lim}\left(I^*(a_3)_{uq}\right) = \frac{1}{M_{U(a_3)}} \sum G^*(Y) \middle| Y \in \overline{Apr}\left(I^*(a_3)_{uq}\right);\ \left(1 \le q \le k\right) \tag{12}$$

where $M_{U(a_1)}$, $M_{U(a_2)}$ and $M_{U(a_3)}$ respectively represent the number of objects that are contained in the upper approximation of the classes of objects $I^*(a_1)_{uq}$, $I^*(a_2)_{uq}$ and $I^*(a_3)_{uq}$.

As we see, each class of objects $I(a_1)_q$, $I(a_2)_q$ and $I(a_3)_q$ is defined by means of its own lower and upper limits, which make up the interval fuzzy-rough number $\overset{=}{A}$ Figure 1, defined as

$$\overline{\overline{A}}=\left[A_q^L,A_q^U\right]=\begin{bmatrix}\left(\overline{Lim}\left(I^*(a_1)_{uq}\right),\underline{Lim}\left(I^*(a_2)_{lq}\right),\overline{Lim}\left(I^*(a_2)_{uq}\right),\underline{Lim}\left(I^*(a_3)_{lq}\right);w_1(A_q^L)\right)\\ \left(\underline{Lim}\left(I^*(a_1)_{lq}\right),\underline{Lim}\left(I^*(a_2)_{lq}\right),\overline{Lim}\left(I^*(a_2)_{uq}\right),\overline{Lim}\left(I^*(a_3)_{uq}\right);w_2(A_q^U)\right)\end{bmatrix} \tag{13}$$

where A_q^L and A_q^U respectively represent the upper and lower trapezoidal fuzzy-rough number which meets the condition that $A_q^L \subset A_q^U$, while $w_1(A_q^L)$ and $w_2(A_q^U)$ respectively represent the maximum values of interval fuzzy-rough number $\overline{\overline{A}}$.

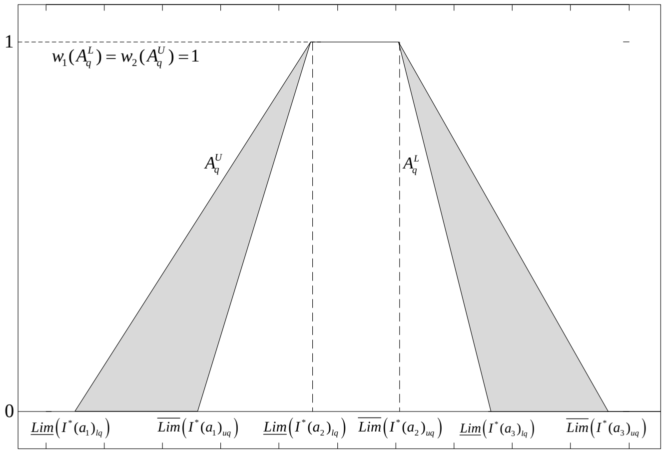

Figure 1. Interval fuzzy-rough number $\overline{\overline{A}}$

From Figure 1 we observe that for interval-valued fuzzy-rough number $\overline{\overline{A}}$ it is valid that $w_1(A_q^L)=w_2(A_q^U)=1$. On this basis we can write equation (13) in the following form:

$$\overline{\overline{A}}=\left[A_q^L,A_q^U\right]=\left[\left(a_{1q}^L,a_{1q}^U\right),\left(a_{2q}^L,a_{2q}^U\right),\left(a_{3q}^L,a_{3q}^U\right)\right] \tag{14}$$

where $a_{jq}^L=\underline{Lim}\left(I^*(a_j)_{lq}\right)$ and $a_{jq}^U=\overline{Lim}\left(I^*(a_j)_{uq}\right)$; $\left(j=1,2,3;\ 1\le q\le k\right)$.

If there is consensus among the decision makers on the assignment of specific values from the linguistic fuzzy scale then $a_{1q}^L=a_{1q}^U$, $a_{2q}^L=a_{2q}^U$ and $a_{3q}^L=a_{3q}^U$. Then interval-valued fuzzy-rough number $\overline{\overline{A}}$ becomes fuzzy number *A* type-1.

3. CRITIC method

The CRITIC method (CRiteria Importance Through Intercriteria Correlation) (Žižović et al., 2020) is a correlation method. Standard deviations of ranked criteria values of options in columns, as well as correlation coefficients of all paired columns are used to determine criteria contrasts.

Step 1: Starting from an initial decision matrix, $X=\left[\xi_{ij}\right]_{m\times n}$, we normalize the element of the initial decision matrix and form the normalized matrix $X=\left[\hat{\xi}_{ij}\right]_{m\times n}$.

$$X = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1n} \\ \xi_{21} & \xi_{22} & \cdots & \xi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{m1} & \xi_{m2} & \cdots & \xi_{mn} \end{bmatrix}_{m\times n} \end{array} \tag{15}$$

The normalization of matrix elements $X = \left[\xi_{ij}\right]_{m\times n}$ is done by applying (16) and (17):

a) for maximizing criteria:

$$\xi_{ij} = \frac{\xi_{ij} - \xi_j^{\min}}{\xi_j^{\max} - \xi_j^{\min}}, \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,m; \tag{16}$$

b) for minimizing criteria:

$$\xi_{ij} = \frac{\xi_j^{\max} - \xi_{ij}}{\xi_j^{\max} - \xi_j^{\min}}, \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,m; \tag{17}$$

where $\xi_j^{\max} = \max\limits_j \left\{\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj}\right\}$; $\xi_j^{\min} = \min\limits_j \left\{\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj}\right\}$.

Upon normalizing criteria of the initial decision matrix, all elements ξ_{ij} are reduced to interval values [0, 1], so it can be said that all criteria have the same metrics.

Step 2: For criterion C_j $\left(j = 1,2,\ldots,n\right)$ we define the standard deviation σ_j, that represents the measure of deviation of values of alternatives for the given criterion of average value. Standard deviation of a given criterion is the measure considered in the further process of defining criteria weight coefficients.

Step 3: From the normalized matrix $X = \left[\xi_{ij}\right]_{m\times n}$ we separate the vector $\xi_j = \left(\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj}\right)$ that contains the values of alternatives A_i $\left(i = 1,2,\ldots,m\right)$ for the given criterion C_j $\left(j = 1,2,\ldots,n\right)$. After forming the vector $\xi_j = \left(\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj}\right)$, we construct the matrix $L = \left[l_{jk}\right]_{n\times n}$, that contains coefficients of linear correlation of vectors ξ_j and ξ_k.

The quantity of data W_j contained within criterion j is determined by combining previously listed measures σ_j and l_{jk} as follows:

$$W_j = \sigma_j \cdot \varphi_j = \sigma_j \sum_{k=1}^{n} (1 - l_{kj}) \tag{18}$$

Based on the previous analysis we can conclude da a higher value W_j means a larger quantity of data received from a given criterion, which in turn increases the relative significance of the given criterion for the given decision process.

Step 4: Objective weights of criteria are reached by normalizing measures W_j:

$$w_j = \frac{W_j}{\sum_{k=1}^{m} W_k} \tag{19}$$

4. Fuzzy rough CRITIC method

The modification of the CRITIC method presented in this section is based on three starting points: 1) Modification of the initial decision matrix data normalization method, 2) Modification of the expression for determining the final values of criterion weights, and 3) Extension of the modified CRITIC method using fuzzy rough numbers. In the following part, the modified fuzzy rough CRITIC method algorithm is presented and testing is performed on an example from the literature.

Step 1. Construct the basic fuzzy rough decision matrix ($\Im$). We will assume that the evaluation of alternatives was performed by e experts using the fuzzy scale. Also, we will assume that expert preferences are presented in the home matrix $\Im^b = \left[\vartheta_{ij}^b \right]_{m\times n}$ where $1 \le b \le e$; *i=1,...,m; j=1,...,n*; and $\vartheta_{ij}^b = \left(\vartheta_{ij}^{b(l)}, \vartheta_{ij}^{b(m)}, \vartheta_{ij}^{b(u)} \right)$ represent linguistic variables from the fuzzy scale used by expert *e*. For each element $\theta_{ij}^{e(l)}$, $\theta_{ij}^{e(m)}$ and $\theta_{ij}^{e(u)}$ from $\Im^b = \left[\vartheta_{ij}^b \right]_{m\times n}$ we form matrices of the aggregated sequences of experts $\Im^{b(l)} = \left[\vartheta_{ij}^{b(l)} \right]_{m\times n}$, $\Im^{b(l)} = \left[\vartheta_{ij}^{b(m)} \right]_{m\times n}$ and $\Im^{b(l)} = \left[\vartheta_{ij}^{b(u)} \right]_{m\times n}$. Using expressions (1)-(12) sequence $\vartheta_{ij}^{e(l)}$, $\vartheta_{ij}^{e(m)}$ and $\vartheta_{ij}^{e(u)}$ are transformed into fuzzy rough number $\overline{\vartheta}_{ij}^b = \left(\left[\overline{\vartheta}_{ij}^{b(l)-}, \overline{\vartheta}_{ij}^{b(l)+} \right], \left[\overline{\vartheta}_{ij}^{b(m)-}, \overline{\vartheta}_{ij}^{b(m)+} \right], \left[\overline{\vartheta}_{ij}^{b(u)-}, \overline{\vartheta}_{ij}^{b(u)+} \right] \right)$; $1 \le b \le e$. For fusion fuzzy rough values $\overline{\vartheta}_{ij}^b$ ($1 \le b \le e$)the fuzzy rough weighted geometric Bonferroni function was used. This is how the aggregated fuzzy rough matrix $\Im = \left[\overline{\vartheta}_{ij} \right]_{m\times n}$ is defined.

Step 2. The elements of the matrix $\Im = \left[\overline{\vartheta}_{ij} \right]_{m\times n}$ are normalized as follows:

$$\overline{\xi}_{ij} = \begin{cases} \left[\left(\dfrac{\overline{\vartheta}_{ij}^{(l)-}}{\vartheta_{j+}^U}, \dfrac{\overline{\vartheta}_{ij}^{(l)+}}{\vartheta_{j+}^U} \right), \left(\dfrac{\overline{\vartheta}_{ij}^{(m)-}}{\vartheta_{j+}^U}, \dfrac{\overline{\vartheta}_{ij}^{(m)+}}{\vartheta_{j+}^U} \right), \left(\dfrac{\overline{\vartheta}_{ij}^{(u)-}}{\vartheta_{j+}^U}, \dfrac{\overline{\vartheta}_{ij}^{(u)+}}{\vartheta_{j+}^U} \right) \right]; & if \quad j \in B, \\ \left[\left(\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(u)+}}, \dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(u)-}} \right), \left(\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(m)+}}, \dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(m)-}} \right), \left(\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(l)+}}, \dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(l)-}} \right) \right]; & if \quad j \in C \end{cases} \tag{20}$$

where $\vartheta_{i+}^U = \max_{1\le i\le m} (\vartheta_{ij}^{(u)+})$ and $\vartheta_{i-}^L = \min_{1\le i\le m} (\vartheta_{ij}^{(l)-})$.

Step 3: Construct a matrix of linear correlations. For each criterion C_j from the normalized matrix $\Im^N = \left[\overline{\xi}_{ij} \right]_{m\times n}$, the vector $\overline{\xi}_j = \left(\overline{\xi}_{1j}, \overline{\xi}_{2j}, ..., \overline{\xi}_{mj} \right)$ is defined, and linear correlations of the vectors $\overline{\xi}_j$ and $\overline{\xi}_k$ are calculated. By summing the linear correlations by criteria, we obtain the measure of the conflict of criteria $\varphi_j = \sum_{k=1}^{n} (1 - l_{jk})$. The amount of information W_j contained in criterion *j* is determined by applying expression (21):

$$W_j = \sigma_j \sum_{k=1}^{n} (1 - l_{kj}) \tag{21}$$

Korak 4: Determination of weight coefficients of criteria. Objective weights of criteria are obtained by applying expression (22):

$$w_j = \frac{\frac{\bar{\xi}_j}{1-\bar{\xi}_j} \cdot W_j}{\sum_{j=1}^{n} \left(\frac{\bar{\xi}_j}{1-\bar{\xi}_j} \cdot W_j \right)} \tag{22}$$

Example:

We will assume that the multi-criteria model considers the evaluation of three alternatives under five criteria. We will also assume five experts evaluated the alternatives using the fuzzy scale presented in Table 1.

Table 1. Fuzzy scale

Linguistic terms	Membership function
Absolutely low (AL)	(1, 1.5, 2.5)
Very low (VL)	(1.5, 2.5, 3.5)
Low (L)	(2.5, 3.5, 4.5)
Medium low (ML)	(3.5, 4.5, 5.5)
Equal (E)	(4.5, 5.5, 6.5)
Medium high (MH)	(5.5, 6.5, 7.5)
High (H)	(6.5, 7.5, 8.5)
Extremly high (EH)	(7.5, 8.5, 9.5)
Absolutely high (AH)	(8.5, 9, 10)

Experts' assessments of alternatives are presented in Table 2.

Table 2. Expert evaluation of alternatives

	A1	A2	A3
C1	EH,EH,EH,AH,AH	H,EH,H,MH,H	VL,L,L,L,L
C2	AH,AH,AH,AH,EH	E,ML,ML,ML,E	AH,AH,AH,H,AH
C3	EH,AH,AH,EH,AH	H,EH,H,EH,H	EH,H,H,EH,AH
C4	EH,AH,AH,H,AH	H,H,H,H,EH	MH,MH,E,MH,E
C5	VL,VL,AL,VL,VL	E,E,ML,ML,ML	AL,AL,VL,VL,AL

By applying expressions (1) - (12) the expert estimates were transformed into fuzzy rough values, Table 3.

Table 3. Fuzzy rough home matrix

Crit.	A1	A2
C1	([7.56,8.28],[8.53,8.89],[9.53,9.89])	([5.96,6.75],[6.97,7.76],[7.97,8.76])
C2	([7.97,8.50],[8.74,9.00],[9.74,10.0])	([3.56,4.13],[4.56,5.13],[5.56,6.14])
C3	([7.70,8.43],[8.60,8.97],[9.60,9.97])	([6.56,7.14],[7.56,8.14],[8.56,9.14])
C4	([7.16,8.43],[8.07,8.97],[9.07,9.97])	([6.50,6.81],[7.5,7.81],[8.50,8.810])
C5	([1.22,1.50],[1.93,2.50],[2.95,3.50])	([3.56,4.13],[4.56,5.13],[5.56,6.14])

Crit.	A3
C1	([1.93,2.44],[2.95,3.44],[3.96,4.45])
C2	([7.41,8.39],[8.19,8.92],[9.20,9.92])
C3	([6.70,7.64],[7.70,8.49],[8.70,9.49])
C4	([4.70,5.33],[5.70,6.33],[6.70,7.33])
C5	([1.03,1.31],[1.55,2.11],[2.56,3.12])

Using the expression (20), the elements from Table 3 were normalized. Then, using the expressions (21) and (22), the matrices of linear correlations of fuzzy rough elements were defined and the final values of the weighting coefficients were determined as follows:

$w_1 = 0.153;$

$w_2 = 0.380;$

$w_3 = 0.189;$

$w_4 = 0.118;$

$w_5 = 0.160.$

5. Conclusion

This research presents a modification of the CRITIC method using fuzzy rough numbers. Fuzzy rough numbers are applied because part of the uncertainty and subjectivity are neglected in the classic fuzzy and rough models. Given the well-known performance of fuzzy sets in representing uncertainties and confirmed advantages of rough numbers in subjectivity manipulation, a modification of the CRITIC method based on information processing using hybrid fuzzy rough numbers is proposed. Also, the application of the fuzzy rough CRITIC method is shown in an example that considers the evaluation of three alternatives under five criteria.

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