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Equivalence of MCDM Methods and Synthesis of Solution Based on Ratings Obtained in Different Models

Irik Z. Mukhametzyanov[1], Dragan Pamucar[2*]

[1] Institute of Socio-Economic Research – subdivision of the Ufa Federal Research Centre of the Russian Academy of Sciences, Ufa, Russia
[2] Széchenyi István University, Győr, Hungary

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ABSTRACT

Synthesis of solutions based on a set of models is a modern trend in the field of multi-criteria choice. It is assumed that a solution based on many methods increases the reliability of the decisions made. One of the important tasks is to select an independent set of models. Comparison of various multi-criteria methods is performed using two lists: rank and rating. To compare the rating of alternatives obtained using different MCDM models, the article uses the Relative Performance Indicator (RPI). Using RPI, six identical methods for aggregating private attributes of alternatives are established: Weighted Sum Model (WSM), Ratio System approach (RS), Multi-Attributive Border Approximation area Comparison (MABAC), Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) with L_1 metric, Multi Atributive Ideal-Real Comparative Analysis (MAIRCA) and Ranking of Alternatives with Weights of Criterion (RAWEC) provided that each aggregation method combines the same method of linear normalization of attributes. This allows avoiding duplication of equivalent methods in the Multi-Method Model (3M) approach combining different MCDM models. When solving MCDM problems, it is recommended to use the simplest and most easily interpreted of them: WSM. The presented methodology is recommended as mandatory for the analysis of new or hybrid MCDM methods to eliminate duplication of existing methods. A synthesis of a solution based on ratings obtained in different MCDM models within the 3M approach is proposed. The method includes coordinating the common goal of several models and bringing the ratings obtained in different MCDM models to a common scale, which allows comparing and aggregating the ratings. The resulting rating is more informative than a rating based on ranks, such as Borda rules or similar, since it reflects the real proportions of the effectiveness of alternatives in different models.

*Corresponding author.
E-mail address: dpamucar@gmail.com

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1 Introduction

The problem of multi criteria decision making (MCDM) on a discrete set of alternatives has the following description [1, 2]: there are *m* objects (alternatives) A_i, each of which is characterized by *n* attributes (features) defined within the framework of the selected criteria C_j (j=1,…, *n*). The attributes are quantitative estimates (a_{ij}) of the selected (important) properties of the objects.

On the other hand, the attributes are criteria for choosing the best alternative. The selection problem is complicated by a conflict of criteria, when one of the alternatives surpasses a certain set of other alternatives by one (or several) features, but not by the remaining features. It is required to make an optimal choice on a finite set of alternatives. One of the common approaches to solving MCDM problems is to transform the feature vector of each alternative into a scalar feature — the alternative rating (assessment score, performance indicator). Rating of the alternative is determined using an aggregation function, the arguments of which are the normalized values of features and the weights of the criteria. A specially constructed aggregation function defines the MCDM method. According to the method of construction, the MCDM methods can be a classification into three groups:

(1) Value Measurement methods, such as WSM (Weighted Sum Model) [1]; RS (the Ratio System approach) by Brauers and Zavadskas [3], WPM (Weighted Product Model) by Chakraborty and Zavadskas [4], MABAC (Multi-Attributive Border Approximation area Comparison) by Pamučar and Ćirović [5], etc.,

(2) Goal or Reference Level models, such as TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) by Opricovic [6] and VIKOR (VIse Kriterijumska Optimizacija kompromisno Resenje, in Serbian) by Opricovic and Tzeng [7],

(3) Outranking Techniques, such as PROMETHEE (Preference Ranking Organization METHod for Enhancement of Evaluations) by Brans *et al.,* [8], ORESTE (Organization, Rangement Et Synthesis de donnéEs relarionnelles, in French) [9].

Based on the ranking of alternatives, a ranking list is constructed.

Normalization and weighting of criteria are standard MCDM methods, independent of the aggregation method. Hence, there is a tendency in MCDM to create hybrid models that combine the best methods of normalization/inversion, weighting and subsequent aggregation of attributes of the alternatives [10-14]. For example, the combination of the Max normalization method, the AHP/EV weighting method and the TOPSIS aggregation represent one of the hybrid models. This approach aims to provide more reliable results. By integrating different methods, hybrid MCDM methods can more effectively handle diverse information, conflicting criteria and different stakeholder preferences.

As a justification for their effectiveness, the authors of hybrid methods use a comparison of the results with those obtained using basic MCDM methods, such as cost measurement methods or the proximity to the “ideal” model. The comparison is made based on the Spearman rank correlation (for rank lists) or Pearson correlation (for rating lists) see, for example [15, 16], or on some general arguments and principles [17].

It is obvious that the large number of available methods creates a problem of choosing a solution method. Different methods of solving a problem (different models) lead in some cases to different results. Sometimes this difference is significant. At the same time, there is no objective criterion for choosing an effective method for solving a problem. In such a situation, when choosing a method, researchers adhere to the principle of the success of a particular method in solving problems of a certain class in the application area of interest Zavadskas *et al.* [18, 19]. The frequency of using a method is also important.

In many cases, the ranks of alternatives in different methods are consistent with each other, which increases the reliability of solutions. This can be easily established using rank correlation.

Meanwhile, the ratings of alternatives (assessment score) for the same set of methods are less correlated. It is obvious that a rating list, unlike a ranked list, reflects the "fine" structure of relationships. The difference between the rating value of two (or more) alternatives may be insignificant. Given the high sensitivity of ratings to the parameters of the MCDM model, such alternatives should have the same preference status (the same ranks) Mukhametzyanov and Pamucar [20]. The availability of many methods (including hybrid models) at the disposal of researchers has determined the modern trend in the field of MCDM: the synthesis of solutions based on many models, designated in the study [20] as the Multi-Method Model (3M). It is assumed, that a solution based on many methods increases the reliability of the decisions made. This is consistent with the majority principle as the main method of making collegial decisions. Although, historical experience shows that this principle is not fulfilled in some cases.

The main problems of decision making based on the 3M approach, which currently do not have a final solution, are the following [20]:

(i) which MCDM methods to include in the list of methods used to solve a specific problem - qualitative and quantitative composition,
(ii) how to compare the results obtained in different methods,
(iii) how to evaluate the significance (weight) of methods,
(iv) whether to group methods and how to form groups,
(v) what is the solution synthesis method.

In terms of grouping MCDM methods the current result is a ranking obtained by aggregating the results of three ranking methods: value measurement methods, reference level models and the full multiplicative form within the MULTIMOORA concept [21-23].

In terms of solution synthesis, various tools can be used, such as dominance theory, arithmetic/geometric mean, Borda rule Hafezalkotob *et al.*, [23-24] or similar ones, for example, the Copeland method Özdağoğlu *et al.*, [25], dominance-oriented graph, optimization model, ORESTE method, rank position method and exact order preference method.

This study solves two of the above problems. Namely, the selection of an independent set of models in the list of methods used to solve a specific problem. This study presents a methodology for comparing various multi-criteria methods based on comparing the relative performance indicator (RPI) of alternatives. Another problem solved in this paper is a method for synthesizing the results of various models. The authors propose an approach that preserves information about the ratio of ratings in each individual model as much as possible.

The paper has the following structure. The second section presents an interpretation of MCDM methods in the form of an MCDM model that combines various methods for normalizing the decision matrix, various methods for estimating the weight of criteria and various methods for aggregating the attributes of alternatives. A brief description of the MCDM methods for which the equivalence of the models has been established is given, and some author's comments on the use of the MAIRCA (Multi Atributive Ideal-Real Comparative Analysis) and RAWEC (Ranking of Alternatives with Weights of Criterion) methods are given. Section 2.3 presents a description of various linear procedures for normalizing the initial decision matrix included in various MCDM models in the subsequent analysis.

The third section presents a comparison of the methods. Using the relative performance indicator (RPI) of alternatives, six identical methods for aggregating private attributes of alternatives are established: WSM, RS, MABAC, TOPSIS (L_1), MAIRCA and RAWEC, provided that each aggregation method combines the same method of linear normalization of attributes. This allows avoiding duplication of equivalent methods in the synthesis procedures of various methods. When solving MCDM problems, the simplest and most easily interpreted of them is recommended for use: WSM. The presented methodology is recommended as mandatory for the analysis of new MCDM ranking methods to eliminate duplication of existing methods.

The fourth section presents the harmonization of the common goal of several models and the reduction of ratings obtained in different MCDM models to a common scale, which allows comparing and aggregating ratings. Based on the linear transformation of the rating of alternatives obtained in different models, a synthesis of the results in the form of a weighted sum is performed. The resulting rating is more informative than the rating based on ranks (Borda's rule and similar), since it reflects the real proportions of ratings.

Each of the properties and statements of this article is provided with a corresponding numerical example. This is an example of the equivalence of 6 different MCDM models and examples of synthesizing a solution based on ratings obtained in different MCDM models.
The conclusion and findings follow.

2 Methods

2.1 Designation

A_i	alternatives (objects) (i=1,…, m)
C_j^+, C_j^-	criteria or objects properties (j=1,…, n), (+) benefit, (-) cost
a_{ij}	natural value of the jth attribute of the ith alternative (elements of the decision matrix D)
x_{ij}	normalized elements of decision matrix a_{ij}
$\bar{x}_j$	average value of jth criterion
a_j^{max}	maximum element in criteria j
a_j^{min}	minimum element in criteria j
w_j	weight or importance of criteria (j=1,…, n)
Q_i	the performance indicator (assessment score) of alternatives (objects) (i=1,…, m)
dQ_i	*Relative Performance Indicator* (RPI) *of alternatives*

2.2 MCDM rank model

One of the common approaches to solving MCDM problems is to transform the feature vector of each alternative into a scalar feature — the alternative rating. The alternative rating is determined using an aggregation function whose arguments are the normalized feature values and criterion weights:

$$Q_i=F(x_{ij}, w_j). \quad (1)$$

The rating list Q is sorted in descending (or ascending) order, based on which a ranking list of alternatives is formed. Without loss of generality, we will assume that the sorting and renumbering of these two lists is performed in the form:

$$Q_1 \geq Q_2 \geq \ldots \geq Q_m \ , \quad (2)$$
$$A_1 \succ A_2 \succ \ldots \succ A_m \ , \quad (3)$$

Where, the sign “≻” means preferable, and the subscripts determine the rank of the alternative.

Historically, the MCDM method is primarily understood as the procedure for aggregating private features of alternatives F according to (1). However, it is well known that three main components of MCDM methods: the method for assessing the importance of attributes (criteria weight), the method for normalizing the decision matrix, and the distance metric for Reference Level methods (TOPSIS, etc.) are essential and in many cases (tasks) determine the ranking result.

Researchers have at their disposal:

- more than 10 main methods for determining the weight of criteria in the absence of a preference criterion in choosing a method,
- more than 5 main methods for normalizing the decision matrix, the choice of which is based on principles,
- 3 main distance metrics in the n-dimensional feature space for MCDM methods based on the distance from the "ideal" (L_1 - CityBlock; L_2 - 'Eucludean', L_∞ - 'Chebyshev').

The choice of the three methods (F, w, x) is not formalized and is carried out based on some general principles by Mukhametzyanov [26]. It follows that the number of MCDM problem solution options for only one aggregation method is determined by the number of possible (admissible) combinations of model arguments [13, 14, 20]. Using WSM aggregation, it is possible to implement 50 different options or models within the framework of only the 3 main arguments. Obviously, the ratings of alternatives in different models will differ, and in some models, the ranks will also differ. It should be noted that none of the arguments used in the model (1) has priority over another. This means that all models are equal. Considering the above arguments, instead of the term MCDM method, it is advisable to use the term" MCDM model" with the obligatory indication of the argument methods used in solving the problem. For example, model:

Q_i=TOPSIS(L_1,*'w'*= AHP-EV, *'Norm'*=Max)

uses the TOPSIS method with L_1-distance metric (CityBlock), and the Max-method of normalization, and implements a weight estimate based on a matrix of paired comparisons of criteria in the AHP process and eigenvector method.

The MCDM rank model includes a method for assessing the significance of criteria w_j, a method for normalizing the decision matrix x_{ij} =*Norm*(a_{ij}), and a method F for aggregating private features of alternatives into the performance indicator Q_i of each alternative. The model can also contain additional parameters determined by the method for constructing the aggregation function.

In many cases, combining model parameters does not significantly change the ratings, and the ranking of alternatives does not change. This situation characterizes the stability of the solution to variations in the model parameters. In this situation, the final choice is not complicated by a multitude of options. Another situation, on the contrary, characterizes the instability of the solution, in which variations in the model parameters change the ranking and the final choice is not determined. For example, in the studies of Mukhametzyanov and Pamucar [20, 26] examples of decision matrices are presented, for which the alternatives of rank 1 for 5 different normalization methods are different. Structurally, each admissible triple: normalization, weighting and aggregation method} is interpreted as one of the MCDM models. To use the "MCDM model" approach, it is necessary to determine which normalization, weighting and aggregation methods should be selected and how to further integrate the ranking results. For subsequent integration, it is important to analyze the independence of the models to ensure equal "voting" conditions for each of the models in the final selection of the contender.

2.2.1 Weighted Sum Method (WSM)

WSM has a simple form:

$$Q_i = \sum_{j=1}^{n} w_j x_{ij} \ , \tag{4}$$

Where, x_{ij} are the normalized values of the decision matrix, w_j are the weights of the criteria.

The best alternative corresponds to the highest value of the performance indicator *Q*.

2.2.2 The Ratio System approach [3]

In Ratio System, the non-beneficial sum is subtracted from the beneficial sum:

$$Q_i = \sum_{j=1}^{n} sign(C_j) \cdot w_j \cdot x_{ij} \ , \tag{5}$$

$$sign(C_j) = \begin{cases} 1, & if \ j \in C_j^+ - \text{beneficial criteria} \\ -1, & if \ j \in C_j^- - \text{non beneficial criteria} \end{cases}, \tag{6}$$

The best alternative corresponds to the highest value of the performance indicator *Q*.

2.2.3 Multi-Attributive Border Approximation area Comparison (MABAC) [4]

MABAC is a cost measurement method. The performance indicator of alternatives is defined as:

$$Q_i = \sum_{j=1}^{n} (v_{ij} - g_j) \ , \tag{7}$$

$$v_{ij} = (x_{ij} + 1) \cdot \omega_j, \ i = 1,..., m; \ j = 1,...,n \ , \tag{8}$$

$$x_{ij} = \text{Max-Min}(a) = \begin{cases} \dfrac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}, \text{for benefit criteria} \\ \dfrac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}, \text{for cost criteria} \end{cases}, \tag{9}$$

$$g_j = \left(\prod_{i=1}^{m} v_{ij} \right)^{1/m} . \tag{10}$$

The best alternative corresponds to the highest value of the performance indicator *Q*.

2.2.4 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [7]:

To determine the performance indicator of the *i*th alternative Q_i, a homogeneous function was used:

$$Q_i = \frac{S_i^-}{S_i^+ + S_i^-} \ , \tag{11}$$

$$v_{ij} = x_{ij} \cdot w_j, \ S_i^+ = d(v_{ij}, v_j^+), \ S_i^- = d(v_{ij}, v_j^-) \ , \tag{12}$$

$$v_j^+ = \{\max_i v_{ij} \left| if \ j \in C_j^+; \ \min_i v_{ij} \right| if \ j \in C_j^- \} \ , \tag{13}$$

$$v_j^- = \{\min_i v_{ij} \left| if \ j \in C_j^+; \ \max_i v_{ij} \right| if \ j \in C_j^- \} \ , \tag{14}$$

S_i^+ and S_i^- were the distances *d* between the ideal and anti-ideal objects respectively. Whereas, the alternative A_i in the *n*-dimension attributes space, which are defined in one of the L_p-metrics.

$$L_p\left(X,Y\right)=\left[\sum_{i=1}^{m}(x_i-y_i)^p\right]^{1/p},\ 1\le p\le\infty; L_\infty\left(X,Y\right)=\max_i\left|x_i-y_i\right|, \tag{15}$$

The TOPSIS ranking result depends on the choice of distance metric. Let us denote the TOPSIS method with the L_1 City Block metric as TOPSIS(L_1).

The best alternative corresponds to the highest value of the performance indicator *Q*.

2.2.5 Multi Atributive Ideal-Real Comparative Analysis (MAIRCA)

The basic version of the method is as follows Pamucar *et al*. [27]:

Step 1. Define *m* alternatives and *n* criterion. Define decision matrix: $D=(a_{ij})$ [$m\times n$]. Define weights of criterion w_j [$1\times n$].

Step 2. Defining preferences for the choice of alternatives P_{Ai}. If decision maker is neutral to the selection probability of each alternative, then $P_{Ai}=1/m$.

Step 3. Calculation of the elements of the theoretical ratings matrix T_p :

$$t_{p_{ij}}=P_{Ai}\cdot w_j\,. \tag{16}$$

Step 4. Definition of the elements of the real ratings matrix T_r :

$$t_{r_{ij}}=t_{p_{ij}}\cdot x_{ij}\,, \tag{17}$$

$$x_{ij}=Norm(a_{ij})\,.$$

As a normalization method, the authors of [27] use Max-Min normalization:

$x_{ij}=\dfrac{a_j^{\max}-a_{ij}}{a_j^{\max}-a_j^{\min}}$, for benefit criteria, and

$x_{ij}=\dfrac{a_{ij}-a_j^{\min}}{a_j^{\max}-a_j^{\min}}$, for cost criteria.

Step 5. The calculation of the total gap matrix *G*:

$$G=T_p-T_r=t_{p_{ij}}-t_{r_{ij}}\,. \tag{18}$$

Step 6. The calculation of the final values of criteria functions (Q_i) by alternatives.

$$Q_i=\sum_{j=1}^{n}g_{ij} \tag{19}$$

The best alternative corresponds to the highest value of the performance indicator *Q*.

Comments on the MAIRCA method

1) Setting the priority P_{Ai} for a part of alternatives in Step 2 is an essential parameter of the MAIRCA model and affects the ratings and ranking. For the MAIRCA method, the equivalence of WSM is fulfilled in the absence of alternative priority, If DM is neutral to selection probability of each alternative, then $P_{Ai} = 1/m$. The MAIRCA method is equivalent to WSM in the absence of priority of alternatives: $P_{Ai} = 1/m$, $\forall i$, i.e. the decision maker is neutral to the probability of choosing each alternative.

2) Max-Min normalization transforms the values of all attributes to [0, 1]. It may seem that normalization with different range of attribute values, for example, Max, will affect the ranking. The answer is negative. The ratings change for different aggregation methods, but PRI and ranking remain unchanged.

2.2.6 Ranking of Alternatives with Weights of Criterion (RAWEC)

The basic version of the method is as follows Puška *et al*. [28]:

Step 1. Define m alternatives and n criterion. Define decision matrix: $D=(a_{ij})$ $[m\times n]$. Define weights of criterion w_j $[1\times n]$.

Step 2. Normalization. Define two decision matrix r_{ij} and r'_{ij} :

$r_{ij}=a_{ij}/a_j^{max}$ for benefit criteria, and $r_{ij}= a_j^{min}/a_{ij}$ for cost criteria (inversion), (20)

$r'_{ij}= a_j^{min}/a_{ij}$ for benefit criteria (inversion), and $r'_{ij}=a_{ij}/a_j^{max}$ for cost criteria. (21)

Since strictly monotone functions are used in normalization and inversion of values, then for each j, the alternative i^*, which has the maximum value of the attribute $\max_i(r_{ij})$, has the minimum value $r'_{i^*j} = \min_i(r'_{ij})$, and vice versa.

Step 3. Determine the weighted ratings of the ith alternative:

$$v_i = \sum_{j=1}^{n} w_j \cdot (1 - r_{ij}), \tag{22}$$

$$v_i^{'} = \sum_{j=1}^{n} w_j \cdot (1 - r_{ij}^{'}). \tag{23}$$

The deviation from 1 (the best value) for the first standardized list (r_{ij}) should preferably be as small as possible, and the deviation for the second value (r'_{ij}) should be as large as possible.

Step 4. Determine the rating of alternatives:

$$Q_i = \frac{v_i^{'} - v_i}{v_i^{'} + v_i}. \tag{24}$$

The best alternative corresponds to the highest value of the performance indicator Q.

Comments on the RAWEC method

1) As a normalization function $Norm(a_{ij})$, the authors of the method use the transformation $Max(a_{ij})=a_{ij} / a_j^{max}$ for cost criteria and the inverse transformation $iMax(a_{ij})= a_j^{min} / a_{ij}$,, which is nonlinear and, therefore, for normalized data, the dispositions of attribute values do not correspond to natural values. Thus, benefit attributes have a linear normalization, and cost attributes have a nonlinear one for r_{ij} (and vice versa for r'_{ij}), which leads to a distortion of the original data. We use and recommend the linear inverse transformation $ReS(r_{ij})= - r_{ij} + r_j^{max} + r_j^{min}$ [29].

2) Calculating the deviations in step 3 from the best value equal to 1 seems to require that the best normalized value for each criterion should be equal to 1. These are the Max, dSum, Max-Min methods. However, dQ will also be the same when using methods such as Sum, Vec, Z[0,1], i.e. the RAWEC method is equivalent to the WSM method when using linear normalization methods (including ReS inversion).

3) From the formulas of step 3 it follows that $v_i \in (0, 1)$, $v'_i \in (0, 1)$, since $\sum w_j=1$ and $r_{ij} \leq 1$. The limit values 0 and 1 are not considered, since they represent complete dominance of one of the attributes. From which it follows that $Q_i \in (-1, 1)$.

2.3 Linear normalization/inversion methods

The general formula for linear normalization is as follows [29]:

$$x_{ij} = Norm(a_{ij}) = \frac{a_{ij} - a_j^*}{k_j}, \; i = 1,...,m, \; j = 1,...,n \; . \tag{25}$$

Typical methods are defined by the following parameters of displacement and compression stretching:

Max:	Sum:	Vec:	dSum:	Max-Min:
$k_j = a_j^{\max}$,	$k_j = \sum_{i=1}^{m} a_{ij}$,	$k_j = \sqrt{\sum_{i=1}^{m} a_{ij}^2}$,	$k_j = \sum_{i=1}^{m} (a_j^{\max} - a_{ij})$,	$k_j = a_j^{\max} - a_j^{\min}$
$a_j^* = 0$	$a_j^* = 0$	$a_j^* = 0$	$a_j^* = a_j^{\max} - k_j$	$a_j^* = a_j^{\min}$

Z[0,1]: $a_{ij} = -a_{ij} + a_j^{min} + a_j^{max}$, for $j \in C^-$ – cost criteria (inversion)

$$u_{ij} = \frac{a_{ij} - a_j^*}{s_j}, \; a_j^* = mean(a_{ij}), \; s_j = \sqrt{\frac{1}{m} \cdot \sum_{i=1}^{m} (a_{ij} - a_j^*)^2}$$

$$x_{ij} = (u_{ij} - u^{min}) / \left(u^{max} - u^{min}\right)$$

The universal ReS algorithm [30] for transforming cost criteria into benefit criteria (inversion) is as follows:

Step 1: Linear normalization $x_{ij}=Norm(a_{ij})$

Step 2: Reverse sorting algorithm (ReS)

$$ReS\left(x_{ij}\right) = -x_{ij} + x_j^{min} + x_j^{max} \; , \; \text{for } j \in C^- - \text{ cost criteria} \, . \tag{26}$$

3 Equivalence of MCDM methods

3.1 Relative Performance Indicator (RPI)

The presence of many multi-criteria decision-making methods requires the development of a tool that can be used to compare them with each other and to synthesize solutions. As noted in the introduction, comparison of different MCDM methods is performed to a greater extent based on Spearman's rank correlation (for a rank list) or Pearson's correlation (for a rating list).
Comparison of ranks of different methods is simple, but it is rather rough, since ranks do not reveal the degree of superiority of alternatives among themselves. A rating list, in contrast to a rank list, reflects the "fine" structure of relationships between alternatives [20]. However, ratings of different methods are defined in different scales determined by the method of aggregating private attribute values.

As a result of applying two different MCDM models, two rating lists are obtained: $Q_i^{(1)}$ and $Q_i^{(2)}$, i=1,..., m. The subscript corresponds to the ith alternative; the superscript corresponds to the model number. If the goal of both methods is consistent, then we order both rating lists by decreasing rating for the goal Larger-The-Better (LTB), or by increasing rating for the goal Smaller-The-Better (STB). If the goal of two methods conflicts, for example, LTB vs STB, then it is necessary to reconcile the goals. To do this, it is sufficient to apply the ReS transformation to one of the list [30]:

$$Q_i = -Q_i + Q_i^{\max} + Q_i^{\min} . \quad (27)$$

As a result of the ordering (considering the coordination of the goals of the two methods), two ranking lists are obtained: $R_i^{(1)}$ and $R_i^{(2)}$, i=1,..., m. The subscript corresponds to the rank of the alternative. Rank 1 determines the best rating value. The value of R_i determines the number of the alternative that has the ith rank.

Definition 1: Two rank lists are identical if $R_i^{(1)} \equiv R_i^{(2)}$, $\forall i$.

Definition 2: Two rating lists are equivalent if they have identical rank lists and if they are transformed into each other using a linear transformation:

$$Q_i^{(2)} = k \cdot Q_i^{(1)} + b,\ \forall i,\ k, b - Const . \quad (28)$$

To show that the two rating lists are equivalent, it is necessary to determine the constants k and b by solving the system of equations (19), which is not rational. Another way is to determine the pairwise correlation coefficient (Pearson) of the two rating lists, which should be equal to 1. For comparison, it is rational to transform both lists (after ordering and considering the coordination of goals) to the form of the Relative Performance Indicator (RPI) [20].

$$\mathrm{d}Q_i = \frac{|Q_{i+1} - Q_i|}{Q_m - Q_1} \cdot 100\%,\ i = 1,...,m-1 , \quad (29)$$

In this case, the indicator dQ is the relative (given in the Q scale) increase or decrease in the efficiency indicator for the ordered list of alternatives and the property is satisfied:

$$\sum_{p=1}^{m-1} dQ_p = 100. \quad (30)$$

Definition 3: Two rating lists are equivalent if they have identical rank lists and if they have the same RPI:

$$dQ_i^{(1)} = dQ_i^{(2)}, \forall\ i=1,\ldots, m\text{-}1. \quad (31)$$

Example: Consider two rating lists:

$Q_i^{(1)}$={0.2948 0.7995 0.3337 0.4127 0.3956 0.5646 0.4573 0.5299} (32)
$Q_i^{(2)}$={0.6017 0.7279 0.6114 0.6312 0.6269 0.6692 0.6423 0.6605} (33)

Offhand, it is difficult to establish a relationship between these two sets. Let's sort them in descending order and simultaneously set the ranks of the elements (element number in the ordered list):

$Qs_i^{(1)}$={0.7995 0.5646 0.5299 0.4573 0.4127 0.3956 0.3337 0.2948}
$Qs_i^{(2)}$={0.7279 0.6692 0.6605 0.6423 0.6312 0.6269 0.6114 0.6017}
$R_i^{(1)}$={2 6 8 7 4 5 3 1}
$R_i^{(2)}$={2 6 8 7 4 5 3 1}

It turned out that both ranking lists $Q_i^{(1)}$ and $Q_i^{(2)}$ produce identical ranking lists $R_i^{(1)} \equiv R_i^{(2)}$.Let us present a graphical illustration for $Q_i^{(1)}$ and $Q_i^{(2)}$ regardless of the scale (Figure 1).

Fig. 1. Positions of the ratings of alternatives $Q_i^{(1)}$ and $Q_i^{(2)}$ after reduction to a common numerical scale

Only by carefully analyzing the figures can we assume that the distances between adjacent elements for the two lists are proportional. Indeed, using formula (29) we obtain:

d$Q_i^{(1)}$={46.5444 6.8698 14.3956 8.8325 3.3856 12.2668 7.7053}
d$Q_i^{(2)}$={46.5444 6.8698 14.3956 8.8325 3.3856 12.2668 7.7053}

Note that the two presented rating lists (32) and (33) are related by a linear dependence: $Q_i^{(2)}=0.2501\cdot Q_i^{(1)}+0.528$ and the correlation coefficient is equal to 1. That is, these two lists are equivalent by definition 2.

Initially, the first list represents the ratings Q_i of alternatives of the decision-making problem with the matrix D_0 by (34) for the WSM(Max-Min(D_0), *w*) model with the ReS inversion and weight coefficients *w*=(0.1 0.2 0.4 0.2 0.1). The decision matrix D_0 of this example defines 8 alternatives for 5 attributes, of which the 2rd and 5th criteria are cost criteria:

$$D_0 = \begin{pmatrix} 71 & 4500 & 150 & 1056 & 478 \\ 85 & 5800 & 145 & 2680 & 564 \\ 76 & 5600 & 135 & 1230 & 620 \\ 74 & 4200 & 160 & 1480 & 448 \\ 82 & 6200 & 183 & 1350 & 615 \\ 81 & 6000 & 173 & 1565 & 580 \\ 80 & 5900 & 160 & 1650 & 610 \\ 85 & 4700 & 140 & 1750 & 667 \end{pmatrix}. \tag{34}$$

The second list represents the rankings of alternatives of the same problem, for a similar model, but using the normalization proposed by Lai and Hwang method [31].

$$x_{ij} = \frac{2}{m} \cdot \frac{a_{ij}}{a_i^{\max} - a_i^{\min}}, \tag{35}$$

The coefficient $2/m$ in the previous formula represents a scaling factor that does not affect the RPI. The shift coefficient in the formula for Max-Min normalization also does not affect the RPI. Thus, the two ranking lists are equivalent and both ranking models are equivalent, as demonstrated in the presented example.

The equivalence of two rating lists can also be established using the usual Pearson pair correlation coefficient. Unlike the pair correlation coefficient, the dQ vector contains important information about the degree of distinguishability of alternatives by rating [20], i.e. it is more informative.

3.2 Equivalence of the MABAC, Ratio System approach, TOPSIS(L_1), MAIRCA, and RAWEC methods of the MCDM to the Weighted Sum Method

Despite the significant difference in the aggregation formulas of the WSM, RS, MABAC, TOPSIS(L_1), MAIRCA and RAWEC methods, these methods are identical in RPI values, provided that each model uses the same linear normalization method and the same vector of criteria weight coefficients. The difference in the aggregation formulas (4), (5)-(6), (7)-(10), (11)-(15), (16)-(19), (20)-(24) naturally lead to different values of the performance indicator of alternatives Q_i. Why then do the dQ_i values coincide? This is a consequence of the scaling effect: transformations of the decision matrix D_0 (aggregation formulas of features) do not change the relative distances (invariance of dispositions of the Q_i values). In all cases, the authors failed to obtain rigorous proof of this effect. The presence of long chains in the formulas for calculating the rating of alternatives for different methods complicates rigorous calculations. However, the authors conducted numerous computational experiments with variations in the problem dimension, the solution matrix, normalization methods (performed only for linear transformations) and weight coefficients in a wide range. In all cases, dQ_i coincide.

Figure 2 shows the results of one of the numerous tests confirming the equivalence of the WSM, RS, MABAC, TOPSIS(L1), MAIRCA and RAWEC methods.

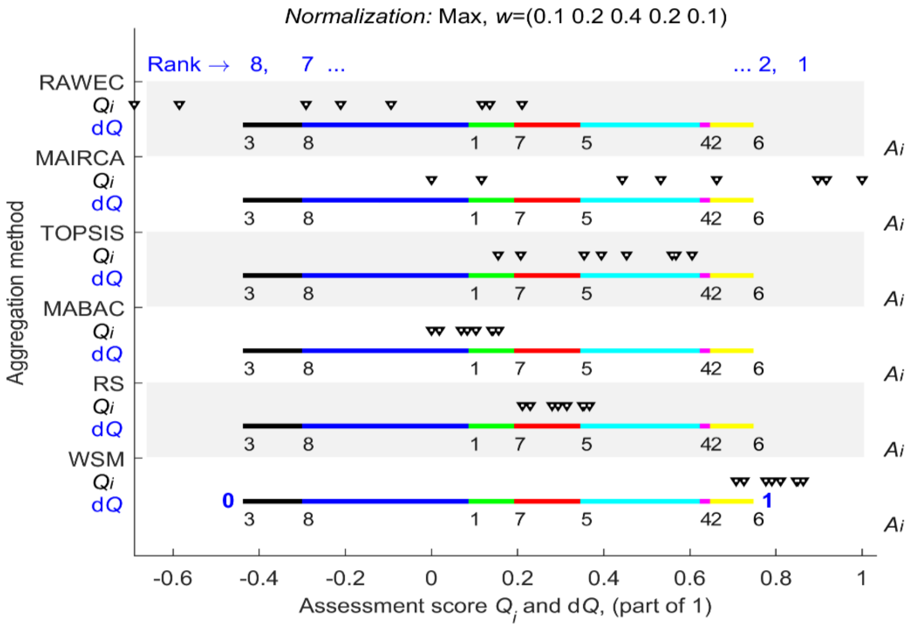

Fig. 2. Equivalence of WSM, RS, MABAC, TOPSIS(L_1), MAIRCA and RAWEC methods. Decision matrix by (34)

The models include 6 aggregation methods and 6 normalization methods:

Agg(*k*)={WSM, RS, TOPSIS(L_1), MABAC, MAIRCA, RAWEC}, *k*=1, …, 6.
Norm(*s*)={Max, Sum, Vec, dSum, Max-Min, Z[0,1]}, *s*=1,…,6.

The linear inversion method ReS [30] was used for normalization of cost criteria. Number of models: $N=k{\cdot}s=36$.

The calculation results for each model are presented in Figure 2 in three scales: ratings Q_i (numeric scale), dispositions dQ_i (fractions from 1) and ranks R_i (alternative numbers in ascending rank from 8 to 1). The rating values of each alternative in Figure 2 are indicated by triangular pixels. In each of the methods, the ratings of alternatives are defined in their own measurement scale and are difficult to compare. There is a shift and stretching-compression of the results for different methods. But the calculation of dQ_i for each of the methods shows their equality. In Figure 1, dQ_i are presented as segments in fractions from 1. dQ_i denotes the relative distance between the rating values of alternatives A_i and A_{i+1}, i.e. alternatives *i* and *i*+1 of rank. Thus, all rating lists are equivalent, which also means the equivalence of the MCDM models. What are the implications of the equivalence of the aggregation methods:

- the MABAC, RS, TOPSIS(L_1), MAIRCA and RAWEC methods should be excluded from the MCDM methods collection since the simpler WSM method yields the same results.
- the identity of RS and WSM shows that the ReS inversion [27] used in WSM to transform the cost criterion values includes the –*r* inversion and ensures the continuity of the aggregation method,
- the significance of the additive WSM method (Value Measurement method) increases since its results coincide with the results of the TOPSIS(L_1) method using the anti-ideal approach (Reference Level models) and coincide with the results of the non-linear MABAC aggregation method using a multiplicative component. In particular, the equivalence of WSM and TOPSIS(L_1) means that both approaches have the same character and shows that Value Measurement models and Reference Level models are interrelated.

The first statement is true only if the model uses a linear normalization method. For example, in the basic descriptions of the methods under study, the Max normalization method is used in combination with a nonlinear method of inversion of cost criteria of the x_j^{min}/x_{ij}. type. In this case, the relative dQ ratings obtained in different methods differ. However, as noted in the study [29], the use of nonlinear transformations leads to data (information) distortion. The use of nonlinear normalization should be motivated.

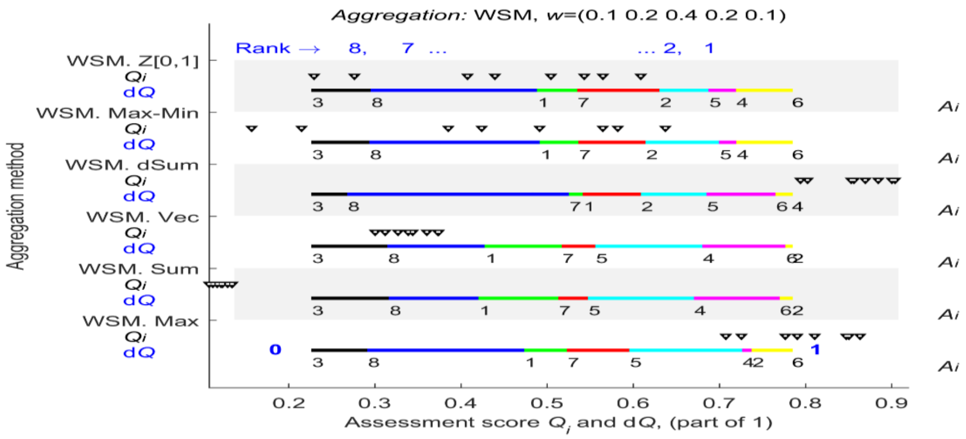

Fig. 3. Ranking of alternatives for the WSM model in combination with normalization methods Max, Sum, Vec, dSum, Max-Min, Z[0,1]. Decision matrix by (34)

Thus, the example shows that, out of 36 different MCDM models, only 6 are relevant. When combining the WSM method and various normalization methods, the relative dQ ratings obtained in different methods differ. Figure 3 shows the results of one of the many tests confirming this thesis.

The presentation of the results in this figure is like Figure 2. For each model, the results are presented in three scales: Q_i ratings (numeric scale), dQ_i dispositions (proportions from 1) and R_i ranks (alternative numbers in ascending rank from 8 to 1). It is easy to see that the dispositions of the alternatives (dQ scale) are different in each of the methods. In three different models (WSM.Sum, WSM.Vec, WSM.dSum) out of 6, the first-rank alternatives are different. But the analysis of the results shows that the distances between the rating values of the 2nd and 4th alternatives and the 6th alternative are insignificant (dQ is small). Following the study [20], these alternatives are poorly distinguishable. Due to this, alternative A_6 most likely has formal priority. Since there are no rational criteria for choosing a normalization method, each of the results reflects the ratings of the alternatives considering the transformation of the measurement scales and the distribution of values for each attribute. All results are equivalent. In this situation, it is necessary to perform a solution synthesis based on the results of several models, which is presented in Section 4.

Comparison tests were also performed for the aggregation methods WPM, WASPAS, CODAS, COPRAS, VIKOR, TOPSIS(L_2), PROMETHEE, ORESTE. The effect of RPI equality was not detected.

4 Synthesis of solutions obtained for different MCDM rank models

4.1 Converting the rating of different MCDM rank models to a single scale

Let us use the solution of the MCDM problem using the 3M approach (the set of admissible rank models of MCDM). Let k models be selected.

Synthesis of the solution based on the rating scale is preferable to the synthesis of the solution based on the rank scale. In the latter case, there is a loss of information. Namely, the degree of closeness-distance of the integral indicator Q_i of different alternatives is lost. Earlier, in the study [20], the possibility of adjusting the rank list was shown based on dQ_i.

Different MCDM models have their own scale, and some have a different focus, for example, in the VIKOR method, a lower value of Q_i is better.

We will show how to easily transform different scales into one common scale:

Step 1. The general goal of synthesis, when a higher value of the efficiency indicator Q_i is better, is taken as the base one.

Step 2. For the methods included in 3M for which a lower Q_i value is better, we perform an inversion of the values using the ReS algorithm [30]:

$$Q_i = -Q_i + Q_i^{max} + Q_i^{min}. \quad (36)$$

This transformation preserves the proportions between the Q_i values.

Step 3. Transform the Q_i values for all methods included in 3M to [0, 1] using the linear Max-Min transformation:

$$\overline{Q}_i = \frac{Q_i - Q_i^{min}}{Q_i^{max} - Q_i^{min}}, \quad (37)$$

According to [26, 29], for linear transformations, the proportions between the Q_p and Q_q values, $\forall$ p, q, are preserved:

$$\frac{\overline{Q}_p - \overline{Q}_q}{\overline{Q}_i^{max} - \overline{Q}_i^{min}} = \frac{Q_p - Q_q}{Q_i^{max} - Q_i^{min}}, \quad (38)$$

The best $\overline{Q}_i$ value for all models is 1 and is indifferent to the rating scale of each model.

Thus, for all methods included in 3M, the values of the rating of alternatives $\overline{Q}_i \in [0, 1]$ with preservation of the ordering and proportions of the original values.

Let us present an example for a problem defined by the decision matrix according to (34). The rating of alternatives for the WSM model in combination with the normalization methods Max, Sum, Vec, dSum, Max-Min (M-M), Z[0,1] is defined in the previous section (Figure 3). We will transform the ratings in accordance with formula (38). The results of calculating the ratings in the individual scale of the model and in a single scale (after transformation) are presented in Table 1.

Table 1
Rating of alternatives in the model scales and in a single scale (after transformation)

A_i	Q_i (in the model scales)						$\overline{Q}_i$ (in a single scale)					
	Max	Sum	Vec	dSum	M-M	Z[0,1]	Max	Sum	Vec	dSum	M-M	Z[0,1]
1	0.776	0.117	0.327	0.856	0.385	0.407	0.443	0.348	0.361	0.565	0.475	0.470
2	0.850	0.134	0.374	0.869	0.491	0.504	0.917	1	1	0.686	0.696	0.725
3	0.707	0.107	0.300	0.794	0.157	0.229	0	0	0	0	0	0
4	0.847	0.129	0.360	0.904	0.582	0.565	0.897	0.797	0.814	1.000	0.885	0.884
5	0.811	0.123	0.344	0.884	0.565	0.543	0.662	0.576	0.591	0.822	0.849	0.827
6	0.863	0.134	0.373	0.900	0.637	0.609	1	0.975	0.987	0.967	1	1
7	0.790	0.121	0.339	0.853	0.424	0.439	0.532	0.515	0.521	0.536	0.556	0.553
8	0.726	0.112	0.312	0.802	0.215	0.276	0.116	0.161	0.158	0.075	0.121	0.123

This transformation allows us to compare ratings and subsequently aggregate ratings obtained in different models. In this example, for example, it is interesting to compare the ratings of the single scale of alternative A_2. In different models, the rank changes from 4 positions to 1. Linear transformation (38) preserves the ranks and dispositions of the alternatives dQ in each of the models. The results are presented in Table 2.

Table 2
Ranks and dispositions of dQ alternatives in different models

rank	*Number of alternative A_i*						dQ_i, %					
	Max	Sum	Vec	dSum	M-M	Z[0,1]	Max	Sum	Vec	dSum	M-M	Z[0,1]
1	6	2	2	4	6	6	-	-	-	-	-	-
2	2	6	6	6	4	4	8.3	2.5	1.3	3.3	11.5	11.6
3	4	4	4	5	5	5	2.0	17.8	17.3	14.4	3.6	5.7
4	5	5	5	2	2	2	23.5	22.1	22.3	13.6	15.3	10.2
5	7	7	7	1	7	7	13.0	6.2	7.0	12.1	14.0	17.2
6	1	1	1	7	1	1	8.9	16.7	16.1	2.9	8.1	8.4
7	8	8	8	8	8	8	32.7	18.7	20.3	46.2	35.5	34.6
8	3	3	3	3	3	3	11.6	16.1	15.8	7.5	12.1	12.3

Dispositions dQ_i complement the ranking information by showing how far apart the ranking values of adjacent alternatives are. For example, in models using the Sum and Vec normalization methods, the rankings of rank 1 and 2 alternatives (these are alternatives A_2 and A_6) are poorly distinguishable (~2.5 and 1.3%).

4.2 Synthesis of the solution of different methods based on a single rating scale

Provided that for different methods the rating of alternatives is reduced to a single scale, the synthesis of the solution for *k* MCDM models is possible in the form of a weighted sum:

$$Qs_i = \sum_{j=1}^{k} \theta_j \overline{Q}_i^{(j)} , \tag{40}$$

Where, Qs_i is the integral indicator of the *i*th alternative, ϑ_j is the weight of the *j*th MCDM model in the selected 3M structure, $\overline{Q}_i^{(j)}$ is the normalized rating of the *i*th alternative for the *j*th model, *j*=1,…, *k*.

The best alternative corresponds to the highest value of Qs_i.

We denote the proposed approach as Synthesis of the Rating based on transformation to a Single Scale (SRSS).

The resulting rating is more informative than the Borda rating or similar ones, since it reflects the real proportions of ratings ("fine" structure of relations). With the same ordering, the Borda rating assigns Borda points proportionally to the rank list, which do not correspond to the real proportions of the alternatives' efficiency indicator. In this method, if there are *m* alternatives, the first-ranked alternative gets *m* votes and the second ranked gets one vote less, and so on. Continuing the analysis of the problem defined by the decision-making matrix according to (34), we will synthesize the solution using the WSM model in combination with 6 normalization methods Max, Sum, Vec, dSum, Max-Min, Z[0,1]. The $\overline{Q}_i$ ratings in a single scale were calculated earlier and are presented in Table 1. In this example, all models have the same weights. The results of the rating synthesis for 6 models are presented in Table 3.

Table 3
Synthesis of solutions of 6 models according to the Borda rule and using the aggregation of ratings of different models (SRSS)

Rank	Borda rules			Scale C		
	Score	# of A_i	dQ	Score	# of A_i	dQ
1	39	6	-	5.929	6	-
2	34	4	12.8	5.277	4	11.0
3	32	2	5.1	5.024	2	4.3
4	27	5	12.8	4.328	5	11.7
5	17	7	25.6	3.214	7	18.8
6	13	1	10.3	2.662	1	9.3
7	6	8	17.9	0.754	8	32.2
8	0	3	15.4	0.000	3	12.7

The results of Table 3 show a good agreement between the synthetic rating by the Borda rule and the rating obtained using the SRSS approach. The disposition indicator dQ is approximately the same in both cases.

The following example demonstrates the difference between the Borda synthesis and the SRSS synthesis. The original problem is borrowed from the study Boyaci and Tüzemen [32]. In this example, all models also have the same weights. The results of the rating synthesis of 6 models are presented in Table 4.

Table 4
Synthesis of solutions of 6 models according to the Borda rule and using the aggregation of ratings of different models (SRSS)

Rank	Borda rules			Scale coordination		
	Score	# of A_i	dQ	Score	# of A_i	dQ
1	49	2	-	5.775	9	-
2	48	9	2.2	5.683	2	1.9
3	43	5	11.1	5.218	5	9.4
4	36	1	15.6	5.123	1	1.9
5	26	4	22.2	4.661	4	9.4
6	25	3	2.2	4.642	3	0.4
7	25	8	0.0	4.496	8	3.0
8	8	10	37.8	1.513	10	60.5
9	6	7	4.4	1.010	7	10.2
10	4	6	4.4	0.849	6	3.3

For this example, the first-rank alternatives in the Borda and SRSS methods are different. In fact, the distance between the 1st and 2nd ranking values is insignificant (dQ is small). In this case, the additional information provided by the dQ indicator shows that quantitative estimates do not allow one to give preference to any alternative. Decision making in this case is the prerogative of the decision maker.

4 Conclusion

The tendency to synthesize solutions based on many MCDM models assumes that each method includes different relationships between alternatives, which increases the overall information content and improves the reliability of the result []. This premise suggests defining a list of acceptable methods for solving a specific problem. These methods should be independent and reflect different relationships between alternatives.

To compare the ranking of alternatives obtained using different MCDM models, the article uses the Relative Performance Indicator (RPI). Using RPI, six methods of aggregation of private attributes of alternatives with identical results were established: WSM, RS, MABAC, TOPSIS(L_1), MAIRCA and RAWEC, provided that each aggregation method combines the same method of linear normalization of attributes. This allows avoiding duplication of equivalent methods in the procedures for synthesizing different methods. When solving MCDM problems, it is recommended to use the simplest and most easily interpreted of them: WSM. The presented methodology is recommended as mandatory for analyzing new or hybrid MCDM methods to eliminate duplication of existing methods. Within the 3M approach, a synthesis of a solution based on ratings obtained in different MCDM models is proposed. The method includes coordinating the common goal of several models and bringing the ratings obtained in different MCDM models to a common scale, which allows comparing and aggregating the ratings. Bringing the ratings to a common scale is achieved using the Max-Min linear normalization. This is based on the property of invariance of the dispositions of the dQi rating under linear transformations. This preserves the original information content of the alternatives rating. After reduction to a single scale, the solution synthesis is performed as a weighted linear combination of the alternatives rating obtained in different MCDM models. The weights determine the significance of the models [35,36]. The synthesis of solutions based on the rating of alternatives obtained in various models, in contrast to the synthesis of solutions based on the ranks of alternatives, reflects the real proportions of the performance indicator of alternatives. The examples provided demonstrate the effectiveness of the proposed approaches.

According to the authors, the following important questions require further research:

- which MCDM methods should be included in the list of methods. These methods should be independent and reflect different relationships between alternatives in the context of the selected criteria,
- how to evaluate the significance (weight) of the methods,
- whether to group the methods and how to form the groups.

Author Contributions

Conceptualization, methodology, validation, I.Z.M. and D.P.; software, visualization, writing—original draft preparation, I.Z.M.; formal analysis, writing—review and editing, D.P. All authors have read and agreed to the published version of the manuscript.

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Data availability statement

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Conflicts of Interest

The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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