

GAME THEORETIC MODELS FOR A CLOSED-LOOP SUPPLY CHAIN WITH STOCHASTIC DEMAND AND BACKUP SUPPLIER UNDER DUAL CHANNEL RECYCLING

Bibhas Chandra Giri ^{1*} and Sushil Kumar Dey ¹

¹ Department of Mathematics, Jadavpur University, Kolkata, West Bengal, India

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Abstract: *In this paper, a dual-channel closed-loop supply chain is considered for waste recycling. The manufacturer produces the finished product using recycled and recyclable waste materials as well as fresh raw materials. The recyclable wastes collected by the collector are supplied to the manufacturer directly or indirectly via a third party (recycler). Two different game models are considered for two different cases of recycling: recycling by the manufacturer and recycling by the recycler. If the collector fails to supply the required amount of waste materials, the backup supplier meets up the shortfall by supplying fresh raw materials. The customer demand is assumed to be stochastic. Optimal results for the two game models are obtained through numerical examples. It is seen that ex-ante pricing commitment i.e, fixed markup strategy is beneficial for the whole supply chain as well as the supply chain entities, compared to the decentralized policy. From the numerical study, it is also observed that when the recyclability degree of wastes increases, the expected total profit increases for the whole supply chain. A higher price sensitivity of customer demand leads to lower profit for the chain members.*

Keywords: *Supply chain management, Closed-loop supply chain, Recycling, Markup-strategy.*

1. Introduction

One of the biggest concerns of our society today is degradation of environment. Increasing consumption, richer lifestyle, higher level of logistics and transportation have led to higher carbon emissions and as a consequence, all these are raising important questions about environmental sustainability. Most of the supply chains in today's business scenario are attentive to sustainability, not only for the present age but also for the future generations. Various ways like remanufacturing, reusing, green

* Corresponding author.

E-mail addresses: bcgiri.jumath@gmail.com (B.C. Giri), sushildevyu@gmail.com (S.K. Dey)

Game theoretic models for a closed-loop supply chain with stochastic demand and backup... purchasing, recycling etc. are being used to achieve environmental sustainability. Recycling is one of the most suitable way to adopt in manufacturing industries because recovery of materials and recycling from used product is one of the major avenues to reduce the usage of fresh materials. For example, plastic is the third highest manufacturing sector in the United States where over million of workers are working but for the conscience of environmental sustainability they have installed about 30,000 recycling drop-off points nationwide and plastic film recycling is continuing to grow (www.nytimes.com). Kreiger et al. (2014) studied about the recycling of hybrid polyethelene for 3-D painting. Bing et al. (2015) also studied in the same line concerning the household plastics of different types. The UK has a recycling rate of approximately 60% for iron and steel. Most of this waste comes from scrap vehicles, cooker, fridges and other kitchen appliances and, in Germany, the recycling rate for plastic is 70% (Giri & Dey, 2019). Measuring of reduction limit of repeated recycling for paper flow was analyzed by Chen et al. (2016). Sheu and Chen (2012) analyzed the effect of governmental financial intervention on a green supply chain management using a three stage game-theoretic model. Strategically low wholesale price is suggested to recycled-component suppliers to stimulate the manufacturer's intention of green production under green taxation. In case of waste recycling, some European countries like Sweden and Germany achieved great results of success even though recycling rate is lower than most of the other countries (Zhang et al., 2012). Jafari et al. (2017) studied dual channel recycling in a three-echelon supply chain with game theoretic approach. Ragert et al. (2017) studied about mechanical and chemical recycling of solid plastic waste. Their discussion was about the main challenges and some potential remedies to the recycling strategies. Wan et al. (2017) reviewed solid state recycling of aluminium chips. Sultan et al. (2017) studied an integrated model for product recycling desirability. Texas Instruments makes significant investments to efficiently use, reuse, or recycle materials across its operations, and reduces its potential environmental impact by sourcing materials responsibly, as well as appropriately managing waste handling and disposal (Giri & Dey, 2019)

In this paper, for a multi-echelon closed-loop supply chain with price dependent stochastic customer demand, we investigate the optimal decisions for pricing and corresponding profit for each player using game theoretic approach. Dual channel recycling (recycling by the manufacturer and recycling by the third party i.e, recycler) is adopted in this paper in two different models. The paper is organised as follows: Review of relevant literature is given in the next section. Notations and problem description are provided in Section 3. Two different game models and their analytical results are discussed in Section 4. In Section 5, numerical demonstration along with sensitivity analysis is given. Finally, the paper is concluded with future research directions in Section 6.

2. Literature review

In this section, a brief review of three different streams of research such as dual channel supply chain, sustainable development in supply chain and markup policies are given.

2.1. Dual channel supply chain

In traditional dual-channel supply chain, researchers customarily use online channel and offline channel. Yao and Liu (2005), Mukhopadhyay et al. (2008), Liu et al. (2010), Zhang et al. (2012), Cao et al. (2013) examined the optimal pricing

decisions for asymmetric information scenario through a dual channel structure. A profit maximization strategy in a dual-channel was derived by Batarfi et al. (2016). Zhang et al. (2017) studied about the retailer's channel structure choice; whether he would chose a online channel, offline channel or dual-channel. Some recent works on pricing, service and quality decisions in dual channel have been done by Wang et al. (2017) and Li et al. (2017). Chen et al. (2017) studied the impact of adding a new channel on price, quality and profit's change. Zhao et al. (2017) analyzed pricing policies for complementary products in a dual-channel supply chain where one among the two manufacturers uses dual channel. Wei et al. (2018) analyzed a dual collecting channel with dynamic nature of life-cycle of wastes. The effects of profit discount and collection competition on firms pricing decisions, collection rates and profits were studied and the remanufacturer's optimal strategy of maximizing its profit or maximizing the collection rate was revealed. Recently, Zhang et al. (2019) developed a new strategy for a dual-channel retailer to identify whether the strategy is always beneficial for improving the dual-channel retailer's profit or market share.

2.2. Sustainable development in supply chain

In recent years, mainly in the last decade, sustainability has become one of the biggest global business issues. Business environment has become more complex in nature. Environmental complexity due to unsustainable resources and activities is raising high. Hence, in this situation, large industries as well as small business firms are concerned for sustainability due to Governmental pressure and instructions, as well as for their own concern about a greener world. Navinchandra (1990) first proposed the idea of green product design. This means the improvement of the product's compatibility with the environment, without harming its quality or its function. An empirical study for sustainable supply chain management was proposed by Ageron et al. (2012). Ahi and Searcy (2013) analyzed a competitive literature of definitions for green and sustainable supply chain management. Impacts of lean, resilient and green practices on social, economic and environmental sustainability of supply chains were proposed by Govindan et al. (2014). An optimization oriented brief review of social and environmental sustainability was presented by Eskandarpour et al. (2015). Li et al. (2016) determined the pricing policy in a competitive dual-channel green supply chain. Yu and Solvang (2017) discussed the recycling with environmental considerations. Jafari et al. (2017) studied dual-channel waste recycling under deterministic scenario.

2.3. Markup pricing strategy

A simple but often used pricing policy is to include a fixed markup over the wholesale price of each item. According to Liu et al. (2006) retail fixed markup (RFM) simply exists as an "agreement" more than a formal written code. Markup can also be defined as the difference between the wholesale price and the retail price. Two types of markup are commonly used by the retailers: (i) Fixed-price markup and (ii) Fixed percentage markup. Wang et al. (2017) analyzed the performance of this two types of markup startegy under a chain to chain competition with dominant retailer. For instance, under a keystone markup, the retailer simply doubles the production cost to settle the retail price. So markup actually specifies pricing policies among different entities involved in a supply chain. In general, a contract is for long term and it varies over time and product but retail price markup remains fixed for the duration of the specified supply chain. Gasoline dealers or some grocers also use traditional fixed markup policy. Liu et al. (2006) studied vertically restrictive pricing using markup

Game theoretic models for a closed-loop supply chain with stochastic demand and backup... strategy. For an integrated supply chain with price dependent demand, they could not find a closed form solution under any general distribution of the stochastic customer demand. They also showed that Pareto-improving RFM solution exists in a deterministic scenario but it is not always possible to find when demand is stochastic. A two-way price commitment for the retailer and the manufacturer was studied by Liu et al. (2006). They assumed fixed markup contract for the retailer and price protection contract for the manufacturer. Maiti and Giri (2017) proposed a model with both variable and fixed markup. Giri et al. (2017) analyzed pricing policies for a three-echelon supply chain with sub-supply chain and RFM strategy.

3. Notations and Problem Description

We use the following notations throughout the paper:

- C_c unit collection cost of recyclable wastes to the collector.
- C_r unit recycling cost of recyclable wastes to the recycler.
- C_s unit procurement cost of recycled waste to the back-up supplier.
- C_m unit recycling cost of recyclable wastes to the manufacturer.
- C_p unit production cost of the finished product to the manufacturer.
- u per unit shortage penalty cost of the manufacturer.
- v per unit salvage value of the manufacturer.
- ϵ random part of the demand.
- θ recyclability degree of waste denoting the portion of waste that can be recovered and turned into new products. ($0 < \theta < 1$)
- γ quantity of recycled materials required to produce one unit of the finished product. ($\gamma > 1$)
- q quantity of finished items produced by the manufacturer.
- $\frac{\gamma}{\theta}$ quantity of recyclable waste required to produce one unit of the finished product.
- a maximum possible demand faced by the manufacturer for the finished product.
- b price sensitivity of the customer's demand. ($b > 0$)
- λ fractional part of the manufacturer's requirement of recycled materials supplied by the collector. ($0 < \lambda < 1$)
- z stocking factor for the stochastic demand.
- P_d wholesale price charged by the collector to the manufacturer for one unit of recyclable waste.
- P_c wholesale price charged by the collector to the recycler for one unit of recyclable waste.
- P_r wholesale price charged by the recycler to the manufacturer for one unit of recycled material.
- P_s wholesale price charged by the supplier to the manufacturer for one unit of fresh raw material.
- P retail price charged by the manufacturer to the customers for one unit of finished product.
- D customer demand of the finished product at the manufacturer.
- D_c quantity of raw materials supplied by the collector to the recycler.
- D_r quantity of recycled waste supplied by the recycler to the manufacturer.
- D_s quantity of fresh raw materials supplied by the backup supplier to the manufacturer.
- Π_c collector's profit.

- Π_r recycler’s profit.
- Π_s supplier’s profit.
- Π_m manufacturer’s profit.
- Π_{cr} profit obtained from the co-ordination established between the collector and the recycler

The proposed closed-loop supply chain consists of one manufacturer, one collector, one recycler and one backup supplier. The manufacturer may get the recycled materials from the recycler as well as recyclable wastes from the collector. A dual channel is considered to receive recyclable wastes and recycled materials from the collector and the recycler, respectively (see Figure 1). When the collector or the recycler fails to satisfy the manufacturer’s need ($q\gamma$ units), the manufacturer needs help of a backup supplier. The manufacturer then buys fresh raw materials from the backup supplier at a high price to make up the shortfall.

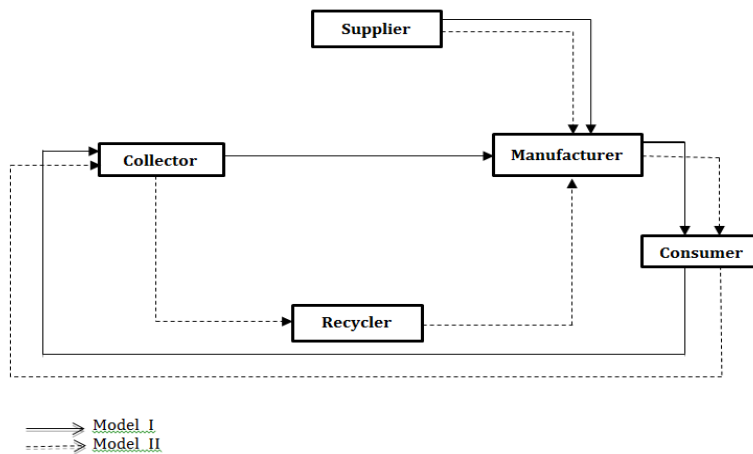


Figure 1. Material flow diagram

We assume that the customer’s demand D is linear, price-dependent and random in nature. We take $D = a - bP + \epsilon$ where $b > 0, P < \frac{a}{b}$ and ϵ is the random part of the customer demand. Here a denotes market’s total potential demand but actual demand is D and b represents the price sensitivity for customer demand. In supply chain literature, this type of demand function is common where the customer demand depends on retail price and demand decreases with the increment of retail price (Jafari et al., 2017; Petruzzi & Dada , 1999). Unit shortage penalty cost and salvage value are also incurred in the model setting as well.

4. Model Development

Under the problem scenario mentioned above, we develop two models (see Figure 1) depending upon two different situations :

Model I: In this model, the collector collects the recyclable wastes from the end customers and then supplies to the manufacturer. The manufacturer first recycles the waste materials and then produces finished goods for the end customers. Any shortfall of wastes is meet up by a backup supplier by supplying fresh raw materials.

Model II: This model includes a recycler. The collector collects the wastes but the recycling is done by the recycler. The recycler recycles the wastes and then sends to the manufacturer for production of finished goods. Any shortfall of recycled material is meet up by a backup supplier by supplying fresh raw materials.

4.1. Model I: The manufacturer gets recyclable wastes from the collector

Here, we assume that the collector may or may not satisfy the manufacturer's demand of recyclable wastes. The natural disasters, communication problems or unavailability of resources may be the reasons behind this. We assume that the manufacturer estimates an amount of $q\gamma$ units of raw materials to produce q units of finished product. Let us suppose that the collector can supply $\frac{q\gamma\lambda}{\theta}$ units of wastes where $0 < \lambda \leq 1$ and θ ($0 < \theta < 1$) is the recyclability degree of waste. So, we have in this case, $D_c = \frac{q\gamma\lambda}{\theta}$, $0 < \lambda \leq 1$ and $D_s = q\gamma(1 - \lambda)$. When $\lambda = 1$, the manufacturer's demand for recyclable wastes is completely meet up by the collector and hence there is no need of any action from the backup supplier. In this model, we develop three game theoretic approaches, viz. centralized game, decentralized game and fixed-markup game.

4.1.1. Centralized game

Since the market demand is stochastic, so sometimes the estimated inventory of the manufacturer may be less than the market demand or sometimes there may be some left over inventory in hand. We assume that the manufacturer sells the left over inventory in a secondary market with a salvage value v per unit. Unit shortage penalty cost is u . Then the expected profits of the manufacturer, the collector and the supplier are given by

$$\Pi_m = E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p], \quad (1)$$

$$\Pi_c = E[(P_d - C_c)D_c], \text{ and} \quad (2)$$

$$\Pi_s = E[(P_s - C_s)D_s], \quad (3)$$

respectively, where $X^+ = \max(X, 0)$ and the subscripts c , s and m stand for the collector, the supplier and the manufacturer, respectively. We replace $z = q - y(P)$, where z is the stocking factor on which the shortage or overage depends. Stocking factor is also sometimes called safety stock factor. Our objective is to find the optimal selling price, stocking factor rather than selling price and the stocking quantity. The expected total profit in the centralized game is given by

$$\begin{aligned} \Pi &= \Pi_m + \Pi_c + \Pi_s \\ &= E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s (P_d + C_m)D_c \\ &\quad + (P_d - C_c)D_c + (P_s - C_s)D_s] \\ &= (P - C_p)[y(p) + \mu] - (C_p - v)\phi(z) - (P + u - C_p)\psi(z) \\ &\quad - (C_m + C_c) [z + y(P)] \frac{\gamma\lambda}{\theta}, \end{aligned}$$

where $\phi(z) = \int_0^z (z - t)f(t)dt$ and $\psi(z) = \int_z^\infty (t - z)f(t)dt$.

Now, our problem becomes $Maximize_{z;P} \Pi$. We consider the first and second order partial derivatives of Π with respect to z and P . When z is fixed, we get the optimal value of P as $P^*(z) = \frac{a+bC_p+\mu+(C_m+C_c)\frac{b\gamma\lambda}{\theta} - \psi(z)}{2b}$ and optimal value of z for a fixed P is $z^*(P) = F^{-1}\left\{1 - \frac{(C_p-v)+(C_m+C_c)\frac{\gamma\lambda}{\theta}}{P+u-v}\right\}$.

Corollary 1. The profit function Π is concave in z for a given value of P and concave in P for a given value of z .

Proof: See Appendix A.

Proposition 1. (i) The optimal retail price increases with the stocking factor and (ii) the optimal stocking factor of the manufacturer is also an increasing function of the retail price.

Proof: (i) We have the optimal retail price $P^*(z) = \frac{a+bC_p+\mu+(C_m+C_c)\frac{b\gamma\lambda}{\theta} - \psi(z)}{2b}$.

Then clearly, $\frac{dP^*(z)}{dz} = -\left(\frac{1}{2b}\right)\frac{d}{dz}\psi(z) = \frac{1}{2b}\int_z^\infty f(t)dt > 0$, since $f(t) \geq 0$ for all t .

(ii) For the optimal stocking factor z^* , we have $F(z^*) = 1 - \frac{(C_p-v)+(C_m+C_c)\frac{\gamma\lambda}{\theta}}{P+u-v}$.

Differentiating partially with respect to P , we get $f(z^*)\frac{dz}{dP} = \frac{(C_p-v)+(C_m+C_c)\frac{\gamma\lambda}{\theta}}{(P+u-v)^2}$ which implies

$$\frac{dz}{dP} = \frac{1}{f(z^*)} \frac{(C_p - v) + (C_m + C_c)\frac{\gamma\lambda}{\theta}}{(P + u - v)^2} > 0, \text{ since } f(z) \geq 0.$$

Proposition 2. Under linear additive demand function, the supply chain's centralized solution is to set quantities z^* , P^* and to order $a - bP^* + z^*$ such that

(i) if $F(\cdot)$ is an arbitrary distribution, then the entire support must be searched to find z^* , (ii) if $F(\cdot)$ satisfies $2r(z)^2 + \frac{dr(z)}{dz} > 0$ where $r(z) = \frac{f(z)}{1-F(z)}$ is the hazard rate, then z^* is the largest z satisfying the first order condition.

Proof: For details of the proof see Appendix A.

4.1.2. Decentralized game

Here our objective is to maximize separately the expected profits of the manufacturer, the supplier and the collector, which are as follows:

$$\Pi_m = E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p]$$

$$\Pi_c = E[(P_d - C_c)D_c]$$

$$\Pi_s = E[(P_s - C_s)D_s]$$

Now, we suppose that the profit margins for the players in this game are same i.e. $P_d = \frac{P+C_c}{2}$ and $P_s = \frac{P+C_s}{2}$. Using these relations, we derive the optimal values of P and

$$z \text{ as, } P^*(z) = \frac{a+\mu+bC_p-\psi(z)+b[(P_d+C_m)\frac{\gamma\lambda}{\theta}+P_s\gamma(1-\lambda)]}{2b}$$

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$$\text{and, } z^*(P) = F^{-1} \left\{ 1 - \frac{(C_p - v) + (P_d + C_m) \frac{\gamma\lambda}{\theta} + P_s \gamma(1 - \lambda)}{P + u - v} \right\}$$

4.1.3. Fixed markup strategic game

In the fixed markup strategic game, we assume that the supplier's wholesale price $P_s = (1 - \alpha_1)P$ where $0 < \alpha_1 < 1$ and the collector's wholesale price is $P_d = (1 - \alpha_2)P$ where $0 < \alpha_2 < 1$ and that $0 < \alpha_1 \leq \alpha_2 < 1$. Using these relations in the profit functions, we get

$$\Pi_m = E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p],$$

$$\Pi_c = E[(P_d - C_c)D_c], \text{ and}$$

$$\Pi_s = E[(P_s - C_s)D_s].$$

We get the optimal price of the manufacturer as

$$P^*(z) = \frac{a + \mu + bC_p - \psi(z) + b[C_m \frac{\gamma\lambda}{\theta} - C_s \gamma(1 - \lambda)] - (z - a)[(1 - \alpha_2) \frac{\gamma\lambda}{\theta} + (1 - \alpha_1) \gamma(1 - \lambda)]}{2b - 2b(1 - \alpha_2) \frac{\gamma\lambda}{\theta} - 2b(1 - \alpha_1) \gamma(1 - \lambda)}$$

and optimal stocking factor as

$$z^*(P) = F^{-1} \left\{ 1 - \frac{(C_p - v) + ((1 - \alpha_2)P + C_m) \frac{\gamma\lambda}{\theta} + ((1 - \alpha_1)P - C_s) \gamma(1 - \lambda)}{P + u - v} \right\}$$

The optimal wholesale prices of the supplier and the collector are given by the relations $P_s^* = (1 - \alpha_1)P^*$ and $P_d^* = (1 - \alpha_2)P^*$.

4.2. Model II: The manufacturer gets the recycled materials from the recycler

Here, we consider the situation where the collector supplies recyclable wastes to the recycler for recycling. However, the recycler may or may not satisfy the manufacturer's demand of recycled materials. In case of any shortfall of recycled materials, the manufacturer purchases the required amount of fresh raw materials from the backup supplier. Therefore, in this case we have

$$D_c = \frac{q\gamma}{\theta} \lambda, \quad 0 < \lambda \leq 1$$

$$D_r = q\gamma\lambda$$

$$D_s = q\gamma(1 - \lambda)$$

4.2.1. Centralized game

Like the previous model, here we assume that the manufacturer needs total q units of finished product to satisfy customer demand. If the market demand exceeds the order quantity, shortage occurs and the shortage penalty cost of the manufacturer is then $u(D - q)$. On the other hand, if the market demand is less than the total quantity q , the leftover inventory is sold in a secondary market at a lower cost v . Then the total revenue from the leftover inventory is $v(q - D)$. Thus the expected profits of the manufacturer, the collector, the recycler and the supplier are given by

$$\Pi_m = E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p], \quad (4)$$

$$\Pi_c = E[(P_c - C_c)D_c], \quad (5)$$

$$\Pi_r = E[P_r D_r - (P_c + C_r)D_c], \quad (6)$$

$$\Pi_s = E[(P_s - C_s)D_s], \quad (7)$$

where $X^+ = \max(X, 0)$ and the subscripts c, s, r and m stand for the collector, the supplier, the recycler and the manufacturer, respectively. The expected total profit in the centralized game is

$$\begin{aligned} \Pi &= \Pi_m + \Pi_c + \Pi_r + \Pi_s \\ &= E[P \min(q, D) - u(D - q)^+ \\ &\quad + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c + (P_d - C_c)D_c + (P_s - C_s)D_s] \\ &= (P - C_p)[y(P) + \mu] - (C_p - v)\phi(z) - (p + u - C_p)\psi(z) - (C_r + C_c)[z + y(P)] \frac{\gamma\lambda}{\theta}, \end{aligned}$$

where $\phi(z) = \int_0^z (z - t)f(t)dt$ and $\psi(z) = \int_z^\infty (t - z)f(t)dt$.

When z is fixed, we derive the optimal value of P as $P^*(z) = \frac{a + bC_p + \mu + (C_r + C_c)\frac{\gamma\lambda}{\theta} - \psi(z)}{2b}$ and for a fixed P , the optimal value of z as $z^*(P) = F^{-1}\left\{1 - \frac{(C_p - v) + (C_r + C_c)\frac{\gamma\lambda}{\theta} - \psi(z)}{P + u - v}\right\}$.

Corollary 2. The profit function Π is concave in z for a given value of P and concave in P for a given value of z .

Proof: See Appendix B.

4.2.2. Decentralized game

The decentralized game is considered when all the members in the supply chain have similar decision powers and they are not interested for a collaborative business together. There may be some mutual agreements between a pair of members but they will never collaborate all together like a centralized model. So, our problem is now to maximize separately the expected profits of the manufacturer, the collector and the supplier, which are

$$\Pi_m = E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - (P_d + C_m)D_c - qC_p],$$

$$\Pi_c = E[(P_d - C_c)D_c],$$

$$\Pi_s = E[(P_s - C_s)D_s].$$

Similar to the previous model, we now suppose that the profit margins for the players in this game are same i.e., $P_d = \frac{P + C_c}{2}$ and $P_s = \frac{P + C_s}{2}$. Then the optimal values of P and

$$\begin{aligned} z \text{ are given by } P^*(z) &= \frac{a + \mu + bC_p - \psi(z) + b[(P_d + C_m)\frac{\gamma\lambda}{\theta} + P_s\gamma(1 - \lambda)]}{2b} \text{ and} \\ z^*(P) &= F^{-1}\left\{1 - \frac{(C_p - v) + (P_d + C_m)\frac{\gamma\lambda}{\theta} + P_s\gamma(1 - \lambda)}{P + u - v}\right\}. \end{aligned}$$

Proposition 3. The joint profit for all the members in the supply chain in Model II is greater than that of Model I if $C_m > C_r$.

Proof: In Model I, the expected total profit of the supply chain is

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$$\begin{aligned}\Pi_D^I &= \Pi_m + \Pi_s + \Pi_c \\ &= (P - C_p)[y(P) + \mu] - (C_p - v)\phi(z) - (P + u - C_p)\psi(z) - [(C_m + C_c) \frac{\gamma\lambda}{\theta} \\ &\quad + C_s \gamma(1 - \lambda)][z + y(P)],\end{aligned}$$

and the expected total profit of the supply chain in Model II is

$$\begin{aligned}\Pi_D^{II} &= \Pi_m + \Pi_s + \Pi_c + \Pi_r \\ &= (P - C_p)[y(P) + \mu] - (C_p - v)\phi(z) - (P + u - C_p)\psi(z) - [(C_r + C_c) \frac{\gamma\lambda}{\theta} + \\ &\quad C_s \gamma(1 - \lambda)][z + y(p)].\end{aligned}$$

Clearly, $\Pi_D^{II} > \Pi_D^I$ whenever $C_m > C_r$.

4.2.3. Fixed markup strategic game

In the fixed markup strategic game also, each of the players wants to maximize its own profit individually. Each downstream player wants to fix his wholesale price grater than the preceding upstream member. Hence we assume that the collector's wholesale price $P_c = (1 - \alpha_3)P_r$, the recycler's wholesale price $P_r = (1 - \alpha_4)P$, and the supplier's wholesale price $P_s = (1 - \alpha_5)P$, and that $0 < \alpha_5 \leq \alpha_4 \leq \alpha_3 < 1$.

Using the above relations in the profit functions

$$\begin{aligned}\Pi_m &= E[P \min(q, D) - u(D - q)^+ + v(q - D)^+ - P_s D_s - P_r D_r - q C_p] \\ \Pi_c &= E[(P_c - C_c)D_c] \\ \Pi_r &= E[P_r D_r - (P_c + C_r)D_c] \\ \Pi_s &= E[(P_s - C_s)D_s],\end{aligned}$$

we get the optimal price of the manufacturer as

$$P^*(z) = \frac{a + \mu + b C_p - \psi(z) - (a + z)[(1 - \alpha_4)\gamma\lambda + (1 - \alpha_5)\gamma(1 - \lambda)]}{2b[1 - (1 - \alpha_4)\gamma\lambda - (1 - \alpha_5)\gamma(1 - \lambda)]}$$

and the optimal value of the stocking factor z as

$$z^*(P) = F^{-1} \left\{ 1 - \frac{(C_p - v) + P[(1 - \alpha_4)\gamma\lambda + (1 - \alpha_5)\gamma(1 - \lambda)]}{P + u - C_p} \right\}.$$

5. Numerical Examples

5.1. Example 1 for Model I

In this example, we set the parameter-values for Model I. We assume that the random demand follows (i) exponential distribution i.e., $f(\alpha, x) = \alpha e^{-\alpha x}$, $x > 0$ with $\alpha = 0.02$, mean $\mu = 50$; and (ii) uniform distribution i.e., $f(z) = \frac{1}{100}$, $0 \leq z \leq 100$, with mean $\mu = 50$. We consider the other parameter-values as follow: $C_p = 5$, $\theta = 0.7$, $\gamma = 1.3$, $a = 1000$, $b = 1.3$, $C_c = 15$, $C_m = 65$, $C_s = 100$, $\mu = 10$, $\alpha_1 = 0.65$, $\alpha_2 = 0.65$, $\lambda = 0.6$, $u = 3$, $v = 4$ in appropriate units. For this set of data, we obtain the optimal price, optimal stocking factor and profit for each player in different games. The optimal results for exponential and uniform demand distributions are shown in Tables 1 and 2, respectively.

Table 1. Optimal results in Model I for exponential distribution

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	471.103	498.87	479.14
z^*	59.81	15.10	15.96
Π_m^*	-	45336.70	48973.6
Π_c^*	-	63836.90	66882.3
Π_s^*	-	18989.40	13837.7
Expected total profit	1,33,691.0	1,28,163.0	1,29,693.6

Table 2. Optimal results in Model I for uniform distribution

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	475.19	501.98	482.24
z^*	70.024	26.18	27.42
Π_m^*	-	46103.2	49776.7
Π_c^*	-	65495.7	68630.8
Π_s^*	-	19555.7	14325.2
Expected total profit	1,37,255.0	1,31,154.6	1,32,732.7

Tables 1 and 2 show the optimal results of each of the players under exponential and uniform distributions. Expected total profits for all the gaming approaches are higher in case of uniform demand distribution compared to the respective models in exponential demand distribution. The optimal retail price of the product is lower in case of fixed markup strategy which results in higher customer demand and higher profit. The optimal profits of the manufacturer and the collector are higher in case of the fixed markup strategy than those in decentralized policy.

5.2. Example 2 for Model II

Here also we consider two types of demand distribution as given below :
 (i) exponential distribution i.e., $f(x) = \alpha e^{-\alpha x}$, $x > 0$ with $\alpha = 0.02$, mean $\mu = 50$;
 (ii) uniform distribution i.e., $f(z) = \frac{1}{100}$, $0 \leq z \leq 100$, with same mean $\mu = 50$. We consider the parameter-values as follow: $C_p = 5$, $\theta = 0.7$, $\gamma = 1.3$, $a = 1000$, $b = 1.3$, $C_c = 15$, $C_m = 65$, $C_r = 10$, $C_s = 100$, $\mu = 10$, $\alpha_3 = 0.45$, $\alpha_4 = 0.40$, $\alpha_5 = 0.35$, $\lambda = 0.6$, $u = 3$, $v = 4$ in appropriate units. For this set of data, we obtain the optimal price, optimal stocking factor and expected profit of each player in different gaming approaches, as shown in Tables 3 and 4.

Table 3. Optimal results in Model II for exponential distribution

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	442.75	465.98	402.32
z^*	84.90	8.24	8.15
Π_m^*	-	26788.0	27448.9
Π_c^*	-	67415.1	70915.1
Π_s^*	-	38296.1	40743.6
Π_r^*	-	20463.4	14526.3

Expected total profit	1,62,928.0	1,52,964.6	1,53,633.9
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Table 4. Optimal results in Model II for uniform distribution

Optimal results	Centralized game	Decentralized game	RFM strategy
P^*	445.639	465.98	403.72
z^*	81.80	15.21	15.04
Π_m^*	-	27042.9	27663.9
Π_c^*	-	68464.8	65064.7
Π_s^*	-	38928.0	41584.4
Π_r^*	-	20808.0	14469.9
Expected total profit	1,66,501.0	1,55,244.0	1,56,534.0

Tables 3 and 4 depict the optimal results for each player as well as for the whole supply chain in Model II. Optimal retail prices are lower in this model compared to those in Model I which corresponds to higher demand. Here also optimal values of the profits are greater for the uniform distribution and, for both the distributions, the expected total profit of the supply chain is improved in the markup policy, compared to the decentralized game. The expected total profits of the supply chain for the two decentralized cases in this model are higher than those of the respective cases in Model I due to the lower recycling cost of the recycler (Proposition 3).

5.3. Sensitivity analysis

Now, in particular for the exponential distribution, we examine the sensitivity of the key parameters θ , b and γ on the optimal prices as well as the expected profit of the supply chain in different strategies of both the game models.

5.3.1. Sensitivity with respect to θ

As the value of θ increases, in Model I, the supply chain's expected total profit increases for the centralized, decentralized and markup strategic game models. This happens because higher recyclability degree results in higher quality value of the used wastes and this reduces the recycling cost and also the usage of total wastes. We see that the profit of the manufacturer increases as θ increases but the collector and the backup supplier's optimal profits are obtained for their respective specific values of θ .

In Model II also, the expected total profit increases with θ . The expected total profit is maximum in the centralized model, which is the benchmark case. For the markup strategy, the expected total profit is higher compared to that of the decentralized gaming strategy (see Figure 2).

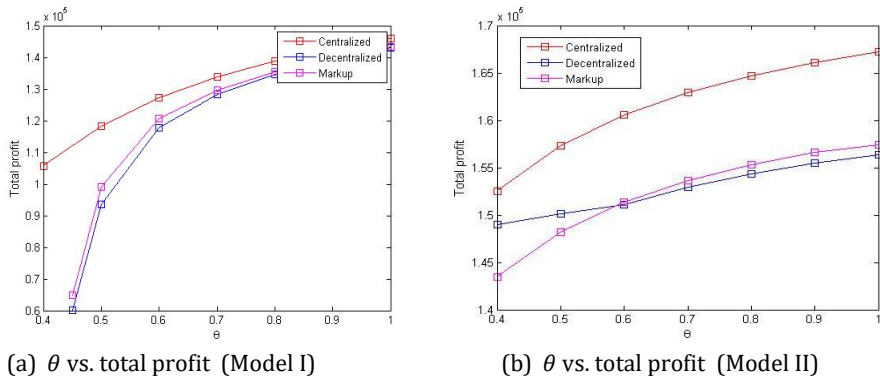


Figure 2. Sensitivity w. r. to θ

5.3.2. Sensitivity with respect to γ

As the value of γ increases, the manufacturer requires more recyclable wastes to produce q units of finished product. Hence the production cost will increase for the manufacturer and that leads to lower profit. However, the collector and the backup supplier attain higher profits for increasing γ , as they will have to supply more raw materials.

If the value of γ increases, the amount of recycled materials to be supplied by the recycler to the manufacturer increases. So, in that case, the expected profit decreases in all the three types of gaming approaches. Because of ex-ante price markup commitment, the expected total profit in case of markup policy is higher compared to that of the decentralized policy (see Figure 3)

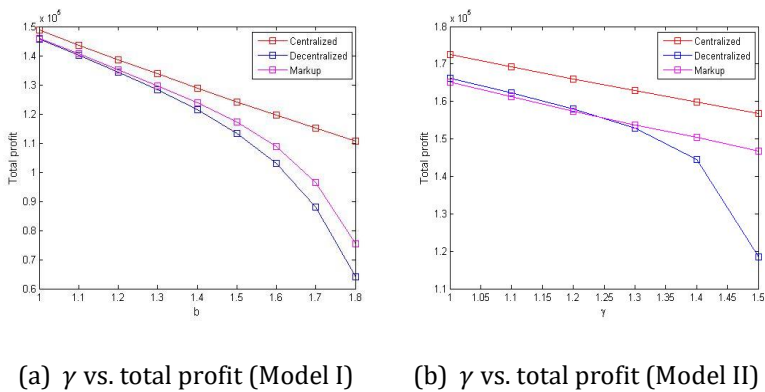


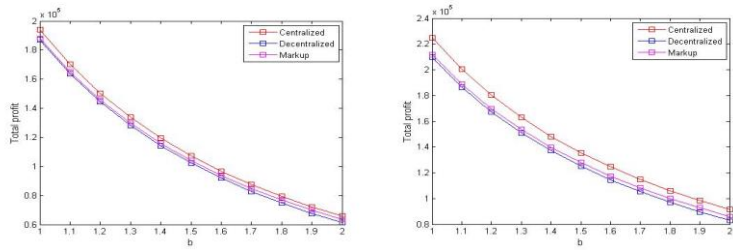
Figure 3. Sensitivity w. r. to γ

5.3.3. Sensitivity with respect to b

For higher values of b (price sensitivity of customer demand), the customer demand is lower. As a result, the profits of all individual entities decrease for higher values of b . In the markup policy, the supply chain's optimal expected profit becomes higher than that in the decentralized game (see Figure 4).

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Similar observation is made for the game Model II. The expected total profit of the supply chain as well as individual profits gradually decrease with higher values of b .



(a) b vs. total profit (Model I)

(b) b vs. total profit (Model II)

Figure 4. Sensitivity w. r. to b

6. Conclusion

In this paper, we have studied a closed-loop supply chain scenario where recycling is the main concern for environmental sustainability. A manufacturer performs recycling using two different channel of recycling, directly by his own and also by the help of another recycler. For two different game models, depending on different ways of recycling, we have analyzed the optimal pricing strategy of all the supply chain members. For stochastic demand, it is not always easy to get closed form solution of the model. So, numerically we obtain the optimal solutions for two types of demand distribution - uniform and exponential. From the sensitivity analysis, we have the following observations:

- I. Ex-ante markup strategy is beneficial (compared to decentralized model) for the supply chain entities, specially the manufacturer. However, profit is not always pareto-improving in case of stochastic demand scenario (here specially for the backup supplier in Model I and recycler in Model II), which supports the result of Liu et al. (2006).
- II. For higher value of θ , the supply chain will gain higher profit. The individual entities will also gain higher profit for a particular range of θ .
- III. When the recycler recycles the wastes at a cost lower than the manufacturer, the expected total profit of the supply chain is higher.
- IV. A higher price sensitivity of customer's demand decreases customer demand, and hence it leads to lower profit for the manufacturer.

Several future studies can be done using different contract policies among the members. One can also assume multiplicative form of stochastic demand. Instead of fixed markup, the entities can go for variable markup policy also.

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Proof of Corollary 1:

The expected total profit in the centralized Model I is,

$$\Pi = (P - C_p)[y(p) + \mu] - (C_p - v)\phi(z) - (P + u - C_p)\psi(z) - (z + y(P))(C_m + C_c) \frac{\gamma\lambda}{\theta}$$

Taking first and second order partial derivatives of Π with respect to z and P we get,

$$\frac{\partial \Pi}{\partial z} = -(C_p - v) + (P + u - v)[1 - F(z)] - (C_m + C_c) \frac{\gamma\lambda}{\theta}$$

$$\frac{\partial^2 \Pi}{\partial z^2} = -(P + u - v)f(z) < 0, \text{ since } v < P.$$

$$\frac{\partial \Pi}{\partial p} = (P - C_p)(-b) + (a - bP + \mu) - (C_m + C_c) \frac{\gamma\lambda}{\theta}(-b) - \psi(z)$$

$$\frac{\partial^2 \Pi}{\partial p^2} = -2b < 0, \text{ since } b > 0.$$

Proof of Proposition 2:

We have $\frac{d\Pi}{dz} = -(C_p - v) + (P + u - v)[1 - F(z)] - (C_m + C_c) \frac{\gamma\lambda}{\theta}$

Let $R(z) = \frac{d\Pi}{dz}$

Now, $\frac{dR(z)}{dz} = \frac{d}{dz} \left[\frac{d\Pi}{dz} \right]$

$$= \frac{d}{dz} [-(C_p - v) - (C_m + C_c) \frac{\gamma\lambda}{\theta}] + \frac{d}{dz} [(P + u - v)(1 - F(z))],$$

where $P(z) = \frac{a+bC_p+\mu+\frac{b\gamma\lambda}{\theta} - \psi(z)}{2b} = P^0 - \frac{\psi(z)}{2b}$, where $P^0 = \frac{a+bC_p+\mu}{2b} + (C_m + C_c) \frac{\gamma\lambda}{2\theta}$

So, $\frac{dR(z)}{dz} = \frac{d}{dz} [(P^0 - \frac{\psi(z)}{2b} + u - v)(1 - F(z))]$

$$= \frac{1}{2b} [1 - F(z)]^2 - (P^0 + u - v - \frac{\psi(z)}{2b})f(z)$$

$$= \frac{f(z)}{2b} \left\{ 2b(P^0 + u - v) - \psi(z) - \frac{1-F(z)}{r(z)} \right\}, \text{ where } r(z) = \frac{f(z)}{1-F(z)}, \text{ the hazard rate.}$$

Again, $\frac{d^2R(z)}{dz^2} = \frac{d}{dz} \left\{ \frac{dR(z)}{dz} \right\}$

$$= \frac{dR(z)/dz}{f(z)} \cdot \frac{df(z)}{dz} - \frac{f(z)}{2b} \left\{ (1 - F(z)) + \frac{f(z)}{r(z)} + \frac{(1-F(z)) \left[\frac{dR(z)}{dz} \right]}{[r(z)]^2} \right\}$$

Hence, $\frac{d^2R(z)}{dz^2} \Big|_{\frac{dR(z)}{dz}=0} = \frac{-f(z)(1 - F(z))}{2b[r(z)]^2} [2[r(z)]^2 + \frac{dr(z)}{dz}]$

Now if $F(\cdot)$ be a probability distribution function which satisfies, $2r(z)^2 + \frac{dr(z)}{dz} > 0$ then it follows that $R(z)$ is either monotone or unimodal implying that $R(z) = \frac{d\Pi[z,P(z)]}{dz}$ has at most two roots.

Again, $R(z) = -(C_p - v) - (C_m + C_c) \frac{\gamma\lambda}{\theta} < 0$. So, if $R(z)$ has only one root then it gives the maximum value of $\Pi(z, P)$ and if it has two roots then the larger of them corresponds to the maximum value of $\Pi(z, P)$.

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Appendix B

Proof of Corollary 2 :

The expected total profit in the centralized Model II is

$$\Pi = (P - C_p)[y(P) + \mu] - (C_p - v)\phi(z) - (P + u - C_p)\psi(z) - (C_r + C_c)(z + y(P))\frac{\gamma\lambda}{\theta}$$

$$\text{So, } \frac{\partial \Pi}{\partial z} = -(C_p - v) + (P + u - v)[1 - F(z)] - (C_r + C_c)\frac{\gamma\lambda}{\theta}$$

$$\frac{\partial^2 \Pi}{\partial z^2} = -(P + u - v)f(z) < 0, \text{ since } v < P \text{ and } f(z) \geq 0$$

$$\frac{\partial \Pi}{\partial p} = (P - C_p)(-b) + (a - bP + \mu) - (C_r + C_c)\frac{\gamma\lambda}{\theta}(-b) - \psi(z)$$

$$\frac{\partial^2 \Pi}{\partial p^2} = -2b < 0, \text{ since } b > 0.$$

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