SCIENTIFIC OASIS

Decision Making: Applications in Management and Engineering

Journal homepage: www.dmame-journal.org
ISSN: 2560-6018, eISSN: 2620-0104

M-OPARA: A Modified Approach to the OPARA Method

Do Duc Trung[1] , Nazlı Ersoy[2,*], Tran Van Dua[1], Duong Van Duc[1], Manh Thi Diep[1]

1 Hanoi University of Industry, Cau Dien, Bac Tu Liem, Hanoi, Vietnam
2 "Osmaniye Korkut Ata University, Osmaniye, 80000, Türkiye

ARTICLE INFO

Article history:
Received 15 July 2024
Received in revised form 19 Oct 2024
Accepted 7 December 2024
Available online 17 February 2025

Keywords:
Multi-criteria decision-making; MCDM; OPARA method; M-OPARA method

ABSTRACT

The process of selecting the optimal option among multiple conflicting criteria is a fundamental task across various disciplines and is known as multi-criteria decision-making (MCDM). This study seeks to enhance the Objective Pairwise Adjusted Ratio Analysis (OPARA) method, which is limited by its inability to rank alternatives when any criterion within the decision matrix assumes a zero value. To address this limitation, an additional transformation step is introduced, resulting in the development of the M-OPARA method. The effectiveness of the M-OPARA method has been rigorously assessed through diverse case studies, incorporating different sets of criterion weights derived from 20 scenarios, and by comparing its performance against several MCDM techniques, including OPARA, PIV, ROV, SAW, WASPAS, MARCOS, and FUCA. The findings indicate that the M-OPARA method achieves high accuracy in ranking alternatives and successfully mitigates the constraints of the OPARA method. The methodological advancements introduced by M-OPARA constitute a substantial improvement in the rank ability of alternatives within decision-making frameworks. This novel approach facilitates more precise and dependable decision-making in practice, equipping decision-makers with a more flexible and robust analytical tool.

1. Introduction

Owing to their broad applicability across diverse disciplines, multi-criteria decision-making (MCDM) methods have recently attracted considerable scholarly attention [1-3]. New methods are frequently introduced, with incomplete statistics suggesting that over two hundred distinct approaches currently exist [4]. The application of MCDM methods across diverse fields continues to expand steadily [5]. Beyond the development of novel methods, the refinement of existing techniques to enhance their applicability has also become a focal point of academic research. Various strategies have been employed to modify MCDM methods for specific applications, with five commonly adopted approaches summarized as follows.

One significant research direction involves extending MCDM methods into fuzzy MCDM methods to accommodate cases where the elements within the decision matrix are represented as fuzzy sets

[*] *Corresponding author.*
E-mail address: nazliersoy@osmaniye.edu.tr

https://doi.org/10.31181/dmame8120251334

[6-7] or when data concerning the consistency of decision-makers' opinions is insufficient [8]. Another approach focuses on adapting MCDM methods to address uncertain conditions within the decision matrix, often referred to as grey situations [9]. Additionally, some recent studies have explored the application of a single MCDM method for dual purposes, both ranking alternatives and determining criteria weights, representing a distinctive methodological advancement. A further strategy entails modifying the algorithms within established MCDM methods [10-11], with various examples documented in the literature.

Although five key trends in modifying MCDM methods have emerged as primary areas of scholarly interest, the literature presents a wide range of approaches for altering the algorithms within existing MCDM frameworks. This diversity highlights the challenge of identifying a universal modification strategy applicable to all MCDM methods. Instead, any modifications should be tailored to the inherent characteristics of each specific method, where feasible. However, the development of entirely new MCDM methods does not appear to be the dominant research trajectory. The vast number of existing MCDM methods may, in part, contribute to a diminished interest among researchers in pursuing this direction [12].

OPARA is a recently developed method introduced in 2024, with no published studies currently available in the literature. Unlike conventional normalization techniques, OPARA employs pairwise adjusted ratios, ensuring that the evaluation of each alternative considers not only its specific data but also the entire decision dataset. Within the OPARA framework, pairwise adjusted ratios play a critical role in determining the dominance or relative significance of each alternative [13]. However, a detailed analysis reveals that the pairwise adjusted ratio steps cannot be executed when the decision matrix contains zero elements, as the presence of a zero in the denominator renders the ratios undefined. This study aims to address this limitation by modifying the OPARA method, resulting in the development of the M-OPARA method. The proposed modification introduces an additional step that facilitates the application of OPARA even in cases where the decision matrix includes zero values.

To validate the accuracy and reliability of the enhanced approach, extensive analyses have been conducted in multiple case studies with various MCDM techniques and various criterion weight sets. Consequently, the proposed methodology enables researchers to apply the OPARA method without any restrictions on zero values in the decision matrix. Besides, comparative studies have been conducted with various MCDM approaches having different methodological backgrounds, offering an overall investigation of the effects of various distributions of criterion weights on MCDM results.

The significance of this study lies in several key contributions. (i) M-OPARA eliminates the issue of zero values obstructing the ranking process, ensuring a more precise and reliable evaluation of alternatives. (ii) This advancement introduces a more flexible and comprehensive approach to decision-making, making substantial contributions, particularly in complex MCDM scenarios. (iii) The performance of the proposed model has been rigorously assessed under varying conditions through a two-stage sensitivity analysis, reinforcing the robustness and reliability of decision-making processes. (iv) Additionally, this study addresses the inherent limitations of OPARA, extending its applicability and enhancing its effectiveness as a decision-support tool. (v) To demonstrate the efficacy of the M-OPARA method, various MCDM techniques with distinct methodological foundations have been employed, incorporating different weighting scenarios. A comprehensive analysis has also been conducted using four case studies, each featuring distinct numbers of criteria and alternatives, as well as varying criteria directions. (vi) Ultimately, this study is expected to be particularly beneficial for practitioners dealing with complex decision-making challenges in fields such as operations research and business decision-making, as well as for researchers and scholars in decision science.

The remainder of this paper is structured as follows. Section 2 presents examples of previous studies related to similar concepts. Section 3 introduces the mathematical formulations of the Entropy and OPARA methods and examines scenarios in which the OPARA method is unsuitable for ranking alternatives. Additionally, proposed modifications to the OPARA method are outlined. Section 4 provides numerical examples to assess the effectiveness of the M-OPARA method. Finally, the concluding section summarizes the findings and their implications.

2. Literature review

The application of MCDM techniques involves the alteration of existing methods to more closely fit environments or issues. The alterations are generally employed to improve the accuracy, effectiveness, and applicability of the decision-making process, and numerous instances of these variations are well-documented within academic literature. One of the most prominent examples of such a variation is the conversion of MCDM techniques into fuzzy MCDM models. This approach has been used in numerous investigations across various disciplines. For example, the fuzzy COPRAS technique has helped customers in choosing hotels while booking online, according to Roy et al. [14], and the fuzzy VIKOR method has aided in deciding the abrasive materials for use while producing grinding wheels [15]. The other applications include the evaluation of the degree of equipment failure by fuzzy AHP and fuzzy MAIRCA methods. Boral et al. [16], the selection of last-mile delivery options by the fuzzy WASPAS method [17], the assessment of Indian mobile wallet service providers by the fuzzy EDAS method [18], the identification of variables that influence the Romanian economy by fuzzy SAW, fuzzy BWM, and fuzzy WASPAS methods [19], among others.

Adapting MCDM methods to handle uncertain (grey) scenarios has led to the development of grey MCDM (MCDM-G) techniques. Examples include PIV-G and PSI-G for warehouse location selection Ulutaş et al. [20], EDAS-G for ranking European smart cities [21], SWARA-G and MOORA-G for evaluating logistics performance [22], and MCRAT-G and COBRA-G for supplier ranking [23]. Another modification involves changing normalization methods in techniques like SAW [24], CRADIS [25], etc. Another approach integrates ranking and weighting simultaneously. For instance, PSI was combined with TOPSIS and MABAC to rank Indonesian companies' financial health, with PSI-TOPSIS and PSI-MABAC outperforming PSI alone [26]. Additionally, PSI-CoCoSo was used to rank Turkish transportation companies, proving more effective than standalone PSI [27].

The fifth approach to enhancing MCDM methods entails modifying specific algorithms within these techniques. For instance, the EDAS method has been refined based on cumulative prospect theory (CPT) to incorporate psychological factors influencing decision-makers, leading to its application in interval-valued intuitionistic fuzzy sets (IVIFS) [28]. The WASPAS method has been enhanced through the integration of weights derived from objective weight calculation techniques [29]. Adjustments to the AHP method have involved modifying the structure of the relative criterion scoring matrix [30]. The CoCoSo method has been improved through the application of the Hamming distance measure Wang et al. [31], while the CODAS method has been refined by constructing a weighted fuzzy aggregation operator [32]. The VIKOR method has undergone modifications to facilitate the ranking of alternatives with differing numbers of criteria Anvari et al. [33] and has been further adapted to simultaneously consider benefit-type quantitative attributes, cost-type quantitative attributes, significant qualitative attributes, and less significant qualitative attributes [34]. Another study introduced modifications to the VIKOR method by incorporating the analysis of intuitionistic fuzzy sets [35]. Similarly, intuitionistic fuzzy sets have been applied to refine the MAIRCA method [36]. Additionally, transforming the original decision matrix into a new matrix has been employed to enhance the EDAS method's capacity to manage scenarios where the initial decision matrix contains negative elements [37], among other refinements.

3. Methodology Approach

3.1 Entropy Method

The Entropy method provides the advantage of reducing subjective bias among decision-makers, thereby improving the objectivity of the evaluation process [38]. This method comprises the following steps [38].

Step 1. Normalize the decision matrix.

$$v_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad (1)$$

v_{ij} indicates the normalized value of an alternative, whereas x_{ij} denotes the criteria.

Step 2. Determine the entropy value for the j_{th} criterion.

$$e_j = -k\sum_{i=1}^{m} v_{ij}\, \mathrm{In}(v_{ij}) = -\frac{1}{\mathrm{In}(m)}\sum_{i=1}^{m} v_{ij}\mathrm{In}(v_{ij}) \quad (2)$$

m indicates the number of alternatives

Step 3. Determine the degree of diversification d_j.

$$d_j = 1 - e_j, j \in [1, \ldots, n] \quad (3)$$

Step 4. Calculation of criteria weights.

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \quad (4)$$

3.2 OPARA Method

To determine the ranking of alternatives using the OPARA method, the following steps should be followed [13].

Step 1. Construct a decision matrix as shown in Equation (5).

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{m1} \\ x_{21} & x_{22} & \cdots & x_{m2} \\ \cdots & \cdots & \ddots & \cdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix} \quad (5)$$

x_{ij} represents the value of criterion j for alternative i. Let w_j denote the weight of the j-th criterion.

Step 2. Determine the range-based pairwise adjusted ratio (RPAR) between alternatives k and l using equation (6). The symbols BC and NC represent benefit criteria and cost criteria, respectively.

$$RPAR_{kl} = \sum_{j \in BC} w_j \cdot \left(\frac{x_{kj}}{x_{lj}}\right)^{p_j} + \sum_{j \in NC} w_j \cdot \left(\frac{x_{lj}}{x_{kj}}\right)^{p_j}, \quad k, l \in \{1, 2, \ldots, n\} \quad (6)$$

In Equation (6), p_j is the adjustment coefficient in RPAR, which is calculated according to equation (7).

$$\rho_j = \begin{cases} \dfrac{(\alpha - 1)\max(x_{ij}) + \min(x_{ij})}{\alpha \cdot \max(x_{ij})} & if \quad \dfrac{\max(x_{ij}) - \min(x_{ij})}{\max(x_{ij}) + \min(x_{ij})} > \beta \\ 1 & otherwise \end{cases} \quad (7)$$

In equation (7), the values of the parameters α and β are chosen to be 5 and 0.8, respectively [13].

Step 3. Determine the linearly adjusted pairwise ratio (LPAR) between alternatives k and l using Equation (8).

$$LPAR_{kl} = \sum_{j \in BC} w_j \cdot \left(\frac{x_{kj}}{x_{lj}}\right)^{\tau_j} + \sum_{j \in NC} w_j \cdot \left(\frac{x_{lj}}{x_{kj}}\right)^{\tau_j}, \quad (8)$$

The value of τj in Equation (8) is decided by the user. If a criterion is linear, τj equals 1. To increase

LPAR, choose τj greater than 1; to decrease LPAR, choose τj less than 1.

Step 4: Calculate the aggregated pairwise adjusted ratios (APARkl) using Equation (9).

$$APAR_{kl} = \omega \cdot RPAR_{kl} + (1-\omega)LPAR_{kl} \tag{9}$$

In Equation (9), ω ∈ [0,1] and is typically chosen to be 0.5.

Step 5. Compute the alternative scores using Equation (10).

$$S_i = \frac{1}{n}\left(\sum_{l=1}^{n}\left(\frac{APAR_{il}}{\sum_{k=1}^{n} APAR_{kl}}\right)\right) \tag{10}$$

Step 6. The alternative with the highest score is ranked first.

An analysis of Equations (6) and (8) shows that if a zero value is present in the decision matrix, these equations cannot be applied. To illustrate this, the data in Table 1 is used to demonstrate cases where certain RPARkl and LPAR values cannot be calculated. Suppose there are five alternatives, A_1, A_2, A_3, A_4, and A_5, to be ranked based on four criteria: C_1-C_4. C_1 and C_2 are benefit criteria (+), while C_3 and C_4 are cost criteria (-). Table 1 provides the relevant data. The weights for criteria are represented as w_1, w_2, w_3, and w_4, respectively. In this example, the values of C_1 for A_3 and C_4 for A_2 are intentionally set to zero to highlight that Equations (6) and (8) cannot be applied in such cases.

Table 1
A Numerical Example

Alt.	Nature			
	+	+	-	-
	C_1	C_2	C_3	C_4
A_1	12	43	3	7
A_2	18	37	5	0
A_3	0	28	11	14
A_4	32	19	6	21
A_5	26	55	8	6

To confirm this observation, the following proof is presented:

Applying Equation (6) to calculate the RPARkl values:

$$RPAR_{13} = w_1 \cdot \left(\frac{12}{0}\right)^{p_1} + w_2 \cdot \left(\frac{43}{28}\right)^{p_2} + w_3 \cdot \left(\frac{11}{3}\right)^{p_3} + w_4 \cdot \left(\frac{14}{7}\right)^{p_4}$$
$$RPAR_{21} = w_1 \cdot \left(\frac{18}{12}\right)^{p_1} + w_2 \cdot \left(\frac{37}{43}\right)^{p_2} + w_3 \cdot \left(\frac{3}{5}\right)^{p_3} + w_4 \cdot \left(\frac{7}{0}\right)^{p_4}$$
$$RPAR_{23} = w_1 \cdot \left(\frac{18}{0}\right)^{p_1} + w_2 \cdot \left(\frac{37}{28}\right)^{p_2} + w_3 \cdot \left(\frac{11}{5}\right)^{p_3} + w_4 \cdot \left(\frac{14}{0}\right)^{p_4}$$
$$RPAR_{24} = w_1 \cdot \left(\frac{18}{32}\right)^{p_1} + w_2 \cdot \left(\frac{37}{19}\right)^{p_2} + w_3 \cdot \left(\frac{6}{5}\right)^{p_3} + w_4 \cdot \left(\frac{21}{0}\right)^{p_4}$$
$$RPAR_{25} = w_1 \cdot \left(\frac{18}{26}\right)^{p_1} + w_2 \cdot \left(\frac{37}{35}\right)^{p_2} + w_3 \cdot \left(\frac{8}{5}\right)^{p_3} + w_4 \cdot \left(\frac{6}{0}\right)^{p_4}$$
$$RPAR_{43} = w_1 \cdot \left(\frac{32}{0}\right)^{p_1} + w_2 \cdot \left(\frac{19}{28}\right)^{p_2} + w_3 \cdot \left(\frac{11}{6}\right)^{p_3} + w_4 \cdot \left(\frac{14}{21}\right)^{p_4}$$
$$RPAR_{53} = w_1 \cdot \left(\frac{26}{0}\right)^{p_1} + w_2 \cdot \left(\frac{55}{28}\right)^{p_2} + w_3 \cdot \left(\frac{11}{8}\right)^{p_3} + w_4 \cdot \left(\frac{14}{6}\right)^{p_4}$$

Clearly, in this case, the values for RPAR13, RPAR21, RPAR23, RPAR24, RPAR25, RPAR43, and RPAR53 cannot be computed. This issue also arises when applying Equation (8) to compute the LPAR values. To address this limitation, an extension of the OPARA method is proposed below.

3.3 A Simple Modification to the OPARA Method

A straightforward approach is proposed to modify the OPARA method as follows.

After transforming the decision matrix according to Equation (11), a new decision matrix is generated as follows:

$$X' = \begin{bmatrix} x'_{11} & x'_{12} & \cdots & x'_{n1} \\ x'_{21} & x'_{22} & \cdots & x'_{n2} \\ \cdots & \cdots & \ddots & \cdots \\ x'_{n1} & x'_{n2} & \cdots & x'_{nm} \end{bmatrix} \qquad (12)$$

In Equation (12):

$$\mathrm{x}'_{\mathrm{ij}} = \mathrm{x}_{\mathrm{ij}} + \frac{\max(\mathrm{x}_{\mathrm{ij}})}{\max(\mathrm{x}_{\mathrm{ij}}) - \min(\mathrm{x}_{\mathrm{ij}})} \qquad (11)$$

In (11), $\max(x_{ij})$ and $\min(x_{ij})$ are the maximum and minimum values of criterion j among all alternatives. The modified OPARA method is named the M-OPARA method. Adding the maximum value, $\max(x_{ij})$ to the numerator ensures that, in almost all cases, x'_{ij} will be greater than zero. In practical MCDM problems, it is highly unlikely for the maximum value of a criterion to be zero. For instance, when ranking chemicals based on factors such as chemical composition, chemical properties, and reactivity, no chemical can exhibit a maximum value of zero for any of these criteria. However, certain criteria, such as toxicity or pollution potential, may have a minimum value of zero, indicating a completely harmless substance.

Similarly, when evaluating construction materials based on attributes such as strength, hardness, and heat resistance, no material can possess a maximum value of zero for these properties. Nevertheless, some criteria, such as thermal conductivity or sound insulation, may have a minimum value of zero. This demonstrates that, in practice, it is rare for a criterion to have a maximum value of zero, whereas it is more common for a criterion to have a minimum value of zero. In the example presented in Table 1, the application of Equations (11) and (12) produces the transformed matrix X′, as shown in Table 2.

Table 2
Matrix X′

Alt.	C_1	C_2	C_3	C_4
A_1	13	44.528	4.375	8
A_2	19	38.528	6.375	1
A_3	1	29.528	12.375	15
A_4	33	20.528	7.375	22
A_5	27	56.528	9.375	7

Once matrix X′ is constructed, the subsequent steps of the M-OPARA method follow those of the original OPARA method. This implies that, in comparison to the OPARA method, the M-OPARA method introduces only one additional step immediately after Step 1 of the original approach. Consequently, implementing the M-OPARA method requires the sequential application of Equations (5), (11), (12), (7), (6), (8), (9), and (10). However, it is crucial to validate whether the proposed M-OPARA method maintains accuracy. To ensure its reliability, the four case studies presented in the following section aim to assess and confirm the method’s accuracy.

4. Performance Evaluation of the M-OPARA Method

This study assessed the effectiveness and applicability of the M-OPARA method through empirical tests conducted on four distinct case studies. These case studies provide practical insights into how M-OPARA can be utilized in addressing real-world, complex MCDM problems. The research approach integrates theoretical foundations with the development of a novel model, which is subsequently tested and validated through applied experiments and detailed case study analyses. The robustness of the proposed model was verified through a two-stage comprehensive sensitivity analysis for each case study. In the first stage, the results obtained from the M-OPARA method were compared with those derived from the OPARA, PIV, ROV, SAW, WASPAS, MARCOS, and FUCA methods. These methods were selected due to their frequent application, computational simplicity, and reliable outcomes. Notably, the OPARA method could not be applied in the first two case studies due to the presence of zero values in the decision matrix.

In the second stage, the impact of variations in the weight of the most critical criterion on the ranking outcomes was analyzed using Equation (13) [36], leading to the development of multiple scenarios.

$$w_{m\gamma} = (1 - w_{m\pi}).\frac{w_\gamma}{(1 - w_m)} \tag{13}$$

Where m represents any criterion, $w_{m\gamma}$ denotes the modified value of $w_{m\pi}$ indicates the reduced value of the best criterion, w_γ refers to the original value of m, and w_m represents the original value of the best criterion [36].

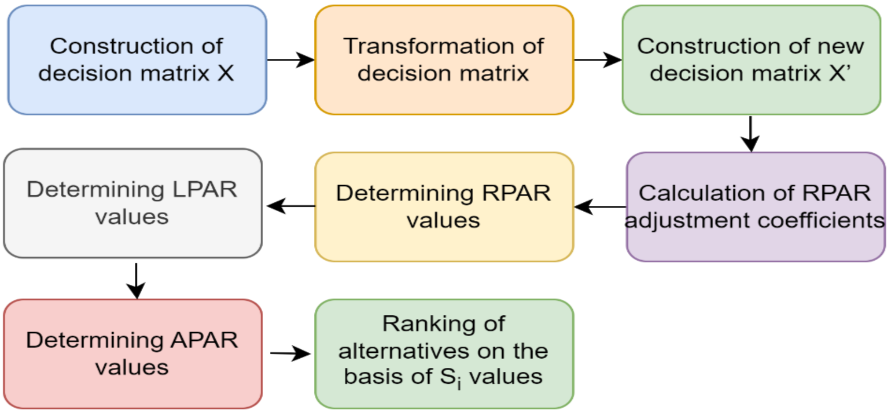

Fig. 1. The Framework of the Proposed M-OPARA Method
Source: Created by Researchers

4.1 Case 1

In this section, the data from Table 1 is utilized to apply the M-OPARA method. The criterion weights were determined using the Entropy technique, yielding the following values: w_1 = 0.3865, w_2 = 0.0896, w_3 = 0.0960, and w_4 = 0.4278. The ranking of alternatives using the M-OPARA method was carried out through the following sequential steps. First, Equation (11) was applied to eliminate zero values in the decision matrix, producing a transformed matrix (see Table 2). Next, Equation (7) was used to compute the values of ρ_1, ρ_2, ρ_3, and ρ_4, which were found to be 0.806, 1, 1, and 0.809, respectively. Among these, only C_1 and C_4 exceeded the threshold value. Finally, the RPAR and LPAR values were computed using Equations (6) and (8), with the results presented in Tables 3 and 4. In calculating the LPAR values, τj was chosen as τj=1 [13].

Table 3
RPAR Values

Alt.	A_1	A_2	A_3	A_4	A_5
A_1	1	0.6077	4.1738	1.5086	0.8748
A_2	2.9695	1	8.2791	5.7439	2.5590
A_3	0.3995	0.2020	1	0.7925	0.3776
A_4	1.1060	0.7690	7.0117	1	0.7784
A_5	1.3319	0.7985	6.5985	1.7317	1

Table 4
LPAR Values

Alt.	A_1	A_2	A_3	A_4	A_5
A_1	1	0.5614	6.2339	1.6851	0.8368
A_2	4.1310	1	14.0649	9.9141	3.4691
A_3	0.3513	0.1670	1	0.8253	0.3335
A_4	1.2351	0.8215	13.2708	1	0.7631
A_5	1.4503	0.8072	11.6516	1.9832	1

To compute the APAR values, Equation (9) was applied with ω = 0.5 Mehdi et al. [13] (see Table 5). Subsequently, Equation (10) was used to determine the Si values, and the final ranking outcomes for the alternatives are presented in Table 6.

Table 5
APAR Values

Alt.	A_1	A_2	A_3	A_4	A_5
A_1	1	0.5846	5.2038	1.5968	0.8558
A_2	3.5503	1	11.1720	7.8290	3.0140
A_3	0.3754	0.1845	1	0.8089	0.3556
A_4	1.1705	0.7953	10.1412	1	0.7708
A_5	1.3911	0.8028	9.1250	1.8575	1

Table 6
Si Values and Rankings of the Alternatives

Alt.	Si	Rank
A_1	0.1428	4
A_2	0.4353	1
A_3	0.0507	5
A_4	0.1748	3
A_5	0.1964	2

Due of their popularity, the PIV, ROV, SAW, WASPAS, MARCOS, and FUCA methods were used for comparison after evaluating the alternatives using M-OPARA. Note that the PIV, ROV, and FUCA methods can be used with zero decision matrix members [39-40]. PIV, ROV, and FUCA were used to rank matrix X alternatives, whereas M-OPARA, SAW, WASPAS, and MARCOS were used to rank matrix X′ alternatives. The results are summarized in Table 7.

Table 7
Ranking of Alternatives in Case 1

	M-OPARA		PIV		ROV		SAW		WASPAS		MARCOS		FUCA	
Alt.	S	R	S	R	S	R	S	R	S	R	S	R	S	R
A_1	0.143	4	0.290	3	0.293	3	0.372	4	0.326	4	0.369	4	3.105	3
A_2	0.435	1	0.147	1	0.381	1	0.777	1	0.765	1	0.771	1	2.048	1
A_3	0.051	5	0.565	5	0.083	5	0.121	5	0.095	5	0.120	5	4.483	5
A_4	0.175	3	0.390	4	0.223	4	0.495	3	0.363	3	0.491	3	3.262	4
A_5	0.196	2	0.175	2	0.373	2	0.512	2	0.443	2	0.507	2	2.102	2

To evaluate ranking algorithms' consistency, r_s was determined. The coefficient is calculated using Equation (14), where D_i is the difference in ranking of option i across different approaches [41-43]. Table 8 shows the analysis results.

$$S = 1 - \frac{6D_i^2}{\mathrm{m}(m^2 - 1)} \quad (14)$$

The S value between the M-OPARA and SAW-WASPAS-MARCOS methods is 1, indicating that these methods produce identical rankings for the alternatives. This perfect correlation is also reflected in Table 8. Additionally, the S value between the M-OPARA and PIV-ROV-FUCA methods is 0.9, demonstrating minimal differences in the rankings produced by these two sets of methods. During the initial phase of the sensitivity analysis, the M-OPARA method was compared with various other methods, as summarized in Table 7. In the subsequent phase, the impact of different criterion weights on the results was examined using the scenarios outlined in Table 9. In the first scenario (S_1), equal weights were assigned to all criteria (w_j = 0.25). The w_j values in the first row of Table 9 represent the actual criterion weights. To analyze the effect of changes in the weight of the most significant criterion (w_4 = 0.4278) on the ranking results, Equation (13) [36] was applied, generating 17 different scenarios (S_2–S_{18}). Finally, to prioritize specific criteria, the positions of C_2–C_4 and C_3–C_4 were swapped, creating two additional scenarios (S_{19}–S_{20}). As a result, a total of 20 scenarios were developed for analysis.

Table 9
Different Scenarios of Criteria Weights for Case 1

	C_1	C_2	C_3	C_4
w_j	0.3865	0.0896	0.0960	0.4278
S_1	0.25	0.25	0.25	0.25
S_2	0.3894	0.0903	0.0967	0.4235
S_3	0.3923	0.0909	0.0974	0.4193
S_4	0.3951	0.0916	0.0981	0.4151
S_5	0.3979	0.0922	0.0988	0.4109
S_6	0.4007	0.0929	0.0995	0.4068
S_7	0.4034	0.0935	0.1002	0.4028
S_8	0.4061	0.0942	0.1009	0.3987
S_9	0.4088	0.0948	0.1015	0.3948
S_{10}	0.4115	0.0954	0.1022	0.3908
S_{11}	0.4141	0.0960	0.1029	0.3869
S_{12}	0.4167	0.0966	0.1035	0.3830
S_{13}	0.4193	0.0972	0.1042	0.3792
S_{14}	0.4219	0.0978	0.1048	0.3754
S_{15}	0.4244	0.0984	0.1054	0.3716
S_{16}	0.4269	0.0990	0.1060	0.3679
S_{17}	0.4294	0.0996	0.1067	0.3643
S_{18}	0.4319	0.1001	0.1073	0.3606
S_{19}	0.3865	0.4278	0.0960	0.0896
S_{20}	0.3865	0.0896	0.4278	0.0960

Figure 2 presents a summary of the alternative rankings obtained using the M-OPARA method across different scenarios. Despite significant variations in the criterion weights, the rankings of the alternatives remained largely consistent across the 20 scenarios. Notably, in all scenarios, A_1 consistently ranked first, while A_5 consistently ranked fifth. The rankings of the remaining alternatives exhibited minimal fluctuations, with only scenarios S_{19} and S_{20} displaying differences in the rankings of A_1, A_4, and A_5. This consistency suggests that the M-OPARA method is capable of producing highly stable rankings even when criterion weights vary. Consequently, the M-OPARA method not only performs comparably to other MCDM methods but also demonstrates its robustness in ranking alternatives under varying weight distributions. In other words, the modification of the OPARA method has proven to be effective in this context.

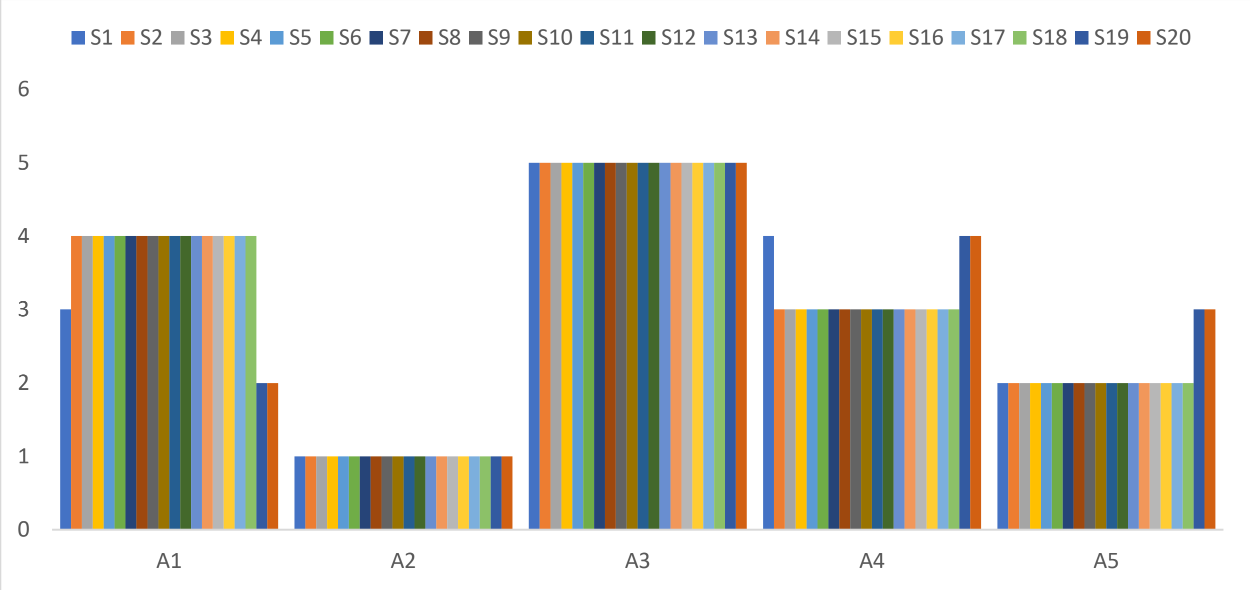

Fig. 2. Ranking of Options in Different Scenarios for Case 1

4.2 Case 2

In this case, a distinct scenario was intentionally created in which the application of the original OPARA method is not feasible. Specifically, the values for C_1 in A_3 and C_3 in A_2 were deliberately set to zero, as shown in Table 10.

Table 10
Numerical Example for Case 2

Alt.	Nature		
	+	+	-
	C_1	C_2	C_3
A_1	7	10	21
A_2	8	6	0
A_3	0	8	7
A_4	9	6	8
w_j	0.3550	0.270	0.6179

The bottom row of Table 10 presents the criterion weights determined using the Entropy method. By applying Equation (7), the values of ρ_1, ρ_2, and ρ_3 were computed as 0.82, 1, and 0.81, respectively. The threshold value τ_j was set to $\tau_j = 1$ [13]. Following the approach used in Case 1, the alternatives were ranked using seven different methods: M-OPARA, PIV, ROV, SAW, WASPAS, MARCOS, and FUCA. The ranking results are summarized in Table 11.

Table 11
Ranking of Alternatives in Case 2

	M-OPARA		PIV		ROV		SAW		WASPAS		MARCOS		FUCA	
Alt.	S	R	S	R	S	R	S	R	S	R	S	R	S	R
A_1	0.132	3	0.602	4	0.152	4	0.339	3	0.238	3	0.337	3	3.564	4
A_2	0.589	1	0.033	1	0.467	1	0.956	1	0.955	1	0.950	1	1.423	1
A_3	0.092	4	0.417	3	0.213	3	0.135	4	0.129	4	0.135	4	2.710	3
A_4	0.187	2	0.217	2	0.369	2	0.442	2	0.348	2	0.439	2	2.303	2

Additionally, r_s were calculated for this case, with the results presented in Table 12.

Table 12
Spearman's Rank Correlation Coefficient in Case 2

	PIV	ROV	SAW	WASPAS	MARCOS	FUCA
M-OPARA	0.8	0.8	1	1	1	0.8

The Spearman coefficient between the M-OPARA and SAW-WASPAS-MARCOS methods was found to be 1, indicating complete consistency in the alternative rankings produced by these methods. Additionally, the rankings obtained using the M-OPARA method exhibited a strong agreement with those generated by the PIV and ROV methods, with an S value of 0.8. These results confirm that, in this case, ranking alternatives with the M-OPARA method is effectively equivalent to employing the aforementioned MCDM methods. Furthermore, the stability of alternative rankings under varying criterion weights was examined. Table 13 presents the criterion weights across 20 different scenarios, where w_j represents the actual criterion weights, while S_1 denotes equal weighting. Equation (13) was applied to generate 17 weight scenarios (S_2–S_{18}), in which the importance of the highest-priority criterion (w_3 = 0.6179) was reduced by 1% in each scenario, while the weights of the other criteria were incrementally increased. In the final stage, two additional scenarios (S_{19}–S_{20}) were created by swapping the positions of the actual weights in the first row (C_1–C_3 and C_2–C_3). This adjustment aimed to assess the impact of altering the position of the key criterion on the ranking results. Figure 3 summarizes the rankings of the alternatives obtained using the M-OPARA method under different weight scenarios, as presented in Table 13.

Table 13
Different Scenarios of Criteria Weights for Case 2

	C_1	C_2	C_3		C_1	C_2	C_3
wj	0.3550	0.0270	0.6179	S11	0.4099	0.0312	0.5588
S_1	0.3333	0.3333	0.3333	S12	0.4151	0.0316	0.5533
S_2	0.3608	0.0275	0.6117	S13	0.4203	0.0320	0.5477
S_3	0.3665	0.0279	0.6056	S14	0.4254	0.0324	0.5422
S_4	0.3721	0.0283	0.5996	S15	0.4304	0.0328	0.5368
S_5	0.3777	0.0288	0.5936	S16	0.4353	0.0332	0.5315
S_6	0.3832	0.0292	0.5876	S17	0.4402	0.0335	0.5261
S_7	0.3886	0.0296	0.5818	S18	0.4451	0.0339	0.5209
S_8	0.3940	0.0300	0.5759	S19	0.6179	0.0270	0.3550
S_9	0.3994	0.0304	0.5702	S20	0.3550	0.6179	0.0270
S_{10}	0.4047	0.0308	0.5645				

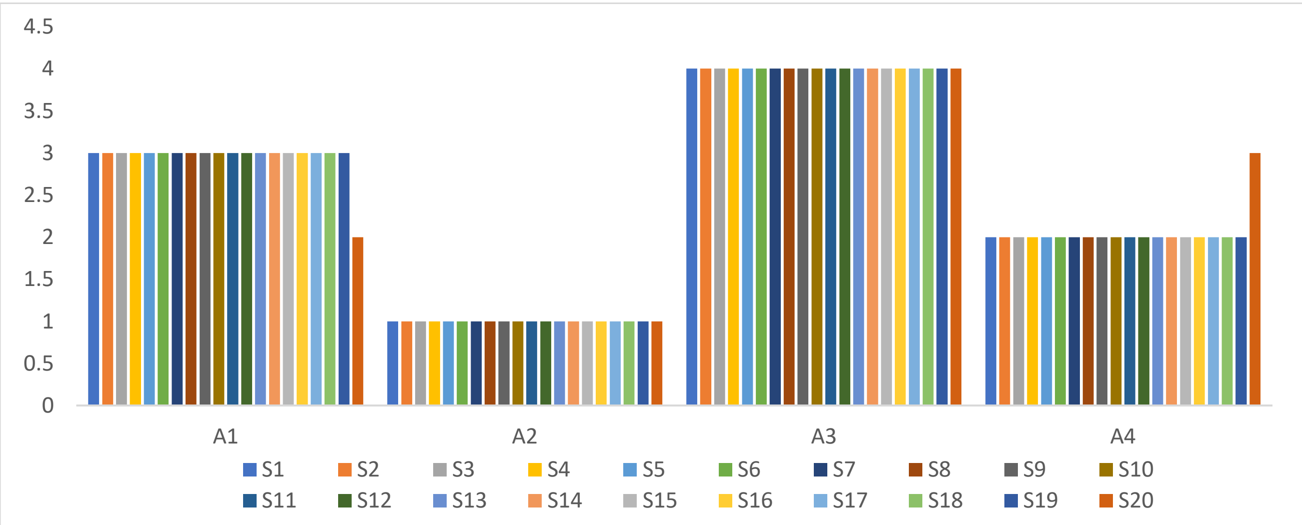

Fig. 3. Ranking of Options in Different Scenarios for Case 2

In 19 of the 20 established scenarios, the rankings of the alternatives remained unchanged, with the overall order being $A_2 > A_4 > A_1 > A_3$. Only in the 20th scenario did the positions of alternatives A_1 and A_4 switch. This consistency highlights the stability of the rankings across different scenarios, reinforcing the M-OPARA method's ability to generate reliable rankings even when the criteria weights are adjusted. These findings confirm that the M-OPARA method not only performs comparably to various MCDM methods but also maintains its stability when ranking alternatives under fluctuating criteria weights. This further validates the effectiveness of modifying the OPARA method in this context. Through the two cases examined above, it is evident that modifying the OPARA method to develop the M-OPARA method has successfully addressed scenarios where the original OPARA method could not be applied. However, to further determine whether the modification is truly effective, it is necessary to compare the performance of M-OPARA with other MCDM methods, including the OPARA method itself. To explore this further, the following two case analyses will be conducted.

4.3 Case 3

This section presents an MCDM case study adapted from the study by Mehdi et al. [13] to apply the M-OPARA method and compare its results with those obtained using various MCDM methods, including the OPARA method (Table 14).

Table 14
Numerical Example for Case 3 [13]

Alt.	Nature						
	+	+	+	-	-	-	-
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	23	264	2.37	0.05	167	8900	8.71
A_2	20	220	2.2	0.04	171	9100	8.23
A_3	17	231	1.98	0.15	192	10800	9.91
A_4	12	210	1.73	0.2	195	12300	10.21
A_5	15	243	2	0.14	187	12600	9.34
A_6	14	222	1.89	0.13	180	13200	9.22
A_7	21	262	2.43	0.06	160	10300	8.93
A_8	20	256	2.6	0.07	163	11400	8.44
A_9	19	266	2.1	0.06	157	11200	9.04
A_{10}	8	218	1.94	0.11	190	13400	10.11
w_j	0.25	0.214	0.179	0.143	0.107	0.071	0.036

The criteria weights are provided in the final row of this table. The values of ρ1-7 were calculated as 1, and τj was set to τj=1, as per [13]. Table 15 summarizes the alternative rankings determined using the M-OPARA method in this study, alongside the rankings produced by the OPARA, PIV, ROV, SAW, WASPAS, MARCOS, and FUCA methods.

Table 15
Rankings of Alternatives in Case 3

	M-OPARA		OPARA		PIV		ROV		SAW		WASPAS		MARCOS		FUCA	
Alt.	S	R	S	R	S	R	S	R	S	R	S	R	S	R	S	R
A_1	0.114	1	0.125	1	0.013	1	0.450	1	0.946	1	0.472	1	0.785	1	2.108	1
A_2	0.106	5	0.121	2	0.040	2	0.325	5	0.893	3	0.446	3	0.741	3	4.159	5
A_3	0.098	6	0.089	6	0.110	6	0.191	7	0.721	6	0.350	6	0.599	6	6.859	7
A_4	0.086	9	0.075	10	0.169	10	0.042	10	0.614	9	0.295	10	0.509	9	9.537	10
A_5	0.095	7	0.088	7	0.113	7	0.201	6	0.709	7	0.345	7	0.588	7	6.607	6
A_6	0.092	8	0.084	8	0.122	8	0.152	8	0.678	8	0.331	8	0.563	8	7.607	8
A_7	0.111	2	0.117	3	0.027	2	0.428	2	0.901	2	0.449	2	0.748	2	2.572	2
A_8	0.109	3	0.113	4	0.035	3	0.412	3	0.878	4	0.436	4	0.729	4	3.444	3
A_9	0.107	4	0.111	5	0.045	5	0.381	4	0.857	5	0.426	5	0.711	5	3.502	4
A_{10}	0.083	10	0.076	9	0.144	9	0.085	9	0.613	10	0.296	9	0.509	10	8.606	9

Additionally, the Spearman rank correlation coefficient between the M-OPARA method and the other methods has been computed using Eq. (14) and is presented in Table 16.

Table 16
Spearman's Rank Correlation Coefficient in Case 3

	OPARA	PIV	ROV	SAW	WASPAS	MARCOS	FUCA
M-OPARA	0.915	0.935	0.976	0.964	0.952	0.964	0.976

The Spearman's rho between the M-OPARA method and the other methods is close to 1, indicating a strong positive correlation. In particular, the correlation between the M-OPARA method and the ROV and FUCA methods is notably high, demonstrating a strong similarity in the rankings of alternatives. Furthermore, when compared to the original OPARA method, the Spearman coefficient is 0.915, which is very close to 1. This confirms that the rankings produced by the M-OPARA method are nearly identical to those obtained using the OPARA method. As in the previous sections, this section also examines the impact of different criterion weights on the M-OPARA results across 20 scenarios. The first scenario assumes equal weights (w_j: 0.143), while the weight of the key criterion (w_1: 0.25) is gradually reduced to generate 14 scenarios (S_2–S_{15}) using Equation 13 [36]. Scenarios S_{16}–S_{20} are created by swapping the actual criterion weights ("C_1–C_3, C_1–C_4, C_1–C_5, C_1–C_6, C_1–C_7"). The rankings of the alternatives obtained using the M-OPARA method across all scenarios are summarized in Figure 4.

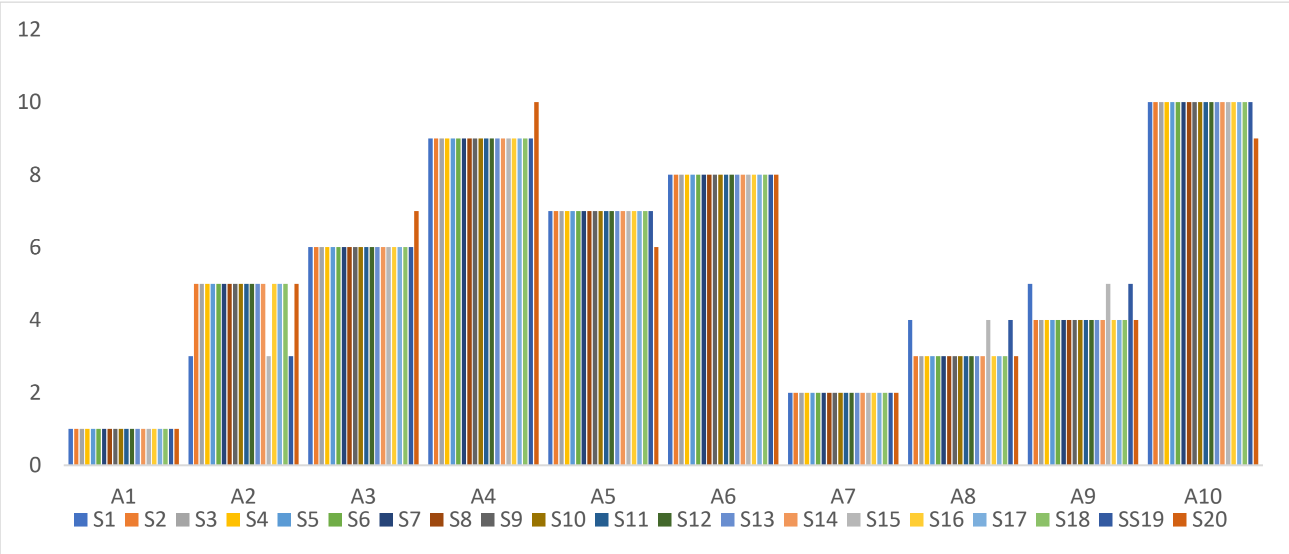

Fig. 4. Ranking of Options in Different Scenarios for Case 3

Across all scenarios with varying criterion weights, A_1 consistently holds the first position, while A_6 remains in the eighth position. The rankings of the remaining alternatives exhibit minimal fluctuations across different scenarios. These findings confirm that the M-OPARA method ensures a high degree of stability in ranking alternatives despite changes in criterion weights. In this regard, the M-OPARA method not only achieves performance comparable to other MCDM methods—including the original OPARA method, PIV, ROV, SAW, WASPAS, MARCOS, and FUCA—but also demonstrates a strong capability to maintain ranking consistency across 20 different scenarios with varying criterion weights. In conclusion, the M-OPARA method has proven to be highly accurate in ranking alternatives in this case.

4.4 Case 4

This section applies the M-OPARA method to an MCDM case study adapted from [13], comparing its results with other methods like OPARA, PIV, ROV, SAW, WASPAS, MARCOS, and FUCA. The criteria weights are listed in the final row of Table 17.

Table 17
Numerical Example for Case 4 [13]

	Nature						
	+	+	+	+	+	+	+
Alt.	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	70	50	50	40	30	40	56
A_2	70	40	50	60	40	40	54
A_3	50	40	60	50	30	50	58
A_4	80	10	70	50	50	60	67
A_5	80	30	70	40	40	50	73
A_6	100	60	60	70	80	70	79
A_7	50	30	50	60	20	40	61
A_8	40	10	60	50	10	40	55
A_9	50	10	50	40	30	60	57
A_{10}	40	20	60	40	30	50	68
A_{11}	70	40	60	60	40	40	51
A_{12}	70	70	50	60	50	60	78
w_j	0.2933	0.2015	0.1669	0.1499	0.0553	0.0343	0.099

The values of ρ1-7 were set to 1, and τj was chosen as τj=1 [13]. Table 18 presents the alternative rankings from M-OPARA and the other methods, while Table 19 summarizes the r_s between M-OPARA and the other methods, calculated using equation (14).

Table 18
Rankings of Alternatives in Case 4

	M-OPARA		OPARA		PIV		ROV		SAW		WASPAS		MARCOS		FUCA	
Alt.	S	R	S	R	S	R	S	R	S	R	S	R	S	R	S	R
A_1	0.0879	5	0.0881	5	0.1448	6	0.1572	8	0.6646	7	0.6609	6	0.5854	7	6.8859	8
A_2	0.0876	6	0.0877	6	0.1403	5	0.1908	6	0.6831	6	0.6794	5	0.6016	6	6.2441	6
A_3	0.0803	7	0.0803	7	0.1658	8	0.1675	7	0.6298	8	0.6217	8	0.5548	8	6.7532	7
A_4	0.0802	8	0.0799	8	0.1502	7	0.2617	4	0.6853	5	0.6295	7	0.6036	5	4.9856	4
A_5	0.0882	4	0.0883	4	0.1318	3	0.2713	3	0.7172	3	0.7013	3	0.6317	4	4.8481	3
A_6	0.1233	1	0.1248	1	0.0232	1	0.4416	1	0.9476	1	0.9462	1	0.8346	1	1.8693	1
A_7	0.0732	9	0.0729	9	0.1853	9	0.1296	9	0.5906	9	0.5767	9	0.5202	9	7.4645	9
A_8	0.0564	12	0.0554	12	0.2343	12	0.0738	11	0.4917	12	0.4477	12	0.4331	12	9.4699	12
A_9	0.0594	11	0.0584	11	0.2255	11	0.0544	12	0.5019	11	0.4696	11	0.4420	11	9.0387	11
A_{10}	0.0645	10	0.0638	10	0.2121	10	0.1022	10	0.5341	10	0.5125	10	0.4704	10	8.6668	10
A_{11}	0.0894	3	0.0896	3	0.1333	4	0.2272	5	0.7032	4	0.6980	4	0.6193	4	5.7873	5
A_{12}	0.1096	2	0.1108	2	0.0771	2	0.2990	2	0.8162	2	0.8108	2	0.7189	2	4.1687	2

Table 19
Spearman's Rank Correlation Coefficient in Case 4

	OPARA	PIV	ROV	SAW	WASPAS	MARCOS	FUCA
M-OPARA	1	0.9790	0.8881	0.9441	0.9790	0.9469	0.8951

The rankings from the M-OPARA method closely align with those of other MCDM methods, showing complete similarity with the original OPARA method (Spearman coefficient of 1). The coefficients between M-OPARA and other methods are also high, with the lowest at 0.89. To assess ranking consistency, the M-OPARA method was applied across 20 scenarios for criteria weights. The first scenario used equal weights (w=0.143), while 13 additional scenarios (S_2-S_{14}) were generated using equation 13 [36]. Six more scenarios were created by prioritizing each criterion individually ("C_1-

C_2; C_1-C_3; C_1-C_4; C_1-C_5; C_1-C_6; C_1-C_7"), rearranging the actual weights. Figure 5 summarizes the alternative rankings across these scenarios.

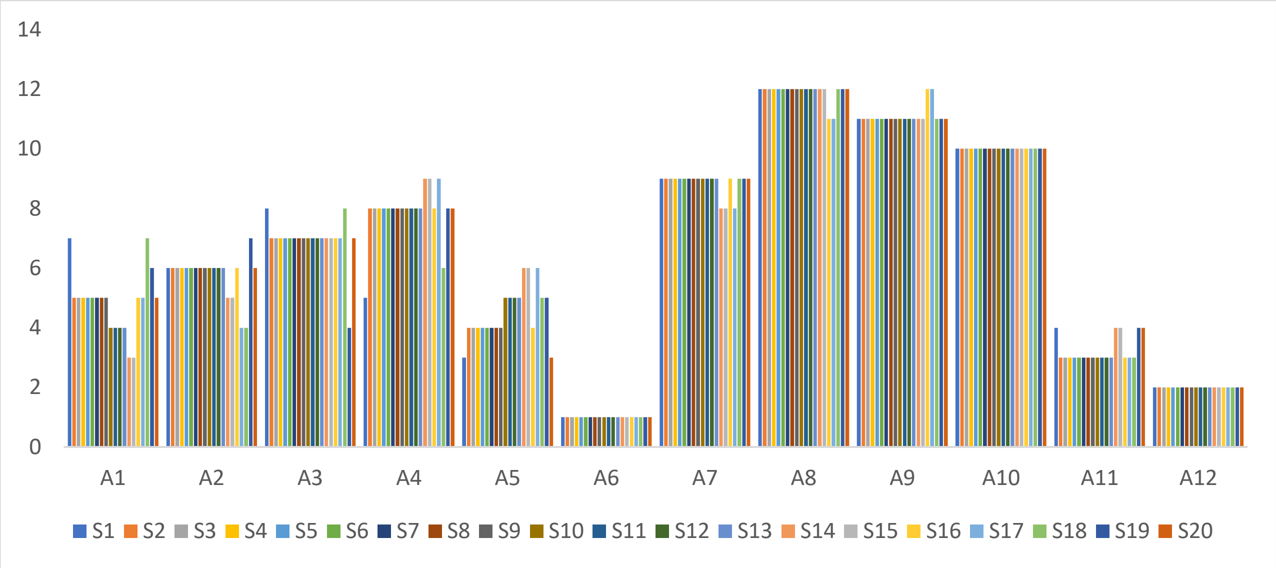

Fig 5. Ranking of Options in Different Scenarios for Case 4

Despite minor changes in criteria weights across 20 scenarios, the M-OPARA method consistently ranked A_6 1st and A_{10} 10th, with minimal variations in other alternatives' rankings. This demonstrates M-OPARA's high consistency and stability in ranking alternatives across diverse scenarios, confirming its accuracy and reliability. The study applied M-OPARA to four case studies. In Cases 1 and 2, where zero values in the decision matrix prevented the use of OPARA, M-OPARA was compared with PIV, ROV, SAW, WASPAS, MARCOS, and FUCA, showing high accuracy and strong correlation with these methods. In Cases 3 and 4, M-OPARA was compared with OPARA and other methods, revealing very high correlations (0.915 for Case 3 and 1 for Case 4). These results highlight M-OPARA's robust performance and consistency across different scenarios and methods.

Table 20
Comparison of M-OPARA and Other MCDM Methods

Case 1							
	OPARA	PIV	ROV	SAW	WASPAS	MARCOS	FUCA
M-OPARA	-	0.9	0.9	1	1	1	0.9
Case 2							
M-OPARA	-	0.8	0.8	1	1	1	0.8
Case 3							
M-OPARA	0.915	0.935	0.976	0.964	0.952	0.964	0.976
Case 4							
M-OPARA	1	0.9790	0.8881	0.9441	0.9790	0.9469	0.8951

Despite the variations observed across the four cases, the M-OPARA method exhibits a high degree of consistency in alternative rankings compared to other MCDM methods. Notably, in the two cases where the original OPARA method was applicable, the M-OPARA method effectively and accurately ranked the alternatives. A key strength of the M-OPARA method is its robustness in maintaining stable rankings even when criterion weights are altered. This stability, combined with its comparable performance to established MCDM methods, underscores the effectiveness of the proposed modification. Consequently, the development of the M-OPARA method represents a significant scientific contribution, enhancing the precision and applicability of the OPARA method.

5. Conclusion

MCDM methodologies are extensively employed across various sectors to address complex decision-making challenges, enabling decision-makers to make well-informed and strategic choices by considering multiple criteria. This study introduces a modification to the OPARA method, aimed at mitigating information loss associated with normalization processes in conventional decision-making techniques, thereby developing an enhanced approach termed M-OPARA. To validate the proposed model, its performance has been examined in comparison with the original OPARA method (for Cases 3 and 4) and seven established MCDM techniques. Furthermore, the impact of variations in criterion weights, tested across 20 different scenarios, has been systematically analyzed. In alignment with established MCDM research, where modifications are typically introduced through parameter adjustments, the effectiveness of the M-OPARA method has been rigorously evaluated through sensitivity analyses. The results indicate a high correlation between M-OPARA and other MCDM techniques, with the lowest observed correlation being 0.80. Specifically, the correlation coefficient between M-OPARA and OPARA was 1 in Case 4 and 0.92 in Case 3. Additionally, when criterion weights were altered, M-OPARA exhibited a strong capacity to maintain stable alternative rankings. These findings demonstrate that M-OPARA achieves a level of accuracy comparable to other MCDM methods. Moreover, in scenarios where the original OPARA method is inapplicable due to the presence of zero values in the decision matrix, M-OPARA provides a viable and reliable alternative for ranking alternatives.

The M-OPARA method addresses key limitations of the OPARA method, particularly its inability to handle criteria with zero values, which compromises decision-making accuracy. By resolving this issue, M-OPARA ensures more reliable and efficient alternative rankings, even when zero values are present. This modification is significant for advancing MCDM and analysis, offering both theoretical and practical benefits for researchers in decision support systems, optimization, and related fields. Its applications are particularly valuable in engineering, logistics, healthcare, and finance, where accurate decision-making is critical. However, the study highlights certain limitations. Equation (8) shows that if a criterion's maximum value is zero, both OPARA and M-OPARA become inapplicable. Future work could explore alternative mathematical formulations to address this. Additionally, neither method currently supports qualitative elements in decision matrices. Converting linguistic variables to numerical values could enable their application in such cases. Further research could also test M-OPARA on more complex problems and compare its performance with other MCDM methods like CURLI, CRADIS, or CoCoSo.

Author Contributions

Conceptualization, D.D.T, T.V.D, D.V.D, and M.T.D; methodology, D.D.T, N.E; software, D.D.T, T.V.D, D.V.D, and M.T.D; validation, D.D.T, and N.E; formal analysis, D.D.T, and N.E; investigation, D.D.T, T.V.D, D.V.D, and M.T.D; resources, D.D.T, T.V.D, D.V.D, and M.T.D; data curation, D.D.T, and N.E; writing—original draft preparation, D.D.T, T.V.D, D.V.D, N.E and M.T.D; writing—review and editing, D.D.T, T.V.D, D.V.D, N.E; visualization, D.D.T, and N.E; supervision, D.D.T, T.V.D, D.V.D, and M.T.D; project administration, D.D.T, T.V.D, D.V.D, and M.T.D; funding acquisition, D.D.T, T.V.D, D.V.D, and M.T.D. All authors have read and agreed to the published version of the manuscript.

Funding

This research received no external funding

Data Availability Statement

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Hussain, S. A. I., Chandra, H., & Mandal, U. K. (2022). Comparison of Cross-Entropy Based MCDM Approach for Selection of Material in Sugar Industry. *Chapters*. http://doi.org/10.5772/intechopen.98242

[2] Sahoo, S. K., Goswami, S. S., & Halder, R. (2024). Supplier selection in the age of industry 4.0: a review on MCDM applications and trends. *Decision Making Advances*, *2*(1), 32-47. https://doi.org/10.31181/dma21202420

[3] Mzougui, I., Carpitella, S., Certa, A., El Felsoufi, Z., & Izquierdo, J. (2020). Assessing supply chain risks in the automotive industry through a modified MCDM-based FMECA. *Processes*, *8*(5), 579. https://doi.org/10.3390/pr8050579

[4] Baydaş, M., Eren, T., Stević, Ž., Starčević, V., & Parlakkaya, R. (2023). Proposal for an objective binary benchmarking framework that validates each other for comparing MCDM methods through data analytics. *PeerJ Computer Science*, *9*, e1350. https://doi.org/10.7717/peerj-cs.1350

[5] Jalhoom, R. J. K., & Mahjoob, A. M. R. (2024). An MCDM approach for evaluating construction-related risks using a combined fuzzy grey DEMATEL method. *Engineering, Technology & Applied Science Research, 14*(2), 13572-13577. https://doi.org/10.48084/etasr.6959

[6] Rezakhani, P. (2012). Fuzzy MCDM model for risk factor selection in construction projects. *Engineering Journal*, *16*(5), 79-94. https://doi.org/10.4186/ej.2012.16.5.79

[7] Vasant, P., & Bhattacharya, A. (2007). Sensing degree of fuzziness in MCDM model using modified flexible S-curve MF. *International journal of systems science*, *38*(4), 279-291. https://doi.org/10.1080/00207720601117108

[8] Saghafian, S., & Hejazi, S. R. (2005). Multi-criteria group decision making using a modified fuzzy TOPSIS procedure. International Conference on Computational Intelligence for Modelling, Control and Automation and International Conference on Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'06), 215-221. https://doi.org/10.1109/CIMCA.2005.1631471

[9] Adıgüzel Mercangöz, B., Yıldırım, B. F., & Kuzu Yıldırım, S. (2020). Time period based COPRAS-G method: application on the Logistics Performance Index. *LogForum*, *16*(2), 239-250. http://doi.org/10.17270/J.LOG.2020.432

[10] Matin, A. O., & Misagh, F. (2019). A modified MCDM algorithm with cumulative entropy weights for selecting the winner of the tender. *Стратегические решения и риск-менеджмент*, *10*(1), 47-51. http://doi.org/10.17747/2618-947X-2019-1-46-51

[11] Tsuei, H.-J., Tsai, W.-H., Pan, F.-T., & Tzeng, G.-H. (2020). Improving search engine optimization (SEO) by using hybrid modified MCDM models. *Artificial Intelligence Review*, *53*, 1-16. https://doi.org/10.1007/s10462-018-9644-0

[12] Abdulaal, R. M., & Bafail, O. A. (2022). Two New Approaches (RAMS-RATMI) in Multi-Criteria Decision-Making Tactics. *Journal of mathematics*, *2022*(1), 6725318. https://doi.org/10.1155/2022/6725318

[13] Mehdi, K.-G., Abdolghani, R., Maghsoud, A., Zavadskas, E. K., & Antuchevičienė, J. (2024). Multi-Criteria personnel evaluation and selection using an objective pairwise adjusted ratio analysis (OPARA). *Economic computation and economic cybernetics studies and research.*, *58*(2), 23-45. http://doi.org/10.24818/18423264/58.2.24.02

[14] Roy, J., Kumar Sharma, H., Kar, S., Kazimieras Zavadskas, E., & Saparauskas, J. (2019). An extended COPRAS model for multi-criteria decision-making problems and its application in web-based hotel evaluation and selection. *Economic research-Ekonomska istraživanja*, *32*(1), 219-253. https://doi.org/10.1080/1331677X.2018.1543054

[15] Chatterjee, P., & Chakraborty, S. (2016). A comparative analysis of VIKOR method and its variants. *Decision Science Letters*, *5*(4), 469-486. http://dx.doi.org/10.5267/j.dsl.2016.5.004

[16] Boral, S., Howard, I., Chaturvedi, S. K., McKee, K., & Naikan, V. N. A. (2020). An integrated approach for fuzzy failure modes and effects analysis using fuzzy AHP and fuzzy MAIRCA. *Engineering Failure Analysis*, *108*, 104195. https://doi.org/10.1016/j.engfailanal.2019.104195

[17] Simić, V., Lazarević, D., & Dobrodolac, M. (2021). Picture fuzzy WASPAS method for selecting last-mile delivery mode: a case study of Belgrade. *European Transport Research Review*, *13*, 1-22. https://doi.org/10.1186/s12544-021-00501-6

[18] Biswas, S., & Pamucar, D. (2023). A modified EDAS model for comparison of mobile wallet service providers in India. *Financial Innovation*, *9*(1), 41. https://doi.org/10.1186/s40854-022-00443-5

[19] Ionașcu, A. E., Goswami, S. S., Dănilă, A., Horga, M.-G., Barbu, C. A., & Şerban-Comănescu, A. (2024). Analyzing Primary Sector Selection for Economic Activity in Romania: An Interval-Valued Fuzzy Multi-Criteria Approach. *Mathematics*, *12*(8), 1157. https://doi.org/10.3390/math12081157

[20] Ulutaş, A., Balo, F., Sua, L., Demir, E., Topal, A., & Jakovljević, V. (2021). A new integrated grey MCDM model: Case of warehouse location selection. *Facta Universitatis, Series: Mechanical Engineering*, *19*(3), 515-535. http://doi.org/10.22190/FUME210424060U

[21] Adali, E. A., Öztaş, G. Z., Öztaş, T., & Tuş, A. (2022). Assessment of European cities from a smartness perspective: An integrated grey MCDM approach. *Sustainable cities and society*, *84*, 104021. https://doi.org/10.1016/j.scs.2022.104021

[22] Ulutaş, A., & Karaköy, Ç. (2021). Evaluation of LPI values of transition economies countries with a grey MCDM model. In *Handbook of research on applied AI for international business and marketing applications* (pp. 499-511). IGI Global. http://doi.org/10.4018/978-1-7998-5077-9.ch024

[23] Ulutaş, A., Krstić, M., Topal, A., Agnusdei, L., Tadić, S., & Miglietta, P. P. (2024). A Novel Hybrid Gray MCDM Model for Resilient Supplier Selection Problem. *Mathematics*, *12*(10), 1444. https://doi.org/10.3390/math12101444

[24] Biswas, T., & Chaki, S. (2022). Applications of modified simple additive weighting method in manufacturing environment. *International Journal of Engineering*, *35*(04), 830-836. http://doi.org/10.5829/ije.2022.35.04a.23

[25] Ha, L. (2023). Selection of suitable data normalization method to combine with the CRADIS method for making multi-criteria decision. *Applied Engineering Letters*, *8*(1), 24-35. https://doi.org/10.18485/aeletters.2023.8.1.4

[26] Wardany, R. N. (2024). A study comparative of PSI, PSI-TOPSIS, and PSI-MABAC methods in analyzing the financial performance of state-owned enterprises companies listed on the Indonesia stock exchange. *Yugoslav Journal of Operations Research*(00), 17-17. https://doi.org/10.2298/YJOR240115017W

[27] Ulutaş, A., Popovic, G., Radanov, P., Stanujkic, D., & Karabasevic, D. (2021). A new hybrid fuzzy PSI-PIPRECIA-CoCoSo MCDM based approach to solving the transportation company selection problem. *Technological and Economic Development of Economy*, *27*(5), 1227-1249. https://doi.org/10.3846/tede.2021.15058

[28] Wang, J., Cai, Q., Wei, G., & Liao, N. (2022). Modified EDAS Method Based on Cumulative

Prospect Theory for Multiple Attributes Group Decision Making with Interval-valued Intuitionistic Fuzzy Information. *arXiv preprint arXiv:2211.02806*. https://doi.org/10.48550/arXiv.2211.02806

[29] Singh, S. P., Kundu, T., Adhikari, A., & Basu, S. (2020). An integrated weighting-based modified WASPAS methodology for assessing patient satisfaction. 2020 International Conference on Decision Aid Sciences and Application (DASA), 592-596. https://doi.org/10.1109/DASA51403.2020.9317282

[30] Watanasungsuit, A., Sangkhiew, N., Inthawongse, C., & Jomtong, P. (2023). A Modified AHP for Large-scale MCDM: a Case of Power Station Construction Project Selection. *Science and Technology, Asia*, *28*, 220-230. https://www.researchgate.net/publication/374418746

[31] Wang, H., Mahmood, T., & Ullah, K. (2023). Improved CoCoSo method based on Frank softmax aggregation operators for T-spherical fuzzy multiple attribute group decision-making. *International Journal of Fuzzy Systems*, *25*(3), 1275-1310. https://doi.org/10.1007/s40815-022-01442-5

[32] Deveci, K., Cin, R., & Kağızman, A. (2020). A modified interval valued intuitionistic fuzzy CODAS method and its application to multi-criteria selection among renewable energy alternatives in Turkey. *Applied Soft Computing*, *96*, 106660. https://doi.org/10.1016/j.asoc.2020.106660

[33] Anvari, A., Zulkifli, N., & Arghish, O. (2014). Application of a modified VIKOR method for decision-making problems in lean tool selection. *The international journal of advanced manufacturing technology*, *71*, 829-841. https://doi.org/10.1007/s00170-013-5520-x

[34] Qi, J., Hu, J., & Peng, Y. (2021). Modified rough VIKOR based design concept evaluation method compatible with objective design and subjective preference factors. *Applied Soft Computing*, *107*, 107414. https://doi.org/10.1016/j.asoc.2021.107414

[35] Shahriari, M. (2017). Soft computing based on a modified MCDM approach under intuitionistic fuzzy sets. *Iranian Journal of Fuzzy Systems*, *14*(1), 23-41. https://doi.org/10.22111/ijfs.2017.3035

[36] Ecer, F. (2022). An extended MAIRCA method using intuitionistic fuzzy sets for coronavirus vaccine selection in the age of COVID-19. *Neural Computing and Applications*, *34*(7), 5603-5623. https://doi.org/10.1007/s00521-021-06728-7

[37] Keshavarz-Ghorabaee, M. (2021). A simple modification to the EDAS method for two exceptional cases. *BOHR International Journal of Advances in Management Research*, *1*(1), 36-39. https://www.preprints.org/manuscript/202104.0111

[38] Zardari, N. H., Ahmed, K., Shirazi, S. M., & Yusop, Z. B. (2015). Weighting Methods and their Effects on Multi-Criteria Decision Making Model Outcomes in Water Resources Management. https://doi.org/10.1007/978-3-319-12586-2

[39] Mufazzal, S., & Muzakkir, S. (2018). A new multi-criterion decision making (MCDM) method based on proximity indexed value for minimizing rank reversals. *Computers & Industrial Engineering*, *119*, 427-438. https://doi.org/10.1016/j.cie.2018.03.045

[40] Madić, M., Radovanović, M., & Manić, M. (2016). Application of the ROV method for the selection of cutting fluids. *Decision Science Letters*, *5*(2), 245-254. https://dx.doi.org/10.5267/j.dsl.2015.12.001

[41] Van Thien, N., & Son, N. H. (2024). Material Selection for PMEDM Process. *International Journal of Mechanical Engineering and Robotics Research*, *13*(3). https://doi.org/10.18178/ijmerr.13.3.315-324

[42] Kizielewicz, B., & Bączkiewicz, A. (2021). Comparison of Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy WASPAS and Fuzzy MMOORA methods in the housing selection problem. *Procedia Computer Science*, *192*, 4578-4591. https://doi.org/10.1016/j.procs.2021.09.236

[43] Aazagreyir, P., Appiahene, P., Appiah, O., & Boateng, S.(2024). Comparative analysis of fuzzy multi-criteria decision-making methods for quality of service-based web service selection. I*AES International Journal of Artificial Intelligence, 13*(2), 1408-1419. https://doi.org/10.11591/ijai.v13.i2.pp1408-1419