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A Probabilistic Decision-Support Framework for Contract Duration Assessment in Repetitive Construction Projects Using Integrated Line of Balance and PERT

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ABSTRACT

Uncertainty in activity durations presents a major challenge to construction project management, particularly when assessing the probability of completing repetitive construction projects within contractual deadlines. To address this challenge, the present study develops an integrated computational framework that combines the Line of Balance (LOB) scheduling technique, accommodating both single-crew and multiple-crew configurations, with the probabilistic principles of the Program Evaluation and Review Technique (PERT). The proposed framework is implemented through a newly developed Java-based application, designated the Repetitive-Projects Evaluation and Review Technique (RPERT). By incorporating three-point duration estimates—optimistic, most likely and pessimistic—for recurring activities, RPERT enables planners to quantify expected project completion probabilities while maintaining compatibility with conventional scheduling calculations. The software architecture comprises four interconnected functional modules: user interface, database, execution and data processing. A comprehensive illustrative case study is employed to evaluate the proposed system, demonstrating its computational accuracy, practical feasibility and methodological validity. The findings establish RPERT as a reliable decision-support tool for incorporating duration uncertainty into repetitive construction scheduling and facilitating probability-based assessments of contractual completion targets.

1. Introduction

Developed in 1957 by the United States Navy Special Projects Office to support the coordination of the Polaris nuclear submarine missile programme, PERT represented a significant advancement in the planning, monitoring and probabilistic scheduling of complex, large-scale projects. Unlike conventional management approaches, which depended largely on historical performance records,

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PERT established a forward-looking analytical framework that employed statistical methods to estimate the probability of achieving predetermined project milestones. Its application enables project managers to assess ongoing activities, identify critical dependencies, anticipate potential scheduling constraints and redistribute resources from activities with greater scheduling flexibility towards those subject to tighter operational restrictions. Through systematic representation of project activities and their precedence relationships, PERT provides an integrated perspective on project progress while revealing how managerial interventions may influence interconnected operations.

A fundamental component of this methodology is the probabilistic treatment of activity duration uncertainty through three-point time estimation. Specifically, the optimistic duration (a) represents the shortest feasible completion time under exceptionally favourable conditions, with an estimated occurrence probability of approximately one percent. Conversely, the pessimistic duration (b) corresponds to the longest anticipated completion time under highly unfavourable circumstances, likewise associated with a one percent probability. The most likely duration (m) reflects the statistically typical time required to complete an activity under normal operating conditions. These three parameters are mathematically synthesized to calculate the expected completion duration (μ) utilizing the weighted average formula $\mu = \{a + 4m + b\}/\{6\}$, while the statistical dispersion or standard deviation (s) of the activity is derived through the equation $s = \{b - a\}/\{6\}$, thereby establishing a robust mathematical foundation for analyzing overall project completion risks and scheduling durations.

1.1 Terminology

The fundamental terminology and analytical concepts underlying PERT are established in previous studies [8; 15] and can be summarised as follows:

2. PERT Event: A distinct milestone indicating the commencement or completion of one or more activities. Although an event requires neither time nor resources, its occurrence depends on the successful completion of every preceding activity connected to that milestone.
3. Predecessor Event: An event that directly precedes another event within the project network, without any intermediate event separating them. A predecessor event may result from the completion of multiple activities.
4. Successor Event: An event occurring immediately after another event, with no intervening events. Its occurrence may depend on the completion of more than one preceding activity.
5. PERT Activity: An operational task connecting two events within a project network. Unlike an event, an activity requires a measurable duration and consumes resources, including labour, materials, equipment and workspace. Its execution depends on the occurrence of the preceding event.
6. Optimistic Time: The shortest feasible duration for completing an activity when operating conditions are more favourable than normally anticipated.
7. Pessimistic Time: The longest reasonably anticipated activity duration under adverse operating circumstances, excluding exceptional catastrophic events.
8. Most Likely Time: The most realistic estimate of activity duration when execution proceeds under normal working conditions.
9. Expected Time: The estimated average duration required to complete an activity, reflecting the mean completion time anticipated across numerous repetitions over an extended period, rather than the duration of a single execution under normal conditions.

10. Float or Slack: The permissible delay in completing an activity without affecting subsequent activities or the overall project completion date. Free float represents the allowable delay without postponing successor activities, whereas total float indicates the maximum delay that does not extend overall project duration.
11. Critical Path: The longest continuous sequence of dependent activities connecting the initial and terminal project events. This sequence establishes the overall project completion duration, meaning that delays affecting critical-path activities directly extend the project schedule by at least the corresponding delay unless corrective measures are implemented.
12. Critical Activity: An activity characterised by zero total float. However, the presence of zero float does not necessarily establish that an activity belongs to the identified critical path.
13. Lead Time: The required time by which a predecessor event must be completed to ensure sufficient duration for intervening activities before a specified PERT event is achieved.
14. Lag Time: The earliest permissible time at which a successor event may occur following a specified PERT event.
15. Slack: A measure of the additional time and resources available for achieving a particular event relative to its scheduled requirements. Positive slack indicates progress ahead of schedule, negative slack signifies a delay, and zero slack reflects adherence to the established schedule.
16. Fast Tracking: A schedule-compression strategy involving the concurrent execution of a greater number of critical activities that would otherwise be undertaken sequentially.
17. Crashing Critical Path: A schedule-acceleration approach that reduces overall project duration by shortening the execution times of activities situated along the critical path [8; 15].

2. RESEARCH BACKGROUND

Construction scheduling methodologies are selected according to project-specific characteristics, particularly operational scale, structural complexity, execution duration, resource availability and stakeholder expectations. These considerations generally distinguish construction projects into repetitive and non-repetitive categories, as illustrated in Figure 1. Historically, non-repetitive construction planning employed graphical scheduling techniques, notably the Gantt chart, introduced during World War I, which represented activities vertically against a horizontal timescale. Subsequent developments in 1957 established more advanced network-oriented approaches, including the Critical Path Method (CPM) and PERT. Initially developed to coordinate extensive, non-recurring defence and aerospace programmes, these methodologies facilitated the management of interdependent activities, synchronisation of concurrent operations and reduction of scheduling inefficiencies.

Over the past three decades, PERT has evolved into a widely accessible, computer-supported planning approach that assists in allocating labour, financial resources and construction materials more efficiently. In contrast, repetitive construction projects encompass linear and non-linear operations, with linear projects involving successive, comparable production stages, as commonly encountered in multi-storey buildings and residential developments. Their recurring operational characteristics necessitate specialised scheduling approaches, including the Vertical Production Method (VPM) and LOB. Originally introduced by Goodyear in 1941 and subsequently applied extensively during World War II, LOB provides a systematic means of monitoring production progress, evaluating operational status and controlling the timing of repetitive activities.

Drawing upon the complementary capabilities of these established approaches, the present research develops RPERT as an integrated computational framework for probabilistic scheduling in repetitive construction environments. The proposed software combines LOB-based scheduling

parameters for single-crew and multiple-crew arrangements with the stochastic analytical principles of PERT. This integration enables the estimation of project completion probabilities under uncertain durations of identical recurring activities, particularly where completion must satisfy predetermined contractual deadlines. The computational performance and practical applicability of RPERT are evaluated through an illustrative case study, in which software-generated probability estimates are compared with corresponding manually derived analytical results. The study concludes by consolidating the principal findings concerning the proposed system's functionality, computational accuracy and potential contribution to repetitive construction project scheduling.

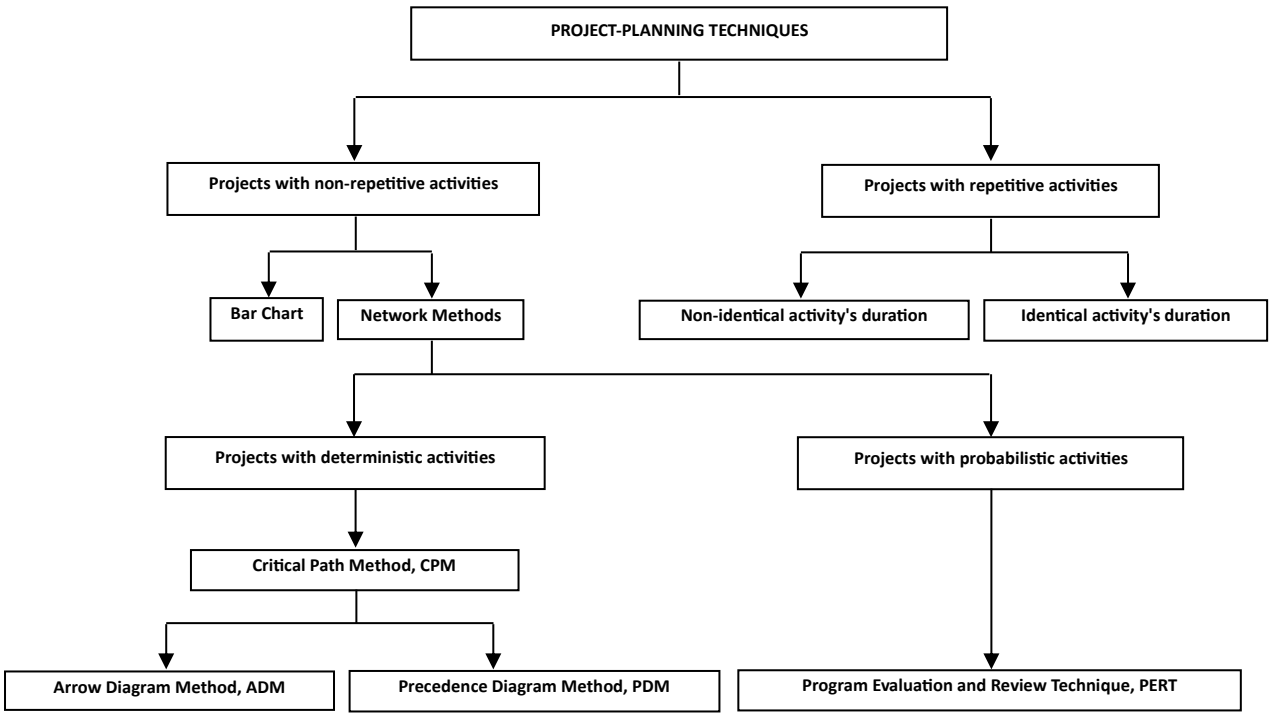


Fig.1: Taxonomy of the Existing Planning Techniques for Construction Projects

The extensive application of CPM and PERT in project scheduling has generated considerable interest among construction practitioners and academic researchers. Within PERT networks, reliable estimation of project completion time is essential for effective planning and competitive bidding. However, the computational complexity associated with estimating completion durations has encouraged the development of numerous approximation techniques, many of which overlook dependencies between network paths. Such dependencies arise when different paths share common activities, thereby influencing their respective completion times. To address this limitation, a simplified approximation procedure was introduced that explicitly incorporates inter-path dependencies while retaining the conventional critical-path concept [26].

The approach separates critical-path activities into independent and dependent components, with the latter comprising activities shared across multiple critical paths. Order statistics are subsequently employed to estimate the duration associated with these dependent components. Initially demonstrated for normally and exponentially distributed activity durations, the underlying methodology is potentially applicable to other probability distributions. Simulation outcomes were adopted as reference estimates of project completion time for evaluating approximation accuracy. The findings indicated that mixture distributions provide a more representative characterisation of

completion-time behaviour, while the proposed approximation produces estimates more closely aligned with simulation results [26].

Despite more than five decades of development following the introduction of CPM and PERT, conventional project scheduling systems have continued to exhibit limitations in providing calibrated and empirically validated completion-time distributions without demanding complex user inputs. Although contemporary Decision Support Systems (DSS) incorporate increasingly sophisticated project management capabilities, comparable improvements in stochastic scheduling analysis have remained limited. In response, PERT 21 was developed as an alternative probabilistic analytical framework grounded in validated theoretical principles [36]. Designed for integration with existing DSS platforms, this approach enables improved stochastic analysis without requiring organisations to replace their established project management infrastructure. Furthermore, sequencing and schedule-crashing formulations originally associated with CPM were adapted within PERT 21 to accommodate uncertainty in project execution [36].

Another important methodological development concerns the reduction of unnecessary dummy arcs in PERT networks. A network-generation algorithm was proposed that sequentially applies seven transformation rules to eliminate redundant dummy arcs while preserving the precedence constraints specified in the scheduling table [27]. Drawing upon graph-theoretical properties of line graphs, the final transformation produces a network containing the minimum required number of dummy arcs. Implementation through C++ Builder demonstrated that the procedure could generate minimal PERT networks within relatively short computational periods. Its applicability was also maintained when the original network contained transitive arcs, thereby extending its practical usefulness across different precedence structures [27].

Research into project acceleration has additionally examined alternative strategies for crashing stochastic activity networks. A simulation-based crashing procedure was evaluated using three representative PERT networks and three hypothetical datasets, with the computational model implemented in C++ [19]. Across the examined datasets, the proposed procedure substantially reduced the mean combined cost of crashing and schedule overruns relative to the conventional PERT approach. Reported cost savings ranged from 60.6% to 56.2%, indicating the potential economic advantages of incorporating simulation-based decision rules into project acceleration strategies [19]. The probabilistic representation of project evolution has also been investigated through analytical formulations based on alternative sequences of events. One such approach examined stochastic project scheduling from the perspective of possible event sequences, designated PERT-paths, extending from project initiation to completion [30]. Rather than relying exclusively on individual activity durations, the methodology considers the mean duration, variance and occurrence probability associated with each possible path. Analytical expressions were developed to estimate the mean and variance of overall project duration from these path-specific characteristics. Application to two illustrative cases enabled comparison with conventional PERT estimates and simulation outcomes, demonstrating differences in the accuracy of the competing estimation procedures [30].

For projects characterised by poorly structured objectives and ambiguous implementation requirements, Programme Analysis, Control and Evaluation (PACE) was introduced as a complementary extension of PERT [29]. The methodology addresses situations in which uncertainty concerns either the intended project objectives or the procedures required to achieve them. Its participatory structure facilitates the identification and monitoring of milestones, clarification of expected deliverables and specification of the activities necessary to accomplish agreed objectives. Particle-swarm optimization has been applied to balance duration, cost, resource fluctuation, quality, and schedule flexibility in construction planning [29]. The reliability of conventional PERT approximations has also been questioned through simulation-based investigations of

project completion estimates. By examining illustrative examples drawn from two established instructional texts, simulation analysis demonstrated the nature and magnitude of approximation errors associated with traditional PERT calculations [38]. These findings raised concerns regarding the practical reliability of simplified project-duration estimates and highlighted opportunities to improve the teaching and application of probabilistic scheduling methods. Although simulation offers a more comprehensive means of evaluating project completion uncertainty, introductory treatments frequently acknowledge its relevance before returning to conventional analytical approximations, potentially creating an overly favourable impression of their estimation accuracy [38].

Resource limitations introduce further computational difficulties into stochastic project scheduling. A metaheuristic solution was developed for the Resource-Constrained Project Scheduling Problem (RCPSP), considering PERT networks containing activities with uncertain durations and requirements for multiple resource categories [7]. Because the underlying optimisation problem is NP-hard, a Hybrid Scatter Search (HSS) algorithm was employed to obtain computationally feasible solutions. Project completion times were evaluated through Monte Carlo (MC) simulation, while algorithmic performance was assessed against optimal solutions for smaller networks and established benchmark instances representing larger practical problems. The computational findings indicated satisfactory solution quality for both relatively small project networks and more realistic scheduling environments [7].

Beyond resource-constrained formulations, activity sequencing has been examined under generalised cost structures. A scheduling problem involving dependent activities, unrestricted resource availability and arbitrary convex cost functions was investigated through algorithmic optimisation [11]. A generic solution procedure was developed for the general formulation, while a computationally efficient polynomial-time algorithm was established for a particular restricted case. These contributions extended the mathematical treatment of project scheduling beyond conventional duration-oriented network calculations [11]. The statistical assumptions underlying PERT duration estimation have received considerable attention, particularly regarding the interpretation of beta-distribution parameters. Stochastic project-crashing studies have combined PERT/CPM, Monte Carlo simulation, and mathematical programming to represent correlated duration and cost uncertainty [21]. regarding the conventional assumption of constant variance for a specified duration range [23]. The revised formulation retains the original PERT expression for expected duration while modifying the variance calculation. Importantly, it ensures consistency between the elicited most likely activity duration and the mode of the corresponding beta distribution. This resolves an inconsistency in conventional parameter estimation, whereby deriving beta-distribution parameters from the standard PERT mean and variance does not necessarily preserve the intended modal value [23].

Applications of stochastic scheduling have also extended to production management, where accurate completion-time estimates influence organisational competitiveness and timely market delivery. A computational approach was developed to support production scheduling decisions by examining the effects of alternative standard-deviation estimation procedures on predicted completion times [34]. Within the PERT framework, stochastic completion-time estimation incorporates the approximate standard deviation σ and expected activity duration Texp, while the scheduled project duration Tsch is particularly sensitive to the assumed variability. The widely adopted N-method was evaluated under two estimation conditions: situations in which Tmin and Tmax are relatively close to Tmean, and situations involving extreme minimum and maximum duration estimates. Different permutations and combinations of Tmin, Tmean and Tmax were processed using purpose-developed software to calculate Tsch. The resulting estimates from three investigated methods were subsequently compared with observed project completion durations over time [34].

Multi-objective optimisation has provided another means of addressing uncertainty in resource allocation. Multi-objective scheduling research has addressed uncertain resource allocation and project priorities through

mathematical optimization of time–cost trade-offs [4]. The quantities of resources assigned to individual activities constitute the decision variables. The resulting optimal-control formulation incorporates four competing objectives: minimising total direct project cost, reducing mean completion time, minimising completion-time variance and maximising the probability of completing the project before a specified threshold. Multi-objective scheduling research has addressed uncertain resource allocation and project priorities through mathematical optimization of time–cost trade-offs [4].

Related optimisation research considered resource allocation across service stations within finite-capacity dynamic PERT networks [40]. The proposed model incorporated three objectives: minimising direct service-station costs per period, reducing mean project completion time under steady-state conditions and minimising the steady-state probability that the system is empty. The network was represented through interconnected service stations, each containing a single server, with independent exponentially distributed service times corresponding to activity durations. Contractual penalties, early-completion incentives, and quality costs have been integrated into crashing-based time–cost trade-off models [40]. The allocation of available schedule flexibility has similarly been examined as a formal scheduling problem. Robust line-of-balance scheduling has been investigated to protect repetitive project deadlines while balancing cost and schedule stability [10]. Under this procedure, supplementary time is assigned according to activity duration, subject to the requirement that no individual path exceeds the overall project completion time. The resulting allocation supports the construction of a feasible PERT schedule. Robust line-of-balance scheduling has been investigated to protect repetitive project deadlines while balancing cost and schedule stability [10].

For dynamic project environments involving shared facilities, the relationship between project duration and expenditure becomes more complex. Chance-constrained and expected-value formulations have been developed to balance construction duration, cost, and resource use under uncertainty [5]. Because successive projects utilise common facilities, their execution cannot be treated as independent. The scheduling environment was therefore represented as a queueing network, with service times corresponding to activity durations and node arrivals governed by renewal processes. Initially, the dynamic network was transformed into an appropriate stochastic representation, allowing project completion-time distributions to be obtained through a continuous-time Markov chain. The subsequent optimisation stage formulated the time–cost relationship as a problem involving four conflicting objective functions, addressed through a discrete-time approximation. Chance-constrained and expected-value formulations have been developed to balance construction duration, cost, and resource use under uncertainty [5].

Mathematical programming has also been applied to the acceleration of critical-path activities by reducing pessimistic duration estimates. Simulation-based optimization has been used to examine project crashing scenarios and quantify time–cost trade-offs under uncertain durations and budget constraints [1]. The optimisation objective was to minimise pessimistic completion times through strategically allocated expenditure. Simulation-based optimization has been used to examine project crashing scenarios and quantify time–cost trade-offs under uncertain durations and budget constraints [1]. Cost allocation associated with project delays represents another area of PERT-related research. A weighted serial cost-sharing rule was developed to distribute delay-related costs among relevant activities or participants within a project network [9]. The proposed allocation mechanism was supported by a formal mathematical characterisation and a polynomial-time computational algorithm. Its application demonstrated how cooperative cost-sharing principles could be incorporated into network-based project delay management [9].

Conventional PERT risk assessment has also been modified to improve the representation of engineering project progress. An enhanced scheduling procedure was proposed to address limitations in estimating activity durations and identifying representative project paths within interconnected networks [37]. The approach first revised activity-duration calculations to obtain estimates more closely reflecting actual operating conditions. An equivalent-weight probability procedure was then employed to adjust the principal project path, thereby improving the correspondence between calculated and anticipated project durations. An engineering case study was used to demonstrate the feasibility of the revised methodology [37] Further statistical investigations have identified restrictions associated with the probability distributions implicitly adopted in conventional PERT calculations. In particular, standard formulations tend to produce relatively light-tailed distributions and

impose constant variance conditional on the specified activity-duration range. Stochastic project-crashing studies have combined PERT/CPM, Monte Carlo simulation, and mathematical programming to represent correlated duration and cost uncertainty [21]. Stochastic project-crashing studies have combined PERT/CPM, Monte Carlo simulation, and mathematical programming to represent correlated duration and cost uncertainty [21].

The analytical treatment of stochastic project networks has additionally been extended to independently distributed activity durations without restricting their underlying probability distributions. A network-based procedure was developed in which, for each selected path, all associated activities were identified to construct a subnetwork whose completion time matched that of the corresponding path [39]. Elapsed time was introduced as a supplementary variable, enabling the subnetwork to be represented as a Markov skeleton process with a state space determined by its structural configuration. The backward equation was subsequently employed to derive the subnetwork completion-time distribution, providing an analytical basis for evaluating probabilistic scheduling outcomes [39]. The independence assumptions commonly adopted in project-duration estimation have also been subjected to theoretical and computational examination. Conventional approaches frequently assume that activity durations are mutually independent random variables, while some additionally treat completion times across different paths as independent [6]. Using multivariate analytical techniques, the consequences of disregarding correlation were investigated, particularly regarding the direction of estimation errors in the classical PERT mean project-duration calculation. Simulation experiments further examined how dependence between activities influences the normality of path-duration distributions. The investigation highlighted the importance of distinguishing activity-level independence from path-level independence when evaluating stochastic project completion times [6].

The implementation of established network algorithms in accessible computational environments has received attention as an alternative to specialised scheduling software. A spreadsheet-based implementation of the conventional PERT/CPM procedure was developed to identify critical paths within project networks [33]. This approach reflected growing interest in using spreadsheet platforms to deliver quantitative management techniques directly to end users, thereby improving the accessibility of established project scheduling calculations [33]. Stochastic dynamic programming has likewise been employed to establish bounds on project completion durations under changing external conditions. Markov and semi-Markov decision-process approaches have been used to estimate project makespan distributions and assess resource-sensitive time–cost trade-offs under stochastic activity durations [3]. Unlike conventional stationary formulations, the activity-duration distributions were assumed to vary in response to external social and economic uncertainties, including strikes and inflation. These changes were represented through a separate continuous-time Markov process, allowing the model to incorporate evolving environmental conditions. Markov and semi-Markov decision-process approaches have been used to estimate project makespan distributions and assess resource-sensitive time–cost trade-offs under stochastic activity durations [3].

Efforts to simplify probabilistic scheduling have also produced integrated CPM/PERT simulation procedures. A computational model was introduced and evaluated through comparison with conventional CPM/PERT analytical calculations [25]. The results indicated that the simplified simulation framework retained computational robustness while providing a relatively straightforward means of examining project network behaviour. Its principal contribution was to facilitate scenario evaluation through simulation and risk analysis, offering project managers an accessible alternative for assessing scheduling decisions under uncertainty [25]. Reducing the information required for probabilistic estimation constitutes another important direction in PERT development. A simplified formulation was proposed in which the conventional three-point duration estimation procedure was replaced by a two-point approach [12]. The method assumes normally distributed activity durations rather than the beta distribution employed in traditional PERT,

requiring only the most likely and pessimistic duration estimates. By reducing the number of input parameters, the procedure decreases the effort involved in project scheduling. Evaluation across 12 project networks produced duration, variance and completion-probability estimates comparable to those generated by conventional PERT [12].

Simulation-based optimisation has further been used to determine appropriate activity-crashing sequences and expenditure strategies. A computational model was developed to identify the order in which activities should be accelerated and the corresponding crashing policy that minimises expected total expenditure, comprising crashing costs and schedule-overrun penalties [20]. The optimisation incorporates a specified penalty function for delayed project completion, allowing alternative acceleration strategies to be assessed in terms of their combined financial consequences. The investigation extended earlier doctoral research on stochastic project crashing [18]. Alternative statistical approximations have also been developed for estimating activity-duration moments from three-point inputs. Revised expressions for the mean and variance were proposed using pessimistic, optimistic and most likely duration estimates [31]. Numerical comparisons with actual values indicated that the proposed approximations achieved greater accuracy than both conventional PERT estimates and previously modified formulations. An additional methodological advantage was that the expressions did not require assumptions concerning the parameters of the beta distribution [31].

The probability that individual activities or complete paths become critical has been investigated through exact analytical formulations. Recent repetitive construction scheduling models accommodate multiple crews and uncertain activity durations using probabilistic distributions including PERT, triangular, and normal formulations [14]. The approach assumes that individual activity durations are discrete random variables and derives exact expressions for the corresponding criticality measures. Recent repetitive construction scheduling models accommodate multiple crews and uncertain activity durations using probabilistic distributions including PERT, triangular, and normal formulations [14]. Project evaluation has also been extended beyond completion-time estimation to incorporate implementation quality and outcome assessment. An EXemplary Program Evaluation and Review Technique (EXPERT) framework was introduced to support the evaluation of project outcomes and the quality of implementation processes [24]. This development broadened the application of network-oriented evaluation principles to additional dimensions of project management performance [24].

Uncertainty in project execution has prompted reconsideration of the conventional interpretation of non-critical path float. A revised path-float concept was developed for PERT networks to address the potentially misleading scheduling information generated by deterministic CPM calculations [43]. The proposed formulation evaluates non-critical path flexibility in relation to specified project completion probabilities and required durations. Application to an illustrative network demonstrated consistent path-float estimates under the prescribed probabilistic and temporal conditions. The approach therefore provides project managers and construction planners with additional information for assessing scheduling flexibility under uncertain activity durations [43]. Further refinement of PERT activity-duration estimation has been achieved by revisiting the assumptions underlying established approximation procedures. An improved formulation was proposed based on a modification of an earlier approximation [16], using a more general assumption concerning the relevant distributional parameters [28]. The resulting expressions estimate the mean and variance of PERT activity durations while relaxing selected restrictions of the original formulation. Numerical comparisons indicated that the revised and conventional PERT methods produced approximately equivalent mean and variance estimates under the examined conditions [28].

More recent investigations demonstrate a gradual transition from conventional analytical refinements towards computationally integrated, data-driven and hybrid probabilistic scheduling methodologies. Initial contributions concentrated on correcting limitations in established network

calculations through mathematically formulated alternatives, including the Manual Project Evaluation and Review Technique [37]. Complementary investigations examined the influence of alternative probability distributions on overall project-duration characteristics [22], while subsequent methodological revisions reconsidered the assumptions underlying conventional three-point estimation procedures in contemporary project management applications [36]. As computational capabilities expanded, probabilistic scheduling research increasingly incorporated predictive modelling to address operational uncertainty and improve duration forecasting. Data-mining techniques and machine learning algorithms were employed to identify potential sources of delay and estimate project completion risks [17]. Hybrid artificial intelligence (AI) models were subsequently introduced to represent uncertain environmental influences within stochastic scheduling frameworks [41]. In parallel, Building Information Modelling (BIM) was integrated with genetic algorithms to automate project network generation and support more efficient scheduling processes [13]. These developments marked an increasing emphasis on computational techniques capable of processing complex project information and supporting automated decision-making.

Further methodological advances addressed the interactions between construction activities, resource flows and competing project objectives. Discrete-event simulation was combined with system dynamics to represent activity interdependencies and material movements within construction operations [2]. Multi-objective metaheuristic optimisation was employed to investigate trade-offs among project duration, expenditure and labour utilisation [35], while dynamic risk-path modelling frameworks were developed to characterise structural complexity and uncertainty propagation across interconnected project networks [32]. Collectively, these approaches extended probabilistic scheduling beyond isolated activity-duration estimation towards a broader representation of operational interactions and competing management priorities. Recent contributions have continued to refine completion-time forecasting through improved analytical and simulation-based formulations. Modified PERT (M-PERT) approaches have been developed to strengthen probabilistic duration estimation [46], while discrete-event simulation has been integrated with PERT-based risk assessment to examine uncertainty in project execution [42]. Hybrid MC simulation frameworks embedded within CPM networks have additionally provided alternative mechanisms for evaluating possible project completion outcomes under variable activity durations [48]. These developments illustrate the increasing integration of established network scheduling principles with computational procedures designed to represent stochastic project behaviour.

Parallel progress in digital construction management has strengthened the automation and predictive capabilities of scheduling systems. BIM-based probabilistic scheduling engines have been developed to incorporate uncertainty assessment into digitally coordinated project workflows [49]. Advanced machine learning models have been applied to forecast construction delay risks [17], while AI-supported resource allocation frameworks have facilitated computational assessment of alternative resource deployment strategies [51]. For repetitive construction operations, ensemble machine learning methods have been specifically designed to predict variations in labour productivity across recurring work sequences [52]. Broader predictive analytics frameworks have further extended data-driven approaches to project performance forecasting and scheduling decision support [53]. Within repetitive construction scheduling, particular attention has been directed towards integrating uncertainty analysis with production-oriented planning techniques. Stochastic LOB formulations have been combined with metaheuristic search algorithms to improve scheduling decisions under variable activity conditions [54]. The practical performance of LOB has also been examined in large-scale repetitive housing projects, providing application-based evidence concerning its suitability for monitoring and coordinating recurring construction operations [55]. These

investigations reinforce the importance of scheduling methods that account for both the sequential organisation of repetitive activities and the uncertainty affecting their execution durations.

The convergence of probabilistic network analysis and repetitive production scheduling represents a further development in this research area. Integrated approaches combining LOB and PERT have been investigated for risk assessment in linear infrastructure projects, allowing uncertainty in activity durations to be considered alongside the spatial and sequential characteristics of repetitive operations [56]. More recently, discrete-event simulation has been coupled with multiple-crew LOB scheduling mechanisms to evaluate the probability of completing repetitive construction projects within contractual deadlines [57]. These contributions establish a methodological foundation for integrating crew deployment configurations, recurring activity relationships and stochastic completion-time analysis within a unified computational scheduling framework.

3. EMPLOYED TECHNIQUES

The proposed model is formulated by integrating six methodological components that establish the scheduling relationships and probabilistic characteristics of repetitive construction activities: (1) The Precedence Diagram Method (PDM) is employed to define activity dependencies and represent the operational sequence within each project stage; (2) LOB is applied to establish the scheduling configuration of each activity (k), where k = 1, 2, …, K, across all stages of the repetitive project, thereby representing its progression within the overall project timeframe; (3) The conventional LOB scheduling representation is transformed into a modified CPM framework to facilitate the required analytical calculations; (4) PERT is incorporated to estimate the probability of completing a repetitive construction project, characterised by identical activity durations, within a predetermined contractual timeframe; (5) For each activity (k), where k = 1, 2, …, K, a time buffer (TBk,kk) is defined at every project stage (s), where s = 1, 2, …, S, representing the interval between the completion of activity (k) and the commencement of its succeeding activity (kk); and (6) A stage buffer (SBk,kk) may be specified between any two consecutive activities to maintain a predetermined separation in the number of stages at a given time, thereby satisfying practical or technological requirements. The magnitude of this stage buffer is determined by the project planner according to the operational constraints governing the corresponding activities.

4. CONSIDERED ASSUMPTIONS

The proposed scheduling framework is developed under seven fundamental assumptions governing crew deployment, activity durations and project execution conditions: (1) Continuous crew utilisation is maintained throughout each assigned activity, such that once a crew commences work at the first stage, it proceeds through all subsequent stages at an unchanged production rate without experiencing idle periods until completion of the final stage; (2) Activity-duration estimation is treated as an essential component of network analysis and incorporates three temporal scenarios: (I) the optimistic duration (Dmin), representing the shortest anticipated completion time; (II) the pessimistic duration (Dmax), indicating the longest anticipated completion time; and (III) the most likely or normal duration (Dnor), reflecting completion under typical operating conditions; (3) Each recurring activity is assigned a uniform average duration across all project stages to preserve a constant production rate. Where adjustments are necessary to achieve a feasible overall project duration, identical modifications must be applied to the corresponding activity duration at every stage; (4) The scheduling methodology accommodates both single-crew and multiple-crew deployment configurations; (5) Learning effects associated with repeated activity execution, whereby

successive repetitions progressively reduce actual completion durations, are excluded from the analysis; (6) Each activity is executed on only one construction unit at any given time; and (7) Project execution is assumed to proceed without disruptions arising from unforeseen incidents during construction.

5. MATHEMATICAL FORMULATIONS

First phase, the required mathematical formulations are established to determine the total activity duration at each stage of the construction project. These calculations incorporate the corresponding activity quantity and its maximum, minimum and normal production rates, as expressed in the following equations:

$$\text{Dmin} = \text{BQ} \div \text{PRmax} \quad \text{----------} \quad (1)$$

$$\text{Dmax} = \text{BQ} \div \text{PRmin} \quad \text{----------} \quad (2)$$

$$\text{Dnor} = \text{BQ} \div \text{PRnor} \quad \text{----------} \quad (3)$$

$$\text{Te} = (\text{Dmin} + \text{Dmax} + 4 \times \text{Dnor}) \div 6 \quad \text{----------} \quad (4)$$

$$\sigma = (\text{Dmax} - \text{Dmin}) \div 6 \quad \text{----------} \quad (5)$$

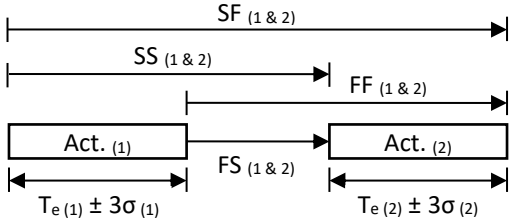


Fig.2: Network Sample Per Stage

Where Dmin, Dmax and Dnor denote the optimistic, pessimistic and most likely activity durations, respectively, expressed in days; BQ represents the budgeted quantity of the activity, measured in units; and PRmax, PRmin and PRnor indicate the optimistic, pessimistic and most likely production rates, respectively, expressed in units/day. Te denotes the equivalent activity duration in days, whereas σ represents the corresponding standard deviation. The precedence relationships within any path of the construction project network are characterised by four temporal constraints: FS (1 & 2) specifies the finish-to-start lag, measured in days, between the completion of a predecessor activity and the commencement of its successor; SS (1 & 2) represents the start-to-start lag between the commencement times of preceding and succeeding activities; FF (1 & 2) defines the finish-to-finish lag between their respective completion times; and SF (1 & 2) indicates the start-to-finish lag between the commencement of the predecessor and the completion of the successor, with all lags expressed in days.

Equations (1-3) establish the optimistic, pessimistic and most likely durations of individual activities at each project stage. Subsequently, Equation (4) determines the equivalent activity duration, while Equation (5) calculates its standard deviation. These relationships are illustrated in Figure (2). Second phase, the mathematical formulations are extended to determine total activity durations by incorporating the repetitive execution of activities throughout the construction project. The corresponding calculations are presented in Equations (6&7) as follows:

$$\text{Te}^* = \text{Te} \times \{1 + [(\text{N-1}) \div (\text{NC})]\} \quad \text{----------} \quad (6)$$

$$\sigma^* = \text{N} \times \sigma \quad \text{----------} \quad (7)$$

Where Te* is the modified equivalent duration by (days) for each activity at all stages as one unit; N is the number of repetitive stages into studied construction project; NC is the number of crews; and σ* is the modified standard deviation of studied activity at all stages as one unit.

Third phase, the mathematical framework is further developed to calculate the overall project duration by accounting for the repetitive nature of construction activities across successive stages. The required analytical relationships are expressed in Equations (8-16) as follows:

$$TB\ (1\&2) \geq FS\ (1\ \&\ 2) \quad ------ (8)$$

$$TB\ (1\&2) \geq SS\ (1\ \&\ 2) - Te\ (1) \quad ------ (9)$$

$$TB\ (1\&2) \geq FF\ (1\ \&\ 2) - Te\ (2) \quad ------ (10)$$

$$TB\ (1\&2) \geq SF\ (1\ \&\ 2) - Te\ (1) - Te\ (2) \quad ------ (11)$$

$$FS^*\ (1\ \&\ 2) \geq TB\ (1\&2) + Te\ (1) - Te^*(1) \quad ------ (12)$$

$$FS^*\ (1\ \&\ 2) \geq TB\ (1\&2) + Te\ (2) - Te^*\ (2) \quad ------ (13)$$

$$FS^*\ (1\ \&\ 2) \geq Te\ (1) \times SB\ (1\&2) - Te^*(1) \quad ------ (14)$$

$$FS^*\ (1\ \&\ 2) \geq Te\ (2) + Te\ (2) \times SB\ (1\&2) - Te^*(2) - Te\ (1) \quad ------ (15)$$

$$PD\ mean \geq \sum_{k=1}^{k=K} \{Te^*(k) + FS^*\ (k\ \&\ k+1)\} \quad ------ (16)$$

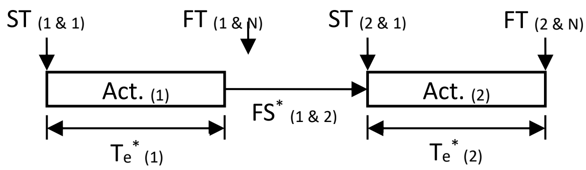


Fig.3: Network Sample at all Stages

Where TB (1&2) denotes the time buffer, expressed in days, separating the completion of a predecessor activity from the commencement of its successor at each stage along any construction project network path; SB (1&2) represents the stage buffer, measured in stages, between the commencement of two consecutive activities across all project stages; FS* (1 & 2) defines the modified finish-to-start relationship between successive activities, measured from the completion of the predecessor to the commencement of the successor, with all project stages treated as a single unit; and PD mean indicates the mean project duration, expressed in days, encompassing all stages along each network path, from the initial activity (k = 1) to the final activity (k = K). These relationships are illustrated in Figure (3).

Last phase, the mathematical formulations are applied to estimate the probability of completing the construction project within a specified duration, incorporating the effects of activity repetition across successive stages. The corresponding probabilistic calculations are presented in Equations (17-21) as follows:

$$PSD \geq \{\sum_{k=1}^{k=K} (\sigma^*k)\ \hat{}\ 2\}\hat{}\ 0.5 \quad ------ (17)$$

$$PD\ min = PD\ mean - (3 \times PS) \quad ------ (18)$$

$$PD\ max = PD\ mean + (3 \times PS) \quad ------ (19)$$

$$PD\ max \geq PD\ required \geq PD\ min \quad ------ (20)$$

$$Z = (\text{PD required} - \text{PD mean}) \div \text{PSD} \quad \text{----------} \quad (21)$$

Where PSD represents the standard deviation of the overall construction project duration across all stages for each network path, extending from the initial activity (k = 1) to the final activity (k = K). PD min and PD max denote the minimum and maximum project durations, respectively, with all construction stages considered as a single unit. Similarly, PD required specifies the required project completion duration, corresponding to the contractual timeframe. Z represents the standardised difference between the required and mean project durations, calculated using the standard deviation of the overall project duration across all stages.

By combining the Z value obtained from Equation (21) with the cumulative probability tables of the normal distribution, planners can estimate the probability of completing a repetitive construction project within a specified contractual duration. Conversely, a predetermined completion probability can be used alongside the same probability tables to determine the corresponding Z value through Equation (21) and subsequently establish the contractual duration required to achieve that probability. These complementary calculations are performed within the proposed RPERT framework, which integrates LOB scheduling for single-crew and multiple-crew configurations with the probabilistic analytical capabilities of PERT.

6. SOFTWARE IMPLEMENTATION

Figure 4 presents the architecture of the proposed RPERT software, developed in Java to support probabilistic scheduling through four interconnected functional modules. The user interface module facilitates interaction through two principal components: an input interface for specifying activity characteristics, precedence relationships and contractual completion requirements, and an output interface for displaying the calculated probability of completing the project within the prescribed timeframe. Data management is undertaken by a relational database module implemented using Derby, which maintains both user-defined parameters and computational results. The database organises information into six categories: general project characteristics, holidays, working exceptions, activity-specific parameters, activity dependencies and contractual completion deadlines.

The running module constitutes the analytical component of RPERT, employing purpose-developed Java algorithms to perform the required scheduling calculations. These include estimating optimistic, pessimistic and most likely activity durations at individual stages, determining equivalent durations and associated standard deviations, and aggregating the corresponding measures across the overall project.

The module also evaluates modified finish-to-start relationships between successive repetitive activities to establish minimum, maximum and mean project durations. These calculations subsequently provide the basis for estimating the probability of satisfying the specified contractual completion period. Communication between the software components is managed through the processing module, which incorporates Java and Derby database drivers to coordinate bidirectional information transfer. The Derby driver transfers user-defined inputs from the interface to the database and communicates the final analytical outputs to the display interface. Meanwhile, the Java driver retrieves stored parameters for execution within the running module and returns the resulting computational outputs to the central database. Collectively, these four modules provide an integrated computational environment for implementing RPERT-based probabilistic scheduling in repetitive construction projects.

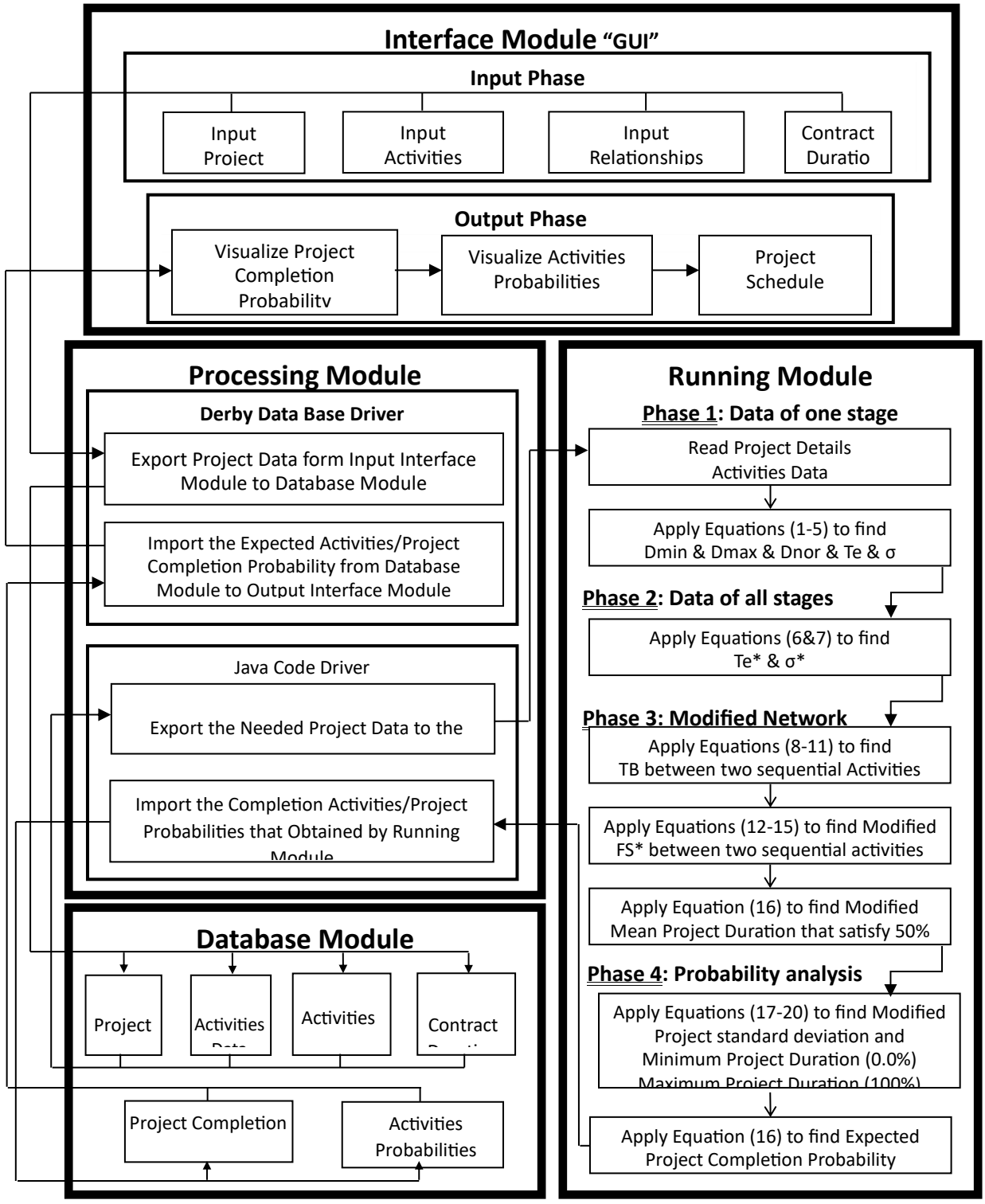


Fig.4: RPERT Main Modules

7. VERIFICATION OF SOFTWARE

This section evaluates the computational accuracy and practical applicability of RPERT through a small illustrative construction project, for which independently derived manual calculations are compared with software-generated results. The analysis is intended to verify the consistency of both approaches while demonstrating the system's ability to estimate project completion probabilities within specified contractual durations according to stakeholder requirements. The illustrative project comprises four activities connected through different precedence relationships, with the input parameters for a single construction stage presented in Figure (5) and Table (1).

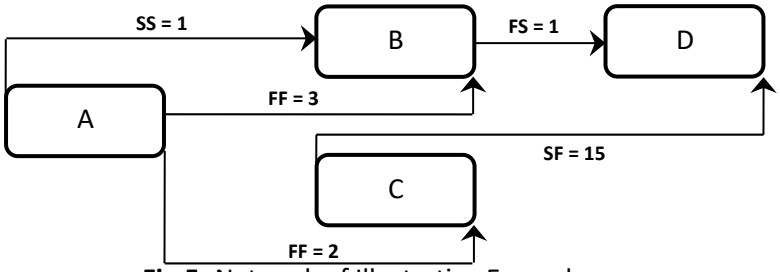


Fig.5: Network of Illustrative Example

Each activity offers three execution modes, characterised by distinct production rates (units/day) under a single-crew configuration. The activity-specific parameters associated with each mode are provided in Table (1) for one representative stage.

Table (1)
Available Project Activities Data and Mode Options Per Stage

Activity ID	Depends On	Relation Type	Lag Value	Stage Buffer	Mode Options	Quantity (units)	Production Rate (units / day)
			--	--	1	1,000	500
					2		400
					3		200
			1	2	1	2,000	600
			3		2		400
					3		300
			2	1	1	500	100
					2		50
					3		30
				0	1	200	80
			5	3	2		40
					3		40

The project consists of five identical repetitive stages, commencing on Thursday, 1st October 2026. Fridays and Sundays are designated as non-working days, with additional holidays scheduled for 10th October 2026 and 12th November 2026. However, Friday, 16th October 2026, and Sunday, 15th November 2026, are specified as working-day exceptions.

Table (2)
Analysed Project Activities Data

Activity ID	Dmin (days)	Dnor (days)	Dmax (days)	Te (days)	Te* (days)	σ	σ*
A	2.0	2.5	5.0	2.8	14.0	0.5	2.5
B	2.0	5.0	6.7	4.8	24.0	0.8	3.9
C	5.0	10.0	16.7	10.3	51.5	2.0	10.0
D	2.5	5.0	5.0	4.6	23.0	0.4	2.0

Based on these scheduling conditions, the analysis addresses three principal requirements: (1) determining the expected probability of project completion by 31th December 2026; (2) estimating the completion probability within 60 working days; and (3) identifying the expected project completion date corresponding to a 90% completion probability.

Table (3)
Modified Project Activities Relations

Activity ID	A	B	C	D
A		TB = -1.80	TB = -8.30	----
		FS* = -8.40	FS* = -11.2	----
B	TB = -1.80	2	----	TB = +1.00
	FS* = -8.40		----	FS* = -17.4
C	TB = -8.30	----		TB = +0.10
	FS* = -11.2	----		FS* = -14.6
D	----	TB = +1.00	TB = +0.10	
	----	FS* = -17.4	FS* = -14.9	

The application of Equations (1-16), as detailed in Tables (2&3) and Figure (6), yields a mean project duration of 62.4 working days.

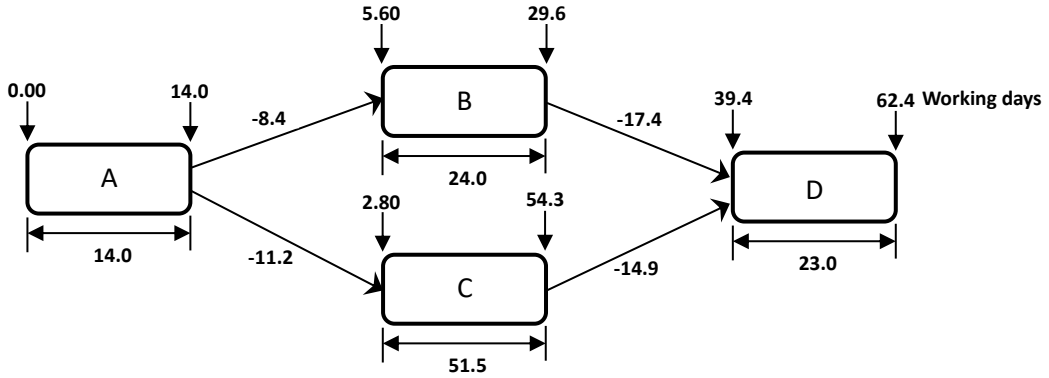


Fig.6: Solved Network of Illustrative Example

Subsequent calculations using Equations (17-20) establish a project duration standard deviation of 10.5 days, with minimum and maximum completion durations of 30.9 and 93.9 working days, respectively. Based on Equation (21), three scheduling outcomes are obtained: (1) a Z value of 0.34, corresponding to an expected completion probability of 63.3% for the first specified deadline; (2) a Z value of -0.23, indicating a completion probability of 40.9% within the second specified duration; and (3) a target completion probability of 90%, associated with a Z value of 1.28. Achieving the third target requires a contractual duration of 75.8 working days, corresponding to an expected completion date of 14th January 2027. These results provide the respective probabilistic estimates and scheduling requirements for the three scenarios examined.

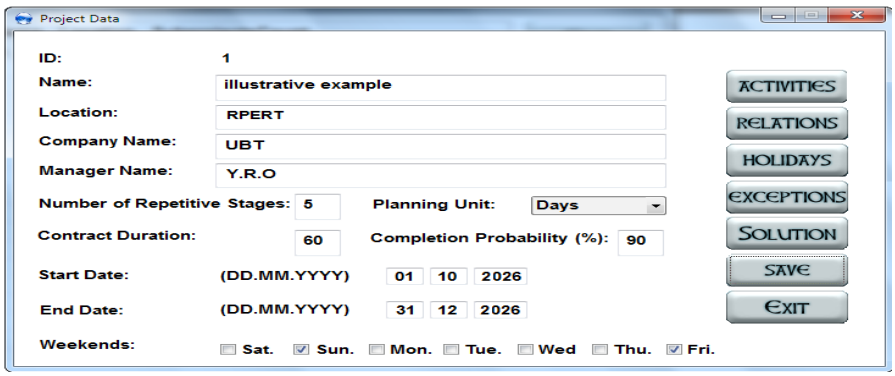


Fig.7: Project Details Form

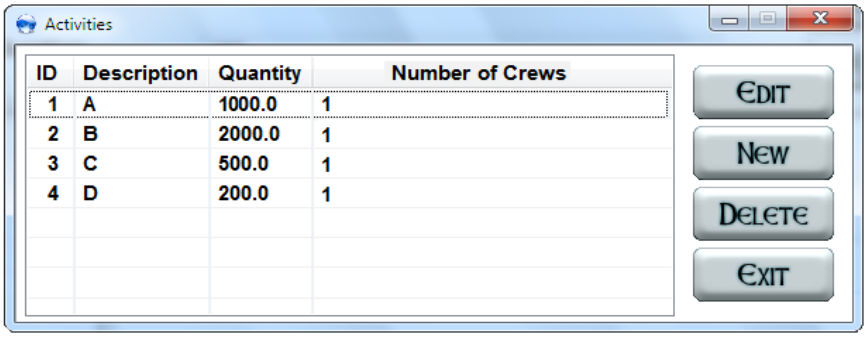


Fig.8: Project Activities Form

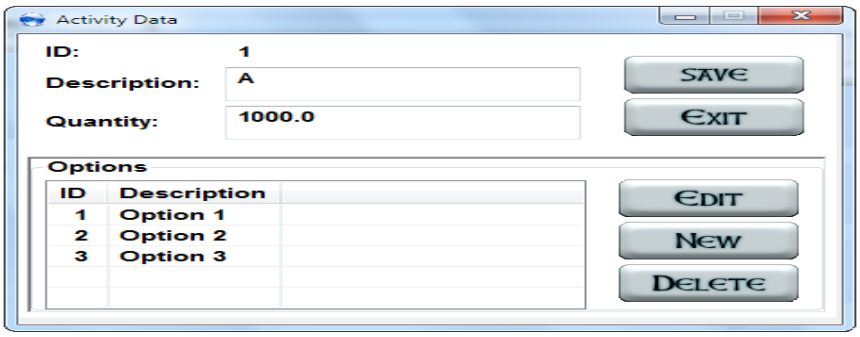


Fig.9: Project Activities Details Form



Fig.10: Project Activities Options Form

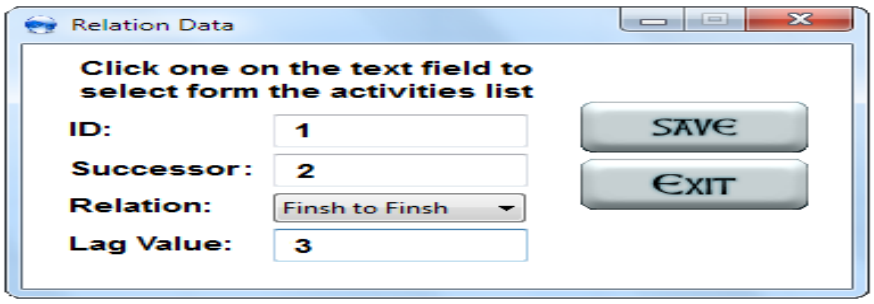


Fig.11: Project Activities Relationships Details Form

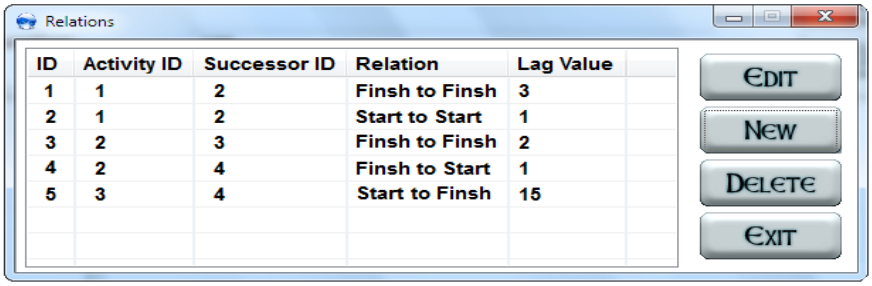


Fig.12: Project Activities Relationships Form

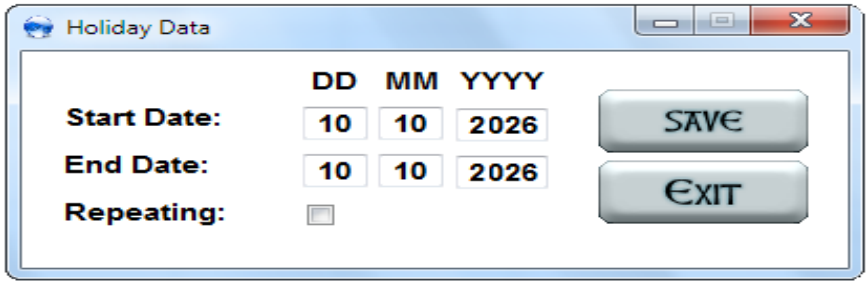


Fig.13: Project Holidays Details Form



Fig.14: Project Exceptions Details Form

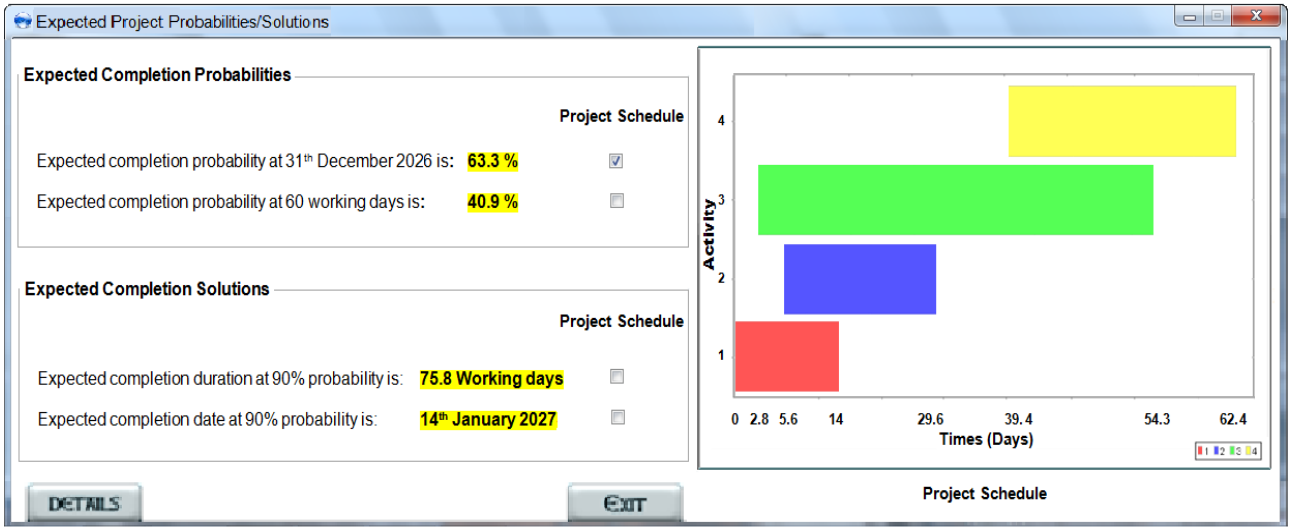


Fig.15: Expected Project Probabilities Solutions Form

The input data are entered through the RPERT input wizards, illustrated in Figures (7, 8, 9, 10, 11, 12, 13 and 14). Figure (15) presents the corresponding output wizard, which displays the final estimated completion probabilities for the repetitive construction project across the specified sets of completion durations considered in the analysis.

8. CONCLUSION

This study developed and validated RPERT, an automated scheduling framework integrating LOB principles with PERT-based probabilistic analysis to estimate the likelihood of completing repetitive construction projects within specified contractual durations. Implemented in Java, the system comprises four interconnected modules: a user interface for data entry and results visualisation, a database for organised information storage and retrieval, a running module for executing mathematical computations, and a processing module for coordinating data exchange between system components. By incorporating optimistic, most likely and pessimistic duration estimates for activities repeated across successive construction stages, RPERT enables probabilistic assessment of project completion while remaining compatible with established scheduling procedures. Its computational performance and practical applicability were examined through an illustrative case study, in which software-generated outputs were compared with manual calculations. The findings demonstrated the framework's ability to support completion probability estimation under scheduling uncertainty, highlighting its potential as a decision-support instrument for duration planning and resource-related decisions in repetitive construction projects.

9. ABBREVIATIONS

For convenience of discussion, the following abbreviations are used without special explanations:

ACI:	**A**ctivity **C**ritical **I**ndex;
ADM:	**A**rrow **D**iagram **M**ethod;
CPM:	**C**ritical **P**ath **M**ethod;
DSS:	**D**ecision **S**upport **S**ystems;
EXPERT:	**EX**emplary **P**rogram **E**valuation and **R**eview **T**echnique;
HSS:	**H**ybrid **S**catter **S**earch;
LOB:	**L**ine **O**f **B**alance;
MC:	**M**onte **C**arlo;
PACE:	**P**rogramme **A**nalysis, **C**ontrol and **E**valuation;

PCI:	**P**ath **C**ritical **I**ndex;
PDM:	**P**recedence **D**iagram **M**ethod;
PERT:	**P**rogram **E**valuation and **R**eview **T**echnique;
RCPSP:	**R**esource-**C**onstrained **P**roject **S**cheduling **P**roblem;
RPERT:	**R**epetitive-**P**rojects **E**valuation and **R**eview **T**echnique;
VPM:	**V**ertical **P**roduction **M**ethod.

References

[1] Senses, S., & Kumral, M. (2024). Trade-off between time and cost in project planning: A simulation-based optimization approach. Simulation, 100(2), 127–143. https://doi.org/10.1177/00375497231196889

[2] Alvanchi, A., Lee, S., & AbouRizk, S. (2011). Modeling Framework and Architecture of Hybrid System Dynamics and Discrete Event Simulation for Construction. *Computer-Aided Civil and Infrastructure Engineering*, *26*(2), 77-91. https://doi.org/10.1111/j.1467-8667.2010.00650.x

[3] Sadri, S., Fatemi Ghomi, S. M. T., & Dehghanian, A. (2024). Analysis of a time–cost trade-off in a resource-constrained GERT project scheduling problem using the Markov decision process. Annals of Operations Research, 338(1), 535–568. https://doi.org/10.1007/s10479-024-05896-8

[4] An, J., Li, S., & Wu, X. P. (2025). Multi-objective planning for time-cost trade-offs in multi-project parallel environment. Engineering, Construction and Architectural Management, 32(6), 4051–4073. https://doi.org/10.1108/ECAM-08-2023-0867

[5] Uncertain Time-Resource-Cost Trade-Off Models for Construction Project Schedule. (2021). KSCE Journal of Civil Engineering, 25(8), 2771–2778. https://doi.org/10.1007/s12205-021-1798-7

[6] Banerjee, A., & Paul, A. (2008). On path correlation and PERT bias. *European Journal of Operational Research*, *189*(3), 1208-1216. https://doi.org/10.1016/j.ejor.2007.01.061

[7] Baradaran, S., Fatemi Ghomi, S. M. T., Mobini, M., & Hashemin, S. S. (2010). A hybrid scatter search approach for resource-constrained project scheduling problem in PERT-type networks. *Advances in Engineering Software*, *41*(7), 966-975. https://doi.org/10.1016/j.advengsoft.2010.05.010

[8] Cagle, R. B. (2003). *Blueprint for Project Recovery--A Project Management Guide*. AMACOM. https://books.google.com.pk/books?id=Fp7FCnW7daAC

[9] Castro, J., Gómez, D., & Tejada, J. (2008). A polynomial rule for the problem of sharing delay costs in PERT networks. *Computers & Operations Research*, *35*(7), 2376-2387. https://doi.org/10.1016/j.cor.2006.11.003

[10] Cao, Q., Zou, X., & Zhang, L. (2022). Multiobjective robust optimization model for generating stable and makespan-protective repetitive schedules. Journal of Construction Engineering and Management, 148(9). https://doi.org/10.1061/(ASCE)CO.1943-7862.0002348

[11] Chrétienne, P., & Sourd, F. (2003). PERT scheduling with convex cost functions. *Theoretical Computer Science*, *292*(1), 145-164. https://doi.org/10.1016/S0304-3975(01)00220-1

[12] Cottrell Wayne, D. (1999). Simplified Program Evaluation and Review Technique (PERT). *Journal of Construction Engineering and Management*, *125*(1), 16-22. https://doi.org/10.1061/(ASCE)0733-9364(1999)125:1(16)

[13] Faghihi, V., Reinschmidt, K. F., & Kang, J. H. (2014). Construction scheduling using Genetic Algorithm based on Building Information Model. *Expert Systems with Applications*, *41*(16), 7565-7578. https://doi.org/10.1016/j.eswa.2014.05.047

[14] Long, L. D. (2024). A non-unit repetitive construction project scheduling with uncertainties. Automation in Construction, 164, 105428. https://doi.org/10.1016/j.autcon.2024.105428

[15] Frame, J. D. (2002). *The New Project Management: Tools for an Age of Rapid Change, Complexity, and Other Business Realities*. Wiley. https://books.google.com.pk/books?id=ZOG2mr6qJ_sC

[16] GINZBURG, D. (1988). On the distribution of activity time in PERT. *The Journal of the Operational Research Society*, *39*(8), 767-771. https://cir.nii.ac.jp/crid/1570572701408130944

[17] Gondia, A., Siam, A., El-Dakhakhni, W., & Nassar Ayman, H. (2020). Machine Learning Algorithms for Construction Projects Delay Risk Prediction. *Journal of Construction Engineering and Management*, *146*(1), 04019085. https://doi.org/10.1061/(ASCE)CO.1943-7862.0001736

[18] Haga, W. A., & Marold, K. A. (2004). A simulation approach to the PERT/CPM time-cost trade-off problem. Project Management Journal, 35(2), 31–37. https://doi.org/10.1177/875697280403500205

[19] Haga, W. A., & Marold, K. A. (2004). A Simulation Approach to the PERT/CPM Time-Cost Trade-Off Problem. *Project Management Journal*, *35*(2), 31-37. https://doi.org/10.1177/875697280403500205

[20] Haga, W. A., & O'Keefe, T. (2001, July 12–14). Crashing PERT networks: A simulation approach [Conference paper]. 4th International Conference of the Academy of Business and Administrative Sciences, Quebec City, Canada. https://www.inf.ufsc.br/~joao.dovicchi/pos-ed/pos/gerti/artigos/Haga_CrashPertNet.pdf,

[21] Kantianis, D. D. (2023). Construction project crashing with uncertain correlated normal and crash task durations and costs: An integrated stochastic practical approach. European Project Management Journal, 13(1), 3–22. https://doi.org/10.56889/pdsd6032

[22] Hajdu, M., & Bokor, O. (2014). The effects of different activity distributions on project duration in PERT networks. *Procedia-Social and Behavioral Sciences*, *119*(19), 766-775. https://doi.org/10.1016/j.sbspro.2014.03.086

[23] Herreraı´as-Velasco, J. M., Herreraı´as-Pleguezuelo, R., & van Dorp, J. R. (2011). Revisiting the PERT mean and variance. *European Journal of Operational Research*, *210*(2), 448-451. https://doi.org/10.1016/j.ejor.2010.08.014

[24] Khan, H. (2002). Exemplary program evaluation and review technique (expert) using projects. 32nd Annual Frontiers in Education,

[25] Lu, M., & AbouRizk, S. M. (2000). Simplified CPM/PERT Simulation Model. *Journal of Construction Engineering and Management*, *126*(3), 219-226. https://doi.org/10.1061/(ASCE)0733-9364(2000)126:3(219)

[26] Mehrotra, K., Chai, J., & Pillutla, S. (1996). A study of approximating the moments of the job completion time in PERT networks. *Journal of Operations Management*, *14*(3), 277-289. https://doi.org/10.1016/0272-6963(96)00002-2

[27] Mouhoub, N. E., Benhocine, A., & Belouadah, H. (2011). A new method for constructing a minimal PERT network. *Applied Mathematical Modelling*, *35*(9), 4575-4588. https://doi.org/10.1016/j.apm.2011.03.031

[28] Nowpada, R. S., & Veeramachaneni, V. S. (2009). An Approximation for the activity duration distribution, supporting original PERT. *Applied Mathematical Sciences*, *3*(57), 2823-2834. https://www.m-hikari.com/ams/ams-password-2009/ams-password57-60-2009/ravishankarAMS57-60-2009.pdf

[29] Turkoglu, H., Arditi, D., & Polat, G. (2024). Augmented time–cost trade-off optimization using particle swarm optimization. Journal of Construction Engineering and Management, 150(5). https://doi.org/10.1061/JCEMD4.COENG-14314

[30] Pontrandolfo, P. (2000). Project duration in stochastic networks by the PERT-path technique. *International Journal of Project Management*, *18*(3), 215-222. https://doi.org/10.1016/S0263-7863(99)00015-0

[31] Premachandra, I. M. (2001). An approximation of the activity duration distribution in PERT. *Computers & Operations Research*, *28*(5), 443-452. https://doi.org/10.1016/S0305-0548(99)00129-X

[32] Project scheduling cost optimization based on resource transfer costs and robustness. (2024). Computers & Operations Research, 161, 106445. https://doi.org/10.1016/j.cor.2023.106445

[33] Seal, K. C. (2001). A generalized PERT/CPM implementation in a spreadsheet. *INFORMS Transactions on Education*, *2*(1), 16-26. https://doi.org/10.1287/ited.2.1.16

[34] Tagore, R. (2007). Permutation and combinations approach to program evaluation and review technique. *ARPN Journal of Engineering and Applied Sciences*, *2*(6), 20-26. Here is the shorter, direct publisher link:
https://www.arpnjournals.com/jeas/research_papers/rp_2007/jeas_1207_71.pdf

[35] Tran, D.-H., Cheng, M.-Y., & Prayogo, D. (2016). A novel Multiple Objective Symbiotic Organisms Search (MOSOS) for time–cost–labor utilization tradeoff problem. *Knowledge-Based Systems*, *94*, 132-145. https://doi.org/10.1016/j.knosys.2015.11.016

[36] Trietsch, D., & Baker, K. R. (2012). PERT 21: Fitting PERT/CPM for use in the 21st century. *International Journal of Project Management*, *30*(4), 490-502. https://doi.org/10.1016/j.ijproman.2011.09.004

[37] Wenying, L., & Xiaojun, L. (2011). Progress Risk Assessment for Spliced Network of Engineering Project Based on Improved PERT. *Systems Engineering Procedia*, *1*, 271-278. https://doi.org/10.1016/j.sepro.2011.08.042

[38] Williams, F. E. (2005). PERT completion times revisited. *INFORMS Transactions on Education*, *6*(1), 21-34. https://doi.org/10.1287/ited.6.1.21

[39] Xiangxing, K., Xuan, Z., & Zhenting, H. (2010). Markov skeleton process in PERT networks. *Acta Mathematica Scientia*, *30*(5), 1440-1448. https://doi.org/10.1016/S0252-9602(10)60136-1

[40] A crashing-based time-cost trade-off model considering quality cost and contract clauses. (2021). International Journal of Construction Management. https://doi.org/10.1080/15623599.2021.1946899

[41] Yaseen, Z. M., Ali, Z. H., Salih, S. Q., & Al-Ansari, N. (2020). Prediction of Risk Delay in Construction Projects Using a Hybrid Artificial Intelligence Model. *Sustainability*, *12*(4), 1514. https://doi.org/10.3390/su12041514

[42] Yudistira, I. G. A. A., Nariswari, R., Arifin, S., Abdillah, A. A., Prasetyo, P. W., & Susyanto, N. (2024). Program Evaluation and Review Technique (PERT) Analysis to Predict Completion Time and Project Risk Using Discrete Event System Simulation Method. *International Journal of Construction Management*, *18*(1), 67-76. https://doi.org/10.21512/commit.v18i1.8495

[43] Zhong Deng, H., & Zhang Jian, S. (2003). New Method for Calculating Path Float in Program Evaluation and Review Technique (PERT). *Journal of Construction Engineering and Management*, *129*(5), 501-506. https://doi.org/10.1061/(ASCE)0733-9364(2003)129:5(501)