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ZEGHDOUDI DISTRIBUTION IN ACCEPTANCE SAMPLING PLANS BASED ON TRUNCATED LIFE TESTS WITH REAL DATA APPLICATION

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Abstract: *The acceptance sampling plan (ASP) is one of the key statistics resources in the industrial sector. It entails the decision-making process for accepting or rejecting the products. The investigated quality parameter is the test unit's mean lifespan. This study develops a new ASP for Zeghdoudi distribution (ZD) when the lifetime is shortened to a specific degree. The optimal plan parameters are accomplished by obtaining the minimum sample size mandatory to ensure the identified mean lifetime for fixing the consumer's risk. Besides, the characteristic operating function (OCF) values for the ASP are displayed, and the producer's risk is determined. Several helpful tables are developed for the suggested ASP based on the ZD for suitable employment. An actual data set is fitted to the Zeghdoudi model and other models to examine the applicability of the suggested ASP in the production sector.*

Key words: *Zeghdoudi distribution; Acceptance sampling plan; Consumer's risk; Truncated lifetime test; characteristic function; Producer's risk Operating.*

1. Introduction

The customer looks for a good, high-quality product that will operate effectively for a very long period. The manufacturer also wants to provide consumers with convenience items at a lower price. The acceptance sampling plan, which helps determine whether to accept or reject submitted lots based on lot quality criteria, is one of the most basic statistical quality control methods. The items' lifetime is considered a quality feature of importance in reliability ASP. The ASP bases decisions on the outcomes of random samples taken from the relevant batch. Since the choice

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of lots is based on the findings of the sample, it is occasionally possible to accept a low-quality lot while rejecting a good-quality lot. The probability that a lot with poor quality will be accepted and a lot with high quality would be rejected, respectively, is known as consumer risk and producer risk.

Numerous academics support the use of truncated lifetime tests in the ASP. For example, Sobel and Tischendrof (1959) investigated the exponential distribution in ASP. The ASP was taken into account by Tsai and Wu (2006) for the generalized Rayleigh distribution. ASP is proposed by Al-Nasser and Al-Omari (2013) using the exponentiated Fréchet distribution. For the exponentiated inverse Rayleigh distribution, Sriramachandran and Palanivel (2014) developed ASP. For the three parameters of the Kappa distribution, Al-Omari (2014) suggested ASP. In ASP based on reduced lifetime time, Al-Omari (2015) took into consideration the generalized inverse exponential distribution. Based on the negative binomial Marshall-Olkin Rayleigh distribution, Jose and Sivadas (2015) created ASP. The log-logistic distribution's ASP in shortened lifetime tests is studied by Kantam et al. in 2001. Lio et al. (2010) investigate the time-truncated ASP for the percentiles of the Burr type XII. Al-Omari et al. (2016) investigated the Half-Normal distribution for time-truncated lifetime tests using a double acceptance sampling scheme.

Al-Omari (2018a) submitted a single ASP according to truncated life tests for Sushila distribution. Li et al. (2018) studied ASP with Bayesian accelerated scheme for the lognormal distribution under Type-I censoring. Transmuted generalized inverse Weibull distribution was taken into account by Al-Omari (2018b) in a single ASP. Shahbaz et al. (2018) considered the power Lindley distribution in SASP for finite and infinite lot size. Testing and Inspection utilizing acceptance sampling plans is a book written by Aslam and Ali (2019). Al-Omari et al. (2019) studied ASP according to truncated life tests for Rama distribution. For the power inverted Topp-Leone distribution, Abushal et al. (2021) considered a single ASP. Isık and Kaya (2021) assumed SASP according to interval type-2 fuzzy sets.

Al-Omari et al. (2021) suggested SASP under a two-parameter Quasi Shanker distribution insuring mean life with an application to industrial data. On the basis of the transmuted Rayleigh distribution, Saha et al. (2021) provided single and double acceptance sampling designs for truncated life tests. Shrahili et al. (2021) developed a single ASP based on percentiles of the new Weibull-Pareto distribution applied to data on the breaking stress of carbon fibers. Based on the power Lomax distribution, Al-Nasser and ul Haq (2021) analyzed ASP from a truncated life test.

Multiple dependent state repeated sampling designs for exponentiated half logistic distribution were examined by Srinivasa et al. (2022). Bayesian group chain sampling is recommended by Hafeez et al. (2022) for Poisson distributions using gamma priors. For consumer protection, Jeyadurga and Balamurali (2022) suggested constructing a variable repetitive group sampling scheme. Applications of the HLMOL-X family of distributions to acceptance sampling and the stress-strength parameter are introduced by Tomy and Jose (2022). Mahmood et al. (2021) explored Topp-Leone Gompertz distribution-based acceptance sampling designs. Plans for acceptance sampling are provided by Nassr et al. (2022) for the three-parameter inverted Topp-Leone model.

According to our knowledge, this work is the first to propose an ASP based on ZD. The following sections make up the remaining text: The Zeghdoudi distribution and some of its features are introduced in Section 2. The suggested acceptance sampling plan for the ZD is described in Section 3 along with its format. In Section 4, along with numerous examples, tables are provided for the minimum sample sizes, operational characteristic values, and the minimal ratio of true mean lifetime. After being fitted to the ZD in accordance with predetermined parameters, an actual data set is applied to the recommended ASP. Recommendations and conclusions are given in Section 5.

2. The Zeghdoudi Distribution

Zeghdoudi distribution (ZD), presented by Messaadia and Zeghdoudi (2018), is a novel lifetime distribution with a single parameter. The probability density function of the ZD is

$$g(x)=\frac{\varphi^{3}}{2+\varphi}x(1+x)e^{-\varphi x},\quad x>0,\varphi>0, \tag{1}$$

which is a mixture of two gamma distributions, $Gamma(2,\varphi)$ and $Gamma(3,\varphi)$ with a mixture coefficient $\frac{\varphi}{2+\varphi}$ as

$$g_{ZD}(x)=\frac{\varphi}{2+\varphi}g_{Gamma(2,\varphi)}(x)+\frac{2}{2+\varphi}g_{Gamma(3,\varphi)}(x).$$

The ZD random variable's cumulative distribution function (cdf) is

$$G(x)=1-\left(1+\frac{\varphi^{2}x^{2}+\varphi(\varphi+2)x}{2+\varphi}\right)e^{-\varphi x},\quad x>0,\varphi>0, \tag{2}$$

For different choices of the distribution parameter, Figure 1 shows the ZD pdf plots. From Figure 1 it can be noted that the Zeghdoudi distribution is positively skewed and the skewness depends on the parameter value.

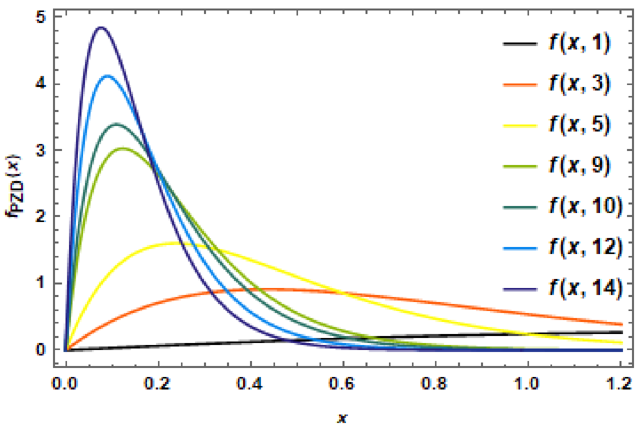

Figure 1. Possible pdf plots of the ZD for some parameters

The *k*th moment of the Zeghdoudi distribution is

$$E(X^{k})=\frac{(k+1)(\varphi+k+2)}{\varphi^{k}(\varphi+2)},k=1,2,3,\dots. \tag{3}$$

Consequently, for $k=1$, the distribution's mean is $E(X)=\frac{2(\varphi+3)}{\varphi(\varphi+2)}$, which will be considered as the quality parameter in this paper.

The ZD's coefficient of skewness and its coefficient of kurtosis are given by

$$SK = \frac{24(\varphi+5)(\varphi+2)^2}{\left(2\varphi^2+12\varphi+12\right)^{\frac{3}{2}}}, \text{ and } Ku = \frac{30(\varphi+6)(\varphi+2)^3}{\left(\varphi^2+6\varphi+6\right)^6}.$$

By resolving the nonlinear equation, $\hat{\varphi}_{MOM} = \hat{\varphi}_{MLE} = \frac{1}{\bar{X}}\left(\sqrt{\bar{X}^2+4\bar{X}+1}+1-\bar{X}\right)$, where $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$, the method of moment (MOM) and maximum likelihood (MLE) estimators of the distribution parameter may be derived. The hazard and survival functions of the ZD random variable, respectively, are

$$H(x,\varphi) = \frac{x(1+x)\varphi^3}{\varphi(\varphi+2)x+2+\varphi+\varphi^2x^2}, \tag{4}$$

and

$$S(x,\varphi) = \left(1+\frac{\varphi^2x^2+\varphi(\varphi+2)x}{2+\varphi}\right)e^{-\varphi x}. \tag{5}$$

Figure 2 characterizes the hazard rate and survival pdf plots for some selections of the distribution parameters. It is obvious that the hazard rate function of the ZD is increasing while its survival function is decreasing.

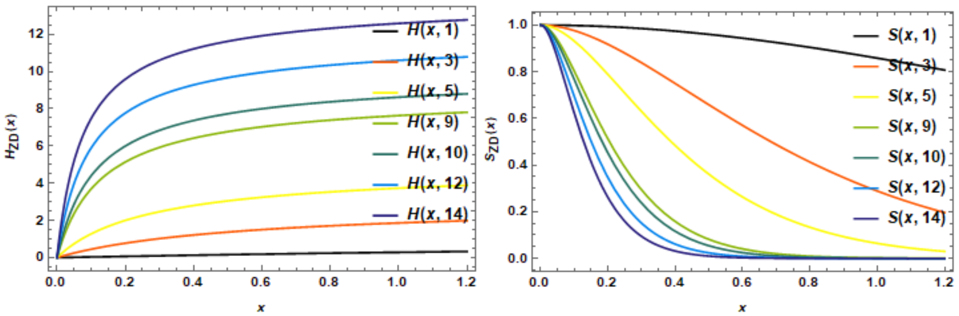

Figure 2. Possible hazard rate and survival plots of the ZD for some parameters

Due to the importance of ZD, Ruidas (2020) proposed a modification of the ZD by transmutation approach called as a transmuted Zeghdoudi distribution. Hamida and Hiba (2021) considered another improvement of the ZD and suggested truncated Zeghdoudi distribution. Hussain and Mohammed (2022) proposed weighted Zeghdoudi distribution.

3. The proposed acceptance sampling plan

In this part, we describe the suggested single acceptance sampling plan strategy and its associated parameters. The ASP can be summed up as follows:

(1) The sample size *m* selected from the lot to be tested,

(2) An acceptance number c that allows the lot to be approved if c or fewer failures are discovered during the test period t.

(3) The ratio t/μ_0, where μ_0 is the aforementioned average lifetime.

In the current study, it is presumed that a product's lifetime complies with the ZD using the pdf provided in (2). In the subsections that follow, the techniques for calculating the operational characteristic values, producer's risk, and minimal sample size are illustrated.

3.1. Minimum sample size

Let's assume that the consumer's risk will be no more than $1-P^*$. Specifically, the probability that the real average lifetime μ is not more than $1-P^*$ and smaller than $\mu_0 = \dfrac{2(\varphi_0+3)}{\varphi_0(\varphi_0+2)}$. For a certain $0<P^*<1$, with ratio of t/μ_0 and an acceptance number *c*, the goal is to find the lowest sample size *m* where the total number of failures observed up until time *t* is not larger than c, using an assumption of $\mu \geq \mu_0$ and a confidence level of P^*.

The fundamental smallest sample size, m, that fulfills the following inequality is the least positive integer, where

$$\sum_{i=0}^{c} \binom{m}{i} p^i (1-p)^{m-i} \leq 1-P^*, \tag{6}$$

given that the lot size is large enough to employ the binomial distribution theory, where $p = G(t;\mu_0)$ given in (2) is the $p = G(t;\mu_0)$ given in (2) that the lifespan will not exceed t when the correct mean is μ_0.

If c or fewer observable failures have occurred up until time t, then from (6) we have that $G(t;\mu) \leq G(t;\mu_0)$ with probability P^*, which suggests $\mu \geq \mu_0$. Note that

$$G(x;\mu,\varphi) = 1-\left[1+4\left(\frac{x}{\mu}\right)^2 \frac{(\varphi+3)^2}{(\varphi+2)^3} + 2\frac{x}{\mu}\left(\frac{\varphi+3}{2+\varphi}\right)\right] e^{-2\frac{x}{\mu}\left(\frac{\varphi+3}{2+\varphi}\right)}, \tag{7}$$

and hence

$$\begin{aligned} G\left(\frac{x}{\mu_0};\frac{\mu}{\mu_0},\varphi\right) &= 1-\left[1+4\left(\frac{x}{\mu_0}\frac{\mu_0}{\mu}\right)^2 \frac{(\varphi+3)^2}{(\varphi+2)^3} + 2\frac{x}{\mu_0}\frac{\mu_0}{\mu}\left(\frac{\varphi+3}{\varphi+\theta}\right)\right] e^{-2\frac{x}{\mu_0}\frac{\mu_0}{\mu}\left(\frac{\varphi+3}{2+\varphi}\right)} \\ &= 1-\left[1+4\left(\frac{z}{w}\right)^2 \frac{(\varphi+3)^2}{(\varphi+2)^3} + 2\frac{z}{w}\left(\frac{\varphi+3}{2+\varphi}\right)\right] e^{-2\frac{z}{w}\left(\frac{\varphi+3}{2+\varphi}\right)}, \end{aligned} \tag{8}$$

where $z = \dfrac{x}{\mu_0}$ and $w = \dfrac{\mu_0}{\mu}$.

For the distribution parameter $\theta = 0.0274$, $t/\mu_0 = 0.628$, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 and $P^* = 0.75$, 0.90, 0.95, 0.99 with $c = 0,1,2,\ldots,10$, the minimum sample sizes that are satisfying (6) are listed in Table 1. According to Kantam et al. (2001), Baklizi (2003), and Al-Omari et al. (2021), the same values are taken into consideration.

3.2. Operating characteristic function

The OCF is a crucial indicator of how well the sampling plan is performed. The operational characteristic function gives the probability that the lot will be approved. The OCF constitutes from the following steps:

1. First, determine the values of plan parameter.
2. Verify the expression of probability of acceptance.
3. Put all the plan parameters and parameters involve in probability of acceptance equation.
4. Calculate the value of probability of acceptance.

For the acceptance sampling plan $\left(m,\, c,\, t/\mu_0\right)$, the OCF is as

$$
\begin{aligned}
OCF(p) &= \sum_{i=0}^{c}\binom{m}{i} p^i (1-p)^{m-i} \\
&= \sum_{i=0}^{c}\binom{m}{i}\left\{1-\left[1+4\left(\frac{z}{w}\right)^2\frac{(\varphi+3)^2}{(\varphi+2)^3}+2\frac{z}{w}\left(\frac{\varphi+3}{2+\varphi}\right)\right]e^{-2\frac{z}{w}\left(\frac{\varphi+3}{2+\varphi}\right)}\right\}^{i} \\
&\quad\times\left\{\left[1+4\left(\frac{z}{w}\right)^2\frac{(\varphi+3)^2}{(\varphi+2)^3}+2\frac{z}{w}\left(\frac{\varphi+3}{2+\varphi}\right)\right]e^{-2\frac{z}{w}\left(\frac{\varphi+3}{2+\varphi}\right)}\right\}^{m-i}
\end{aligned} \tag{9}
$$

where the function $p = G(t;\mu)$ depends on the lots quality parameter μ.

3.3 Producer's risk

Depending on the type of sampling, only part of the lot is inspected and there is always a specific risk. It is possible that a lot is accepted or rejected even though they shouldn't. These risks are often referred to as producer risk and consumer risk, as expressed below.

The likelihood of rejecting a lot with an acceptable quality or better when $\mu > \mu_0$ is known as the producer's risk (PR), and it is determined by

$$P(p) = P(\text{Reject a good lot}) = \sum_{i=c+1}^{m}\binom{m}{i} p^i (1-p)^{m-i}. \tag{10}$$

Using the suggested sample strategy and a predetermined PR value, φ, we want to find the value of μ/μ_0, which keep the PR at most φ. The probability $p = F(t;\mu)$ can be obtained as a function of μ/μ_0. As a result, μ/μ_0 is the lowest positive number that allows $p = F\left(\frac{t}{\mu_0}\frac{\mu_0}{\mu}\right)$ to satisfy the inequality

$$PR(p) = \sum_{i=c+1}^{m}\frac{m!}{i!(m-i)!} p^i (1-p)^{m-i} \le \vartheta.$$

(11)

The lowest values of μ/μ_0 fulfilling (13) for a given acceptance sampling plan $\left(m,\, c,\, t/\mu_0\right)$ and a certain confidence level P^*, are shown in Table 3.

4. Discussion and illustrations

Assume for this section that the lifespan follows the ZD distribution. The OCF values, the minimal difference between the actual and reported mean lifetimes, and the minimal sample size needed to guarantee that the mean lifespan surpasses at least a probability P^* are all provided in Tables 1-3.

Consider a scenario in which the researcher wishes to confirm that the product will have an average lifespan of at least 1000 hours, with probability $P^* = 0.90$, when $c = 2$, such that the test result for the lifetime distribution is at least 1257, i.e., $t / \mu_0 = 1.257$. From Table 1 the tabulated sample size is $m = 9$. Hence, for this example, the ASP is $(m = 6,\ c = 2,\ t / \mu_0 = 1.257)$. This indicates that out of 6 sampled units from the produced lot, the lot is suggested for acceptance with $P^* = 0.90$ and rejected if more than 2 units fail before time t.

The results in Table 2 provide a summary of the OCF values based on the minimal sample size found in Table 1 related to ASP $(m,\ c = 2,\ t / \mu_0)$. According to Table 2, the OCF and the corresponding PR values with $P^* = 0.90$ for the ASP $(m = 6,\ c = 2,\ t / \mu_0 = 1.257)$ are:

μ / μ_0	2	4	6	8	10	12
$OC(p)$	0.755853	0.993757	0.999624	0.999956	0.999992	0.999998
PR	0.244147	0.006243	0.000376	0.000044	0.000008	0.000002

Here, if the true mean lifetime is 6 times the specified one $(\mu / \mu_0 = 6)$, then with a probability of equal to 0.999624, we are confident that the lot can be accepted based on the offered ASP, and the producer's risk is 0.000376. In general, if the true mean lifetime is at least 4000 hours or $\mu / \mu_0 \geq 4$ the producer's risk goes to zero. The smallest ratios μ / μ_0 between the actual mean lifetime and the given average life can be found for the same example from Table 3 ensuring that the PR is lower than or equal to 0.05. For the current case, the smallest value of μ / μ_0 is 2.822 for $c = 2$ with $p^* = 0.90$; hence, the lot can be approved with a probability of 0.95 based on the suggested ASP if the items have a mean lifetime of at least 2.822 times the specified mean lifetime of 1000 hours.

Table 1. Minimum sample sizes of the sampling plans with $\varphi = 0.0274$ in the ZD

		t / μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4	2	2	1	1	1	1	1
	1	9	5	3	3	2	2	2	2
	2	13	7	5	4	3	3	3	3
	3	17	9	6	5	4	4	4	4
	4	21	11	8	6	5	5	5	5
	5	25	13	9	8	6	6	6	6
	6	28	15	11	9	7	7	7	7
	7	32	17	12	10	8	8	8	8
	8	36	19	14	11	9	9	9	9
	9	40	21	15	13	11	10	10	10
	10	43	23	17	14	12	11	11	11
0.90	0	7	3	2	2	1	1	1	1
	1	12	6	4	3	2	2	2	2
	2	17	9	6	5	3	3	3	3
	3	21	11	7	6	5	4	4	4
	4	26	13	9	7	6	5	5	5
	5	30	15	11	9	7	6	6	6
	6	34	18	12	10	8	7	7	7
	7	38	20	14	11	9	8	8	8
	8	42	22	15	12	10	9	9	9
	9	46	24	17	14	11	10	10	10
	10	50	26	18	15	12	11	11	11
0.95	0	9	4	3	2	1	1	1	1
	1	15	7	5	4	3	2	2	2
	2	20	10	7	5	4	3	3	3
	3	24	12	8	6	5	4	4	4
	4	29	15	10	8	6	5	5	5
	5	33	17	12	9	7	6	6	6
	6	38	19	13	11	8	7	7	7
	7	42	21	15	12	9	8	8	8
	8	46	24	16	13	10	9	9	9
	9	50	26	18	14	11	10	10	10
	10	55	28	20	16	12	11	11	11
0.99	0	14	6	4	3	2	1	1	1
	1	20	10	6	5	3	2	2	2
	2	26	13	8	6	4	4	3	3
	3	31	15	10	8	5	5	4	4
	4	36	18	12	9	7	6	5	5
	5	41	20	14	11	8	7	6	6
	6	46	23	15	12	9	8	7	7
	7	50	25	17	13	10	9	8	8
	8	55	28	19	15	11	10	9	9
	9	59	30	20	16	12	11	10	10
	10	64	32	22	17	13	12	11	11

Table 2. OCF values of the ASP $(m, c = 2, t/\mu_0)$ with $\varphi = 0.0274$

P^*	m	t/μ_0	μ/μ_0					
			2	4	6	8	10	12
0.75	13	0.628	0.939065	0.999422	0.999975	0.999997	1	1
	7	0.942	0.897478	0.998512	0.999925	0.999992	0.999999	1
	5	1.257	0.845129	0.996703	0.999808	0.999978	0.999996	0.999999
	4	1.571	0.797426	0.994072	0.999606	0.999952	0.999991	0.999998
	3	2.356	0.678631	0.982007	0.998377	0.999765	0.999952	0.999988
	3	3.141	0.390016	0.926200	0.990647	0.998378	0.999633	0.999898
	3	3.927	0.189559	0.820121	0.968831	0.993563	0.998376	0.999514
	3	4.712	0.083212	0.678631	0.926165	0.982007	0.994953	0.998377
0.90	17	0.628	0.882577	0.998677	0.999940	0.999994	0.999999	1
	9	0.942	0.810560	0.996619	0.999823	0.999981	0.999997	0.999999
	6	1.257	0.755853	0.993757	0.999624	0.999956	0.999992	0.999998
	5	1.571	0.648079	0.986478	0.999048	0.999881	0.999978	0.999995
	3	2.356	0.678631	0.982007	0.998377	0.999765	0.999952	0.999988
	3	3.141	0.390016	0.926200	0.990647	0.998378	0.999633	0.999898
	3	3.927	0.189559	0.820121	0.968831	0.993563	0.998376	0.999514
	3	4.712	0.083212	0.678631	0.926165	0.982007	0.994953	0.998377
0.95	20	0.628	0.831410	0.997846	0.999901	0.999990	0.999998	1
	10	0.942	0.761955	0.995300	0.999750	0.999973	0.999995	0.999999
	7	1.257	0.661273	0.989655	0.999355	0.999925	0.999987	0.999997
	5	1.571	0.648079	0.986478	0.999048	0.999881	0.999978	0.999995
	4	2.356	0.374904	0.942174	0.994080	0.999104	0.999815	0.999952
	3	3.141	0.390016	0.926200	0.990647	0.998378	0.999633	0.999898
	3	3.927	0.189559	0.820121	0.968831	0.993563	0.998376	0.999514
	3	4.712	0.083212	0.678631	0.926165	0.982007	0.994953	0.998377
0.99	26	0.628	0.716035	0.995367	0.999778	0.999977	0.999996	0.999999
	13	0.942	0.609927	0.989672	0.999422	0.999936	0.999989	0.999997
	8	1.257	0.567754	0.984325	0.998989	0.999880	0.999979	0.999995
	6	1.571	0.503623	0.975310	0.998163	0.999766	0.999957	0.999989
	4	2.356	0.374904	0.942174	0.994080	0.999104	0.999815	0.999952
	4	3.141	0.112019	0.797666	0.968500	0.994084	0.998611	0.999606
	3	3.927	0.189559	0.820121	0.968831	0.993563	0.998376	0.999514
	3	4.712	0.083212	0.678631	0.926165	0.982007	0.994953	0.998377

Table 3. The μ / μ_0 values for acceptability of a lot with PR of 0.05 and $\varphi = 0.0274$

		t / μ_0							
P^*	c	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
0.75	0	4.040	4.624	6.170	5.834	8.749	11.663	14.582	17.497
	1	2.533	2.915	2.995	3.743	4.362	5.815	7.270	8.723
	2	2.071	2.299	2.539	2.747	3.268	4.357	5.447	6.535
	3	1.858	2.016	2.093	2.281	2.758	3.676	4.596	5.515
	4	1.732	1.851	2.018	2.005	2.456	3.274	4.093	4.911
	5	1.649	1.743	1.817	2.065	2.253	3.003	3.754	4.505
	6	1.562	1.665	1.801	1.905	2.105	2.806	3.509	4.210
	7	1.521	1.606	1.678	1.783	1.992	2.656	3.320	3.984
	8	1.488	1.560	1.680	1.686	1.902	2.536	3.170	3.804
	9	1.461	1.523	1.593	1.750	2.158	2.437	3.047	3.656
	10	1.422	1.492	1.601	1.675	2.075	2.355	2.944	3.532
0.90	0	5.003	5.421	6.170	7.711	8.749	11.663	14.582	17.497
	1	2.862	3.174	3.487	3.743	4.362	5.815	7.270	8.723
	2	2.334	2.612	2.822	3.173	3.268	4.357	5.447	6.535
	3	2.048	2.247	2.318	2.616	3.420	3.676	4.596	5.515
	4	1.915	2.035	2.184	2.286	3.007	3.274	4.093	4.911
	5	1.800	1.894	2.094	2.271	2.730	3.003	3.754	4.505
	6	1.717	1.854	1.919	2.089	2.530	2.806	3.509	4.210
	7	1.654	1.772	1.884	1.950	2.378	2.656	3.320	3.984
	8	1.606	1.707	1.771	1.840	2.256	2.536	3.170	3.804
	9	1.566	1.655	1.757	1.876	2.158	2.437	3.047	3.656
	10	1.534	1.612	1.675	1.793	2.075	2.355	2.944	3.532
0.95	0	5.502	6.060	7.234	7.711	8.749	11.663	14.582	17.497
	1	3.138	3.403	3.889	4.358	5.614	5.815	7.270	8.723
	2	2.504	2.749	3.067	3.173	4.119	4.357	5.447	6.535
	3	2.174	2.350	2.514	2.616	3.420	3.676	4.596	5.515
	4	2.013	2.196	2.334	2.522	3.007	3.274	4.093	4.911
	5	1.881	2.030	2.214	2.271	2.730	3.003	3.754	4.505
	6	1.809	1.912	2.027	2.251	2.530	2.806	3.509	4.210
	7	1.735	1.822	1.976	2.098	2.378	2.656	3.320	3.984
	8	1.678	1.796	1.856	1.976	2.256	2.536	3.170	3.804
	9	1.631	1.736	1.831	1.876	2.158	2.437	3.047	3.656
	10	1.607	1.686	1.811	1.901	2.075	2.355	2.944	3.532
0.99	0	6.497	7.077	8.086	9.041	11.564	11.663	14.582	17.497
	1	3.525	3.975	4.235	4.861	5.614	5.815	7.270	8.723
	2	2.798	3.106	3.286	3.526	4.119	5.491	5.447	6.535
	3	2.431	2.625	2.850	3.142	3.420	4.559	4.596	5.515
	4	2.217	2.410	2.597	2.730	3.428	4.009	4.093	4.911
	5	2.076	2.212	2.430	2.617	3.096	3.640	3.754	4.505
	6	1.975	2.119	2.222	2.398	2.856	3.373	3.509	4.210
	7	1.882	2.007	2.143	2.232	2.673	3.170	3.320	3.984
	8	1.825	1.957	2.082	2.213	2.528	3.008	3.170	3.804
	9	1.764	1.882	1.969	2.097	2.410	2.876	3.047	3.656
	10	1.727	1.820	1.934	2.000	2.311	2.766	2.944	3.532

Figures 3 and 4 show the OCF plots when $\varphi = 0.0274$ in ZD and $P^* = 0.75$ and $P^* = 0.99$, respectively. It can be noted that the OCF plots are increasing as the mean ratio values are increasing.

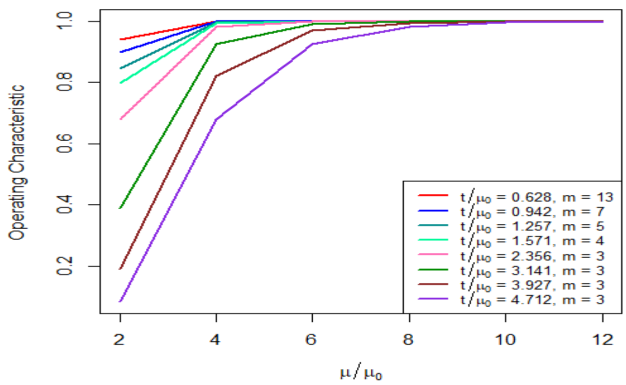

Figure 3. The OCF plots when $P^* = 0.75$ and $\varphi = 0.0274$ in ZD

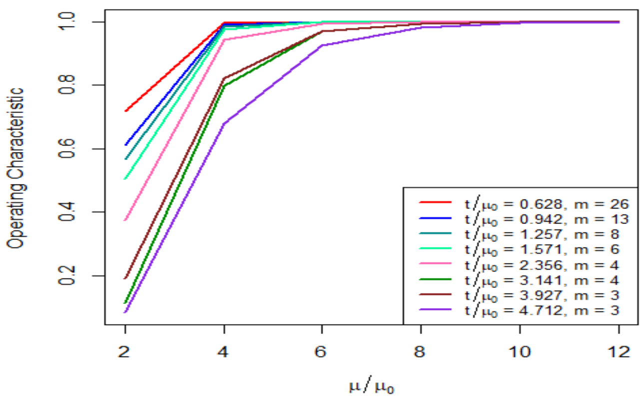

Figure 4. The OCF plots when $P^* = 0.99$ and $\varphi = 0.0274$ in ZD

5. Application of real data

An example of the applicability of the suggested ASP based on the ZD distribution is given using thirty successive values of March precipitation (in inches) in Minneapolis/St. Paul from Hinkley (1977). The data are
0.32, 0.47, 0.52, 0.59, 0.77, 0.81, 0.81, 0.9, 0.96, 1.18, 1.2, 1.2, 1.31, 1.35, 1.43, 1.51, 1.62, 1.74, 1.87, 1.89, 1.95, 2.05, 2.1, 2.2, 2.48, 2.81, 3, 3.09, 3.37, 4.75.

The descriptive statistical measures of these data are $\text{mean} = 1.68, \text{medain} = 1.47,$ $\text{skewness} = 1.03, \text{kurtosis} = 0.93, \text{minimum} = 0.32, \text{maximum} = 4.75$.

Four models of one parameter are considered for the comparison in fitting the data, namely the

1) Rama distribution with pdf $f_{RD}(x;\theta)\frac{\theta^4}{\theta^3+6}(1+x^3)e^{-\theta x},\ x\geq 0, \theta>0.$

2) Rani distribution with pdf $f(x;\theta)=\frac{\theta^5}{\theta^5+24}(\theta+x^4)e^{-\theta x};\theta>0, x>0$.

3) Exponential distribution with pdf $f_{ExpD}(x;\theta)=\theta e^{-\theta x}$.

4) Bilal Distribution (Abd-Elrahman, 2013) with pdf $f_{BD}(x;\theta)=\frac{6}{\theta}\left(1-e^{-\frac{x}{\theta}}\right)e^{-\frac{2x}{\theta}}, x\geq 0, \theta>0$.

The method of maximum likelihood estimation (MLE) is used to estimate the distribution for the unknown parameter in each model. The results are shown in Table 4 and are based on the comparison of the aforementioned models using the Akaike Information Criteria (AIC), Consistent Akaike Information Criteria (CAIC), Baysian Information Criteria (BIC), Hanan Quinn Information Criteria (HQIC), Kolmogorov-Smirov (K-S) statistic, and corresponding p-value. The model that has the lowest AIC, CAIC, BIC, HQIC, and KS values as well as the highest KS score is considered to be the best one. According to the actual data, Figure 5 displays the estimated PDF and CDF for the Zeghdoudi, Rama, Rani, Exponential and Bilal distributions. The total time on test transform (TTT-Transform) plot can be used in various applications to obtain information on the shape of the hazard function of a given data set, which aids in choosing a specific model to fit a given data set. Bergman and Klefsjo (1984) provide more information on the TTT Transform approach.

Figure 6 displays the density, TTT, probability-probability (P-P) plot and quantile-quantile (Q-Q) plots of the PZD to the precipitation data.

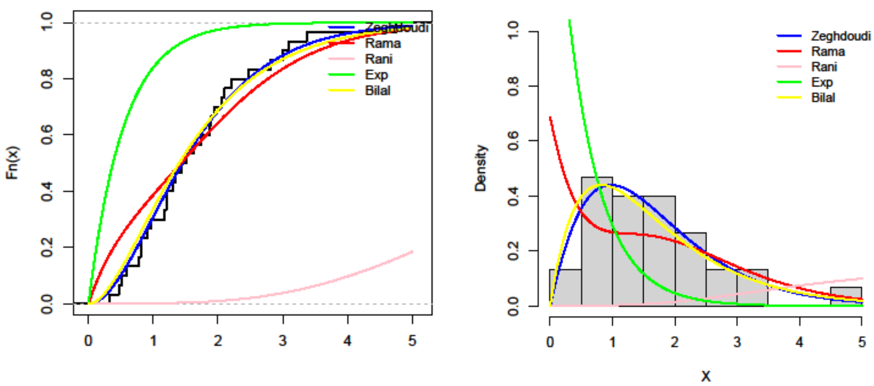

Figure 5. Plots of the estimated pdf and cdf for the real data for the ZD, Rama, Rani, Exp, and Bilal models.

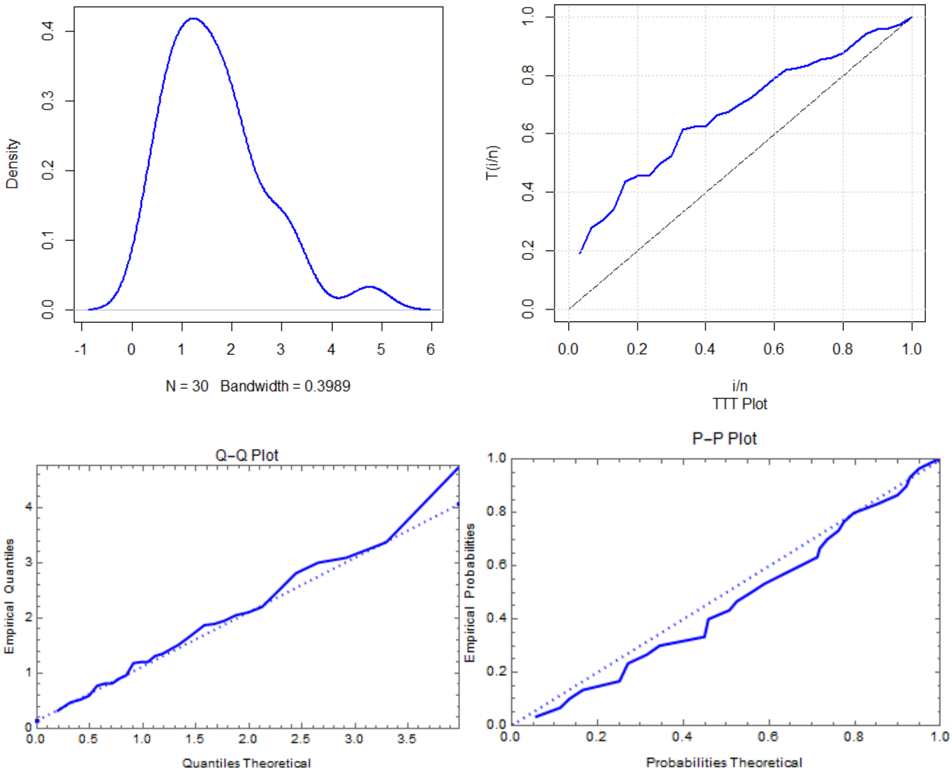

Figure 6. The density, TTT, P-P and Q-Q plots of the PZD to the precipitation data.

Table 4. The goodness-of-fit tests statistics for the precipitation data

	ZD	Rama	Rani	Exp	Bilal
AIC	79.34100	90.88004	96.99700	92.94879	80.73433
CAIC	79.48388	91.02289	97.13984	93.09165	80.87719
BIC	80.74222	92.28123	98.39818	94.34999	82.13553
HQIC	79.78927	91.32829	97.44524	93.39705	81.18259
MML	38.67051	44.44002	47.49850	45.47439	39.36717
Erro	0.16681	0.13435	0.11055	0.10900	0.26525
K-S	0.08774	0.19694	0.23526	0.23520	0.11435
MLE	1.53209	1.63404	1.83337	0.59704	2.01860
P-value	0.97500	0.19500	0.07224	0.07235	0.82760

It is evident from Table 4 that the Zeghdoudi model is the best one because it has the lowest values for AIC, CAIC, BIC, HQIC, and KS and the highest p-value of 0.975. Also, Figure 3 emphasizes the suitability of ZD in fitting these data set. For this data set, the MLE of φ is $\hat{\varphi}=1.53209$. Therefore, the estimated mean of the data is $\hat{\mu}=\dfrac{2(\hat{\varphi}+3)}{\hat{\varphi}(\hat{\varphi}+2)}=1.675$. In Tables 5-6, the offered ASP parameters (minimum sample size and OCF) are obtained based ons $\hat{\varphi}=1.53209$. Assume that the identified average lifetime is $\mu_0 = 1.675$, with testing time $t_0 = 1.0519$. Then, can we accept or reject the lot? For the ratio $t_0 / \mu_0 = 0.628$ with $P^* = 0.90$ from Table 5 we get $n = 30$ when the acceptance number $c = 6$. The decision is accepting the lot with mean lifetime of 1.675 with probability of 0.90 if the number of failures before $t_0 = 1.0519$ is less than or equal 6. But the number of failures before $t_0 = 1.0519$ is 9, and hence the lot should be rejected.

Table 5. Minimum sample sizes of the ASP with $\hat{\varphi} = 1.53209$, $P^* = 0.90$, $t / \mu_0 = 0.628$ for precipitation data

c	0	1	2	3	4	5	6	7	8	9	10
m	6	11	15	19	22	26	30	33	37	40	44

The OCF values based on the minimal sample size given in Table 5 and the corresponding producers' risk based on the ASP $(m = 30,\ c = 6,\ t / \mu_0 = 0.628)$ are given in Table 6. According to Table 5, the OCF and the corresponding PR values with $P^* = 0.90$ for the ASP $(m = 6,\ c = 2,\ t / \mu_0 = 1.257)$ are:

Table 6. The OCDF values and the corresponding producers' risk based on the ASP $(m = 30,\ c = 6,\ t / \mu_0 = 0.628)$ for precipitation data

μ / μ_0	2	4	6	8	10	12
$OC(p)$	0. 99997	0. 99999	1	1	1	1
PR	0.00003	0.00001	0	0	0	0

It is clear that the PR is zero for $\mu / \mu_0 \geq 6$ based on the ASP $(m = 30,\ c = 6,\ t / \mu_0 = 0.628)$.

5. Conclusions

New acceptance sampling plans for the Zeghdoudi distribution are presented in this research and are based on shortened life testing. The important tables come into play when determining the minimal sample size necessary to guarantee a particular mean life of the test units. Additionally, the values of the operating characteristic function and the corresponding producer risk are derived. The indicated acceptance sampling plans strategies are used on an actual data set. The study's findings encourage the researcher to employ the new acceptance sampling plans for Zeghdoudi distribution.

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