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Capacity Decision-Making Under Ownership Structure and Managerial Delegation in a Differentiated Duopoly

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ABSTRACT

This study develops a decision-making framework for analyzing capacity selection under different ownership structures and managerial delegation strategies in a differentiated duopoly market. Using a three-stage Cournot game model, the research investigates how private, state-owned, and mixed-ownership enterprises make strategic capacity and production decisions under full and partial managerial delegation. The model further evaluates the effects of product differentiation, state-owned share proportion, and capital efficiency on firm performance, consumer surplus, and social welfare. The equilibrium outcomes of six competitive scenarios are comparatively examined to identify optimal managerial and ownership strategies. The findings reveal that full delegation generally leads to overcapacity regardless of ownership structure, while partial delegation enables state-owned enterprises to achieve capacity balance and improved social welfare. The results further demonstrate that product differentiation can effectively mitigate overcapacity and enhance welfare outcomes, particularly in mixed-ownership structures. In addition, the study identifies optimal state-owned share proportions under both full and partial delegation mechanisms, highlighting the strategic role of ownership design in managerial decision-making. The research contributes to the literature by integrating ownership structure, managerial delegation, and capacity selection into a unified decision-support framework and provides practical implications for strategic capacity planning, governance design, and operational management in competitive industries.

1. Introduction

In modern enterprises, ownership structure and managerial delegation are always observed as the common instruments in firms that face an oligopolistic market structure. Common wisdom suggests that ownership structure adjustment and managerial delegation have a profound impact on firms' productivity, innovation and overall industry structure, especially capacity selection [7]. In terms of strategic delegation, the related studies beginning with Vickers [28] and Fershtman and Judd [12] are established on the observation that the delegation decision and accompanying actions can serve as commitments that influence competitive interactions with rivals and lead to beneficial

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results. The literature on delegation has been enriched via incorporating various issues, such as mergers [3], collusion [21; 29], product liability and quality control [13; 19], R&D investment [6; 15], trade policy [11-12] and many others. These mainly demonstrate that delegation of decision-making to managers is efficient when managers allocate resources, including their own efforts, in ways that do not divert from owners' goals. However, the firm offering an incentive scheme obtains lower profits compared with that under standard quantity competition. That is, firm owners face the prisoner's dilemma when selecting the delegation contracts for managers, ending up with Pareto inferior results [31]. The decision of delegation contracts is affected by various factors. Fanti et al. [11] suggests that determining in whether the subgame perfect Nash equilibrium is sales delegation or profit maximization depends on bargaining and product differentiation. Kopel and Pezzino [20] shows that hiring a manager is the unique Nash equilibrium solution under both quantity and price competition when endogenizing the choice of delegation. However, an especially considerable number of prior studies mainly focus on the relationship between managerial incentives and firms' profits, and the implication of delegation. By extending managerial delegation to the contexts of different ownership structures, we aim to determine the types of delegation contracts that can achieve the highest utility.

Furthermore, the capacity selection is also a common and significant decision-making in firms' daily operations. In reality, companies in most countries around the world have experienced or are experiencing overcapacity, which arouses the thinking of the government and economists [8]. Delegation allows the owners to empower the professional managers to make strategic decisions (e.g., output arrangement and pricing) by maximizing their owners' benefits. Considering the additional loss caused by overcapacity or under-capacity, the delegated managers should try their best to avoid the unbalance between output and capacity. However, the undeniable fact is that overcapacity and managerial delegation coexist in the development of modern enterprises. Theoretically, most of the existing literature focuses on the formation and optimization of the players' strategies in specific capacity arrangement environments. For instance, Wu et al. [33] investigate the selection of an optimal capacity contract between integrated equipment manufacturers and foundries in the semiconductor industry. Wang et al. [30] optimizes airline capacity allocation through dynamic programming. Wu et al. [32] examines a two-echelon supply chain facing asymmetric capacity constraints. In addition, Schmidt [26] and Richardson et al. [24] believe that managerial incentives act as a disciplinary mechanism, thus reducing managerial slack. In other words, an increase in managerial incentives enhances the competition among managers, who work harder to perform well in production arrangements. Nevertheless, the above studies do not consider the relationship between capacity selection and managerial delegation in corporative production. When the firms adopt full or partial delegation contracts, how the capacity selection is affected remains to be further investigated.

Based on previous works, we introduce ownership structure, managerial delegation and capacity selection into the product market competition and explore the choices of delegation mode and capacity decisions under different ownership structures. As we all know, private and state-owned capitals have different goals described in their objective functions. The firms' shareholders (owners) make strategic decisions of delegation and capacity according to profits or social welfare maximization. Specifically, owners delegate managers to control operating management with separation between the ownership and management. Choosing an appropriate managerial delegation can effectively motivate managers to decide capacity cautiously. The managerial delegation is related to the firm's capacity selection because managers are motivated by profits or sales [1]. In a word, the ownership structure is an important determinant in deciding whether the subgame perfect Nash equilibrium is sales-oriented or profit-oriented delegations. The types of the

delegation would bring out different capacity arrangements. The early work on ownership structure is extensive [2; 4; 9; 16; 17]. However, there is less agreement on exactly how ownership structure affects managers' executive behaviors in capacity decisions. Whether and how ownership structure interacts with managerial delegation and capacity decision-making remains to be further explored.

In reality, ownership structure, managerial delegation, and capacity selection are common issues in firms' daily operations. However, one of the three topics is either overlooked or controlled in the most existing literature. How does ownership structure affect managerial delegation and capacity selection? Whether managerial delegation can alleviate overcapacity? For different ownership structures, what is the optimal delegation contract and the proportion of state-owned shares? To answer these questions, this study is different from the abovementioned works in that we simultaneously consider ownership structure, managerial delegation and capacity selection between the two firms in a differentiated duopoly model and compare the models under the same ownership structure and managerial delegation to find the optimal delegation contract and the proportion of state-owned shares. We find that enterprises always suffer from overcapacity under full delegation, regardless of ownership structure. To realize no overcapacity and the maximization of social welfare, state-owned enterprises (SOEs) should choose partial delegation. Moreover, consumer surplus and social welfare can be enhanced by improving product differentiation under certain competition mode. The partial delegation has the advantage of improving social welfare in both pure and mixed competitions.

The marginal contributions of this article are as follows. First, few studies combine ownership structure, managerial delegation and capacity selection together to analyze their effects on firm performance. Therefore, this paper constructs a duopoly model considering ownership structure, managerial delegation and capacity selection simultaneously. It is helpful to reveal the interactions among them and explore the significant roles of ownership structure and managerial delegation in welfare distribution. Second, we compare the equilibrium outcomes of different competition modes from the perspectives of ownership structures and delegation contracts (full and partial). From the comparison analysis, the comparative advantages of different ownership structures and delegation contracts will be explored under various goals. Third, the effects of product differentiation and the proportion of state-owned shares on equilibrium outcomes under different ownership structures and delegation contracts are investigated, which is beneficial for market players to understand the functions in the competition.

The remainder of this study is organized as follows. Section 2 sets the basic model and Section 3 investigates the equilibrium results under six competition modes and discusses the effects of the proportion of state-owned shares and product differentiation on excess capacity and welfare distributions. Section 4 makes a comparative analysis. The optimal proportion of state-owned shares is determined in Section 5. Section 6 presents the research conclusion.

2. The Model

Suppose a three-stage Cournot competition between Enterprise 1 and Enterprise 2 with the same ownership structure.

Assumption 1. The inverse demand function of each enterprise is $p_i = a - q_i - rq_j, i \neq j, i, j = 1, 2$, where r denotes the degree of product differentiation, $r \in (0, 1)$. The smaller r is, the greater the product differentiation is. The cost function is $c_i = (x_i - q_i)^2 + bq_i$ Chen et al. [5], where x_i is the capacity of Enterprise i. Define $(x_i - q_i)$ as the degree of excess capacity. If $(x_i - q_i) > 0$, the enterprise holds excess capacity, and the idle capacity causes losses. If $(x_i - q_i) < 0$, the enterprise chooses under-capacity, which will lead to a high operation load for the existing capacity and generate additional costs. Only when quantity is equal to capacity, the cost is minimized, Otherwise, additional losses will be generated. The second part bq_i is determined by the scale of production, where b is the

efficiency of enterprises. When two enterprises are private, there is $b = n$; when two enterprises are state-owned, there is $b = m$; when two enterprises are mixed, and $b = \beta m + (1 - \beta)n$ [35]where β is the proportion of state-owned shares. The lower the value of m (n) is, the higher the efficiency of the corresponding state-owned (private) share is. Considering the obvious efficiency advantage of private shares, we suppose $m > n$ [23]. The profit function of Enterprise i is $\pi_i = p_i q_i - c_i$. Inspired by Fanti [10], Hackner [14], and Singh and Vives [27], the consumer surplus is assumed to be $cs = \frac{1}{2}(q_1^2 + q_2^2 + 2rq_1q_2)$. And the social welfare is $sw = \pi_1 + \pi_2 + cs$ [18; 34].

Assumption 2. Considering the separation of ownership and management, this paper investigates the enterprise's inner structure instead of regarding it as a sole agent. Suppose the situation in which the two enterprises may hire a manager and offer him/her incentives to boost the profits. Following the strategic delegation literature [12; 28], the owner enters into the delegation contract combining profits and sales. We assume a linear incentive scheme that may reward (or penalize) the manager for generating revenue beyond the standard profit maximizing level. Formally, the manager is instructed to maximize his/her utility. According to Saha and Sensarma [25], the manager's utility function of a state-owned enterprise is $u_i^S = \pi_i^S + \theta_1 q_i^S$, that of a private enterprise is $u_i^P = \pi_i^P + \theta_2 q_i^P$, and that of a mixed-ownership enterprise is $u_i^M = \pi_i^M + [\beta\theta_1 + (1 - \beta)\theta_2]q_i^M$. The weighted objective function for the mixed-ownership firm is a standard representation of a negotiated corporate governance structure. It effectively captures the internal bargaining process between state and private shareholders regarding the managerial incentive contract [12; 22]. This linear formulation allows for analyzing the continuous impact of ownership structure (β) on strategic interaction, which is critical for identifying optimal state-holding ratios. Depending on the owner's preference, θ_i represents the managerial incentive and has a wide range of values. The standard case of profit maximization is given by $\theta_i = 0$. But if $\theta_i > 0$, the manager is encouraged to pursue more sales. Conversely, $\theta_i < 0$ implies that the manager will be encouraged to pursue profit more than sales. Furthermore, full delegation or partial delegation can be adopted by one enterprise. Specifically, the manager can decide both the quantity and capacity of an enterprise under full delegation, but can only determine the outputs under partial delegation.

Assumption 3. According to the enterprise's ownership structure and managerial delegation, we consider six regimes: (i) Model FPP - both private enterprises adopt full delegation; (ii) Model FSS - both state-owned enterprises take full delegation; (iii) Model FMM - both the two mixed-ownership enterprises choose full delegation; (iv) Model PPP - both private enterprises take partial delegation; (v) Model PSS - both managers of two state-owned enterprises are partially delegated; (vi) Model PMM - both the two mixed-ownership enterprises choose partial delegation. The six cases above are represented by Model FPP, Model FSS, Model FMM, Model PPP, Model PSS, and Model PMM respectively, and their equilibrium results are denoted by corresponding superscripts.

Under the above assumptions, the two enterprises engage in a three-stage sequential game. Specifically, the owners of private and state-owned (or mixed-ownership) enterprises aim to maximize their objective functions V_i. For a private firm, $V_i{}^P = \pi_i^P$; for a state-owned firm, $V_i{}^P = sw_i^S$; for a mixed-ownership enterprise, $V_i{}^M = \beta sw^M + (1 - \beta)\pi_i^M$; the managers' objective functions are u_i^P, u_i^S and u_i^M, respectively. In the first stage, the owners of enterprises decide on the managerial incentive when adopting different delegation contracts (full or partial). In the second stage, the owners or managers of enterprises choose the capacity to maximize profits or the manager's utility. In the third stage, the managers of the two enterprises determine the quantity in Cournot competition. Within each regime, firms are ex ante symmetric and adopt identical delegation contracts. Hence, we restrict attention to symmetric Nash equilibria and denote the managerial incentive as $\theta_{1i} = \theta_1$, and $\theta_{2i} = \theta_2$ for $i = 1, 2$.

3. Model Analysis

The equilibrium outcomes are inversely induced under two managerial delegation contracts (full or partial) and three ownership structures (private, state-owned, or mixed).

Model FPP

If the two private enterprises adopt full delegation, the managers maximize $u_i^P = \pi_i^P + \theta_2 q_i^P$ in Stage 3 by choosing q_i as

$$q_i = \frac{(4-r)(a-n+\theta_2)+2(4x_i-rx_j)}{16-r^2} \quad (1)$$

In Stage 2, substituting (1) into the manager's utility function, and his/her choice of x_i gives the capacity reaction function

$$x_i = \frac{16((4-r)(a-n+\theta_2)-2rx_j)}{r^4-32r^2+128} \quad (2)$$

The reaction curves of the two enterprises as given by (2) are shown in Figure 1. Two thick curves, denoted as RF_1 and RF_2, are drawn with the assumption that $r \in (0,1)$ and $\theta_2 > n-a$. The downward slopes indicate that the product sales are strategic substitutes. If Enterprise 2 chooses zero capacity, Enterprise 1 will choose its monopoly capacity as:

$$\frac{16(4-r)(a-n+\theta_2)}{r^4-32r^2+128}$$

And similarly, if Enterprise 1 chooses zero capacity, Enterprise 2 will choose:

$$x_2 = \frac{16(4-r)(a-n+\theta_2)}{r^4-32r^2+128}$$

Conversely, if Enterprise 2 chooses:

$$x_2 = \frac{(4-r)(a-n+\theta_2)}{2r},$$

Enterprise 1 will simply close down, and similarly, Enterprise 1's choice of $x_1 = \frac{(4-r)(a-n+\theta_2)}{2r}$ will force Enterprise 2 to close down. Therefore, the monopoly and entry-deterring levels of the quantity of each enterprise can be defined in the usual way as the enterprise's capacity. The equilibrium capacities are given by point M comprising x_1^{FPP} and x_2^{FPP}, which we obtain as

$$x_i^{FPP} = \frac{16(a-n+\theta_2)}{r(16-r^2)+4(8-r^2)} \quad (3)$$

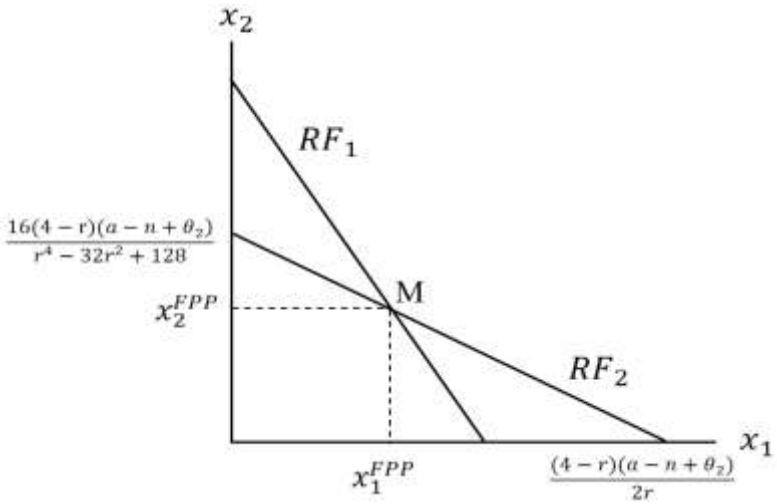

Fig.1: Capacity reaction functions

We now move to the first stage of the game and analyze the strategic interactions in terms of managerial incentives. Utilizing (3) and profit function, the two players determine their managerial incentives simultaneously by maximizing profits as follows:

$$\theta_2^{FPP} = \frac{(a-n)(32(r^2-r-8)-r^4)r}{2r^4(r+2)-64(r^2-8)(r+1)} < 0 \quad (4)$$

A crucial point to note is that for $x_i^{FPP} > 0$, it is necessary that $\theta_2 > n - a$.

Lemma 1 The equilibrium outcomes under Model FPP when $\theta_2 > n - a$ are as follows.

$$q_i^{FPP} = \frac{(a-n)(r^2-16)^2}{2r^4(r+2)-64(r^2-8)(r+1)}, x_i^{FPP} - q_i^{FPP} = \frac{(a-n)r^2(16-r^2)}{2r^4(r+2)-64(r^2-8)(r+1)};$$

$$\pi_i^{FPP} = \frac{(r^2-16)^2(a-n)^2}{4r^4(r+2)-128(r+1)(r^2-8)}; u_i^{FPP} = \frac{(r^4-32(r^2+4))(r^2-16)^2(a-n)^2}{2(r^4(r+2)-32(r+1)(r^2-8))^2};$$

$$cs^{FPP} = \frac{(r^2-16)^4(a-n)^2(r+1)}{4(r^4(r+2)-32(r+1)(r^2-8))^2}; sw^{FPP} = \frac{(r^2-16)^2(a-n)^2(r^4(3r+5)-96(r+1)(r^2-8))}{4(r^4(r+2)-32(r+1)(r^2-8))^2}.$$

We can prove $x_i^{FPP} - q_i^{FPP} > 0$ and thus obtain the following corollary.

Corollary 1. Under the full delegation contract, the two private enterprises engaging in Cournot competition have to face excess capacity.

As shown in Corollary 1, the two private enterprises will suffer from overcapacity under managers' decision-making. Specifically, $\theta_2^{FPP} < 0$ implies that the managers are encouraged to pursue profits more than sales, which conforms to the private enterprise's objective of profit maximization. Thus, the production and operation decisions from managers may receive the board's approval and support easily. Furthermore, the growth rate of capacity is fast than that of quantity. Therefore, the managerial incentives would generate overcapacity finally.

Proposition 1. In Model FPP, the effects of r on the equilibrium outcomes are as follows:

$$\frac{\partial(x_i^{FPP}-q_i^{FPP})}{\partial r} > 0, \frac{\partial \pi_i^{FPP}}{\partial r} < 0, \frac{\partial cs^{FPP}}{\partial r} < 0, \frac{\partial sw^{FPP}}{\partial r} < 0.$$

As shown in Proposition 1, when the two private enterprises choose full delegation, the value of r is positively correlated with $(x_i^{FPP} - q_i^{FPP})$ but negatively correlated with π_i^{FPP}, cs^{FPP}, and sw^{FPP}, indicating that improving product differentiation (i.e., the decrease of r) can alleviate overcapacity and enhance the profits of the two enterprises, consumer surplus, and social welfare. Complete product differentiation is the optimal choice for the market private players. Product differentiation strategy is an effective tool to settle excess capacity by providing more alternatives for consumers and improving market competitiveness.

3.1 Mode FSS

After standard calculation, the equilibrium results can be obtained as follows.

$$q_i^{FSS} = \frac{(r^2-16)^2(a-m)}{r^4(r+3)-32(r+1)(r^2-8)}, x_i^{FSS} = \frac{16(16-r^2)(a-m)}{r^4(r+3)-32(r+1)(r^2-8)},$$

$$\theta_1^{FSS} = \frac{(a-m)(r^4-64(r^2-4))}{r^4(r+3)-32(r+1)(r^2-8)}, x_i^{FSS} - q_i^{FSS} = \frac{(a-m)r^2(16-r^2)}{r^4(r+3)-32(r+1)(r^2-8)},$$

$$\pi_i^{FSS} = \frac{(r-4)^2(r+4)^2(a-m)^2r^4}{(r^4(r+3)-32(r+1)(r^2-8))^2},\ u_i^{FSS} = \frac{2(r-4)^2(r+4)^2(a-m)^2(r^4-32r^2+128)}{(r^4(r+3)-32(r+1)(r^2-8))^2},$$

$$cs^{FSS} = \frac{(r^2-16)^4(a-m)^2(r+1)}{(r^4(r+3)-32(r+1)(r^2-8))^2},\ sw^{FSS} = \frac{(r^2-16)^2(a-m)^2}{(r^4(r+3)-32(r+1)(r^2-8))^2}.$$

It should be noted that there always exist $q_i^{FSS} > 0$, $x_i^{FSS} > 0$ and $\pi_i^{FSS} > 0$ for any value of $r \in (0, 1)$. In addition, we can prove $\theta_1^{FSS} > 0$ and $x_i^{FSS} - q_i^{FSS} > 0$. We obtain Corollary 2 and Proposition 2 by analyzing the equilibrium results.

Corollary 2. When the two state-owned enterprises compete in Cournot competition, the full delegation contract will force them to suffer from overcapacity, which is identical to Corollary 1.

As shown in Corollary 2, the two state-owned enterprises will face overcapacity under full delegation. It indicates that the relationship between capacity and outputs is not affected by the enterprise's ownership structure adjustment between private and state ownerships. On the one hand, although the managers are encouraged to pursue more sales ($\theta_1^{FSS} > 0$), the increase of capacity is faster than that of outputs. On the other hand, most state-owned enterprises are related to national security and social livelihood. It is reasonable to maintain the capacity to exceed market demands to cope with emergencies and unexpected events in reality.

Proposition 2. In Model FSS, the effects of r on the equilibrium outcomes are as follows:

$$\frac{\partial(x_i^{FSS}-q_i^{FSS})}{\partial r} > 0,\ \frac{\partial \pi_i^{FSS}}{\partial r} > 0,\ \frac{\partial cs^{FSS}}{\partial r} < 0,\ \frac{\partial sw^{FSS}}{\partial r} < 0.$$

As shown in Proposition 2, when the two fully delegated state-owned enterprises engage in Cournot competition, the positive correlation between r and $(x_i^{FSS} - q_i^{FSS})$ indicates that producing homogeneous products will exacerbate overcapacity. With the increase in product differentiation, the decrement in enterprise profit is outweighed by the improvement in consumer surplus, which leads social welfare to increase. In a word, complete product differentiation is the optimal choice for the two state-owned enterprises to mitigate overcapacity and maximize social welfare.

Compared with Proposition 1, differ from the negative correlation between r and π_i^{FPP}, increasing product differentiation (i.e., the decrease of r) is adverse for profits in Model FSS. This change lies in the fact that private enterprises pursue profit maximization but state-owned enterprises pursue social welfare maximization, attributing to different ownership structures of market players.

3.2 Model FMM

After standard calculation, the equilibrium results in Model FMM can be obtained as follows.

$$q_i^{FMM} = \frac{(16-r^2)^2(\beta+1)((n-m)\beta+a-n)}{2(r^4(r+\beta+2)-32(r+1)(r^2-8))},\ x_i^{FMM} = \frac{8(\beta+1)(16-r^2)((n-m)\beta+a-n)}{2(r^4(r+\beta+2)-32(r+1)(r^2-8))},$$

$$\theta_i^{FMM} = \frac{((n-m)\beta+a-n)((\beta-1)r(r^4-32r^2+256)+2\beta(r^4+256)-32(1+3\beta)r^2)}{2(r^4(r+\beta+2)-32(r+1)(r^2-8))},$$

$$x_i^{FMM} - q_i^{FMM} = \frac{(\beta+1)(16-r^2)((n-m)\beta+a-n)r^2}{2(r^4(r+\beta+2)-32(r+1)(r^2-8))},$$

$$\pi_i^{FMM} = \frac{(16-r^2)^2(\beta+1)((n-m)\beta+a-n)^2(r^4((\beta-1)r-2)-32(\beta-1)(r+1)(r^2-8))}{4(r^4(r+\beta+2)-32(r+1)(r^2-8))^2},$$

$$u_i^{FMM} = \frac{(\beta+1)^2(16-r^2)^2((n-m)\beta+a-n)^2(r^4-32r^2+128)}{2(r^4(r+\beta+2)-32(r+1)(r^2-8))^2},$$

$$cs^{FMM} = \frac{(r+1)\left(16-r^2\right)^4(\beta+1)^2\left((n-m)\beta+a-n\right)^2}{4(r^4(r+\beta+2)-32(r+1)(r^2-8))^2},$$

$$sw^{FMM} = -\frac{\left(16-r^2\right)^2(\beta+1)\left((n-m)\beta+a-n\right)^2((\beta+5)r^4-(\beta-3)\left(r^5-32(r+1)\left(r^2-8\right)\right))}{4(r^4(r+\beta+2)-32(r+1)(r^2-8))^2}.$$

It should be noted that there always exists $q_i^{FMM} > 0$, $x_i^{FMM} > 0$ for any value of $r \in (0,1)$. In addition, we can prove $x_i^{FMM} - q_i^{FMM} > 0$. We can obtain Corollary 3 and Proposition 3 by analyzing the equilibrium results.

Corollary 3. If the two mixed-ownership enterprises choose full delegation in Cournot competition, they will suffer from overcapacity, which is identical to the cases of Model FPP and Model FSS.

As shown in Corollary 3, the two mixed-ownership enterprises will face overcapacity under full delegation. Although both private property rights and state-owned property rights are contained in the mixed-ownership structure, it still fails to solve the issue of overcapacity effectively. We note that the managerial incentive (θ_i^{FMM}) could be positive or negative under Model FMM, indicating that the manager's pursuit oscillates between sales and profits. However, as the representative of the enterprise's production decision, the manager cannot predict the effective demand of the market appropriately. Considering the heterogeneous goals under different property rights, the enterprise's capacity arrangements fail to cover the market demands accurately. Therefore, it is reasonable to arise overcapacity in Model FMM.

In practice, many companies around the world have experienced or are experiencing overcapacity. No matter whether privatization reform or mixed-ownership reform has been implemented, overcapacity always exists in the process of economic development, which is one of the common economic development issues. Just as the results in Corollary 1, 2, and 3, overcapacity may exist under different ownership structure. Therefore, it is unconscionable in practice to only rely on adjusting the ownership structure to alleviate overcapacity.

Proposition 3. In Model FMM, the effects of β and r on the equilibrium outcomes are as follows.

(i) The effects of β:

There exists m_1 and m_2. If $0 < m < m_1$, then:

$$\frac{\partial(x_i^{FMM}-q_i^{FMM})}{\partial\beta} > 0, \frac{\partial\pi_i^{FMM}}{\partial\beta} > 0, \frac{\partial cs^{FMM}}{\partial\beta} > 0$$

if $m > m_1$, then:

$$\frac{\partial(x_i^{FMM}-q_i^{FMM})}{\partial\beta} < 0, \frac{\partial\pi_i^{FMM}}{\partial\beta} < 0, \frac{\partial cs^{FMM}}{\partial\beta} < 0.$$

If $0 < m < m_2$:

$$\text{then } \frac{\partial sw^{FMM}}{\partial\beta} > 0;$$

if $m > m_2$, then:

$$\frac{\partial sw^{FMM}}{\partial\beta} < 0.$$

(ii) The effects of r:

$$\frac{\partial(x_i^{FMM}-q_i^{FMM})}{\partial r} > 0, \frac{\partial\pi_i^{FMM}}{\partial r} < 0, \frac{\partial cs^{FMM}}{\partial r} < 0, \frac{\partial sw^{FMM}}{\partial r} < 0.$$

As shown in Proposition 3, the influences of β on $(x_i^{FMM} - q_i^{FMM})$, π_i^{FMM}, cs^{FMM}, and sw^{FMM} are

non-monotonic, which depends on the value range of m. When the efficiency of state-owned enterprises is high, taking $0 < m < m_1$ (or m_2) for instance, increasing the proportion of state-owned shares can improve profits, consumer surplus and social welfare. However, when the efficiency of state-owned enterprises is low, especially when $m > m_1$, increasing the proportion of state-owned shares is beneficial for neither enterprises nor consumers. Under this circumstance, it is necessary to introduce private capital to enhance producer surplus and consumer welfare.

Moreover, the effects of r on equilibrium results are the same as those of Proposition 1. Improving product differentiation will alleviate overcapacity and boost enterprise profits, consumer surplus and social welfare. Complete product differentiation is the optimal choice for mixed-ownership enterprises under full delegation. However, the complete positive correlation between r and π_i in Proposition 2 becomes negative, which is attributable to the change in ownership structure.

3.3 Model PPP

After standard calculation, the equilibrium results in Model PPP can be obtained as follows.

$$q_i^{PPP} = \frac{(a-n)(r^4-24r^2+32r+128)}{2(r+2)(r^4-24r^2+32r+64)};\ x_i^{PPP} = \frac{2(a-n)(16(r+2)-r^2\ (r+4))}{(r+2)(r^4-24r^2+32r+64)};$$

$$\theta_2^{PPP} = -\frac{(a-n)r(r^4-32r^2+128)}{2(r+2)(r^4-24r^2+32r+64)});\ x_i^{PPP} - q_i^{PPP} = \frac{(a-n)r(r+4)(8-r^2)}{2(r+2)(r^4-24r^2+32r+64)};$$

$$\pi_i^{PPP} = \frac{(r^4-32r^2+128)(a-n)^2}{4(r+2)(r^4-24r^2+32r+64)};\ u_i^{PPP} = \frac{(a-n)^2(r^4-24r^2+64)(r^4-32r^2+128)}{2(r+2)^2(r^4-24r^2+32r+64)^2};$$

$$cs^{PPP} = \frac{(r^4-24r^2+32r+128)^2(a-n)^2(r+1)}{4(r+2)^2(r^4-24r^2+32r+64)^2};$$

$$sw^{PPP} = \frac{(3r^9+5r^8-160r^7-144r^6+2944r^5+1088r^4-20992r^3-9216r^2+57344r+49152)(a-n)^2}{4(r+2)^2(r^4-24r^2+32r+64)^2}.$$

There always exists $q_i^{PPP} > 0$, $x_i^{PPP} > 0$, and $\pi_i^{PPP} > 0$ for any value of r in the range of $(0, 1)$. In addition, we can prove $\theta_2^{PPP} < 0\ and\ x_i^{PPP} - q_i^{PPP} > 0$. Then Corollary 4 and Proposition 4 can be provided by analyzing the equilibrium results.

Corollary 4. Under the partial delegation contract, the two private enterprises competing in Cournot competition will face overcapacity, which is identical to Model FPP.

As shown in Corollary 4, the two private enterprises will face overcapacity under partial delegation. Although private enterprises are generally more efficient than state-owned enterprises and have more flexibility in production decision-making, they still suffer from overcapacity under partial delegation. The negative managerial incentive (θ_i) indicates that the manager is encouraged to pursue profits more than sales, which leads the enterprises to produce more products than market demands. However, neither the full nor partial delegations can avoid overcapacity effectively for private enterprises. In fact, overcapacity is affected by numerous factors, including market, political policies and social factors. Therefore, the comparison between Model PPP and Model FPP cannot reject that the managerial delegation is an effective tool. On the contrary, it just shows that overcapacity is a common problem for enterprises with different ownership structures and delegation contracts.

Proposition 4. In Model PPP, the effects of r on the equilibrium outcomes are as follows.

$$\frac{\partial(x_i^{PPP}-q_i^{PPP})}{\partial r} > 0,\ \frac{\partial \pi_i^{PPP}}{\partial r} < 0,\ \frac{\partial cs^{PPP}}{\partial r} < 0,\ \frac{\partial sw^{PPP}}{\partial r} < 0.$$

As shown from Proposition 4, when the two private enterprises choose partial delegation, the value of r is positively correlated with $(x_i^{PPP} - q_i^{PPP})$ but negatively correlated with π_i^{PPP}, cs^{PPP} and sw^{PPP}, manifesting that the growth in product differentiation will suppress overcapacity and improve the enterprises' profits, consumer surplus, and social welfare. Therefore, private enterprises are encouraged to increase product innovation under partial delegation, which is beneficial to all the market players. Furthermore, the effects of r on equilibria in Model PPP are the same as those of Proposition 1, which indicates that the choice between the full and partial delegations exerts no impact on the effects of product differentiation.

3.4 Model PSS

After standard calculation, the equilibrium results in Model PSS can be obtained as follows.

$$q_i^{PSS} = \frac{a-m}{r+1};\ x_i^{PSS} = \frac{a-m}{r+1};\ \theta_1^{PSS} = \frac{a-m}{r+1};\ x_i^{PSS} - q_i^{PSS} = 0;$$

$$\pi_i^{PSS} = 0;\ u_i^{PSS} = \frac{(a-m)^2}{(r+1)^2};\ cs^{PSS} = \frac{(a-m)^2}{r+1};\ sw^{PSS} = \frac{(a-m)^2}{r+1}.$$

There always exists $q_i^{PSS} > 0$, $x_i^{PSS} > 0$ for any value of $r \in (0,1)$. In addition, $\theta_1^{PSS} > 0$ and $x_i^{PSS} - q_i^{PSS} = 0$. Then Corollary 5 and Proposition 5 can be obtained by analyzing the equilibrium results.

Corollary 5. The result $x_i^{PSS} - q_i^{PSS} = 0$ in Model PSS indicates that there is no overcapacity in the two partially delegated state-owned enterprises in Cournot competition, differing from Corollary 2.

As shown in Corollary 5, when the two state-owned enterprises adopt partial delegation in Cournot competition, there exists the case that the capacity is equal to the outputs. In theory, the managerial incentive (θ_i) is positive under Model PSS, indicating that the manager is encouraged to pursue sales more than social welfare. Therefore, reasonable production decisions can be made by the manager according to market demand. Under partial delegation, because the goal of managers is different from that of the owners, the combined actions from the managers and owners avoid the mismatch between capacity and outputs.

Contrary to the overcapacity in Corollary 2, no overcapacity happens in Model PSS, which is attributable to the change in the managerial delegations. Combined with Corollary 4 and Corollary 2, it is again confirmed that overcapacity is the result of the comprehensive effects of various factors from the economic environment. Managerial delegations fail to solve the overcapacity problem under Model PPP. However, it is an effective instrument to make well-coordinated capacity-quantity decision and avoid overcapacity in Model PSS.

Proposition 5. In Model PSS, we have:

$$\frac{\partial(x_i^{PSS} - q_i^{PSS})}{\partial r} = 0,\ \frac{\partial \pi_i^{PSS}}{\partial r} = 0,\ \frac{\partial cs^{PSS}}{\partial r} < 0,\ \frac{\partial sw^{PSS}}{\partial r} < 0.$$

As shown in Proposition 5, the overcapacity and the profits of enterprises are not affected by product differentiation. The two state-owned enterprises' profits are zero, revealing that government subsidies are required to support corporate operations. Increasing product differentiation can increase consumer surplus and social welfare. Compared with Proposition 2, the positive relationships between r and $(x_i - q_i)$ (or π_i) are replaced by no impacts in Model PSS, which is induced by the shift of managerial delegations. The effects of r on cs and sw are the same as those in Proposition 2. Consequently, in terms of overcapacity, partial delegation is the better choice for state-owned enterprises.

3.5 Model PMM

After standard calculation, the equilibrium results in Model PMM can be obtained as follows.

$$q_i^{PMM} = \frac{((n-m)\beta+a-n)(\beta+1)(r^3(r-2\beta)+\left(6\beta^2-8(\beta+3)\right)r^2+8(\beta(\beta+1)+4)r-32(\beta-4))}{2(r^5+(2-3\beta)r^4+2(3(\beta^2-4)-5\beta)r^3+(\beta^3+5\beta^2+16(2\beta-1))r^2+8(\left(\beta^2(\beta-5)+2(3\beta+8)\right)r+2(\beta^3-3\beta^2+8)))};$$

$$x_i^{PMM} = \frac{((n-m)\beta+a-n)(2\beta r^4+(5\beta(1-\beta)-4)r^3+2\left(2\left(\beta^2(\beta-2)-2\right)-7\beta\right)r^2+8(\beta^3+\beta^2+4)r+32(\beta+2))}{2((r+(2-3\beta))r^4+2(3(\beta^2-4)-5\beta)r^3+(\beta^2(\beta+5)+16(2\beta-1))r^2+8\left(\beta^2(\beta-5)+2(3\beta+8)\right)r+16((\beta-3)\beta^2+8))};$$

$$\theta_i^{PMM} = \frac{((n-m)\beta+a-n)(\left(6r^3+14r^2-16(r+2)\right)\beta^3+2(r^2(3(5-2r)-r^2)+8(5r-2))\beta^2+(r+4)\left(r^4-30r^2+8(3r+8)\right)\beta-r^5+32r(r^2-4))}{2((r+4)^2\beta^3+\left(6r^3+5r^2-8(5r-6)\right)\beta^2+(16(2r+3)-(3r+10)r^2)r\beta+(r+2)(r^4-24r^2+32(r+2)))};$$

$$x_i^{PMM} - q_i^{PMM} = \frac{(r^2-2\beta r-8)((n-m)\beta+a-n)(r+4)(r-\beta)(\beta-1)}{2\left(r+(2-3\beta)\right)r^4+4(3(\beta^2-4)-5\beta)r^3+2((\beta+5)\beta^2+16(2\beta-1))r^2+16(\left((\beta-5)\beta^2+6\beta+16\right)r+2\left((\beta-3)\beta^2+8\right))};$$

$$\pi_i^{PMM} = \frac{\begin{array}{c}((n-m)\beta+a-n)^2(1-\beta)((\beta+1)r^9+2\left(1-2\beta^2\right)r^8+2\left(\beta(8\beta^2-29)-7(\beta^2+4)\right)r^7+(3\beta^3(21-8\beta)+\\ 109\beta^2+64\beta-80)r^6+\left(4\beta^4(9\beta-31)-8\beta^2(34\beta-15)+16(81\beta+64)\right)r^5+(4\beta^4(31\beta+1)-304\beta^2(2\beta+5)-\\ 64(17\beta-14))r^4+\left(80\beta^5+352\beta^4+1552\beta^3+384\beta^2-9728\beta-7168\right)r^3+(192\beta^4(2-\beta)+64\beta^2(35\beta+118)\\ +1024(\beta-6))r^2+\left(256(\beta^4(2-\beta)+4(\beta(\beta+24)+16))-2304\beta^3\right)r+1024(\beta+2)(\beta^3-5\beta^2+4(\beta+2)))\end{array}}{\begin{array}{c}4(r^4\left(r+(2-3\beta)\right)+\left(2(3(\beta^2-4)-5\beta)r+\left(\beta^2(\beta+5)+16(2\beta-1)\right)\right)r^2\\ +8(\left(\beta^2(\beta-5)+2(\beta^2(\beta-3)+8)\right)+2(3\beta+8))r)^2\end{array}};$$

$$u_i^{PMM} = \frac{\begin{array}{c}((n-m)\beta+a-n)^2(8r^2\left(r^2-8(r+2)\right)\beta^6-16r\left(4\left(16-9r^2+12r\right)-r^3(2r+19)+\right)\beta^5-(2(512(r-2)+\\ 8r^2(r^3-92r-192))+5r^4(153+5r^2))\beta^4-2(16(49r^4+2\left(r^2(29r-124)+16(r+12)\right))-(6r^2+73r+\\ 124)r^5)\beta^3-(2(64(43r^3-5r^2-120r+16)+18r^7-4r^4(83r+71)+r^8)+81r^6)\beta^2-4(10r^5(3r-4)\\ -r^7(r+2)-416r^4+256\left(r^2(6r+11)-8(3r+4)\right))\beta+2(r^8-56r^6-64(5r^2(16-3r^2)-128)))\end{array}}{4((r+4)^2\beta^3+(6(r^3-8)+5r(r-8))\beta^2+(16r(r+3)-r^3(3r+10))\beta+(r+2)(r^4+8(4(r+2))-3r^2))^2};$$

$$cs^{PMM} = \frac{\left((n-m)\beta+a-n\right)^2(\beta+1)^2(r+1)\left(r^4-2\beta r^3+2\left(3\beta^2-4\beta-12\right)r^2+8\left(\beta^2+\beta+4\right)r+32(4-\beta)\right)^2}{4(r^5+(2-3\beta)r^4+2(3\beta^2-5\beta-12)r^3+(\beta^3+5\beta^2+16(2\beta-1))r^2+8(\beta^3-5\beta^2+6\beta+16)r+16(\beta^3-3\beta^2+8))^2};$$

$$sw^{PMM} = \frac{\begin{array}{c}\left((n-m)\beta+a-n\right)^2\left(16(165\beta+184)\right)r^5-\left(\beta^2-2\beta-3\right)r^9-\left(2\beta(3-2\beta^2)+5(3\beta^2-1)\right)r^8-8(2(\beta^2(\beta^2-1)\\ +10)+3\beta(5-3\beta^2))r^7-\left(24\left(6-\beta^5\right)-4\beta(27\beta+104)+2\beta^2\left(79\beta^2-181\right)\right)r^6-(36\beta^4\left(5+\beta^2\right)+8\beta^2(131\beta\\ +132-30\beta^3)-\left(116\beta^6-344\beta^5-780\beta^4+32\left(2\left(\beta^3-17\right)+\beta(133\beta+206)\right)\right)r^4-(256\beta^3(2\beta^2-23)\\ +64\beta^2\left(3\beta^2-209\right)+128(155\beta+164))r^3-64\left((7\beta^5(4-\beta)+48\beta^3-317\beta^2+144)+16\beta(\beta^3-20)\right)r^2\\ -\left(512\beta^2\left(\beta\left(4\left(\beta^2+5\right)-\beta^3\right)+\left(71-14\beta^2\right)\right)-57344(\beta+1)\right)r-1024(2\beta^5-9\beta^4+43\beta^2-24\beta-48))\end{array}}{\begin{array}{c}4(r^5+(2-3\beta)r^4+2(3\beta^2-5\beta-12)r^3+\left(\beta^2(\beta+5)+16(2\beta-1)\right)r^2\\ +8((\beta^3-5\beta^2+6\beta+16)r+2(\beta^3-3\beta^2+8)))^2\end{array}}.$$

There always exists $q_i^{PMM} > 0$, $x_i^{PMM} > 0$, and $\pi_i^{PMM} > 0$ for any value of $r \in (0,1)$. Then Corollary 6 and Proposition 6 can be obtained by analyzing the equilibrium results.

Corollary 6. The two partially delegated mixed-ownership enterprises may choose overcapacity or under-capacity in Cournot competition, which depends on the values of β and r. When $\beta < r$, there is overcapacity. When $\beta > r$, there is under-capacity.

As shown in Corollary 6, when adopting the partial delegation in Cournot competition, the two mixed-ownership enterprises may face overcapacity or under-capacity. Specifically, when $\beta < r$, there is overcapacity. Otherwise, there is under-capacity for mixed-ownership enterprises.

Theoretically, the managerial incentive (θ_i) may be positive or negative under Model PMM, suggesting that it is uncertain whether managers are more interested in the weighted sum of profits and sales. When product differentiation is lower than a certain level ($\beta < r$), the products produced by Enterprise i are close substitutes in the market, which causes excess supply and overcapacity.

Different from that in Model FMM, the change is caused by the choice of delegation contract. The capacity and quantity are both decided by the manager in the full delegation for utility maximization. However, in the partial delegation, the capacity and quantity are decided by owners and managers, respectively, driven by heterogeneous objectives. Therefore, under different goals, the game between private and state ownerships leads to the uncertainty of the relationship between capacity and outputs.

Proposition 6. In Model PMM, the effects of β and r on the equilibrium results are as follows.

(i) The effects of β:

There exists m_3, m_4, m_5 and m_6.

When $\mathrm{m}_3 > 0$, if $0 < \mathrm{m} < \mathrm{m}_3$, then $\frac{\partial(x_i^{PMM} - q_i^{PMM})}{\partial \beta} < 0$; if $m > m_3$, then $\frac{\partial(x_i^{PMM} - q_i^{PMM})}{\partial \beta} > 0$.

When $m_4 > 0$, if $0 < m < m_4$, then $\frac{\partial \pi_i^{PMM}}{\partial \beta} > 0$; if $m > m_4$, then $\frac{\partial \pi_i^{PMM}}{\partial \beta} < 0$.

When $m_5 > 0$, if $0 < m < m_5$, then $\frac{\partial cs^{PMM}}{\partial \beta} > 0$; if $m > m_5$, then $\frac{\partial cs^{PMM}}{\partial \beta} < 0$.

When $m_6 > 0$, if $0 < m < m_6$, then $\frac{\partial sw^{PMM}}{\partial \beta} > 0$; if $m > m_6$, then $\frac{\partial sw^{PMM}}{\partial \beta} < 0$.

(ii) The effects of r:

$\frac{\partial(x_i^{PMM} - q_i^{PMM})}{\partial r} > 0$, $\frac{\partial \pi_i^{PMM}}{\partial r} < 0$, $\frac{\partial cs^{PMM}}{\partial r} < 0$, $\frac{\partial sw^{PMM}}{\partial r} < 0$.

As shown in Proposition 6, when the two mixed-ownership enterprises choose partial delegation, the influences of β on equilibrium results depend on the value ranges of parameters (m, n and r). It indicates that the welfare effects of the proportion of state-owned shares on different market participants are influenced by product differentiation and capital efficiency. The relationship between β and π_i (cs and sw) shows an inverted-U shape. Specifically, with the decline of the state-owned shares, profits, consumer surplus and social welfare first increase and then decrease. Therefore, the stereotype that increasing the proportion of state-owned shares is beneficial to improving social welfare is not wise. Determine an appropriate efficiency of state-owned shares to realize the growth of social welfare, which is helpful to promote the efficiency of resource utilization.

Furthermore, the correlations between r and the equilibrium results indicate that a complete differentiation strategy can reduce overcapacity or under-capacity to the greatest extent and realize optimal profits and social welfare. In practice, improving product differentiation can enhance the firm's market competitiveness and avoid overcapacity (or under-capacity) effectively. Therefore, differentiation strategy is significant in market competition, which has complex influences on the goals of different players (private ownership, state ownership, consumers, government, and so on). The impacts of r on equilibrium results are the same as those in Proposition 3, failing to be affected by the different delegation contracts.

4. Comparative Analysis

In this section, we carry out the comparison analysis from the perspectives of the ownership structure and managerial delegation, respectively, to explore the welfare effects of ownership structure and managerial delegation in Cournot competition.

4.1 Same Ownership Structure

4.1.1 Two private enterprises

This subsection compares the equilibrium results under Model FPP and Model PPP.

Corollary 7. $(x_i^{FPP} - q_i^{FPP}) < (x_i^{PPP} - q_i^{PPP})$, $\pi_i^{FPP} > \pi_i^{PPP}$, $cs^{FPP} < cs^{PPP}$, $sw^{FPP} < sw^{PPP}$.

As shown in Corollary 7, there is lower overcapacity and more profits when the two private enterprises choose a full delegation contract. Therefore, the higher managerial incentive can alleviate the inefficiency of resource allocation and improve private enterprises' revenues. However, consumer surplus and social welfare in the partial delegation are better than those in the full delegation. Since the production decision from the partially delegated managers needs to be coordinated with the capacity decision from the owners, which has a higher internal cost compared with the full delegation enterprise, and thus the overcapacity is higher and the profit is lower. In conclusion, taking the full delegation can affect the supply side positively but hurt the demand side.

4.1.2 Two state-owned enterprises

This subsection compares the equilibrium results under Model FSS and Model PSS.

Corollary 8. $(x_i^{\mathrm{FSS}} - q_i^{\mathrm{FSS}}) > (x_i^{\mathrm{PSS}} - q_i^{\mathrm{PSS}})$, $\pi_i^{FSS} > \pi_i^{PSS}$, $cs^{FSS} < cs^{PSS}$, $sw^{FSS} < sw^{PSS}$.

As shown in Corollary 8, in terms of overcapacity, the two state-owned enterprises will choose partial delegation because no overcapacity happens to them. Partial delegation has an advantage over full delegation in improving consumer surplus and social welfare. Consequently, state-owned enterprise's optimal choice to realize no overcapacity and maximal social welfare is the partial delegation. However, the profit under Model FSS is better than that under Model PSS, indicating the profit is affected by the managerial incentive and that the full delegation promotes profit. In conjunction with Corollary 7, partial delegation is always helpful to improve consumer surplus and social welfare in both pure private competition and pure state-owned competition.

4.1.3 Two mixed-ownership enterprises

This subsection compares the equilibrium results under Model FMM and Model PMM.

Corollary 9. $(x_i^{\mathrm{FMM}} - q_i^{\mathrm{FMM}}) > (x_i^{\mathrm{PMM}} - q_i^{\mathrm{PMM}})$; $\pi_i^{FMM} < \pi_i^{PMM}$; $cs^{FMM} > cs^{PMM}$; $sw^{FMM} < sw^{PMM}$.

As shown in Corollary 9, when two mixed-ownership enterprises are in Cournot competition, the overcapacity is more serious under the full delegation than that of the partial delegation, which is consistent with Corollary 3 and Corollary 6. Because of the additional cost of overcapacity, the profit in Model PMM is higher than that of Model FMM. However, the full delegation adopted by mixed-ownership enterprises can produce more surplus for consumers, which is different from the cases of private enterprises and SOEs. The growth of profit outweighs the decrease in consumer surplus, which leads social welfare to increase under partial delegation. Therefore, partial delegation is the better choice for mixed-ownership enterprises to alleviate overcapacity and produce more profits and social welfare.

4.2 Same Managerial Delegation

Because the equilibrium output, capacity, and profit of the enterprise are greater than zero, there exist $(n-m)\beta + a - n > 0$ and $a - n > 0$.

4.2.1 Full delegation

Corollary 10. $\left(x_i^{FSS} - q_i^{FSS}\right) > (x_i^{FMM} - q_i^{FMM}) > (x_i^{FPP} - q_i^{FPP})$. There exists m_7 and m_8. When $m_7 < 0$ and $m_8 > 0$, if $0 < m < m_8$, $\pi_i^{FPP} > \pi_i^{FSS} > \pi_i^{FMM}$; if $m > m_8$, $\pi_i^{FPP} > \pi_i^{FMM} >$

π_i^{FSS}; $cs^{FSS} > cs^{FMM} > cs^{FPP}$; $sw^{FPP} > sw^{FSS} > sw^{FMM}$.

As shown in Corollary 10, from the supply side, under full delegation, private enterprises, state-owned enterprises, and mixed-ownership enterprises all hold excess capacity. Thus, capacity reduction is a key proposition to enterprises with these three kinds of ownership structure. The overcapacity is most serious when the two SOEs engage in Cournot competition. If the ownership is changed, the overcapacity will alleviate. As for the profit, the value of π_i^{FPP} is maximized when the two private enterprises engage in Cournot competition. If the market player changes, the profit π_i^{FMM} will be greater than π_i^{FSS} if $m > m_8$ and $m_7 < 0$. From the demand side, the consumer surplus is maximized under Model FSS, indicating that the state-owned shares promote consumer surplus. In addition, consumer surplus is relatively low when the private shares engage in Cournot competition. Thus, the pursuit of profits will result in a lower consumer surplus. The social welfare ranking is consistent with that of π_i if $0 < m < m_8$, revealing that the strong, positive correlation between social welfare and profit when m is relatively low. From the perspective of social welfare, Model FPP is optimal, indicating that private capital promotes the maximization of social welfare. Under the full delegation, the increase in private enterprise's profit will make up for the lost consumer surplus when the efficiency of state-owned shares is relatively low, thus resulting in an overall increase in social welfare.

4.2.2 Partial delegation

Corollary 11. If $\beta > r$, $(x_i^{PPP} - q_i^{PPP}) > \left(x_i^{PSS} - q_i^{PSS}\right) > (x_i^{PMM} - q_i^{PMM})$; if $\beta < r$, $(x_i^{PPP} - q_i^{PPP}) > (x_i^{PMM} - q_i^{PMM}) > (x_i^{PSS} - q_i^{PSS})$; $\pi_i^{PMM} > \pi_i^{PSS} > \pi_i^{PPP}$; $cs^{PMM} > cs^{PSS} > cs^{PPP}$; $sw^{PSS} > sw^{PMM} > sw^{PPP}$.

As shown in Corollary 11, under partial delegation, the excess capacity in private enterprises is the most serious among the three kinds of ownership structure. And the comparison between SOEs and mixed-ownership enterprises depends on the relationship between β and r. In fact, there is no overcapacity in Model PSS as shown in Corollary 5. But the capacity selection of the two mixed-ownership enterprises depends on the values of β and r. When $\beta > r$, under-capacity will happen in Model PMM. As for profits, the producers will obtain the maximal profits when the two mixed-ownership enterprises engage in Cournot competition. If the ownership structure changes into pure state ownership, the enterprise profits π_i^{PSS} will rank second among the three models. Therefore, from the supply side, the mixed-ownership structure brings benefits to enterprises. And a proper equity allocation in mixed-ownership enterprises will be helpful to alleviate excess capacity. From the demand side, mixed-ownership enterprises perform better than SOEs in terms of consumer surplus. Although the enterprise profits and consumer surplus in Model PMM are both higher than those of Model PSS and Model PPP, sw^{PMM} has not been maximized among the three models. Because either the overcapacity or under-capacity can cause additional costs for mixed-ownership enterprises. It indicates that capacity selection has a significant effect on welfare distributions.

In summary, neither private enterprises, state-owned enterprises, nor mixed-ownership enterprises have a definite absolute advantage. Choosing ownership structures needs to consider the proportion of state-owned shares, product differentiation, and capacity selection under different managerial delegations.

5. Extensions

In this section, under Model FMM and Model PMM, the optimal proportion of state-owned shares of the mixed-ownership enterprise is analyzed by maximizing $[\beta sw + (1-\beta)\pi_i]$, with the aim of further investigating the influence of capital efficiency and product differentiation on strategic ownership structure decisions.

Proposition 7. (i) When maximizing $[\beta sw^{FMM} + (1-\beta)\pi_i^{FMM}]$, the optimal proportion of state-owned shares under Model FMM can be shown as

$$\beta^{FMM} = \frac{\begin{array}{c}4(n-m)r^5+(9m-a+8n)r^4+128(m-n)(r+1)\left(r^2-8\right)+(16(m-n)^2r^{10}+32(m-n)(a-\\ 3m-2n)r^9+((18a+2252n)m-991m^2+a^2-52an-972n^2)r^8-1024(m-n)((a-2m-\\ 3n)r^7+(a-27m+22n)r^6-8(a+2m-7n)r^5-8(a-33m+28n)r^4)-(m-n)^2(524288r^3\\ -786432r^2-2097152r-1048576))^{\frac{1}{2}}\end{array}}{6r^4(m-n)};$$

(ii) If $0 < n < \frac{a}{17}$, then $\frac{\partial \beta^{FMM}}{\partial m} < 0$; if $n > \frac{a}{17}$, then $\frac{\partial \beta^{FMM}}{\partial m} > 0$. $\frac{\partial \beta^{FMM}}{\partial r} < 0$.

As shown in Proposition 7, according to $[\beta sw^{FMM} + (1-\beta)\pi_i^{FMM}]$ maximization, we find that partial nationalization is the optimal ownership structure under full delegation. Model FMM has no absolute advantages in terms of profits, consumer surplus and social welfare compared with Model FPP and Model FSS. Only when state capital is relatively low efficient, the profits of Model FMM is higher than that of Model FSS. However, the weighted sum of profits and social welfare is maximized under Model FMM. It indicates that the negotiation between the private and state-owned shares can reduce the decision failures made by the fully delegated managers. Furthermore, when private capital is sufficiently efficient ($0 < n < \frac{a}{17}$), the decrease in state capital efficiency will reduce the optimal proportion of state-owned shares, manifesting that enhancing the proportion of profits can improve $[\beta sw^{FMM} + (1-\beta)\pi_i^{FMM}]$. The negative correlation between r and β^{FMM} shows that the optimal proportion of state-owned shares can be enhanced by increasing product differentiation. Therefore, the efficiencies of private and state-owned shares and product differentiation influence the optimal proportion of state-owned shares, that is the ownership structure decisions.

Considering the complexity of Model PMM, we assume $a = 10$, $m = 2$, and $n = 1$ to simplify the analysis without losing generality. Figure 2 shows the relationship between β and $[\beta sw^{PMM} + (1-\beta)\pi_i^{PMM}]$ and Proposition 8 is obtained as following.

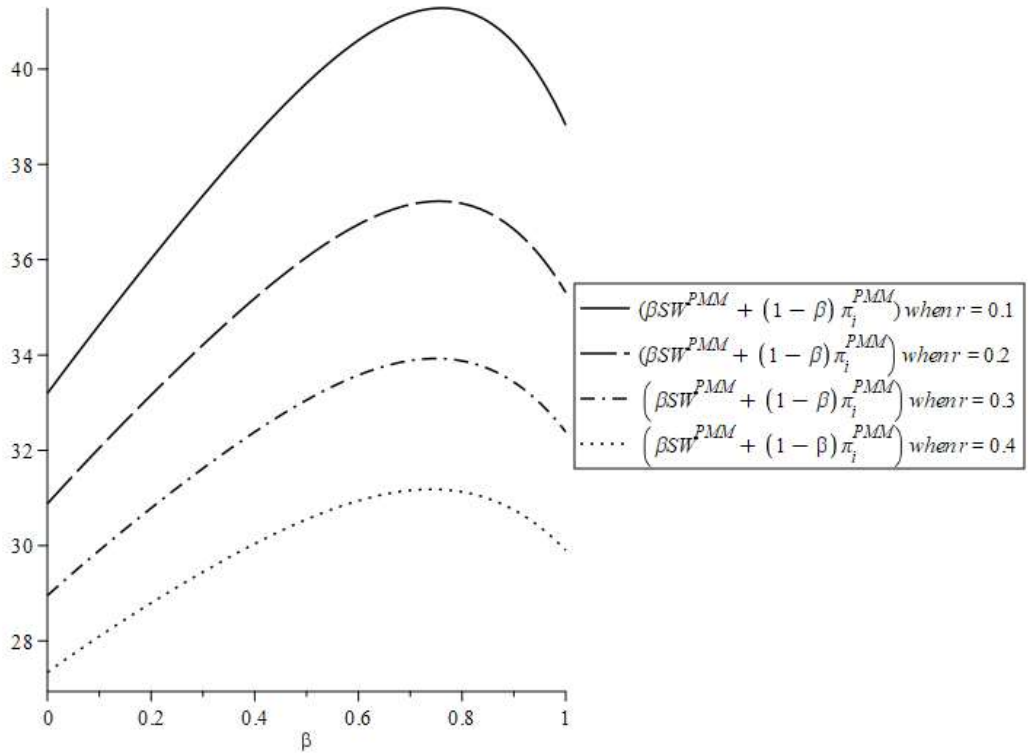

Fig.2: The curve of β and$[\beta sw^{PMM} + (1-\beta)\pi_i^{PMM}]$ when $r = 0.1, 0.2, 0.3, 0.4$, respectively

Proposition 8. (i) Because $F(\beta = 0) < 0$ and $F(\beta = 1) > 0$, there exists $\beta^{PMM} \in (0, 1)$ maximizes $[\beta sw^{PMM} + (1-\beta)\pi_i^{PMM}]$, where

$$
\begin{aligned}
F(\beta) = {} & (2(\beta+1)(2(n-m)\beta + a - m)r^9 + \\
& (29(m-n)\beta^3 + (20m - 15a - 5n)\beta^2 + (13n - 4a - 9m)\beta + 7a - 4m - 3n)r^8 \\
& +2(54(n-m)\beta^4 \\
& +(28a + 21m - 49n)\beta^3 + (140m - 23a - 117n)\beta^2 + (149m - 66a - 83n)\beta \\
& -43a + 56m - 13n)r^7 \\
& +(191(m-n)\beta^5 + (459n - 97a - 362m)\beta^4 \\
& +(228a - 997m + 769n)\beta^3 + (455a - 832m + 377n)\beta^2 + 2(99a + 8m - 107n)\beta \\
& +16(10m + n - 11a))r^6 + 2((92(n-m)\beta^6 + (48a + 384m \\
& -432n)\beta^5 + (980m - 216a - 764n)\beta^4 + (544n - 472a - 72m)\beta^3 \\
& +(344a - 2512m + 2168n)\beta^2 \\
& +(1320a - 2920m + 1600n)\beta + 728a \\
& -1024m + 296n)r^5 + (12(n-m)\beta^7 + (4a - 236m + 232n)\beta^6 + (136a - 572m + 436n)\beta^5 \\
& +(284a + 1660m - 1944n)\beta^4 + (5440m - 1296a \\
& -4144n)\beta^3 + (6560m - 2528a - 4032n)\beta^2 + (1280m - 1520a + 240n)\beta \\
& +512a - 896m + 384n)r^4 \\
& +(120(n-m)\beta^7 + 40(a + 14m - 15n)\beta^6 \\
& +(2448n - 240a - 2208m)\beta^5 + (1472a - 6320m + 4848n)\beta^4 \\
& +(2800a - 3560m + 760n)\beta^3 + (16128m - 872a - 15256n)\beta^2 + (21760m - \\
& 8960a - 12800n)\beta - 5376a + 7168m - 1792n)r^3 + (384(n-m)\beta^7 \\
& +(128a + 2048m - 2176n)\beta^6 + (1920n - 1024a - 896m)\beta^5 + (640a \\
& -5120m + 4480n)\beta^4 + (4736a - 15616m + 10880n)\beta^3 + \\
& (7168a - 23296m + 16128n)\beta^2 + (3456a - 2048m - 1408n)\beta - 2048a \\
& +6144m - 4096n)r^2 + (384(n-m)\beta^7 + (128a + 1792m - 1920n)\beta^6 \\
& +(4352n - 768a - 3584m)\beta^5 + 6400(m-n)\beta^4 + (18304m - 2304a \\
& -16000n)\beta^3 + (51328n - 2176a - 49152m)\beta^2 + (23552a - 67584m + 44032n)\beta \\
& +18432\mathrm{a} - 16384\mathrm{m} - 2048\mathrm{n})\mathrm{r} + 1024(m-n)(\beta^6 \\
& -5\beta^5 + \beta^4) + (27648m - 4096a - 23552n)\beta^3 + (25600n - 9216a - 16384m)\beta^2 \\
& +(14336\mathrm{a} - 49152\mathrm{m} + 34816\mathrm{n})\beta + 16384(\mathrm{a} - \mathrm{m})))
\end{aligned}
$$

(ii) The relationship between $[\beta sw^{PMM} + (1-\beta)\pi_i^{PMM}]$ and β is shown as an inverted-U shape, indicating that $[\beta sw + (1-\beta)\pi_i]^{PMM}$ increases and then decreases with the growth of β.

(iii) The increase of r suppresses the optimal proportion of state-owned shares.

As shown in Proposition 8, according to $[\beta sw + (1-\beta)\pi_i]^{PMM}$ maximization, we find that there exists the optimal proportion of state-owned shares under Model PMM, indicating that mixed ownership is the optimal ownership structure strategy. On the other hand, Figure 2 shows the inverted-U shape between $[\beta sw + (1-\beta)\pi_i]^{PMM}$ and β, which also proves the existence of the optimal proportion of state-owned shares. With the improvement of β, $[\beta sw + (1-\beta)\pi_i]^{PMM}$ will increase and then decrease when other parameters (a, m, n, r) are determined. Compared with Model PPP and Model PSS, the mixed-ownership competition with partial delegation produces the highest enterprise profits. And sw^{PMM} ranks second after sw^{PSS} as shown in Corollary 11. Therefore, the welfare distribution under Model PMM can realize $[\beta sw + (1-\beta)\pi_i]^{PMM}$ maximization. In fact, the combination of private and state-owned shares allows enterprises to diversify their goals beyond the single pursuit of social welfare or profit. The partial delegation contract will lead to the game between enterprise owners and managers according to their own goals, which is conducive to reducing capacity decision-making errors. We also find that the improvement of product differentiation will enhance the optimal proportion of state-owned shares in Model PMM, which is identical to that of Model FMM. It indicates that the effects of product differentiation cannot be changed under different delegation contracts.

6. Conclusions

This study investigates ownership structure, managerial delegation and capacity selection in a differentiated duopoly model and analyzes the effects of product differentiation, the proportion of state-owned shares and capital efficiency on capacity selection and ownership structure under different models. Furthermore, the comparative analysis is provided in terms of ownership structure and managerial delegation, followed by the determination of the optimal proportion of state-owned shares. The main conclusions are as follows.

First, regardless of ownership structures, enterprises always suffer from overcapacity under full delegation. However, the capacity arrangement in Model FPP is profit-oriented, which is different from the sale-oriented decision in Model FSS. Under partial delegation contract, ownership structures exert an effect on enterprises' capacity arrangements. There is no overcapacity in Model PSS. But the capacity depends on the proportion of state-owned shares and product differentiation in Model PMM. In addition, neither the full delegation nor the partial delegation can avoid overcapacity effectively for private enterprises. When conducting managerial delegation, enterprises should consider their ownership structures. The full delegation is better for private enterprises seeking profit maximization. To realize the maximization of social welfare, state-owned enterprises should choose partial delegation.

Second, product differentiation and the proportion of state-owned shares have effects on the equilibrium results under different models. Producing homogeneous products will exacerbate overcapacity under full delegation. The effects of product differentiation on equilibria in Model PPP are the same as those in Model FPP, which indicates that the choice managerial delegation exerts no impact on the role of product differentiation. Consumer surplus and social welfare can be enhanced by improving product differentiation under any competition mode. Complete product differentiation is the optimal choice for mixed-ownership enterprises under full delegation. In Model PSS, the partial delegation can help SOEs realize capacity balance, and the product differentiation fails to affect the capacity arrangements and enterprise profits. Furthermore, the effects of the proportion of state-owned shares on the equilibrium results are complex, which depend on the value ranges of some parameters (m, n and r). Therefore, the enterprise should be prepared to face the side effects when adjusting the ownership to achieve some economic goals.

Third, different ownership structures and delegation contracts have impacts on the comparisons of equilibrium results. In Model FPP, the two private enterprises produce lower overcapacity and more profits than those under Model PPP, which is contrary in terms of consumer surplus and social welfare. SOEs prefer partial delegation to realize no overcapacity and maximal social welfare at the expense of profits. Compared with Model FMM, mixed-ownership enterprises should also choose partial delegation because of the higher profits and social welfare. Partial delegation has the advantage of improving social welfare in both pure and mixed competitions. When choosing full delegation, private, state-owned, and mixed-ownership enterprises all hold excess capacity, which is most serious in SOEs. However, under partial delegation, the overcapacity is most serious in private enterprises instead of SOEs. Compared with pure private and state ownership structures, mixed ownership can produce more welfare from the supply side and demand side under partial delegation. These conclusions have a certain significance for enterprise owners and managers to conduct the right strategies in managerial delegation, capacity selection and ownership structures.

Fourth, there exist the optimal proportions of state-owned shares in Model FMM and Model PMM, which are influenced by product differentiation and capital efficiency. Under either partial or full delegation contracts, partial nationalization maximizes the weighted sum of profits and social welfare and thus becomes the optimal ownership structure strategy for mixed-ownership enterprises. It indicates that the negotiation between the private and state-owned shares can reduce

the decision failures made by the partially or fully delegated managers. In Model FMM, when private capital is sufficiently efficient, the decrease in state capital efficiency will reduce the optimal proportion of state-owned shares. However, the optimal proportion of state-owned shares can be enhanced by increasing product differentiation in both Model FMM and Model PMM. It indicates that the effects of product differentiation cannot be changed under different delegation contracts. Therefore, the efficiencies of private and state-owned shares and product differentiation play key roles in determining the optimal ownership structure strategy.

There are still some limitations that need to be resolved in the future. First, we only study the case of two enterprises in the market competition, which can be extended to multi-enterprise competition. Second, the study does not consider the endogeneity of the managerial delegation. Whether adopt the managerial delegation and choose partial or full delegation should be further analyzed. And the comparative studies on the effects of the endogeneity of managerial delegation on ownership structure and capacity selection should be conducted. Third, the article divides corporate equity into state-owned and private shares, but state-owned and private shares can also be further divided into different types in reality. Therefore, the refinement of equity is worthy of in-depth study in the future.

Author Contributions

Conceptualization, Yufan Zheng and Xinran Xie; methodology, Xinran Xie; software, Yufan Zheng; validation, Yufan Zheng.; formal analysis, Yufan Zheng; writing—original draft preparation, Yufan Zheng and Xinran Xie; writing—review and editing, Yufan Zheng and Xinran Xie; visualization, Yufan Zheng; supervision, Xinran Xie. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Akcigit, U., Alp, H., & Peters, M. (2021). Lack of selection and limits to delegation: Firm dynamics in developing countries. *American Economic Review*, *111(1)*, 231-275. https://doi.org/10.1257/aer20180555

[2] Antal, T., & Számadó, R. (2026). Design and evaluation of a compliance management framework for business operations: A system engineering perspective. *Journal of Engineering Management and Systems Engineering*, *5(2)*, 120-136. https://doi.org/10.56578/jemse050201

[3] Antelo, M., Peón, D., & Martínez-Filgueira, X. M. (2022). Mergers by family firms and managerial delegation: A Cournot model and empirical evidence from Spain. *Management Research Review*, *45(1)*, 1-25. https://doi.org/10.1108/MRR-08-2020-0474

[4] Azar, J., & Vives, X. (2021). General equilibrium oligopoly and ownership structure. *Econometrica*, *89(3)*, 999-1048. https://doi.org/10.3982/ECTA17906

[5] Chen, J., Sun, C., & Liu, J. (2022). Corporate social responsibility, consumer sensitivity, and overcapacity. *Managerial and Decision Economics*, *43(2)*, 544-554. https://doi.org/10.1002/mde.3401

[6] Chou, C. H. (2023). Managerial delegation, product R&D and subsidies on R&D investment costs. *B.E. Journal of Economic Analysis & Policy*, *23(1)*, 55-78. https://doi.org/10.1515/bejeap-2021-0402

[7] Dobson, P. W., & Chakraborty, R. (2020). Strategic incentives for complementary producers to innovate for efficiency and support sustainability. *International Journal of Production Economics*, *219*, 431-439. https://doi.org/10.1016/j.ijpe.2018.02.001

[8] Dong, K., & Sun, W. (2022). Would the market mechanism cause the formation of overcapacity?: Evidence from Chinese listed firms of manufacturing industry. *International Review of Economics & Finance*, *79*, 97-113. https://doi.org/10.1016/j.iref.2022.01.002

[9] Edmans, A., Levit, D., & Reilly, D. (2019). Governance under common ownership. *Review of Financial Studies*, *32(7)*, 2673-2719. https://doi.org/10.1093/rfs/hhy108

[10] Fanti, L. (2016). Social welfare and cross-ownership in a vertical industry: When the mode of competition matters for antitrust policy. *Japan and the World Economy*, *37*, 8-16. https://doi.org/10.1016/j.japwor.2016.02.004

[11] Fanti, L., Gori, L., & Sodini, M. (2017). The bargaining-sales-delegation game. *Managerial and Decision Economics*, *38(6)*, 857-889. https://doi.org/10.1002/mde.2822

[12] Fershtman, C., & Judd, K. L. (1987). Equilibrium incentives in oligopoly. *American Economic Review*, *77*, 927-940.

[13] Friehe, T., Pham, C. L., & Miceli, T. J. (2022). Product liability and strategic delegation: Endogenous manager incentives promote strict liability. *Review of Industrial Organization*, *61(2)*, 149-169. https://doi.org/10.1007/s11151-022-09870-1

[14] Hackner, J. A. (2000). Note on price and quantity competition in differentiated oligopolies. *Journal of Economic Theory*, *93(2)*, 233-239. https://doi.org/10.1006/jeth.2000.2654

[15] Hamada, K. (2021). Cooperative managerial delegation revisited: Including the firm's cost as a managerial objective. *Bulletin of Economic Research*, *73(2)*, 171-178. https://doi.org/10.1111/boer.12240

[16] Itan, I., & Angellina, B. (2023). The mediating role of board size and working capital management in corporate governance on firm performance. *Journal of Accounting, Finance and Auditing Studies*, *9(1)*, 172-198. https://doi.org/10.32602/jafas.2023.008

[17] Jahnke, P. (2019). Ownership concentration and institutional investors' governance through voice and exit. *Business & Politics*, *21(3)*, 327-350. https://doi.org/10.1017/bap.2019.2

[18] Kim, J.-Y. (2021). Public monopoly versus mixed oligopoly: product differentiation and social efficiency. *Economic Research-Ekonomska Istraživanja*, *35*(1), 1306-1321. https://doi.org/10.1080/1331677x.2021.1962385

[19] Komatina, N., & Marinković, D. (2026). An objective multi-attribute decision-making framework for identifying critical phases in automotive component manufacturing. *Journal of Engineering Management and Systems Engineering*, *5(1)*, 1-9. https://doi.org/10.56578/jemse050101

[20] Kopel, M., & Pezzino, M. (2018). *Strategic delegation in oligopoly*.

[21] Lambertini, L., & Trombetta, M. (2002). Delegation and firms' ability to collude. *Journal of Economic Behavior and Organization*, *47(4)*, 359-373. https://doi.org/10.1016/S0167-2681(01)00203-7

[22] Matsumura, T. (1998). Partial privatization in a mixed duopoly. *Journal of Public Economics*, *70(3)*, 473-483. https://doi.org/10.1016/S0047-2727(98)00051-6

[23] Phi, N. T. M., Taghizadeh-Hesary, F., Tu, C. A., Yoshino, N., & Kim, C. J. (2020). Performance Differential between Private and State-owned Enterprises: An Analysis of Profitability and Solvency. *Emerging Markets Finance and Trade*, *57*(14), 3913-3928. https://doi.org/10.1080/1540496x.2020.1809375

[24] Richardson, H. A., Kluemper, D. H., & Taylor, S. G. (2021). Too little and too much authority sharing: Differential relationships with psychological empowerment and in-role and extra-role

performance. *Journal of Organizational Behavior*, *42(8)*, 1099-1119. https://doi.org/10.1002/job.2548

[25] Saha, B., & Sensarma, R. (2011). Mixed ownership, managerial incentives and bank competition. *Bulletin of Economic Research*, *63(4)*, 385-403. https://doi.org/10.1111/j.1467-8586.2010.00354.x

[26] Schmidt, K. (1997). Managerial incentives and product market competition. *Review of Economic Studies*, *64*, 191-213. https://doi.org/10.2307/2971709

[27] Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics*, *15(4)*, 546-554. https://doi.org/10.2307/2555525

[28] Vickers, J. (1985). Delegation and the theory of the firm. *Economic Journal*, *95*, 138-147. https://doi.org/10.2307/2232877

[29] Wang, L. F. S., & Wang, H. (2021). Will managerial delegation impede upstream collusion? *Journal of Economics*, *134*(2), 127-146. https://doi.org/10.1007/s00712-021-00741-z

[30] Wang, W., Tang, O., & Huo, J. (2018). Dynamic capacity allocation for airlines with multi-channel distribution. *Journal of Air Transport Management*, *69*, 173-181. https://doi.org/10.1016/j.jairtraman.2018.02.006

[31] Wang, X., & Wang, L. F. S. (2021). Vertical product differentiation, managerial delegation and social welfare in a vertically-related market. *Mathematical Social Sciences*, *113*, 149-159. https://doi.org/10.1016/j.mathsocsci.2021.06.001

[32] Wu, J., Jiang, F., & He, Y. (2018). Pricing and horizontal information sharing in a supply chain with capacity constraint. *Operations Research Letters*, *46(4)*, 402-408. https://doi.org/10.1016/j.orl.2018.04.009

[33] Wu, X., Kouvelis, P., & Matsuo, H. (2013). Horizontal capacity coordination for risk management and flexibility: Pay ex ante or commit a fraction of ex post demand? *Manufacturing & Service Operations Management*, *15(3)*, 458-472. https://doi.org/10.1287/msom.2013.0435

[34] Xu, L., Zhang, Y., & Matsumura, T. (2023). Competition mode and common ownership in a mixed oligopoly. *The Manchester School*, *91*(4), 306-319. https://doi.org/10.1111/manc.12431

[35] Zhou, W., Huang, W., Hsu, V. N., & Guo, P. (2023). On the Benefit of Privatization in a Mixed Duopoly Service System. *Management Science*, *69*(3), 1486-1499. https://doi.org/10.1287/mnsc.2022.4424