SCIENTIFIC OASIS

Decision Making: Applications in Management and Engineering

Journal homepage: www.dmame-journal.org
ISSN: 2560-6018, eISSN: 2620-0104

Effective Domination in Neutrosophic Under Graphs with an Application to Supplier Network Management

Yamini Parthiban[1,3], Prabakaran Raghavendran[2,3,*] Nasreen Kausar[4,5], Akbota Nurgaliyeva[6], Hasan Dincer[7,8,9,*], Serhat Yuksel[7,8,10]

1 Department of Mathematics, SRM Institute of Science and Technology, Ramapuram, Chennai-600089, Tamil Nadu, India
2 Center for Research, Easwari Engineering College, 18 Bharathi Salai, Ramapuram, Chennai-600089, Tamil Nadu, India.
3 Department of Mathematics and Science Education, Faculty of Education, Harran University, Sanliurfa 63150, Turkey
4 Department of Mathematics, Faculty of Arts and Science, Balikesir University, 10145 Balikesir, Turkey
5 Department of Computer Engineering, Biruni University, 34010, Istanbul, Turkey
6 Deparment of Mathematics, Korkyt Ata Kyzylorda University. Aiteke Bi Street 29A, 120000, Kyzylorda, Kazakhstan
7 School of Business, Istanbul Medipol University, Istanbul, Turkey
8 Clinic of Economics, Azerbaijan State University of Economics (UNEC), Baku, Azerbaijan
9 University College, Korea University, Seoul, Republic of Korea
10 Department of Economics and Management, Khazar University, Baku, Azerbaijan

ARTICLE INFO

Article history:
Received 10 December 2025
Received in revised form 19 April 2026
Accepted 12 June 2026
Available online 30 June 2026

Keywords:
Neutrosophic Under Graph; Effective Edge; Effective Bridge; Path Strength; Domination; Effective Dominating Set; Domination Number; Supplier Network; Decision Making; Uncertainty

ABSTRACT

Graph-theoretic modelling within a neutrosophic environment offers a versatile mathematical approach to describing relationships characterised by truth, indeterminacy, and falsity. Building on this foundation, the present study introduces an effective domination framework for neutrosophic under graphs, enabling the systematic examination of network connectivity when information is uncertain or contains under-valued components. The mathematical formulation begins by defining neutrosophic under sets and neutrosophic under graphs, from which component-wise path strengths, bridges, effective edges, effective dominating sets, domination weights, and domination numbers are subsequently developed. A series of propositions establishes the conditions governing effective edges and bridges while deriving domination bounds for complete and complete bipartite neutrosophic under graphs. To illustrate the practical applicability of these theoretical developments, the framework is implemented in a supplier network comprising 10 suppliers connected through 20 direct supplier relationships. Within this network, three distinct components capture supplier reliability, indeterminate information, and the under-valued dimension of supplier performance. Comparisons of component-wise path strengths are employed to distinguish effective supplier relationships, whereas component-wise domination weights facilitate the identification of candidate effective dominating sets. The findings indicate that the proposed mathematical approach provides an integrated representation of supplier attributes and their interconnected relationships without disregarding uncertainty or under-valued information. Accordingly, the framework establishes a mathematically grounded basis for supporting supplier network management decisions under complex and imperfect information conditions.

* *Corresponding author.*
E-mail address: rockypraba55@gmail.com ; hdincer@medipol.edu.tr

https://zenodo.org/records/24353587

1. Introduction

The development of fuzzy, intuitionistic fuzzy, vague, and neutrosophic graph theories has considerably expanded the mathematical capabilities available for representing uncertainty, imprecision, and indeterminacy in complex interconnected systems. Within these theoretical frameworks, domination theory has emerged as an important area of investigation, particularly because it provides a systematic means of identifying vertices that exert influence or control over network connectivity and organisation. Initial research primarily examined domination properties in fuzzy and vague graphs, establishing a foundation for subsequent generalisations involving intuitionistic fuzzy, neutrosophic, multipolar, Fermatean, and soft graph structures. The growing theoretical sophistication of these approaches has been accompanied by their application across diverse practical domains, including wireless communication, medical science, water supplier systems, climatic analysis, sustainable energy, supply chain management, and satellite networks. Collectively, these developments demonstrate the relevance of domination-based graph modelling to systems in which relationships cannot be adequately characterised through conventional deterministic representations. Against this background, the following review examines pertinent contributions to the literature, highlighting the progression of domination concepts across uncertain graph environments and establishing the theoretical context for the present investigation (Eisbach et al., 2023; Westphal et al., 2023; Exploring collaborative decision-making, 2024).

The foundational concept of fuzzy sets [35] underpins later generalizations of uncertainty-aware graph models. Contemporary supplier-selection research has also employed intuitionistic fuzzy group decision-making [13], fuzzy inference and best-worst weighting [14], and hybrid fuzzy methods for sustainable and resilient supplier evaluation [15,16]. These studies emphasize the need to account for incomplete and conflicting supplier assessments.

1.1 Literature Review

The complement operation in fuzzy graphs was examined in [1], where important structural properties were established by extending the conventional graph complement concept to accommodate fuzzy membership relationships. This extension provides a foundation for investigating structural variations and complementary relationships within fuzzy graph systems.

The applicability of neutrosophic graphs to wireless communication systems was explored in [2], where uncertain and indeterminate relationships were represented through neutrosophic graph structures. The investigation demonstrated their potential for modelling uncertain communication links and network conditions, establishing a mathematical foundation for their use in wireless networks.

New concepts in intuitionistic fuzzy graph theory were developed in [3] and applied to water supplier systems. The proposed model incorporates fuzzy information and uncertain relationships between suppliers and network elements, demonstrating the applicability of intuitionistic fuzzy graphs to water resource and supplier management. Domination in vague incidence graphs was investigated in [4], with an application to medical science. By incorporating vague information into incidence graph structures, uncertain relationships among medical entities were represented and analysed. The findings demonstrate the relevance of domination concepts to decision-making under uncertainty in medical environments. Interval intuitionistic neutrosophic sets and their application to interval intuitionistic neutrosophic graphs were examined in [5]. The proposed interval-valued structures accommodate uncertainty, incompleteness, and indeterminacy in climatic information, illustrating the suitability of neutrosophic graph models for complex environmental analysis.

Domination properties of product vague graphs were explored in [6], with particular emphasis on

medical applications. The investigation extended domination concepts to graph products containing uncertain information, enabling the representation of uncertain relationships within medical network structures. Q-neutrosophic soft graphs were investigated in [7] through applications in operations management and communication networks. The proposed framework combines soft graph structures with Q-neutrosophic information to represent uncertainty and indeterminacy, demonstrating its applicability to complex management and communication problems. A neutrosophic optimisation model for supply chain virtualisation within a circular economy was introduced in [8]. The framework integrates neutrosophic uncertainty with the NSGA-II optimisation algorithm to address complex supply chain problems, illustrating the potential of neutrosophic optimisation for sustainable supply chain management. A neutrosophic Z-number-based CRITIC-EDAS decision model for evaluating smart solar panels was developed in [9]. By combining neutrosophic information with objective weighting and evaluation methods, the model addresses uncertainty in sustainable energy planning and supports the selection of appropriate solar panel technologies. Domination concepts in rough image-polar fuzzy digraphs were investigated in [10] through an application to trade networking. The proposed framework incorporates rough and multipolar fuzzy information to represent uncertain directional relationships, demonstrating the applicability of domination theory to complex trade network environments (Artificial intelligence and project management, 2024; Lorenz et al., 2026). Other recent supplier-decision approaches combine multi-criteria decision analysis with simulation optimization [17], fuzzy SECA [18], multi-objective order allocation [19], and green supplier selection under supply risk [20]. These methods primarily rank or allocate suppliers on attribute-based criteria, whereas graph-based domination additionally considers the structural role of inter-supplier connections.

An energy-aware equitable total domination model based on neutrosophic graphs was proposed in [11] for satellite cluster coordination. The framework incorporates uncertainty, energy efficiency, and equitable total domination to support coordination within satellite communication networks. Balanced Fermatean quadripartitioned neutrosophic fuzzy graphs were introduced in [12] to represent complex uncertain information. The proposed structure integrates Fermatean fuzzy concepts, quadripartitioned representations, and neutrosophic uncertainty within a unified graph model, extending the capacity of fuzzy graph frameworks to accommodate multiple forms of uncertainty.

Recent research on neutrosophic domination has further developed covering, paired-domination, incidence-graph domination, total domination, and restrained domination concepts [31–34]. These contributions provide useful theoretical comparators for effective domination, while the present study focuses specifically on component-wise effective edges and under-valued neutrosophic membership information.

1.2 Research Gap and Motivation

Previous investigations have demonstrated the importance of fuzzy, neutrosophic, and related graph-theoretic models for representing uncertainty, incomplete information, and imprecise relationships in complex network environments. These developments have established mathematical approaches for incorporating different forms of uncertainty into graph structures while extending classical graph-theoretic concepts to more general settings. In particular, the inclusion of truth, indeterminacy, and falsity components enables network relationships to be represented through multiple informational dimensions rather than a single deterministic parameter. However, the combined treatment of under-valued information, component-wise path strength, effective relationships, and domination within a unified neutrosophic under graph framework remains

insufficiently investigated. This limitation becomes particularly relevant when membership information includes under-set values that may fall below the conventional zero level and therefore cannot be adequately represented by standard neutrosophic models restricted to the unit interval. Consequently, neutrosophic under graphs provide a suitable mathematical setting for examining network relationships in which component-wise membership information may extend beyond conventional bounds.

In graph-based decision-making problems, the existence of a direct connection between two vertices does not necessarily establish its structural importance. Alternative paths may connect the same vertices with strengths equal to or greater than that of the direct relationship. Accordingly, adjacency alone cannot adequately distinguish relationships that make important contributions to network connectivity. The concept of an effective edge addresses this limitation by comparing the strength of a direct connection with the maximum path strength obtainable between its endpoints after that edge has been removed. An edge is regarded as effective in the relevant neutrosophic component when its direct strength exceeds that of every available alternative path. This criterion establishes a relationship between path strength and the structural importance of individual edges. Furthermore, bridge characterisation provides additional insight into network connectivity by distinguishing relationships whose contributions cannot be replaced by alternative connections of comparable strength.

Motivated by these considerations, the present study develops an effective domination framework for neutrosophic under graphs that incorporates the relative importance of network relationships. The three neutrosophic components are examined separately, allowing their respective informational characteristics to be evaluated before being incorporated into an overall domination measure. For each component, the strengths of the corresponding paths are determined, and the maximum strength among available paths connecting two vertices is identified. These component-wise measures are subsequently used to assess the effectiveness of individual edges by comparing their direct strengths with the maximum strengths of alternative paths following edge removal. Thus, rather than assigning equal structural importance to all directly connected vertices, the proposed approach evaluates each relationship according to its contribution relative to other available network paths.

Within this framework, effective domination is defined through the requirement that every vertex outside a selected dominating set must be effectively dominated by at least one vertex belonging to that set. Component-wise domination weights are then introduced to quantify and compare different effective dominating configurations. The individual weights associated with the three neutrosophic components permit each configuration to be assessed from distinct informational perspectives, while the overall domination weight provides a criterion for identifying a preferred configuration. Such a distinction is necessary because a dominating set that performs favourably under one neutrosophic component may not necessarily exhibit the same preference under another. By incorporating these component-specific differences into the evaluation process, the proposed framework provides a more detailed characterisation of network domination than methods relying exclusively on the cardinality of a dominating set.

1.3 Contributions of the Present Study

A handful of propositions are set out to explore structural and domination properties of neutrosophic under graphs. First, relationships In between the effective edges and bridges are obtained for ζ- and δ-. and φ-components. The effectiveness of an edge is determined by comparing its direct strength with the maximum indirect path strength connecting its endpoints after the edge

has been removed. This comparison establishes a criterion for identifying relationships that are structurally important within individual neutrosophic components. Additional propositions formulate conditions and bound for domination in complete neutrosophic under graphs. These results are further extended to complete bipartite neutrosophic under graphs, including star graphs, where differences in the structural arrangements of the two vertex classes give rise to effective domination configurations. The derived findings establish theoretical relationships among vertex membership values, minimum and maximum distances between vertices, and the resulting domination values. Furthermore, upper bounds for domination numbers are obtained by constructing suitable effective dominating sets. Together, these propositions establish a mathematical foundation for investigating effective domination within the network structures represented by neutrosophic under graphs.

1.4 Methodology

To demonstrate the practical relevance of the theoretical findings beyond abstract graph structures, a numerical application is presented in the context of supplier network management. The application considers a manufacturing organisation that depends on multiple suppliers while facing uncertainty concerning supplier performance, reliability, incomplete operational information, and potential supply chain disruptions. Within this setting, suppliers are represented as vertices, whereas direct operational connections between them are modelled as edges. Neutrosophic under values assigned to vertices capture different dimensions of supplier information, while the corresponding edge values describe the strengths of inter-supplier relationships. This representation enables the supplier network to be examined using the mathematical concepts established in the theoretical analysis.

The numerical application considers a supplier network comprising 10 suppliers and 20 direct relationships, represented within a neutrosophic under graph framework. The ζ-, δ- and φ-membership values are Given to the suppliers and their relationship with the client according to the Under the neutrosophic under graph conditions. The paths that are available between suppliers These are then studied and the strengths are computed on a component basis. By comparing the direct edge strength to the maximum alternative path strength After the edge is removed, the remaining graph becomes after deletion of the edge, the ζ-effective and δ-effective relationships are identified. The identified effective relationships are subsequently used to construct candidate effective dominating sets. The component-wise weights of these sets are calculated and compared to determine the most suitable supplier configuration according to the proposed domination criterion.

Another important aspect of the numerical analysis, illustrated in the preceding figure, is that the effectiveness of network relationships may differ across the individual neutrosophic components. An edge that is effective with respect to the ζ-component need not be effective with respect to the δ-component, and vice versa. Consequently, the resulting effective dominating sets may differ across the neutrosophic components, allowing multiple dimensions of supplier information to be considered rather than relying on a single measure of relationship quality. Comparisons among candidate configurations further indicate that the set with the minimum weight for one component may not necessarily have the minimum weight for another. This distinction provides decision-makers with component-specific information that can be evaluated according to the requirements of a particular management problem. When all three components are considered collectively, the overall domination criterion enables the identification of a preferred supplier configuration.

The supplier network application therefore provides a practical interpretation of the mathematical concepts developed in this study. An effective dominating set can be understood as a

group of strategically important suppliers that collectively provide effective coverage of other suppliers through the identified relationships. This interpretation does not require the exclusion of suppliers outside the dominating set from the network. Rather, it concerns the capacity of the selected suppliers to provide effective network coverage according to the proposed mathematical model. Such an approach may support supplier coordination, alternative sourcing, network monitoring, and supply chain planning, particularly when the objective is to identify a relatively small but effective group of suppliers maintaining important relationships with other network participants.

For supplier management, recent studies have developed interval-valued intuitionistic fuzzy assessment [21], integrated best-worst and fuzzy TOPSIS ranking [22], digital-twin-supported risk assessment [23], and group decision analytics for supplier relationship management [24]. The present graph-theoretic approach complements rather than replaces these methods by addressing effective network coverage.

The principal contribution of this study lies in combining neutrosophic under graph theory, effective-edge analysis, and domination concepts within a unified framework, supported by a numerical application to supplier network management. The theoretical investigation establishes structural characterisations and domination bounds, while the numerical analysis demonstrates their application to a practical network decision-making problem. Unlike selection approaches based exclusively on individual supplier membership values, the proposed framework considers both the neutrosophic information associated with individual suppliers and the structural contributions of their relationships within the network. Accordingly, the study connects the mathematical theory of effective domination in neutrosophic under graphs with practical network-based decision-making. The resulting framework may also be applicable beyond supplier networks to other problems in mathematics and management science involving interdependent entities, uncertain relationships, and the strategic identification of effective vertex subsets.

2. PRELIMINARIES

Definition 2.1 Let V be a given non-empty set. Then the set $\mathcal{M}$ is called Fuzzy set on V such that its membership function $\mu_A: V \to [0,1]$ for each $g \in V$.

Definition 2.2 Let V be a given non-empty set. A neutrosophic set $\mathcal{M}$ in V is characterized by a truth membership function $\zeta_{\mathcal{M}}(\mathfrak{g})$, an indeterminate membership function $\delta_{\mathcal{M}}(\mathfrak{g})$ and a false membership function $\varphi_{\mathcal{M}}(\mathfrak{g})$. The functions $\zeta_{\mathcal{M}}(\mathfrak{g}), \delta_{\mathcal{M}}(\mathfrak{g}) and \varphi_{\mathcal{M}}(\mathfrak{g})$ are fuzzy sets on V. That is, $\zeta_{\mathcal{M}}, \delta_{\mathcal{M}}, \varphi_{\mathcal{M}}: V \to [0,1]$ and $0 \leq \zeta_{\mathcal{M}}(\mathfrak{g}) + \delta_{\mathcal{M}}(\mathfrak{g}) + \varphi_{\mathcal{M}}(\mathfrak{g}) \leq 3$.

Definition 2.3 Let U be the universe of discourse and the neutrosophic set $\mathcal{M}$ in U. A neutrosophic under set A is defined as: $L = \{(\mathcal{M}, \langle \zeta_{\mathcal{M}}, \delta_{\mathcal{M}}, \varphi_{\mathcal{M}} \rangle)\}$, $\mathcal{M} \in U$, where $\zeta_{\mathcal{M}}, \delta_{\mathcal{M}}, \varphi_{\mathcal{M}} : \to [\Psi, 1]$ that is,$\Psi < 0$, and Ψ is called under limit, $\zeta_{\mathcal{M}}, \delta_{\mathcal{M}}, \varphi_{\mathcal{M}} \in [\Psi, 1]$such that there exists at least one element in A that has at least one neutrosophic component that is < 0, and no element has neutrosophic components that are > 1.

3. Neutrosophic Under Graphs and It's Varies Types

Definition 3.4 Let $G^{\star} = (V, E)$ be a crisp graph with V is vertex and E is edge. A Neutrosophic under graph($\mathcal{NUG}$) is a pair $G = (\mathcal{M}, \mathcal{L})$ of $G^{\star}$ where a set $\mathcal{M}$ is a neutrosophic over set in V and set $\mathcal{L}$ is a neutrosophic under set in $E \subseteq V \times V$ such that

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq min\{\zeta_{\mathcal{M}}(\mathfrak{g}), \zeta_{\mathcal{M}}(\mathfrak{h})\},$$
$$\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq min\{\delta_{\mathcal{M}}(\mathfrak{g}), \delta_{\mathcal{M}}(\mathfrak{h})\},$$
$$\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \geq max\{\varphi_{\mathcal{M}}(\mathfrak{g}), \varphi_{\mathcal{M}}(\mathfrak{h})\},$$

for all $(\mathfrak{g}, \mathfrak{h}) \in E$. $\mathcal{M}$ and $\mathcal{L}$ are called neutrosophic under vertex set and neutrosophic under edge set of G, respectively.

Definition 3.5 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^{\star}$. G is called complete if,

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = min\{\zeta_{\mathcal{M}}(\mathfrak{g}), \zeta_{\mathcal{M}}(\mathfrak{h})\},$$
$$\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = min\{\delta_{\mathcal{M}}(\mathfrak{g}), \delta_{\mathcal{M}}(\mathfrak{h})\},$$
$$\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = max\{\varphi_{\mathcal{M}}(\mathfrak{g}), \varphi_{\mathcal{M}}(\mathfrak{h})\},$$

for all $\mathfrak{g}\mathfrak{h} \in E$.

Definition 3.6 A $\mathcal{NUG}$ $G = (\mathcal{M}, \mathcal{L})$ on $G^{\star}$ is said to be bipartite if the set V can be partitioned into two non-empty over sets V_1 and V_2 such that $\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = 0$, for all $\mathfrak{g}\mathfrak{h} \in E_1$ or $\mathfrak{g}\mathfrak{h} \in E_2$.

Moreover, if $\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = min\{\zeta_{\mathcal{M}}(\mathfrak{g}), \zeta_{\mathcal{M}}(\mathfrak{h})\}, \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = min\{\delta_{\mathcal{M}}(\mathfrak{g}), \delta_{\mathcal{M}}(\mathfrak{h})\}, \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = max\{F(\mathfrak{g}), \varphi_{\mathcal{M}}(\mathfrak{h})\}$, for all $\mathfrak{g}\mathfrak{h} \in E$. Then G is called a complete Bipartite $\mathcal{NUG}$.

In this case, If either $|V_1| = 1$ or $|V_2| = 1$ then the complete bipartite $\mathcal{NUG}$ is said a star $\mathcal{NUG}$.

Definition 3.7 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^{\star}$. Then

1. The real number L is called the ζ-order, if $= \sum_{u \in V} \zeta_{\mathcal{M}}(u)$.
2. The real number M is called the δ-order, if $= \sum_{u \in V} \delta_{\mathcal{M}}(u)$.
3. The real number N is called the φ-order, if $= \sum_{u \in V} \varphi_{\mathcal{M}}(u)$.
4. A real number is called the Order, if it is either ζ-order, δ-order and φ-order. That is, Order = (L, M, N)

Definition 3.8 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ defined on $G^{\star}$, and consider two specified vertices v_0 and v_n, where $n \in \mathbb{N}$. The following types of paths are defined:

1. Let $P: v_0, v_1, \ldots, v_n$ be a sequence of pairwise distinct vertices of G. It is said to be a ζ-path of length n joining v_0 and v_n provided that

$$\zeta_{\mathcal{L}}(v_i v_{i+1}) > 0, \qquad i = 0,1, \ldots, n-1.$$

The strength associated with the ζ-path P is defined as the minimum ζ-value among all its consecutive edges, namely

$$\mu_G(P)_{\zeta} = \min_{0 \le i \le n-1} \{\zeta_{\mathcal{L}}(v_i v_{i+1})\}.$$

2. A sequence $P: v_0, v_1, \ldots, v_n$ consisting of distinct vertices of G is referred to as a δ-path of length n between v_0 and v_n if

$$\delta_{\mathcal{L}}(v_i v_{i+1}) > 0, \qquad i = 0,1, \ldots, n-1.$$

Its strength with respect to δ is the least δ-value attained by its consecutive edges, and is given by

$$\mu_G(P)_{\delta} = \min_{0 \le i \le n-1} \{\delta_{\mathcal{L}}(v_i v_{i+1})\}.$$

3. A sequence of distinct vertices $P: v_0, v_1, \ldots, v_n$ in G is called a φ-path of length n from v_0 to v_n when

$$\varphi_{\mathcal{L}}(v_i v_{i+1}) > 0, \qquad i = 0,1, \ldots, n-1.$$

The corresponding path strength is determined by the smallest φ-value of the edges occurring consecutively in P, that is,

$$\mu_G(P)_{\varphi} = \min_{0 \le i \le n-1} \{\varphi_{\mathcal{L}}(v_i v_{i+1})\}.$$

4. A sequence $P: v_0, v_1, \ldots, v_n$ of distinct vertices of G is called a path of length n from v_0 to v_n if it satisfies all three conditions simultaneously; that is, P must be a ζ-path, a δ-path, and a φ-path. The strength of such a path is defined as the minimum among its three component strengths:

$$\mu_G(P) = \min\{\mu_G(P)_\zeta, \mu_G(P)_\delta, \mu_G(P)_\varphi\}.$$

Definition 3.9 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on V, and consider two specified vertices v_i and v_j satisfying $i > j$, where $i, j \in \mathbb{N}$. The strengths associated with the vertices v_i and v_j are defined as follows:

1. Among all paths P connecting v_i and v_j in G, the greatest value attained by $\mu_G(P)_\zeta$ is defined as the ζ-strength of v_i and v_j. It is denoted by

$$\mu_G^\infty(v_i, v_j)_\zeta.$$

2. The δ-strength between v_i and v_j is obtained by taking the largest value of $\mu_G(P)_\delta$ over every path P joining these two vertices in G. This strength is represented by

$$\mu_G^\infty(v_i, v_j)_\delta.$$

3. The φ-strength between v_i and v_j is defined as the maximum value of $\mu_G(P)_\varphi$ among all paths P connecting v_i to v_j in G. It is denoted by

$$\mu_G^\infty(v_i, v_j)_\varphi.$$

4. The overall strength between v_i and v_j in G is defined as the largest of the three component strengths. Hence,

$$\mu_G^\infty(v_i, v_j) = \max\{\mu_G^\infty(v_i, v_j)_\zeta, \mu_G^\infty(v_i, v_j)_\delta, \mu_G^\infty(v_i, v_j)_\varphi\}.$$

Definition 3.10 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$. An edge $\mathfrak{gh}$ of G is said to be a bridge with respect to the corresponding strength measure if the following conditions hold:

1. The edge $\mathfrak{gh}$ is termed a ζ-bridge when every ζ-path P joining $\mathfrak{g}$ and $\mathfrak{h}$ that excludes the edge $\mathfrak{gh}$ has strength strictly smaller than $\zeta_{\mathcal{L}}(\mathfrak{gh})$.
2. The edge $\mathfrak{gh}$ is called a δ-bridge provided that, for every δ-path P from $\mathfrak{g}$ to $\mathfrak{h}$ that does not contain $\mathfrak{gh}$, its strength is less than $\delta_{\mathcal{L}}(\mathfrak{gh})$.
3. The edge $\mathfrak{gh}$ is referred to as a φ-bridge if the strength of every φ-path P connecting $\mathfrak{g}$ and $\mathfrak{h}$ without using $\mathfrak{gh}$ is strictly below $\varphi_{\mathcal{L}}(\mathfrak{gh})$.
4. An edge $\mathfrak{gh}$ is simply called a bridge whenever it satisfies at least one of the above three conditions; that is, when it is a ζ-bridge, a δ-bridge, or a φ-bridge.

Note: $\mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})$ is the strength between $\mathfrak{g}$ and $\mathfrak{h}$ in the $\mathcal{NUG}$ obtained from G by deleting the edge $\mathfrak{gh}$. This is as the same for the notations $\mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})_\zeta$, $\mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})_\delta$, and $\mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})_\varphi$.

Definition 3.11 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$. Then an edge $\mathfrak{gh}$ in G is called the

1. ζ-effective, if $\zeta_{\mathcal{L}}(\mathfrak{gh}) > \mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})_\zeta$.
2. δ-effective, if $\delta_{\mathcal{L}}(\mathfrak{gh}) > \mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})_\delta$.
3. φ-effective, if $\varphi_{\mathcal{L}}(\mathfrak{gh}) > \mu_{G-\{\mathfrak{gh}\}}^\infty(\mathfrak{g}, \mathfrak{h})_\varphi$.
4. Effective, if it is either of ζ-effective, δ-effective, and φ-effective.

Definition 3.12

1. Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$ and $\mathfrak{g}, \mathfrak{h} \in V$, we say that $\mathfrak{g}$ ζ-effective dominates $\mathfrak{h}$ in G if the edge $\mathfrak{gh}$ be ζ-effective.
2. A subset S of V is called the ζ-effective dominating set in G, if for every$v \in V - S$, there is $u \in S$ such that u dominates v as ζ-effective.

3. The ζ-weight of $\mathfrak{g}$ is defined by $w(\mathfrak{g})_\zeta = \zeta_{\mathcal{M}}(\mathfrak{g}) + \frac{\sum_{\mathfrak{g}\mathfrak{h}\ is\ a\ \zeta-effectiveedge}\ \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h})}{\sum_{\mathfrak{g}\mathfrak{h}\ is\ a\ edge}\ \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h})}$.
4. For any $S \subseteq V$, a natural extension of this concept to a set, is as follows. We also say the ζ-weight of S, it is defined by $w(S)_\zeta = \sum_{u\in S}\ (w(u)_\zeta)$.
5. Let U be the set of all ζ-effective dominating sets in G. The ζ-domination number of G is defined as $(G)_\zeta = min_{D\in U}\big(w(D)_\zeta\big)$. The ζ-effective dominating set that is correspond to $\gamma(G)_\zeta$ is called by ζ-dominating set.

Further, in the similar manner we define $\boldsymbol{\delta}$-dominating set and $\boldsymbol{\varphi}$-dominating set of G, respectively.

The minimum operator is employed as a conservative aggregation mechanism in accordance with the weakest-component criterion. Because the three neutrosophic components capture different dimensions of the available information, the overall domination weight is governed by the least favourable component, without allowing higher values in other components to compensate for a lower value. Within the neutrosophic under framework, the Φ component represents under-valued information and may assume negative values. Consequently, if the Φ component has the lowest value among the three component-wise weights, it determines the overall domination weight. This formulation is consistent with the proposed model, whereby the weakest component determines the overall domination strength associated with an effective dominating set.

Definition 3.13

1. We say that $\mathfrak{g}$ effective dominates $\mathfrak{h}$ in G, if the edge $\mathfrak{g}\mathfrak{h}$ be effective.
2. A subset S of V is called the effective dominating set in G, if for every $v \in V - S$, there is $u \in S$ such that u dominates v as effective.
3. The weight of S, it is defined by $w_v(S) = min\{w_v(D)_\zeta, w_v(D)_\delta, w_v(D)_\varphi\}$.
4. Let U be the set of all effective dominating sets in G. The domination number of G is defined as $\gamma(G) = min_{D\in U}(w(D))$. The effective dominating set that is correspond to $\gamma(G)$ is called by dominating set.

Proposition 3.1 Let $G = (\mathcal{M}, \mathcal{L})$ be a complete $\mathcal{NUG}$ on $G^\star$ such that there is exactly one path between two given vertices, which has

1. ζ-strength. Then $\gamma_v(G)_\zeta \leq min_{u\in V}\zeta_{\mathcal{M}}(u) + 2$
2. δ-strength. Then $\gamma_v(G)_\delta \leq min_{u\in V}\delta_{\mathcal{M}}(u) + 2$
3. φ-strength. Then $\gamma_v(G)_\varphi \leq min_{u\in V}\varphi_{\mathcal{M}}(u) + 2$
4. strength, Then $\gamma_v(G) \leq min_{u\in V}\zeta_{\mathcal{M}}(u), \delta_{\mathcal{M}}(u), \varphi_{\mathcal{M}}(u)) + 2$

Proof. (a) Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$.

The ζ-strength of path P from u to v is of the form

$$\zeta_{\mathcal{M}}(u) \wedge \ldots \wedge \zeta_{\mathcal{M}}(v) \leq \zeta_{\mathcal{M}}(u) \wedge \zeta_{\mathcal{M}}(v) = \zeta_{\mathcal{L}}(uv)$$

So $\mu_G^\infty(u,v)_\zeta < \zeta_{\mathcal{L}}(uv)$. uv is a path from u to v such that $\zeta_{\mathcal{L}}(uv) = \zeta_{\mathcal{M}}(u) \wedge \zeta_{\mathcal{M}}(v)$.

Therefore $\mu_G^\infty(u,v)_\zeta \geq \zeta_{\mathcal{L}}(uv)$.

Hence $\mu_G^\infty(u,v)_\zeta = \zeta_{\mathcal{L}}(uv)$.

Then $\zeta_{\mathcal{L}}(uv) > \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(u,v)_\zeta$. It means that the edge uv is ζ-effective. All edges are ζ-effective and each vertex is adjacent to all other vertices.

So $D = \{u\}$ is a ζ-effective dominating set and $\sum_{\mathfrak{g}\mathfrak{h}\ is\ a\ \zeta-effectiveedge}\ \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \sum_{\mathfrak{g}\mathfrak{h}\ is\ a\ edge}\ \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h})$ for each $u \in V$. The result follows.

Similarly, we can prove for (b), (c), (d).

Proposition 3.2 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$. Then $\mathfrak{g}\mathfrak{h} \in E$ is a

1. ζ-effective edge if and only if $\mathfrak{g}\mathfrak{h}$ is a ζ-bridge

2. δ-effective edge if and only if $\mathfrak{g}\mathfrak{h}$ is a δ-bridge
3. φ-effective edge if and only if $\mathfrak{g}\mathfrak{h}$ is a φ-bridge
4. effective edge if and only if $\mathfrak{g}\mathfrak{h}$ is a bridge

Proof. (a) $\Leftrightarrow$ Suppose $\mathfrak{g}\mathfrak{h}$ is a ζ-effective edge. $\Leftrightarrow$ By Definition 3.8(a), $\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\mathfrak{g}\mathfrak{h}}(\mathfrak{g},\mathfrak{h})_{\zeta}$.

$$\Leftrightarrow \text{Since, } \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \mu^{\infty}_{G}(\mathfrak{g},\mathfrak{h})_{\zeta}.$$

Therefore $\mu^{\infty}_{G}(\mathfrak{g},\mathfrak{h})_{\zeta} > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta}$

$$\Leftrightarrow \text{By Definition of bridge, it means } \mathfrak{g}\mathfrak{h} \text{ is a } \zeta\text{-Bridge.}$$

Therefore, the result follows. Similarly, we can prove for (b), (c), (d).

Proposition 3.3 Let $G = (\mathcal{M}, \mathcal{L})$ be a complete $\mathcal{NUG}$ on $G^{\star}$ such that there exists exactly one path between two given vertices $\mathfrak{g}, \mathfrak{h} \in V$. Then the edge $\mathfrak{g}\mathfrak{h} \in E$ is ζ-effective, δ-effective and φ-effective. Consequently, $\mathfrak{g}\mathfrak{h}$ is effective.

Proof. Let $G = (\mathcal{M}, \mathcal{L})$ be a complete $\mathcal{NUG}$ on $G^{\star}$, and let $\mathfrak{g}, \mathfrak{h} \in V$ be two given vertices such that there exists exactly one path between $\mathfrak{g}$ and $\mathfrak{h}$
Since G is complete, by the definition of a complete $\mathcal{NUG}$,

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \min\{\zeta_{\mathcal{M}}(\mathfrak{g}), \zeta_{\mathcal{M}}(\mathfrak{h})\},$$
$$\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \min\{\delta_{\mathcal{M}}(\mathfrak{g}), \delta_{\mathcal{M}}(\mathfrak{h})\},$$

and

$$\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) = \max\{\varphi_{\mathcal{M}}(\mathfrak{g}), \varphi_{\mathcal{M}}(\mathfrak{h})\}.$$

Since there exists exactly one path between $\mathfrak{g}$ and $\mathfrak{h}$, that path is the edge $\mathfrak{g}\mathfrak{h}$. Therefore, after deleting the edge $\mathfrak{g}\mathfrak{h}$, there is no path between $\mathfrak{g}$ and $\mathfrak{h}$.
Hence,

$$\mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta} < \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}),$$
$$\mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\delta} < \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}),$$

and

$$\mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\varphi} < \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

By Definition of ζ-effective edge,

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta}.$$

Therefore, $\mathfrak{g}\mathfrak{h}$ is a ζ-effective edge.
Similarly, $\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\mu}$, hence $\mathfrak{g}\mathfrak{h}$ is a δ-effective edge.
Also, $\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\varphi}$, and hence $\mathfrak{g}\mathfrak{h}$ is a φ-effective edge.

Thus, $\mathfrak{g}\mathfrak{h}$ is simultaneously ζ-effective, δ-effective and φ-effective. By the definition of an effective edge, it follows that $\mathfrak{g}\mathfrak{h}$ is effective.

Proposition 3.4 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^{\star}$. If $\mathfrak{g}\mathfrak{h} \in E$ is a ζ-effective edge, then $\mu^{\infty}_{G}(\mathfrak{g},\mathfrak{h})_{\zeta} = \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta}$. Similarly, if $\mathfrak{g}\mathfrak{h}$ is a δ-effective edge or a φ-effective edge, then $\mu^{\infty}_{G}(\mathfrak{g},\mathfrak{h})_{\delta} = \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\delta}$ or $\mu^{\infty}_{G}(\mathfrak{g},\mathfrak{h})_{\varphi} = \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\varphi}$, respectively.
Proof. Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^{\star}$ and let $\mathfrak{g}\mathfrak{h} \in E$ be a ζ-effective edge.
By the definition of a ζ-effective edge,

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta}.$$

Now, $\mathfrak{g}\mathfrak{h}$ itself is a ζ-path from $\mathfrak{g}$ to $\mathfrak{h}$ of length one. Hence, by the definition of the strength of a ζ-path,

$$\mu_G(\mathfrak{g}\mathfrak{h})_\zeta = \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

Furthermore, every ζ-path from $\mathfrak{g}$ to $\mathfrak{h}$ other than the path consisting of the edge $\mathfrak{g}\mathfrak{h}$ is also a path in $G - \{\mathfrak{g}\mathfrak{h}\}$. Therefore, the maximum ζ-strength between $\mathfrak{g}$ and $\mathfrak{h}$ in G is attained by the edge $\mathfrak{g}\mathfrak{h}$. Thus,

$$\mu_G^\infty(\mathfrak{g},\mathfrak{h})_\zeta = \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

Combining the above relations, we obtain

$$\mu_G^\infty(\mathfrak{g},\mathfrak{h})_\zeta = \zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\zeta.$$

Similarly, suppose that $\mathfrak{g}\mathfrak{h}$ is a δ-effective edge. By the definition of a δ-effective edge,

$$\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\delta.$$

Since $\mathfrak{g}\mathfrak{h}$ itself is a δ-path from $\mathfrak{g}$ to $\mathfrak{h}$,

$$\mu_G(\mathfrak{g}\mathfrak{h})_\delta = \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

Every other δ-path from $\mathfrak{g}$ to $\mathfrak{h}$ is contained in $G - \{\mathfrak{g}\mathfrak{h}\}$. Hence,

$$\mu_G^\infty(\mathfrak{g},\mathfrak{h})_\delta = \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

Therefore,

$$\mu_G^\infty(\mathfrak{g},\mathfrak{h})_\delta = \delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\delta.$$

Finally, suppose that $\mathfrak{g}\mathfrak{h}$ is a φ-effective edge. By the definition of a φ-effective edge,

$$\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\varphi.$$

Since $\mathfrak{g}\mathfrak{h}$ itself is a φ-path from $\mathfrak{g}$ to $\mathfrak{h}$,

$$\mu_G(\mathfrak{g}\mathfrak{h})_\varphi = \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

Moreover, every other φ-path from $\mathfrak{g}$ to $\mathfrak{h}$ is contained in $G - \{\mathfrak{g}\mathfrak{h}\}$. Hence,

$$\mu_G^\infty(\mathfrak{g},\mathfrak{h})_\varphi = \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}).$$

Consequently,

$$\mu_G^\infty(\mathfrak{g},\mathfrak{h})_\varphi = \varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\varphi.$$

Thus, the result follows for ζ-, δ-, and φ-effective edges.

Proposition 3.5 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$. If $\mathfrak{g}\mathfrak{h} \in E$ is not a ζ-effective edge, then

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\zeta.$$

Similarly, if $\mathfrak{g}\mathfrak{h}$ is not a δ-effective edge or a φ-effective edge, then

$$\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\delta$$

or

$$\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq \mu_{G-\{\mathfrak{g}\mathfrak{h}\}}^\infty(\mathfrak{g},\mathfrak{h})_\varphi,$$

respectively.

Proof. Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$ and let $\mathfrak{g}\mathfrak{h} \in E$ be not a ζ-effective edge.

By the definition of a ζ-effective edge, an edge $\mathfrak{g}\mathfrak{h}$ is ζ-effective if and only if

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) > \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta}.$$

Since $\mathfrak{g}\mathfrak{h}$ is not a ζ-effective edge, the above strict inequality does not hold. Therefore,

$$\zeta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\zeta}.$$

Hence, the first result follows.
Similarly, if $\mathfrak{g}\mathfrak{h}$ is not a δ-effective edge, then, by the definition of a δ-effective edge,

$$\delta_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\delta}.$$

Likewise, if $\mathfrak{g}\mathfrak{h}$ is not a φ-effective edge, then, by the definition of a φ-effective edge,

$$\varphi_{\mathcal{L}}(\mathfrak{g}\mathfrak{h}) \leq \mu^{\infty}_{G-\{\mathfrak{g}\mathfrak{h}\}}(\mathfrak{g},\mathfrak{h})_{\varphi}.$$

Therefore, the required results hold for the ζ-, δ-, and φ-components, respectively.
Proposition 3.6 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^{\star}$. If S is a ζ-effective dominating set of G, then

$$\gamma_v(G)_{\zeta} \leq w(S)_{\zeta}.$$

Similarly, if S is a δ-effective or φ-effective dominating set of G, then

$$\gamma_v(G)_{\delta} \leq w(S)_{\delta}$$

or

$$\gamma_v(G)_{\varphi} \leq w(S)_{\varphi},$$

respectively.
Proof. Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^{\star}$ and let S be a ζ-effective dominating set of G.

Let U_{ζ} denote the collection of all ζ-effective dominating sets of G. By the definition of the ζ-domination number,

$$\gamma_v(G)_{\zeta} = \min_{D \in U_{\zeta}} w(D)_{\zeta}.$$

Since S is a ζ-effective dominating set of G, we have

$$S \in U_{\zeta}.$$

Therefore, the minimum value of $w(D)_{\zeta}$ over all $D \in U_{\zeta}$ cannot exceed the weight of the particular set S. Hence,

$$\gamma_v(G)_{\zeta} = \min_{D \in U_{\zeta}} w(D)_{\zeta} \leq w(S)_{\zeta}.$$

Thus,

$$\gamma_v(G)_{\zeta} \leq w(S)_{\zeta}.$$

Similarly, let U_{δ} denote the collection of all δ-effective dominating sets of G. Since S is a δ-effective dominating set,

$$S \in U_{\delta}.$$

By the definition of the δ-domination number,

$$\gamma_v(G)_{\delta} = \min_{D \in U_{\delta}} w(D)_{\delta}.$$

Consequently,

$$\gamma_v(G)_\delta \leq w(S)_\delta.$$

Therefore,

$$\gamma_v(G)_\delta \leq w(S)_\delta.$$

Finally, let U_φ denote the collection of all φ-effective dominating sets of G. Since S is a φ-effective dominating set,

$$S \in U_\varphi.$$

By the definition of the φ-domination number,

$$\gamma_v(G)_\varphi = \min_{D \in U_\varphi} w(D)_\varphi.$$

Hence,

$$\gamma_v(G)_\varphi \leq w(S)_\varphi.$$

Thus,

$$\gamma_v(G)_\varphi \leq w(S)_\varphi.$$

Hence, the result follows for the ζ-, δ-, and φ-effective dominating sets.

Proposition 3.7 Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$. If S is an effective dominating set of G, then

$$\gamma_v(G) \leq w_v(S).$$

Proof. Let $G = (\mathcal{M}, \mathcal{L})$ be a $\mathcal{NUG}$ on $G^\star$ and let S be an effective dominating set of G.

Let U be the collection of all effective dominating sets of G. Since S is an effective dominating set of G, we have

$$S \in U.$$

By the definition of the domination number of G,

$$\gamma_v(G) = \min_{D \in U} w_v(D).$$

Since $S \in U$, the minimum value of $w_v(D)$ over all $D \in U$ cannot be greater than the weight of the particular effective dominating set S. Therefore,

$$\gamma_v(G) = \min_{D \in U} w_v(D) \leq w_v(S).$$

Hence,

$$\gamma_v(G) \leq w_v(S).$$

Therefore, the result follows.

Proposition 3.8 Let $G = (\mathcal{M}, \mathcal{L})$ be a complete bipartite $\mathcal{NUG}$ on $G^\star$. If $V_1 = \{u\}$ or $V_2 = \{u\}$, then

$$\gamma_v(G)_\zeta \leq \zeta_{\mathcal{M}}(u) + 1,$$

$$\gamma_v(G)_\delta \leq \delta_{\mathcal{M}}(u) + 1,$$

and

$$\gamma_v(G)_\varphi \leq \varphi_{\mathcal{M}}(u) + 1.$$

Consequently,

$$\gamma_v(G) \leq min\{\zeta_{\mathcal{M}}(u), \delta_{\mathcal{M}}(u), \varphi_{\mathcal{M}}(u)\} + 1.$$

Proof. Let $G = (\mathcal{M}, \mathcal{L})$ be a complete bipartite $\mathcal{NUG}$ on $G^{\star}$ and suppose that

$$V_1 = \{u\}$$

or

$$V_2 = \{u\}.$$

Thus, G is a star $\mathcal{NUG}$ with center u.

Since every vertex other than u belongs to the other partite set, every such vertex is adjacent to u. Moreover, for each $v \in V - \{u\}$, the edge uv is the unique path between u and v. Hence, by the definitions of ζ-, δ-, and φ-effective edges, every edge uv is ζ-effective, δ-effective and φ-effective. Therefore, the singleton set

$$S = \{u\}$$

is a ζ-effective dominating set of G. By the definition of the ζ-weight,

$$w(u)_{\zeta} = \zeta_{\mathcal{M}}(u) + \frac{\sum_{uvisa\zeta-effectiveedge}\ \zeta_{\mathcal{L}}(uv)}{\sum_{uvisanedge}\ \zeta_{\mathcal{L}}(uv)}.$$

Since every edge incident with u is ζ-effective, the numerator and denominator are equal. Hence,

$$w(u)_{\zeta} = \zeta_{\mathcal{M}}(u) + 1.$$

Since $\gamma_v(G)_{\zeta}$ is the minimum ζ-weight among all ζ-effective dominating sets, we obtain

$$\gamma_v(G)_{\zeta} \leq w(\{u\})_{\zeta} = \zeta_{\mathcal{M}}(u) + 1.$$

Similarly, every edge incident with u is δ-effective. Therefore,

$$w(u)_{\delta} = \delta_{\mathcal{M}}(u) + 1,$$

and consequently,

$$\gamma_v(G)_{\delta} \leq \delta_{\mathcal{M}}(u) + 1.$$

Likewise, every edge incident with u is φ-effective. Thus,

$$w(u)_{\varphi} = \varphi_{\mathcal{M}}(u) + 1,$$

Hence,

$$\gamma_v(G)_{\varphi} \leq \varphi_{\mathcal{M}}(u) + 1.$$

Finally, since

$$w_v(\{u\}) = min\{w(\{u\})_{\zeta}, w(\{u\})_{\delta}, w(\{u\})_{\varphi}\},$$

we have

$$w_v(\{u\}) = min\{\zeta_{\mathcal{M}}(u), \delta_{\mathcal{M}}(u), \varphi_{\mathcal{M}}(u)\} + 1.$$

Since $\{u\}$ is an effective dominating set, by the definition of the domination number,

$$\gamma_v(G) \leq w_v(\{u\}).$$

Therefore,

$$\gamma_v(G) \leq min\{\zeta_{\mathcal{M}}(u), \delta_{\mathcal{M}}(u), \varphi_{\mathcal{M}}(u)\} + 1.$$

Hence, the result follows.

4. Application of Neutrosophic Under Graphs in Supplier Network Management

Supplier selection constitutes an important decision-making problem in manufacturing, logistics, and supply chain management. Manufacturing organisations generally depend on multiple suppliers for raw materials, components, and other essential resources. However, supplier performance may vary considerably in terms of reliability, uncertainty, and potential failure. Relationships between suppliers may also facilitate alternative material sourcing, information exchange, and logistical coordination.

Recent supplier-selection frameworks increasingly incorporate resilience, sustainability, and organizational context, including Industry 5.0 considerations [25], food supply-chain uncertainty [26], hybrid gray MCDM [27], mining supply chains [28], electronics sourcing [29], and D-AHP–DEMATEL modeling [30]. These studies motivate comparing attribute-based supplier rankings with network-structure-based decisions.

In practical supplier networks, the available information is often incomplete, imprecise, or not entirely reliable. A supplier may demonstrate a high degree of reliability while certain aspects of its performance remain uncertain. Similarly, the possibility of supplier failure cannot always be excluded. These conditions highlight the need for a mathematical representation capable of incorporating different dimensions of supplier information within a unified framework. For this application, the neutrosophic under graph framework developed in the preceding sections is employed to represent the supplier network. Suppliers are modelled as graph vertices, whereas edges represent direct operational relationships between them. The three neutrosophic under components describe the truth, indeterminacy, and falsity aspects of supplier information. The ζ-component is the reliability/satisfaction level of the object. Related to supplier relationship. The δ-component represents the indeterminate information that goes with the relationship, the The false or failure aspect: φ-component. The use of these three components enables the supplier network to be analysed without needing every bit of information to be made one deterministic value.

Effective supplier relationships are identified by comparing the strength of each direct connection with the corresponding maximum path strength in the remaining graph after the considered edge has been removed. A relationship is classified as effective when its direct strength exceeds that of the available alternative paths. The concept of an effective dominating set is subsequently employed to select a group of suppliers capable of effectively dominating the remaining suppliers within the network. Accordingly, the proposed neutrosophic under graph framework provides a mathematical approach for examining supplier relationships and identifying suitable supplier configurations. The following numerical application demonstrates the implementation of this framework in a supplier-network decision-making problem.

4.1 Problem Description

Consider a manufacturing organisation that sources an essential raw material from ten prospective suppliers. Direct operational connections have been established with these suppliers to facilitate material procurement, information sharing, logistical coordination, and alternative sourcing arrangements.

Let

$$V = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}\}$$

be the set of candidate suppliers, where each vertex S_i represents a supplier. An edge between two suppliers represents a direct and feasible operational relationship between them.
The supplier network is represented by the crisp graph

$$G^\star = (V, E).$$

The corresponding neutrosophic under graph is denoted by

$$G = (\mathcal{M}, \mathcal{L}),$$

where $\mathcal{M}$ is the neutrosophic under vertex set and $\mathcal{L}$ is the neutrosophic under edge set.

For each supplier S_i, three components are considered in its assessment. The ζ-membership value represents the degree of reliability associated with the supplier. The δ-membership value represents the degree of indeterminacy arising from incomplete or uncertain information about the supplier. The φ-membership value represents the false or failure component associated with the supplier.

The company's management seeks to determine which supplier relationships satisfy the effectiveness criteria for each of the three neutrosophic components and to identify a minimum-weight effective dominating set. This set comprises selected suppliers that collectively ensure the effective domination of every supplier outside the set, in accordance with the proposed neutrosophic under graph framework. Thus, the decision-making problem considered in this application is to determine a suitable effective dominating set for the supplier network by simultaneously considering the ζ-, δ- and φ-components.

The analysis is carried out by first constructing the underlying supplier network $G^\star$. Neutrosophic under membership values are initially assigned to the graph vertices and edges, followed by the identification of different path types and the evaluation of their respective strengths. Effective edges are then determined by comparing connection strengths after the removal of each corresponding edge. Subsequently, effective dominating sets are constructed, and their associated weights are computed. The effective dominating set with the lowest weight is regarded as the preferred supplier configuration for the manufacturing organisation.

4.2 Representation of the Supplier Network

Let

$$V = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}\}$$

be the set of ten candidate suppliers. The direct operational relationships among the suppliers are represented by the following edge set:

$$\begin{aligned} E = \{ \quad & S_1S_2, S_1S_3, S_1S_4, S_2S_3, S_2S_5, S_2S_6, S_3S_4, S_3S_6, S_3S_7, \\ & S_4S_7, S_4S_8, S_5S_6, S_5S_9, S_6S_7, S_6S_9, S_6S_{10}, \\ & S_7S_8, S_7S_{10}, S_8S_{10}, S_9S_{10}\}. \end{aligned}$$

Thus, the underlying crisp supplier network is

$$G^\star = (V, E),$$

where

$$|V| = 10 \qquad and \qquad |E| = 20.$$

An edge denotes a direct operational connection between two suppliers, encompassing activities

such as material coordination, transportation assistance, information sharing, or alternative sourcing arrangements.

The degree of each supplier in the underlying crisp graph is given by

$Supplier$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
$deg(S_i)$	3	4	5	4	3	6	5	3	3	4

Hence,

$$\sum_{i=1}^{10} deg(S_i) = 3+4+5+4+3+6+5+3+3+4 = 40.$$

By the handshaking property,

$$\sum_{i=1}^{10} deg(S_i) = 2|E|,$$

and therefore

$$2|E| = 40, \qquad |E| = 20.$$

Thus, the supplier network consists of ten suppliers and twenty distinct direct supplier relationships.

The neutrosophic under graph associated with the supplier network is denoted by

$$G = (\mathcal{M}, \mathcal{L}),$$

where $\mathcal{M}$ is the neutrosophic under vertex set and $\mathcal{L}$ is the neutrosophic under edge set.

The network contains several alternative paths between different suppliers. For example, suppliers S_1 and S_6 are connected through

$$S_1 \to S_2 \to S_6$$

and

$$S_1 \to S_3 \to S_6.$$

Similarly, several alternative routes exist between suppliers S_5 and S_{10}. Such alternative paths enable the strength of supplier relationships to be evaluated using the path-strength concepts introduced earlier.

The resulting crisp network $G^\star$ serves as the underlying structure for the neutrosophic under graph. The neutrosophic under membership values are assigned to the vertices in the next subsection.

4.3 Neutrosophic Under Vertex Set

Let

$$V = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}\}$$

be the set of suppliers in the underlying crisp graph $G^\star$. The neutrosophic under vertex set $\mathcal{M}$ assigns three membership values to each supplier. These three components represent the truth, indeterminate and false aspects of the corresponding supplier.

For each $S_i \in V$, let

$$\mathcal{M}(S_i) = \langle \zeta_{\mathcal{M}}(S_i), \delta_{\mathcal{M}}(S_i), \varphi_{\mathcal{M}}(S_i) \rangle.$$

The numerical neutrosophic under assessments of the ten suppliers are given in Table 1.

Table 1
Neutrosophic under Vertex Values of the Suppliers

Supplier	$\zeta_{\mathcal{M}}$	$\delta_{\mathcal{M}}$	$\varphi_{\mathcal{M}}$
S_1	0.86	0.18	-0.12
S_2	0.78	0.24	-0.18
S_3	0.91	0.12	-0.08
S_4	0.72	0.28	-0.22
S_5	0.81	0.20	-0.15
S_6	0.88	0.16	-0.10
S_7	0.69	0.32	-0.27
S_8	0.76	0.26	-0.20
S_9	0.83	0.22	-0.14
S_{10}	0.74	0.30	-0.24

The ζ-membership value represents the degree of reliability associated with each supplier. For instance,

$$\zeta_{\mathcal{M}}(S_3) = 0.91$$

indicates a high degree of reliability for supplier S_3, whereas

$$\zeta_{\mathcal{M}}(S_7) = 0.69$$

represents the lowest ζ-membership value among the considered suppliers.

The δ-membership value represents the indeterminate component of the supplier assessment. In particular,

$$\delta_{\mathcal{M}}(S_3) = 0.12$$

is the lowest indeterminate membership value, while

$$\delta_{\mathcal{M}}(S_7) = 0.32$$

is the highest among the ten suppliers.

The φ-membership values are selected below zero in accordance with the neutrosophic under-set framework. In this dataset,

$$\varphi_{\mathcal{M}}(S_3) = -0.08$$

is the greatest φ-membership value, while

$$\varphi_{\mathcal{M}}(S_7) = -0.27$$

is the smallest.

Therefore, the supplier assessments simultaneously incorporate positive, indeterminate and under-valued information. In particular, a supplier with a high ζ-membership value need not have the smallest δ- or φ-membership value. Hence, the three components provide different aspects of the supplier assessment.

Accordingly, the neutrosophic under vertex set is expressed as

$$\begin{aligned}\mathcal{M} = \{\quad & (S_1, \langle 0.86, 0.18, -0.12\rangle), (S_2, \langle 0.78, 0.24, -0.18\rangle),\\ & (S_3, \langle 0.91, 0.12, -0.08\rangle), (S_4, \langle 0.72, 0.28, -0.22\rangle),\\ & (S_5, \langle 0.81, 0.20, -0.15\rangle), (S_6, \langle 0.88, 0.16, -0.10\rangle),\\ & (S_7, \langle 0.69, 0.32, -0.27\rangle), (S_8, \langle 0.76, 0.26, -0.20\rangle),\\ & (S_9, \langle 0.83, 0.22, -0.14\rangle), (S_{10}, \langle 0.74, 0.30, -0.24\rangle)\}.\end{aligned}$$

The assigned vertex membership values subsequently form the basis for defining the neutrosophic under edge set in accordance with the conditions governing a neutrosophic under graph.

4.4 Neutrosophic Under Edge Set

Let $G^{\star} = (V, E)$ be the crisp supplier network defined above. For each edge $S_iS_j \in E$, we assign a neutrosophic under edge value of the form

$$\mathcal{L}(S_iS_j) = \left\langle \zeta_{\mathcal{L}}(S_iS_j), \delta_{\mathcal{L}}(S_iS_j), \varphi_{\mathcal{L}}(S_iS_j) \right\rangle.$$

The edge membership values are assigned in accordance with the corresponding vertex membership values and satisfy the conditions of a neutrosophic under graph, namely,

$$\zeta_{\mathcal{L}}(S_iS_j) \leq min\{\zeta_{\mathcal{M}}(S_i), \zeta_{\mathcal{M}}(S_j)\},$$
$$\delta_{\mathcal{L}}(S_iS_j) \leq min\{\delta_{\mathcal{M}}(S_i), \delta_{\mathcal{M}}(S_j)\},$$

and

$$\varphi_{\mathcal{L}}(S_iS_j) \geq max\{\varphi_{\mathcal{M}}(S_i), \varphi_{\mathcal{M}}(S_j)\}.$$

The neutrosophic under edge values for the twenty supplier relationships are given in Table 2.

Table 2
Neutrosophic under Edge Values of the Supplier Network

Edge	$\zeta_{\mathcal{L}}$	$\delta_{\mathcal{L}}$	$\varphi_{\mathcal{L}}$
S_1S_2	0.72	0.16	-0.11
S_1S_3	0.82	0.11	-0.08
S_1S_4	0.68	0.17	-0.12
S_2S_3	0.75	0.10	-0.08
S_2S_5	0.70	0.18	-0.15
S_2S_6	0.76	0.14	-0.10
S_3S_4	0.70	0.11	-0.08
S_3S_6	0.84	0.10	-0.08
S_3S_7	0.64	0.12	-0.08
S_4S_7	0.61	0.20	-0.22
S_4S_8	0.69	0.19	-0.20
S_5S_6	0.77	0.15	-0.10
S_5S_9	0.73	0.17	-0.14
S_6S_7	0.65	0.13	-0.10
S_6S_9	0.79	0.14	-0.10
S_6S_{10}	0.68	0.15	-0.10
S_7S_8	0.63	0.21	-0.20
S_7S_{10}	0.60	0.22	-0.24
S_8S_{10}	0.66	0.23	-0.20
S_9S_{10}	0.71	0.20	-0.14

For example, consider the edge S_1S_2. From Table 1,

$$\zeta_{\mathcal{M}}(S_1) = 0.86, \qquad \zeta_{\mathcal{M}}(S_2) = 0.78.$$

Hence,

$$min\{0.86,0.78\} = 0.78.$$

Since

$$\zeta_{\mathcal{L}}(S_1S_2) = 0.72,$$

we have

$$0.72 \leq 0.78.$$

For the δ-component,

$$\delta_{\mathcal{M}}(S_1) = 0.18, \qquad \delta_{\mathcal{M}}(S_2) = 0.24,$$

and therefore

$$min\{0.18,0.24\} = 0.18.$$

Thus,

$$\delta_{\mathcal{L}}(S_1S_2) = 0.16 \leq 0.18.$$

For the φ-component,

$$\varphi_{\mathcal{M}}(S_1) = -0.12, \qquad \varphi_{\mathcal{M}}(S_2) = -0.18.$$

Therefore,

$$max\{-0.12, -0.18\} = -0.12,$$

and

$$\varphi_{\mathcal{L}}(S_1S_2) = -0.11 \geq -0.12.$$

Hence, the edge S_1S_2 satisfies all three required conditions.
As a second verification, consider the edge S_3S_6. We have

$$\zeta_{\mathcal{M}}(S_3) = 0.91, \qquad \zeta_{\mathcal{M}}(S_6) = 0.88,$$

so that

$$\zeta_{\mathcal{L}}(S_3S_6) = 0.84 \leq min\{0.91,0.88\} = 0.88.$$

Similarly,

$$\delta_{\mathcal{L}}(S_3S_6) = 0.10 \leq min\{0.12,0.16\} = 0.12,$$

and

$$\varphi_{\mathcal{L}}(S_3S_6) = -0.08 \geq max\{-0.08, -0.10\} = -0.08.$$

Therefore, S_3S_6 also satisfies the required neutrosophic under graph conditions.

For the remaining edges, the same inequalities are satisfied by the values given in Table 2. Hence,

$$G = (\mathcal{M}, \mathcal{L})$$

is a neutrosophic under graph on the underlying crisp graph $G^\star = (V, E)$.

The constructed vertex and edge membership values will now be used to determine the different types of paths and their corresponding strengths between suppliers.

4.5 Determination of $\boldsymbol{\zeta}$- and $\boldsymbol{\delta}$-Paths

Multiple alternative paths exist between different suppliers within the network. Based on the previously established definitions, a sequence of distinct vertices

$$P: S_{i_0} \to S_{i_1} \to \cdots \to S_{i_k}$$

is a ζ-path if

$$\zeta_{\mathcal{L}}(S_{i_j}S_{i_{j+1}}) > 0, \qquad j = 0,1, \dots, k-1.$$

Since all the $\zeta_{\mathcal{L}}$-values in Table 2 are positive, every path in the underlying supplier network is a ζ-path.

Similarly, a sequence of distinct vertices is a δ-path if

$$\delta_{\mathcal{L}}(S_{i_j}S_{i_{j+1}}) > 0, \qquad j = 0,1, \dots, k-1.$$

Since all the $\delta_{\mathcal{L}}$-values in Table 2 are positive, every path in the underlying supplier network is also a δ-path.

Consider the suppliers S_1 and S_6. From the edge set E, the following paths are available:

$$P_1: S_1 \to S_2 \to S_6,$$
$$P_2: S_1 \to S_3 \to S_6,$$

and

$$P_3: S_1 \to S_4 \to S_3 \to S_6.$$

For the path

$$P_1: S_1 \to S_2 \to S_6,$$

the ζ-strength is

$$\begin{aligned}\mu_G(P_1)_\zeta &= min\{\zeta_{\mathcal{L}}(S_1S_2), \zeta_{\mathcal{L}}(S_2S_6)\}\\ &= min\{0.72,0.76\}\\ &= 0.72.\end{aligned}$$

For the path

$$P_2: S_1 \to S_3 \to S_6,$$

we obtain

$$\begin{aligned}\mu_G(P_2)_\zeta &= min\{\zeta_{\mathcal{L}}(S_1S_3), \zeta_{\mathcal{L}}(S_3S_6)\}\\ &= min\{0.82,0.84\}\\ &= 0.82.\end{aligned}$$

For the path

$$P_3: S_1 \to S_4 \to S_3 \to S_6,$$

the corresponding strength is

$$\begin{aligned}\mu_G(P_3)_\zeta &= min\{\zeta_{\mathcal{L}}(S_1S_4), \zeta_{\mathcal{L}}(S_4S_3), \zeta_{\mathcal{L}}(S_3S_6)\}\\ &= min\{0.68,0.70,0.84\}\\ &= 0.68.\end{aligned}$$

Hence,

$$\mu_G(P_1)_\zeta = 0.72, \quad \mu_G(P_2)_\zeta = 0.82, \quad \mu_G(P_3)_\zeta = 0.68.$$

Therefore, the maximum ζ-strength between S_1 and S_6 is

$$\begin{aligned}\mu_G^\infty(S_1,S_6)_\zeta &= max\{\mu_G(P_1)_\zeta, \mu_G(P_2)_\zeta, \mu_G(P_3)_\zeta\}\\ &= max\{0.72,0.82,0.68\}\\ &= 0.82.\end{aligned}$$

Thus,

$$\mu_G^\infty(S_1,S_6)_\zeta = 0.82.$$

The corresponding δ-strengths of these paths are

$$\begin{aligned}\mu_G(P_1)_\delta &= min\{0.16,0.14\} = 0.14,\\ \mu_G(P_2)_\delta &= min\{0.11,0.10\} = 0.10,\end{aligned}$$

and

$$\mu_G(P_3)_\delta = min\{0.17,0.11,0.10\} = 0.10.$$

Consequently,

$$\begin{aligned}\mu_G^\infty(S_1,S_6)_\delta &= max\{0.14,0.10,0.10\}\\ &= 0.14.\end{aligned}$$

Hence,

$$\mu_G^\infty(S_1,S_6)_\delta = 0.14.$$

Next, consider the suppliers S_2 and S_9. Three possible paths are

$$P_4: S_2 \to S_5 \to S_9,$$
$$P_5: S_2 \to S_6 \to S_9,$$

and

$$P_6: S_2 \to S_3 \to S_6 \to S_9.$$

For P_4,

$$\begin{aligned}\mu_G(P_4)_\zeta &= min\{\zeta_{\mathcal{L}}(S_2S_5), \zeta_{\mathcal{L}}(S_5S_9)\}\\ &= min\{0.70,0.73\}\\ &= 0.70.\end{aligned}$$

For P_5,

$$\begin{aligned}\mu_G(P_5)_\zeta &= min\{\zeta_{\mathcal{L}}(S_2S_6), \zeta_{\mathcal{L}}(S_6S_9)\}\\ &= min\{0.76,0.79\}\\ &= 0.76.\end{aligned}$$

For P_6,

$$\begin{aligned}\mu_G(P_6)_\zeta &= min\{\zeta_{\mathcal{L}}(S_2S_3), \zeta_{\mathcal{L}}(S_3S_6), \zeta_{\mathcal{L}}(S_6S_9)\}\\ &= min\{0.75,0.84,0.79\}\\ &= 0.75.\end{aligned}$$

Therefore,

$$\begin{aligned}\mu_G^\infty(S_2, S_9)_\zeta &= max\{0.70,0.76,0.75\}\\ &= 0.76.\end{aligned}$$

Hence,

$$\mu_G^\infty(S_2, S_9)_\zeta = 0.76.$$

For the δ-component,

$$\begin{aligned}\mu_G(P_4)_\delta &= min\{0.18,0.17\} = 0.17,\\ \mu_G(P_5)_\delta &= min\{0.14,0.14\} = 0.14,\\ \mu_G(P_6)_\delta &= min\{0.10,0.10,0.14\} = 0.10.\end{aligned}$$

Thus,

$$\begin{aligned}\mu_G^\infty(S_2, S_9)_\delta &= max\{0.17,0.14,0.10\}\\ &= 0.17.\end{aligned}$$

Therefore,

$$\mu_G^\infty(S_2, S_9)_\delta = 0.17.$$

The preceding calculations indicate that the strength of connectivity between two suppliers cannot always be assessed solely through their direct relationship, as alternative paths may offer stronger connections between the same suppliers. Accordingly, the maximum path strength provides a basis for evaluating direct supplier relationships relative to other available routes within the network. An equivalent procedure is subsequently employed to examine the remaining relevant pairs of suppliers. The resulting component-wise strengths are used in the following subsection to identify the ζ-effective and δ-effective supplier relationships.

For the φ-component, the present numerical edge values are negative, as required by the chosen neutrosophic under representation. Consequently, the condition

$$\varphi_{\mathcal{L}}(S_iS_j) > 0$$

from the stated definition of a φ-path is not satisfied by these edges. Hence, no φ-path is considered in this numerical application. The subsequent calculations therefore focus on the ζ- and δ-components, for which the path conditions are satisfied by the constructed data.

4.6 Identification of Effective Edges

Supplier relationship effectiveness is evaluated by comparing each edge's membership value with the maximum path strength connecting its endpoints following its removal. This assessment is based directly on the definition of effective edges established in the theoretical section.
For an edge $S_iS_j \in E$, the edge is ζ-effective if

$$\zeta_{\mathcal{L}}(S_iS_j) > \mu^\infty_{G-\{S_iS_j\}}(S_i, S_j)_\zeta,$$

and it is δ-effective if

$$\delta_{\mathcal{L}}(S_iS_j) > \mu^{\infty}_{G-\{S_iS_j\}}(S_i, S_j)_{\delta}.$$

We first illustrate the calculation for the edge S_1S_3.

After deleting the edge S_1S_3, an alternative path between S_1 and S_3 is

$$P: S_1 \to S_2 \to S_3.$$

Its ζ-strength is

$$\begin{aligned}\mu_{G-\{S_1S_3\}}(P)_{\zeta} &= min\{\zeta_{\mathcal{L}}(S_1S_2), \zeta_{\mathcal{L}}(S_2S_3)\}\\ &= min\{0.72, 0.75\}\\ &= 0.72.\end{aligned}$$

The other alternative paths from S_1 to S_3 do not have a strength greater than 0.72. Hence,

$$\mu^{\infty}_{G-\{S_1S_3\}}(S_1, S_3)_{\zeta} = 0.72.$$

Since

$$\zeta_{\mathcal{L}}(S_1S_3) = 0.82,$$

we obtain

$$0.82 > 0.72.$$

Therefore, by the definition of a ζ-effective edge,

$$S_1S_3 \; is \; a \; \zeta - effective \; edge.$$

For the δ-component, after deleting S_1S_3, the maximum alternative path strength is

$$\mu^{\infty}_{G-\{S_1S_3\}}(S_1, S_3)_{\delta} = 0.12.$$

Since

$$\delta_{\mathcal{L}}(S_1S_3) = 0.11 < 0.12,$$

the edge S_1S_3 is not δ-effective.

As another example, consider the edge S_2S_6. After deleting S_2S_6, the strongest alternative ζ-path between S_2 and S_6 has strength

$$\mu^{\infty}_{G-\{S_2S_6\}}(S_2, S_6)_{\zeta} = 0.75.$$

Since

$$\zeta_{\mathcal{L}}(S_2S_6) = 0.76,$$

we have

$$0.76 > 0.75.$$

Hence,

$$S_2S_6 \; is \; \zeta - effective.$$

For the δ-component,

$$\mu^{\infty}_{G-\{S_2S_6\}}(S_2, S_6)_{\delta} = 0.15,$$

whereas

$$\delta_{\mathcal{L}}(S_2S_6) = 0.14.$$

Thus,

$$0.14 < 0.15,$$

and consequently S_2S_6 is not δ-effective.

Table 3 presents a comprehensive summary of the component-wise calculations for all twenty edges.

Table 3
Identification of ζ- and δ-Effective Edges

Edge	$\zeta_{\mathcal{L}}$	$\mu^{\infty}_{G-e\zeta}$	ζ-Eff.	$\delta_{\mathcal{L}}$	$\mu^{\infty}_{G-e\delta}$	δ-Eff.
S_1S_2	0.72	0.76	No	0.16	0.17	No
S_1S_3	0.82	0.72	Yes	0.11	0.12	No
S_1S_4	0.68	0.70	No	0.17	0.16	Yes
S_2S_3	0.75	0.76	No	0.10	0.12	No
S_2S_5	0.70	0.76	No	0.18	0.16	Yes
S_2S_6	0.76	0.75	Yes	0.14	0.15	No
S_3S_4	0.70	0.68	Yes	0.11	0.12	No
S_3S_6	0.84	0.75	Yes	0.10	0.12	No
S_3S_7	0.64	0.65	No	0.12	0.11	Yes
S_4S_7	0.61	0.65	No	0.20	0.19	Yes
S_4S_8	0.69	0.66	Yes	0.19	0.20	No
S_5S_6	0.77	0.73	Yes	0.15	0.15	No
S_5S_9	0.73	0.77	No	0.17	0.16	Yes
S_6S_7	0.65	0.64	Yes	0.13	0.15	No
S_6S_9	0.79	0.73	Yes	0.14	0.15	No
S_6S_{10}	0.68	0.71	No	0.15	0.15	No
S_7S_8	0.63	0.65	No	0.21	0.22	No
S_7S_{10}	0.60	0.65	No	0.22	0.21	Yes
S_8S_{10}	0.66	0.69	No	0.23	0.21	Yes
S_9S_{10}	0.71	0.68	Yes	0.20	0.16	Yes

It follows from Table 3 that the set of ζ-effective edges is

$$E_{\zeta} = \{\ S_1S_3, S_2S_6, S_3S_4, S_3S_6, S_4S_8, S_5S_6, S_6S_7, S_6S_9, S_9S_{10}\}.$$

Thus,

$$|E_{\zeta}| = 9.$$

Similarly, the set of δ-effective edges is

$$E_{\delta} = \{\ S_1S_4, S_2S_5, S_3S_7, S_4S_7, S_5S_9, S_7S_{10}, S_8S_{10}, S_9S_{10}\}.$$

Hence,

$$|E_{\delta}| = 8.$$

An important feature of the numerical analysis is that an edge need not be effective with respect to all neutrosophic components. For example, S_1S_4 is δ-effective but not ζ-effective, whereas S_1S_3 is ζ-effective but not δ-effective. This demonstrates that the three components provide different information about the supplier relationships.

According to the proposition established earlier, an edge is ζ-effective if and only if it is a ζ-bridge. Likewise, an edge is δ-effective if and only if it is a δ-bridge. Accordingly, these calculations also determine the component-wise bridges associated with the supplier network. The effective edges identified through this analysis provide the basis for constructing effective dominating sets in the following subsection.

4.7 Determination of Effective Dominating Sets

From the effective-edge analysis, an edge is considered effective if it is ζ-effective or δ-effective, according to the definition given earlier. Therefore, the effective edge set of the supplier network is

$$E_{eff} = E_\zeta \cup E_\delta.$$

From the sets E_ζ and E_μ obtained above, we have

$$E_{eff} = \{ \quad S_1S_3, S_1S_4, S_2S_5, S_2S_6, S_3S_4, S_3S_6, S_3S_7, \\ S_4S_7, S_4S_8, S_5S_6, S_5S_9, S_6S_7, S_6S_9, S_7S_{10}, S_8S_{10}, S_9S_{10}\}.$$

Thus,

$$|E_{eff}| = 16.$$

We now determine effective dominating sets using the effective edge set E_{eff}.
Consider the set

$$D_1 = \{S_3, S_6, S_{10}\}.$$

The vertices outside D_1 are

$$V - D_1 = \{S_1, S_2, S_4, S_5, S_7, S_8, S_9\}.$$

The domination of each vertex in $V - D_1$ is verified as follows:

$$S_1 \; is \; dominated \; by \; S_3 \quad since \quad S_1S_3 \in E_{eff},$$
$$S_2 \; is \; dominated \; by \; S_6 \quad since \quad S_2S_6 \in E_{eff},$$
$$S_4 \; is \; dominated \; by \; S_3 \quad since \quad S_3S_4 \in E_{eff},$$
$$S_5 \; is \; dominated \; by \; S_6 \quad since \quad S_5S_6 \in E_{eff},$$
$$S_7 \; is \; dominated \; by \; S_6 \quad since \quad S_6S_7 \in E_{eff},$$
$$S_8 \; is \; dominated \; by \; S_{10} \quad since \quad S_8S_{10} \in E_{eff},$$

and

$$S_9 \; is \; dominated \; by \; S_6 \quad since \quad S_6S_9 \in E_{eff}.$$

Hence, every vertex outside D_1 is dominated by at least one vertex in D_1. Therefore,

$$D_1 = \{S_3, S_6, S_{10}\}$$

is an effective dominating set of G.
Consider another set

$$D_2 = \{S_3, S_6, S_8\}.$$

Then

$$V - D_2 = \{S_1, S_2, S_4, S_5, S_7, S_9, S_{10}\}.$$

The vertices S_1, S_4 are effectively dominated by S_3, while S_2, S_5, S_7 and S_9 are effectively dominated by S_6. Finally, S_{10} is effectively dominated by S_8 because

$$S_8S_{10} \in E_{eff}.$$

Therefore,

$$D_2 = \{S_3, S_6, S_8\}$$

is also an effective dominating set.
Next, consider

$$D_3 = \{S_4, S_6, S_{10}\}.$$

The vertices outside D_3 are

$$V - D_3 = \{S_1, S_2, S_3, S_5, S_7, S_8, S_9\}.$$

Here,

$$S_1 \; is \; dominated \; by \; S_4,$$
$$S_2 \; is \; dominated \; by \; S_6,$$
$$S_3 \; is \; dominated \; by \; S_4,$$
$$S_5 \; is \; dominated \; by \; S_6,$$
$$S_7 \; is \; dominated \; by \; S_4 \; or \; S_6,$$
$$S_8 \; is \; dominated \; by \; S_4 \; or \; S_{10},$$

and

$$S_9 \; is \; dominated \; by \; S_6 \; or \; S_{10}.$$

Hence,

$$D_3 = \{S_4, S_6, S_{10}\}$$

is an effective dominating set.

Table 4 summarises the three candidate effective dominating sets examined in the numerical analysis.

Table 4
Candidate Effective Dominating Sets

Set	Vertices	Vertices Dominated Outside the Set
D_1	$\{S_3, S_6, S_{10}\}$	$S_1, S_2, S_4, S_5, S_7, S_8, S_9$
D_2	$\{S_3, S_6, S_8\}$	$S_1, S_2, S_4, S_5, S_7, S_9, S_{10}$
D_3	$\{S_4, S_6, S_{10}\}$	$S_1, S_2, S_3, S_5, S_7, S_8, S_9$

The construction of these sets is based exclusively on the effective relationships established in the preceding subsection. Specifically, every vertex excluded from a selected set is connected to at least one vertex within that set through an effective edge. However, the number of vertices comprising an effective dominating set is not, by itself, sufficient to identify the preferred supplier configuration. Under the proposed neutrosophic under graph framework, the weights associated with the selected suppliers must also be incorporated into the evaluation. Therefore, two effective dominating sets having the same number of suppliers may have different ζ-, δ- and φ-weights.

Hence, the candidate sets

$$D_1, \quad D_2, \quad D_3$$

are considered in the subsequent weight analysis. Their component-wise weights are calculated using the weight definitions introduced in the theoretical section. The set having the minimum corresponding weight will provide the appropriate domination value for the supplier network.

4.8 Calculation of the Component-wise Weights

The weights of the candidate effective dominating sets are now calculated using the weight definitions introduced in the theoretical part. For a supplier S_i, the ζ-weight is given by

$$w(S_i)_\zeta = \zeta_{\mathcal{M}}(S_i) + \frac{\sum_{S_iS_j is a \zeta - effective edge} \zeta_{\mathcal{L}}(S_iS_j)}{\sum_{S_iS_j is an edge} \zeta_{\mathcal{L}}(S_iS_j)}.$$

Similarly, the δ- and φ-weights are obtained from the corresponding components. For a set $D \subseteq V$, the component-wise weight is

$$w(D)_\zeta = \sum_{S_i \in D} w(S_i)_\zeta,$$

$$w(D)_\delta = \sum_{S_i \in D} w(S_i)_\delta,$$

and

$$w(D)_\varphi = \sum_{S_i \in D} w(S_i)_\varphi.$$

We first calculate the ζ-weights of the suppliers belonging to the candidate dominating sets.
For S_3, the incident edges are

$$S_1S_3, \quad S_2S_3, \quad S_3S_4, \quad S_3S_6, \quad S_3S_7.$$

Among these, the ζ-effective edges are

$$S_1S_3, \quad S_3S_4, \quad S_3S_6.$$

Therefore,

$$\begin{aligned} w(S_3)_\zeta &= 0.91 + \frac{0.82 + 0.70 + 0.84}{0.82 + 0.75 + 0.70 + 0.84 + 0.64} \\ &= 0.91 + \frac{2.36}{3.75} \\ &= 1.5393. \end{aligned}$$

For S_6, the incident edges are

$$S_2S_6, \quad S_3S_6, \quad S_5S_6, \quad S_6S_7, \quad S_6S_9, \quad S_6S_{10}.$$

All except S_6S_{10} are ζ-effective. Hence,

$$\begin{aligned} w(S_6)_\zeta &= 0.88 + \frac{0.76 + 0.84 + 0.77 + 0.65 + 0.79}{0.76 + 0.84 + 0.77 + 0.65 + 0.79 + 0.68} \\ &= 0.88 + \frac{3.81}{4.49} \\ &= 1.7286. \end{aligned}$$

For S_{10}, the incident edges are

$$S_6S_{10}, \quad S_7S_{10}, \quad S_8S_{10}, \quad S_9S_{10}.$$

Among these, only S_9S_{10} is ζ-effective. Thus,

$$\begin{aligned} w(S_{10})_\zeta &= 0.74 + \frac{0.71}{0.68 + 0.60 + 0.66 + 0.71} \\ &= 0.74 + \frac{0.71}{2.65} \\ &= 1.0079. \end{aligned}$$

Consequently, the ζ-weight of

$$D_1 = \{S_3, S_6, S_{10}\}$$

is

$$\begin{aligned} w(D_1)_\zeta &= w(S_3)_\zeta + w(S_6)_\zeta + w(S_{10})_\zeta \\ &= 1.5393 + 1.7286 + 1.0079 \\ &= 4.2758. \end{aligned}$$

Now consider the second candidate dominating set

$$D_2 = \{S_3, S_6, S_8\}.$$

For S_8, the incident edges are

$$S_4S_8, \quad S_7S_8, \quad S_8S_{10}.$$

The only ζ-effective edge among them is S_4S_8. Hence,

$$\begin{aligned} w(S_8)_\zeta &= 0.76 + \frac{0.69}{0.69 + 0.63 + 0.66} \\ &= 0.76 + \frac{0.69}{1.98} \\ &= 1.1085. \end{aligned}$$

Therefore,

$$\begin{aligned} w(D_2)_\zeta &= w(S_3)_\zeta + w(S_6)_\zeta + w(S_8)_\zeta \\ &= 1.5393 + 1.7286 + 1.1085 \\ &= 4.3764. \end{aligned}$$

For the third candidate set

$$D_3 = \{S_4, S_6, S_{10}\},$$

the ζ-weight of S_4 is

$$\begin{aligned} w(S_4)_\zeta &= 0.72 + \frac{0.69}{0.68 + 0.70 + 0.61 + 0.69} \\ &= 0.72 + \frac{0.69}{2.68} \\ &= 1.2387. \end{aligned}$$

Hence,

$$\begin{aligned} w(D_3)_\zeta &= 1.2387 + 1.7286 + 1.0079 \\ &= 3.9751. \end{aligned}$$

Thus, among the three candidate sets,

$$w(D_3)_\zeta < w(D_1)_\zeta < w(D_2)_\zeta.$$

Therefore,

$$w(D_3)_\zeta = 3.9751$$

is the minimum ζ-weight among the three candidate sets.
The same procedure is applied to the δ-component. For example, the δ-weight of S_3 is

$$\begin{aligned} w(S_3)_\delta &= 0.12 + \frac{0.12}{0.10 + 0.11 + 0.11 + 0.10 + 0.12} \\ &= 0.12 + \frac{0.12}{0.54} \\ &= 0.3422. \end{aligned}$$

For S_6, the δ-effective incident edges are none of

$$S_2S_6, S_3S_6, S_5S_6, S_6S_7, S_6S_9, S_6S_{10}.$$

Therefore,

$$w(S_6)_\delta = 0.16.$$

For S_8, the δ-effective edge is S_8S_{10}, and hence

$$\begin{aligned} w(S_8)_\delta &= 0.26 + \frac{0.23}{0.19 + 0.21 + 0.23} \\ &= 0.6251. \end{aligned}$$

Thus,

$$\begin{aligned} w(D_1)_\delta &= w(S_3)_\delta + w(S_6)_\delta + w(S_{10})_\delta \\ &= 0.3422 + 0.16 + 1.1125 \\ &= 1.6147, \end{aligned}$$

while

$$\begin{aligned} w(D_2)_\delta &= 0.3422 + 0.16 + 0.6251 \\ &= 1.1273, \end{aligned}$$

and

$$\begin{aligned} w(D_3)_\delta &= w(S_4)_\delta + w(S_6)_\delta + w(S_{10})_\delta \\ &= 0.8322 + 0.16 + 1.1125 \\ &= 2.1047. \end{aligned}$$

Therefore,

$$w(D_2)_\delta = 1.1273$$

is the minimum δ-weight among the three candidate effective dominating sets.

For the φ-component, the numerical edge values are under-valued and none of the edges satisfies the positive φ-path condition adopted in the theoretical definition. Consequently, there are no φ-effective edges in the present numerical network. Hence, the effective-edge contribution to the φ-weight is zero, and

$$w(S_i)_\varphi = \varphi_{\mathcal{M}}(S_i).$$

Therefore,

$$\begin{aligned} w(D_1)_\varphi &= \varphi_{\mathcal{M}}(S_3) + \varphi_{\mathcal{M}}(S_6) + \varphi_{\mathcal{M}}(S_{10}) \\ &= -0.08 - 0.10 - 0.24 \\ &= -0.42, \\ w(D_2)_\varphi &= -0.08 - 0.10 - 0.20 \\ &= -0.38, \end{aligned}$$

and

$$w(D_3)_{\varphi} = -0.22 - 0.10 - 0.24$$
$$= -0.56.$$

The calculated component-wise weights are summarized in Table 5.

Table 5
Component-Wise Weights of Candidate Effective Dominating Sets

Dominating Set	$w(D)_{\zeta}$	$w(D)_{\delta}$	$w(D)_{\varphi}$
$D_1 = \{S_3, S_6, S_{10}\}$	4.2758	1.6147	-0.42
$D_2 = \{S_3, S_6, S_8\}$	4.3764	1.1273	-0.38
$D_3 = \{S_4, S_6, S_{10}\}$	3.9751	2.1047	-0.56

The results show that different candidate effective dominating sets produce different values for the three neutrosophic components. The set D_3 provides the smallest ζ-weight, whereas D_2 provides the smallest δ-weight. For the φ-component, D_3 has the smallest numerical value. The component-wise weights obtained above are used in the next subsection to determine the corresponding domination numbers of the supplier network.

4.9 Determination of the Domination Numbers

The corresponding domination numbers of the supplier network are determined using the component-wise weights associated with the effective dominating sets identified in the preceding subsection.

Recall that the ζ-domination number is defined by

$$\gamma_v(G)_{\zeta} = \min_{D \in U_{\zeta}} w(D)_{\zeta},$$

where U_{ζ} denotes the collection of all ζ-effective dominating sets of G. Similarly,

$$\gamma_v(G)_{\delta} = \min_{D \in U_{\delta}} w(D)_{\delta}$$

and

$$\gamma_v(G)_{\varphi} = \min_{D \in U_{\varphi}} w(D)_{\varphi}.$$

The previously calculated component-wise weights for the candidate effective dominating sets examined in this numerical application are given by

$Dominating set$	$w(D)_{\zeta}$	$w(D)_{\delta}$	$w(D)_{\varphi}$
$D_1 = \{S_3, S_6, S_{10}\}$	4.2758	1.6147	-0.42
$D_2 = \{S_3, S_6, S_8\}$	4.3764	1.1273	-0.38
$D_3 = \{S_4, S_6, S_{10}\}$	3.9751	2.1047	-0.56

Hence, among these candidate sets, the minimum ζ-weight is obtained for D_3. Therefore,

$$\gamma_v(G)_{\zeta} \leq 3.9751.$$

Similarly, the minimum δ-weight among the candidate sets is obtained for D_2. Thus,

$$\gamma_v(G)_{\delta} \leq 1.1273.$$

For the φ-component, the minimum value among the candidate sets is obtained for D_3. Hence,

$$\gamma_v(G)_{\varphi} \leq -0.56.$$

Therefore, the candidate dominating sets provide the following component-wise upper bounds:

$$\gamma_v(G)_\zeta \leq 3.9751, \qquad \gamma_v(G)_\delta \leq 1.1273, \qquad \gamma_v(G)_\varphi \leq -0.56.$$

The results demonstrate that a single dominating set need not be optimal with respect to all three neutrosophic components. In the present example, D_3 gives the smallest ζ-weight and φ-weight, whereas D_2 gives the smallest δ-weight.

Based on the definition of the overall domination number, the weight associated with an effective dominating set is expressed as

$$w_v(D) = min\{w(D)_\zeta, w(D)_\delta, w(D)_\varphi\}.$$

For D_1,

$$\begin{aligned} w_v(D_1) &= min\{4.2758, 1.6147, -0.42\} \\ &= -0.42. \end{aligned}$$

For D_2,

$$\begin{aligned} w_v(D_2) &= min\{4.3764, 1.1273, -0.38\} \\ &= -0.38. \end{aligned}$$

For D_3,

$$\begin{aligned} w_v(D_3) &= min\{3.9751, 2.1047, -0.56\} \\ &= -0.56. \end{aligned}$$

Consequently,

$$w_v(D_3) < w_v(D_1) < w_v(D_2).$$

Thus, among the candidate effective dominating sets considered in the application, D_3 provides the minimum overall weight. Hence, the selected supplier configuration among the considered candidate effective dominating sets is

$$D_3 = \{S_4, S_6, S_{10}\}.$$

The suppliers S_4, $S_6$and , S_{10} therefore constitute the selected effective dominating set among the candidate configurations considered for the supplier network under the proposed neutrosophic under graph model.

The mathematical findings demonstrate that supplier selection is not determined exclusively by individual supplier performance. Rather, the evaluation considers the selected suppliers alongside their effective connections with other suppliers in the network. The proposed method therefore accounts for both the individual neutrosophic attributes of suppliers and the structural relationships linking them. The implications of this finding for supplier management are examined in the subsequent subsection.

4.10 Managerial Interpretation of the Selected Supplier Set

The numerical analysis identifies

$$D_3 = \{S_4, S_6, S_{10}\}$$

as the preferred effective dominating set among the candidate sets considered. This result has a direct interpretation in the context of supplier network management.

The supplier S_4 effectively dominates S_1, S_3, S_7 and S_8 through the effective relationships

$$S_1S_4, \qquad S_3S_4, \qquad S_4S_7, \qquad S_4S_8.$$

Thus, the inclusion of S_4 in the selected set provides effective connectivity with several suppliers in different parts of the network.

The supplier S_6 provides effective relationships with

$$S_2, \quad S_3, \quad S_5, \quad S_7, \quad S_9,$$

through the relationships

$$S_2S_6, \qquad S_3S_6, \qquad S_5S_6, \qquad S_6S_7, \qquad S_6S_9.$$

Consequently, S_6 provides effective coverage for a relatively large portion of the supplier network.

Finally, S_{10} provides effective relationships with S_7, S_8 and S_9 through

$$S_7S_{10}, \qquad S_8S_{10}, \qquad S_9S_{10}.$$

Hence, the inclusion of S_{10} complements the effective coverage already provided by S_4 and S_6.

The domination of all suppliers outside D_3 can therefore be summarized as

$Supplier\ outside\ D_3$	$Effective\ dominating\ supplier$
S_1	S_4
S_2	S_6
S_3	$S_4\ or\ S_6$
S_5	S_6
S_7	$S_4\ or\ S_6\ or\ S_{10}$
S_8	$S_4\ or\ S_{10}$
S_9	$S_6\ or\ S_{10}$

Thus, every supplier not belonging to D_3 has at least one effective relationship with a supplier in D_3. This confirms the effective dominating property of the selected set.

From a managerial standpoint, the findings indicate that achieving effective network coverage does not require the organisation to assign equal importance to all ten suppliers. Rather, the configuration comprising three suppliers

$$\{S_4, S_6, S_{10}\}$$

ensures the effe$ctive$ domination of all remaining suppliers within the proposed neutrosophic under graph framework.

The selection is also supported by the component-wise weight analysis. In particular,

$$w(D_3)_\zeta = 3.9751$$

is smaller than the corresponding ζ-weights of D_1 and D_2, while

$$w(D_3)_\varphi = -0.56$$

is also the smallest $a$mong the three candidate configurations. Although D_2 provides the smallest δ-weight,

$$w(D_2)_\delta = 1.1273,$$

the overall weight criterion gives

$$w_v(D_3) = -0.56 < w_v(D_1) = -0.42 < w_v(D_2) = -0.38.$$

The proposed neutrosophic under graph framework therefore allows decision-makers to evaluate supplier reliability indeterminacy,and under-valued information alongside the structural

connections within the supplier network.

The numerical application demonstrates the potential of effective domination as a network-based decision-support mechanism. Instead of relying exclusively on individual supplier membership values, the framework determines a group of suppliers whose effective connections collectively ensure coverage of the entire network. Accordingly, the numerical findings illustrate the practical relevance of the proposed neutrosophic under graph model for supplier network management.

4.11 Sensitivity Analysis of the Selected Supplier Configuration

A sensitivity analysis is conducted to assess the stability of the selected supplier configuration under variations in the neutrosophic membership values assigned to the selected suppliers.This analysis investigates whether moderate changes in supplier assessments influence the suitability of the identified effective dominating set.

The supplier configuration selected through the preceding analysis is expressed as

$$D_3 = \{S_4, S_6, S_{10}\}.$$

Its component-wise weights are

$$w(D_3)_\zeta = 3.9751, \qquad w(D_3)_\delta = 2.1047, \qquad w(D_3)_\varphi = -0.56.$$

Consider first a variation in the ζ-membership value of S_4. Its original value is

$$\zeta_{\mathcal{M}}(S_4) = 0.72.$$

Suppose that the $reliability$ assessment of S_4 decreases by 0.05. Then the modified value becomes

$$\zeta'_{\mathcal{M}}(S_4) = 0.67.$$

Keeping the edge assessments unchanged, the modified ζ-weight of S_4 is

$$\begin{aligned} w'(S_4)_\zeta &= 0.67 + \frac{0.69}{0.68 + 0.70 + 0.61 + 0.69} \\ &= 0.67 + \frac{0.69}{2.68} \\ &= 1.1887. \end{aligned}$$

Consequently, the modified ζ-weight of the selected set is

$$\begin{aligned} w'(D_3)_\zeta &= w'(S_4)_\zeta + w(S_6)_\zeta + w(S_{10})_\zeta \\ &= 1.1887 + 1.7286 + 1.0079 \\ &= 3.9252. \end{aligned}$$

Thus, even after a decrease of 0.05 in the ζ-membership value of S_4, the selected set remains a feasible effective $dominating\ set$ under the effective-edge relationships used in the analysis.

Next, consider an increase in the indeterminate membership of S_4. Suppose that

$$\delta_{\mathcal{M}}(S_4) = 0.28$$

increases to

$$\delta'_{\mathcal{M}}(S_4) = 0.33.$$

The modified δ-weight of S_4 becomes

$$\begin{aligned} w'(S_4)_\delta &= 0.33 + \frac{0.20 + 0.19}{0.17 + 0.11 + 0.20 + 0.19} \\ &= 0.33 + \frac{0.39}{0.67} \\ &= 0.9121. \end{aligned}$$

Therefore,

$$\begin{aligned} w'(D_3)_\delta &= w'(S_4)_\delta + w(S_6)_\delta + w(S_{10})_\delta \\ &= 0.9121 + 0.16 + 1.1125 \\ &= 2.1846. \end{aligned}$$

The increase in the indeterminate component causes an increase in the corresponding weight of D_3, as expected Nevertheless,the effective domination structure of the supplier network is unchanged because the effective relationships used for domination are determined through the edge-strength comparisons.

$A\ similar\ analysis\ can\ be\ carried\ out$ for S_6, which plays a central role in the selected configuration. Its original ζ-membership value is

$$\zeta_{\mathcal{M}}(S_6) = 0.88.$$

If this value decreases by 0.05, then

$$\zeta'_{\mathcal{M}}(S_6) = 0.83.$$

The corresponding modified weight is

$$\begin{aligned} w'(S_6)_\zeta &= 0.83 + \frac{0.76 + 0.84 + 0.77 + 0.65 + 0.79}{0.76 + 0.84 + 0.77 + 0.65 + 0.79 + 0.68} \\ &= 0.83 + \frac{3.81}{4.49} \\ &= 1.6786. \end{aligned}$$

Hence,

$$\begin{aligned} w'(D_3)_\zeta &= 1.2387 + 1.6786 + 1.0079 \\ &= 3.9252. \end{aligned}$$

The results of these variations are summarized in Table 6.

Table 6
Sensitivity Analysis of the Selected Dominating Set

Variation	Modified Value	Component	Modified Set Weight
Original	–	ζ	3.9751
$\zeta_{\mathcal{M}}(S_4) - 0.05$	0.67	ζ	3.9252
$\delta_{\mathcal{M}}(S_4) + 0.05$	0.33	δ	2.1846
$\zeta_{\mathcal{M}}(S_6) - 0.05$	0.83	ζ	3.9252

The sensitivity analysis reveals that moderate changes in individual supplier assessments affect the numerical weight associated with the selected configuration while leaving its effective domination structure unchanged within the network considered.These findings indicate that the proposed framework reconstruction of the entire network structure.

$In\ particular, the\ suppliers\ S_4, S_6\ and\ S_{10}$ continue to provide effective coverage of the suppliers outside the selected set. Therefore, the selected configuration

$$D_3 = \{S_4, S_6, S_{10}\}$$

shows stability under the considered perturbations.

From a managerial standpoint, such stability is particularly relevant because supplier assessments may evolve as additional performance information becomes available. A decision-making framework that maintains its effectiveness despite moderate variations in these assessments can offer a more dependable basis for supplier network planning than one relying on a single, fixed evaluation.Accordingly,the sensitivity analysis provides further evidence of the applicability of the neutrosophic under graph framework to supplier network decision-making in environments characterised by uncertainty and changing information.

4.12 Comparative Analysis of Candidate Supplier Configurations

The preceding analysis identified three effective dominating sets,

$$D_1 = \{S_3, S_6, S_{10}\}, \qquad D_2 = \{S_3, S_6, S_8\},$$

and

$$D_3 = \{S_4, S_6, S_{10}\}.$$

Although all three sets provide effective domination of the supplier network, their neutrosophic weights are different. Therefore,a comparison of the candidate configurations is necessary to understand the effect of . the three neutrosophic components on the final decision.

The calculated component-wise weights are

$Dominatingset$	$w(D)_\zeta$	$w(D)_\delta$	$w(D)_\varphi$
D_1	4.2758	1.6147	−0.42
D_2	4.3764	1.1273	−0.38
D_3	3.9751	2.1047	−0.56

For the ζ-component, the ordering of the candidate configurations is

$$w(D_3)_\zeta < w(D_1)_\zeta < w(D_2)_\zeta.$$

Hence, D_3 provides the smallest ζ-weight.This indicates that,among the considered configurations,the suppliers

$provides\ the\ smallest\ \zeta -$
$weight.This\ indicates\ that, among\ the\ considered\ configurations, the\ supp$liers

$$\{S_4, S_6, S_{10}\}$$

provide the most favourable configuration with respect to the ζ-component.

For the δ-component, we obtain

$$w(D_2)_\delta < w(D_1)_\delta < w(D_3)_\delta.$$

Therefore, D_2 provides the smallest δ-$weight.In$ terms of indeterminate information, the configuration

$$\{S_3, S_6, S_8\}$$

is consequently preferable among the candidate sets.

For the φ-component,

$$w(D_3)_\varphi < w(D_1)_\varphi < w(D_2)_\varphi.$$

Thus, D_3 gives the smallest value in the φ-component. Since the φ-membership values represent the under-valued or false component in the neutrosophic under framework,this component provides additional information concerning the potential undesirable aspect of the supplier configuration.
The comparative results can be summarized as follows:

$Component$	$Minimum-weightconfiguration$	$Minimumweight$
ζ	$D_3 = \{S_4, S_6, S_{10}\}$	3.9751
δ	$D_2 = \{S_3, S_6, S_8\}$	1.1273
φ	$D_3 = \{S_4, S_6, S_{10}\}$	-0.56

The comparison demonstrates an important feature of the proposed neutrosophic under graph model. The configuration that is optimal $with\ respect$ to one component need not be optimal with respect to another component. In the present application, D_2 has the smallest δ-weight, whereas D_3 has the smallest ζ- and φ-weights.
The overall weight of each candidate set is obtained from

$$w_v(D) = min\{w(D)_\zeta, w(D)_\delta, w(D)_\varphi\}.$$

Thus,

$$w_v(D_1) = -0.42,$$
$$w_v(D_2) = -0.38,$$

and

$$w_v(D_3) = -0.56.$$

Consequently,

$$w_v(D_3) < w_v(D_1) < w_v(D_2).$$

Hence, D_3 is the preferred configuration under the overall weight criterion considered in this application. The comparative analysis also shows why the proposed approach cannot be replaced simply by selecting the supplier having the highest individual ζ-membership value. For example, S_3 has the highest ζ-membership value

$$\zeta_{\mathcal{M}}(S_3) = 0.91,$$

but the configuration containing S_3 need not provide the minimum overall domination weight. The effective relationships among the selected suppliers and the remaining suppliers must also be considered.
Similarly, the configuration D_3 contains S_4, whose individual ζ-membership value is only

$$\zeta_{\mathcal{M}}(S_4) = 0.72.$$

Nevertheless, the effective relationships of S_4 with other suppliers, together with those of S_6 and S_{10}, result in a lower ζ-weight for the complete dominating set.

The proposed model consequently assesses supplier configurations from a network-level perspective rather than considering each supplier in isolation. This approach is particularly relevant to supply chain environments, where supplier effectiveness may be influenced by both individual attributes and relationships with other suppliers. The comparative analysis establishes that the effective dominating-set framework offers a systematic approach to determining a compact supplier configuration while accounting for all three neutrosophic components. Accordingly, the resulting selection incorporates both the structural characteristics of the supplier network and the

corresponding neutrosophic information.

4.13 Application Summary and Practical Implications

The numerical investigation illustrates the practical application of the proposed neutrosophic under graph framework to supplier network management. The network comprises ten suppliers, collectively represented by the vertex set

$$V = \{S_1, S_2, \dots, S_{10}\},$$

and twenty direct supplier relationships represented by the edge set E.
Each supplier is characterized by three membership components,

$$\langle \zeta_{\mathcal{M}}(S_i), \delta_{\mathcal{M}}(S_i), \varphi_{\mathcal{M}}(S_i) \rangle,$$

which allow reliability, indeterminate information and the under-valued component of supplier performance to be considered simultaneously.

Edge membership values were assigned in accordance with the defining conditions of a neutrosophic under graph. Subsequently, effective-edge analysis distinguished those relationships whose direct strengths exceeded the corresponding maximum strengths of alternative paths following the removal of each considered edge.
In particular, the analysis produced the ζ-effective edge set

$$\begin{aligned} E_\zeta = \{ \quad & S_1S_3, S_2S_6, S_3S_4, S_3S_6, S_4S_8, \\ & S_5S_6, S_6S_7, S_6S_9, S_9S_{10}\}, \end{aligned}$$

and the δ-effective edge set

$$\begin{aligned} E_\mu = \{ \quad & S_1S_4, S_2S_5, S_3S_7, S_4S_7, S_5S_9, \\ & S_7S_{10}, S_8S_{10}, S_9S_{10}\}. \end{aligned}$$

Hence, the effective relationships are

$$E_{eff} = E_\zeta \cup E_\mu.$$

The identified relationships form the basis for evaluating effective domination across the supplier network. Subsequently, three candidate effective dominating sets were examined:

$$D_1 = \{S_3, S_6, S_{10}\},$$
$$D_2 = \{S_3, S_6, S_8\},$$

and

$$D_3 = \{S_4, S_6, S_{10}\}.$$

For each selected set, all suppliers outside the set are effectively dominated by at least one of its members. The analysis of component-wise weights further revealed that

$$w(D_3)_\zeta = 3.9751$$

is the smallest ζ-weight among the candidate configurations, while

$$w(D_2)_\delta = 1.1273$$

is the smallest δ-weight. For the under-valued component,

$$w(D_3)_\varphi = -0.56$$

is the smallest value among the considered configurations.

Under the overall weight criterion,

$$w_v(D) = min\{w(D)_\zeta, w(D)_\delta, w(D)_\varphi\},$$

we obtain

$$w_v(D_1) = -0.42, \qquad w_v(D_2) = -0.38, \qquad w_v(D_3) = -0.56.$$

Therefore,

$$w_v(D_3) < w_v(D_1) < w_v(D_2),$$

and the preferred configuration among the candidate effective dominating sets is

$$D_3 = \{S_4, S_6, S_{10}\}.$$

From the supplier-management perspective, the selected set does not imply that the remaining suppliers are excluded from the network. Instead, S_4, S_6 and S_{10} form a representative set through which the remaining suppliers are effectively connected according to the proposed model. The selected suppliers provide effective coverage of the remaining network through the relationships S_1S_4, S_2S_6, S_3S_4, S_5S_6, S_7S_4, S_8S_4, S_9S_6, with additional effective connections available through S_{10}.

The proposed model therefore supports decision-makers in selecting a relatively small group of strategically important suppliers while maintaining effective connectivity throughout the remaining supplier network. The numerical application further highlights the need to account for inter-supplier relationships alongside individual supplier assessments. A supplier exhibiting a high individual membership value may not necessarily form part of the minimum-weight effective dominating configuration. Consequently, the selection process must consider both the effectiveness of network relationships and the combined weights associated with the selected suppliers. By incorporating reliability, indeterminacy, and under-valued information, the proposed neutrosophic under graph framework offers a mathematical basis for supporting supplier network decisions. The numerical example illustrates the application of path strength, effective edges, effective domination, and domination weights to a network-based management problem. These findings consequently demonstrate the relationship between the theoretical formulation of neutrosophic under graphs and their potential applications in mathematics and management science, particularly for supplier network selection and coordination.

Table 7:

Aspect	Proposed NUG Effective Domination	AHP	TOPSIS	VIKOR
Supplier Attributes	✓	✓	✓	✓
Uncertain Information	✓	Requires extension	Requires extension	Requires Extension
Truth/Indeterminacy/Falsity	✓	✗	✗	✗
Negative Under-Valued Information	✓	✗	✗	✗
Supplier-to-Supplier Relationships	✓	Limited	Limited	Limited
Effective/Critical Relationships	✓	✗	✗	✗
Network Structure	✓	Limited	Limited	Limited
Dominating Configuration	✓	✗	✗	✗

5. Conclusion

An effective domination framework for neutrosophic under graphs was formulated in this study

to represent network information encompassing truth, indeterminacy, and under-valued components. The proposed mathematical structure combines component-wise path strengths, bridges, effective edges, effective dominating sets, domination weights, and domination numbers. Theoretical propositions were established to characterise the connections between effective edges and bridges and derive domination bounds for complete and complete bipartite neutrosophic under graphs. The findings establish that domination within these graph structures is determined not solely by the choice of vertices but also by their component-wise weights and the effectiveness of their relationships with other vertices. The proposed framework was applied to a numerical supplier network comprising 10 suppliers and 20 direct supplier relationships. Information associated with the suppliers was expressed through the three neutrosophic components, while effective relationships were determined by analysing path strengths separately for each component. Candidate effective dominating sets were subsequently identified and assessed using their respective component-wise weights and the overall domination criterion established in the proposed model. Among the candidate configurations considered in the application, $D_3 = \{S_4, S_6, S_{10}\}$ provided the minimum overall weight under the proposed criterion. Thus, D_3 was selected as the effective dominating configuration among the evaluated candidates. The numerical investigation demonstrates the capacity of the proposed framework to incorporate individual supplier information alongside the structural relationships connecting suppliers, thereby establishing a mathematical foundation for supplier network evaluation and configuration. Further development of the proposed framework may proceed in several directions. Future investigations could examine supplier networks of greater size and structural complexity, incorporate real-world supplier datasets, and undertake comparative evaluations by integrating established supplier-selection and multi-criteria decision-making methods. Another potential extension involves developing algorithmic procedures to systematically identify minimum-weight effective dominating sets in larger networks. Subsequent research could also investigate dynamic neutrosophic under graphs characterised by time-varying vertex and edge membership values, together with potential applications in transportation, communication, healthcare supply, production, and resource allocation networks.

6. Declarations

5.1.1 Ethical Approval

This article does not contain any studies involving human participants or animals performed by any of the authors. Therefore, ethical approval is not applicable.

5.2 Consent to Participate

Not applicable. This study does not involve human participants.

5.3 Consent to Publish

Not applicable. This study does not contain any individual participant data or identifiable information requiring consent for publication.

5.4 Conflicts of Interest

The authors declare that they have no conflicts of interest regarding this research and manuscript.

References

ADDIN EN.REFLIST [1]Sunitha, M. S., & Kumar, A. (2002). Complement of a fuzzy graph. *Indian Journal of Pure & Applied Mathematics*, *33*(9). https://www.researchgate.net/publication/264994797_Complement_of_a_fuzzy_graph

[2] Fei, Y. (2020). Study on neutrosophic graph with application in wireless network. *CAAI Transactions on Intelligence Technology*, *5*(4), 301–307. https://doi.org/10.1049/trit.2020.0093

[3] Shao, Z., Kosari, S., Rashmanlou, H., & Shoaib, M. (2020). New Concepts in Intuitionistic Fuzzy Graph with Application in Water Supplier Systems. *Mathematics*, *8*(8), 1241. https://doi.org/10.3390/math8081241

[4] Rao, Y., Kosari, S., Shao, Z., Cai, R., & Xinyue, L. (2020). A Study on Domination in Vague Incidence Graph and Its Application in Medical Sciences. *Symmetry*, *12*(11), 1885. https://doi.org/10.3390/sym12111885

[5] Hussain, S. S., Rosyida, I., Rashmanlou, H., & Mofidnakhaei, F. (2021). Interval intuitionistic neutrosophic sets with its applications to interval intuitionistic neutrosophic graphs and climatic analysis. *Computational and Applied Mathematics*, *40*(4), 121. https://doi.org/10.1007/s40314-021-01504-8

[6] Shi, X., & Kosari, S. (2021). Certain Properties of Domination in Product Vague Graphs With an Application in Medicine [Original Research]. *Frontiers in Physics*, *Volume 9 - 2021*. https://doi.org/10.3389/fphy.2021.680634

[7] Uluçay, V. (2021). Q-neutrosophic soft graphs in operations management and communication network. *Soft Computing*, *25*(13). https://doi.org/10.1007/s00500-021-05772-8

[8] Shambayati, H., Shafiei Nikabadi, M., Saberi, S., & Mardani, A. (2024). A neutrosophic optimization model for supply chain virtualization in the circular economy using the non-dominated sorting genetic algorithm II. *Annals of Operations Research*, *342*(1), 687-723. https://doi.org/10.1007/s10479-023-05791-8

[9] Abbas, M. Z., Anjum, R., Hussain, A., Ahmad, M., & Haleemzai, I. (2025). An integrated neutrosophic Z-numbers based CRITIC-EDAS decision model for smart solar panel evaluation in sustainable energy planning. *Scientific Reports*, *15*(1), 38180. https://doi.org/10.1038/s41598-025-22100-4

[10] Fahmi, A., Khan, A., Gull, K., Mukheimer, A., Abdeliawad, A.T., Hashmi, A. and Thinakaran, R. (2025). Domination in Rough \(m\)-Polar Fuzzy Digraphs Based on Trade Networking. *European Journal of Pure and Applied Mathematics*, *18*(3), 6094. https://doi.org/10.29020/nybg.ejpam.v18i3.6094

[11] Sreelakshmi, T. P., & Uma Samundesvari, K. (2026). Neutrosophic graph based energy-aware equitable total domination for efficient satellite cluster coordination. *Knowledge-Based Systems*, *351*, 116669. https://doi.org/10.1016/j.knosys.2026.116669

[12] Venkatesan, D., Rosline Jesuraj, J. and Prabhu Subramanian, M. (2026). Balanced Fermatean quadripartitioned neutrosophic fuzzy graphs and its application, . *Mathematics and Computational Sciences*, *7*(2), 215-227. https://doi.org/10.30511/mcs.2026.735274

[13] Islam, M. M., & Arakawa, M. (2022). Integrated multi-criteria group decision-making model for supplier selection in an uncertain environment. Cogent Engineering, 9(1), 2079220. https://doi.org/10.1080/23311916.2022.2079220

[14] Tavana, M., Sorooshian, S., & Mina, H. (2024). An integrated group fuzzy inference and best-worst method for supplier selection in intelligent circular supply chains. Annals of Operations Research, 342, 803–844. https://doi.org/10.1007/s10479-023-05680-0

[15] Afrasiabi, A., Tavana, M., & Di Caprio, D. (2022). An extended hybrid fuzzy multi-criteria decision model for sustainable and resilient supplier selection. Environmental Science and Pollution Research, 29, 37291–37314. https://doi.org/10.1007/s11356-021-17851-2

[16] Zeng, H., Khokhar, M., Islam, T., & Sharma, A. (2023). A model for green-resilient supplier selection: Fuzzy best-worst multi-criteria decision-making method and its applications. Environmental Science and Pollution Research, 30, 54035–54058. https://doi.org/10.1007/s11356-023-25749-4

[17] Saputro, T. E., Figueira, G., & Almada-Lobo, B. (2023). Hybrid MCDM and simulation-optimization for strategic supplier selection. Expert Systems with Applications, 119624. https://doi.org/10.1016/j.eswa.2023.119624

[18] Eghbali-Zarch, M., Zabihi, S. Z., & Masoud, S. (2023). A novel fuzzy SECA model based on fuzzy standard deviation and correlation coefficients for resilient-sustainable supplier selection. Expert Systems with Applications, 231, 120653. https://doi.org/10.1016/j.eswa.2023.120653

[19] Ali, H., & Zhang, J. (2023). A fuzzy multi-objective decision-making model for global green supplier selection and order allocation under quantity discounts. Expert Systems with Applications, 225, 120119. https://doi.org/10.1016/j.eswa.2023.120119

[20] Saputro, T. E., Rosiani, T. Y., Mubin, A., Dewi, S. K., & Baroto, T. (2024). Green supplier selection under supply risks using novel integrated fuzzy multi-criteria decision making techniques. Journal of Cleaner Production, 449, 141788. https://doi.org/10.1016/j.jclepro.2024.141788

[21] Hendiani, S., & Walther, G. (2025). Towards sustainable futures: Rethinking supplier selection through interval-valued intuitionistic fuzzy decision-making. International Journal of Production Economics, 285, 109620. https://doi.org/10.1016/j.ijpe.2025.109620

[22] Varchandi, S., Memari, A., & Akbari Jokar, M. R. (2024). An integrated best-worst method and fuzzy TOPSIS for resilient-sustainable supplier selection. Decision Analytics Journal, 11, 100488. https://doi.org/10.1016/j.dajour.2024.100488

[23] Hezam, I. M., Ali, A. M., Sallam, K., Hameed, I. A., & Abdel-Basset, M. (2024). Digital twin and fuzzy framework for supply chain sustainability risk assessment and management in supplier selection. Scientific Reports, 14, 17718. https://doi.org/10.1038/s41598-024-67226-z

[24] Li, Y., Tsang, Y. P., Lee, C. K. M., & Wu, C. H. (2024). Multi-criteria group decision analytics for sustainable supplier relationship management in a focal manufacturing company. Journal of Cleaner Production, 476, 143690. https://doi.org/10.1016/j.jclepro.2024.143690

[25] Sarker, M. A., Kar, T. S., Reno, S., & Ahmed, F. (2026). A sustainable and resilient supplier selection in Industry 5.0 era: An integrated fuzzy multicriteria decision-making approach. Engineering Reports, 8(8), e71002. https://doi.org/10.1002/eng2.71002

[26] Sharma, P., & Roy, J. (2026). An integrated fuzzy decision-support framework for sustainable-resilient supplier selection in food supply chains under uncertainty. Journal of Industrial Information Integration, 101228. https://doi.org/10.1016/j.jii.2026.101228

[27] Ulutas, A., Krstic, M., Topal, A., Agnusdei, L., Tadic, S., & Miglietta, P. P. (2024). A novel hybrid gray MCDM model for resilient supplier selection problem. Mathematics, 12(10), 1444. https://doi.org/10.3390/math12101444

[28] Becerra, P., & Diaz, J. (2025). Supplier selection model considering sustainable and resilience aspects for mining industry. Systems, 13(2), 81. https://doi.org/10.3390/systems13020081

[29] Menon, R. R., & Ravi, V. (2022). Using AHP-TOPSIS methodologies in the selection of sustainable suppliers in an electronics supply chain. Cleaner Materials, 5, 100130. https://doi.org/10.1016/j.clema.2022.100130

[30] Gokler, S. H., & Boran, S. (2025). A novel resilient and sustainable supplier selection model based on D-AHP and DEMATEL methods. Journal of Engineering Research, 13(1), 57–67. https://doi.org/10.1016/j.jer.2023.07.015

[31] Khan, S. U. (2021). Graphical analysis of covering and paired domination in the environment of neutrosophic information. Mathematical Problems in Engineering, 2021, 5518295. https://doi.org/10.1155/2021/5518295

[32] Novel concepts on domination in neutrosophic incidence graphs with some applications. (2023). Journal of Advanced Computational Intelligence and Intelligent Informatics, 27(5), 837–847. https://doi.org/10.20965/jaciii.2023.p0837

[33] Sreelakshmi, T. P., & Uma Samundesvari, K. (2024). Total domination in uniform single valued neutrosophic graph. Neutrosophic Sets and Systems, 70, 305–313. https://doi.org/10.5281/zenodo.13175758

[34] Shanmugapriya, R., & Hemalatha, N. C. (2026). On restrained neutrosophic domination number on special graphs. Neutrosophic Sets and Systems, 96, 69–76. https://doi.org/10.5281/zenodo.17096073

[35] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X