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"Thin" Structure of Relations in MCDM Models. Equivalence of the MABAC, TOPSIS(L_1) and RS Methods to the Weighted Sum Method

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ABSTRACT

This paper introduces the conceptual framework of the multi-criteria decision-making (MCDM) rank model, which embodies the integration and harmonization of the aggregation method, the weighing method, the decision matrix normalization technique, and the selection of distance metrics. This definition serves to broaden the spectrum of acceptable MCDM methodologies for problem-solving and specifiing the associated tools. A Multi-Method Model (3M) approach is employed for multi-criteria selection to enhance the reliability of the results. The methodology is outlined for adjusting the rankings of alternatives to account for the distinguishability of ratings in a particular MCDM model using the Relative Performance Indicator (RPI) of alternatives. Through RPI, four methods are established for aggregating individual characteristics of alternatives that yield identical results: Weighted Sum Model (WSM), Multi-Attributive Border Approximation area Comparison (MABAC), Technique for Order Preference by Similarity to Ideal Solutions (TOPSIS (L_1)), and Ratio System approach (RS), eliminating the need to duplicate these methods in the 3M approach. A comprehensive comparison of numerous multi-criteria methods is conducted based on two lists: ranking and rating. Additionally, a method for step-by-step linear transformation of alternative ratings obtained from various MCDM models is defined, facilitating comparison and aggregation of ratings.

1. Introduction

An examination of recent publications indicates a growing trend in addressing multi-criteria selection problems through diverse methodologies, culminating in the subsequent synthesis of solutions [1-10]. It is posited that the reliability of a solution derived from employing a myriad of methods is heightened, rendering it a preferable approach. This approach is hereby designated as the resolution of the Multi-Criteria Decision-Making (MCDM) problem based on a set of admissible rank methods: the Model of MultiPlicity Methods or 3M. The resolution involves the formation of one or more groups of MCDM methods.

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The initial option entails an ensemble solution, while the second involves group analysis and synthesis [11-13]. Addressing the problem in both versions, followed by subsequent analysis and synthesis of the results, is also deemed relevant.

Decision-making in the first option encompasses the analysis and coordination of ranks obtained from various methods, employing, for instance, Spearman's rank correlation [14-16]. A common scenario involves the preference for the same alternative across a larger number of methods, often following the majority principle. Borda's rule and its modifications are frequently applied [17-19]. Additionally, a statistical approach based on a histogram of the distribution of ranks for a selected set of methods is typical [20].

In the second option, several groups of MCDM methods are configured for problem-solving [11]. Within each group of methods, a ranked list of alternatives (or Borda points, or ratings) is established. Subsequently, the ranking problem is addressed - a typical Multi-Stage Decision-Making (MSDM) problem concerning the initial set of alternatives, where the criteria represent groups, and the indicators of alternatives are the ranks (scores) from the first stage for each group.

Rank-based MCDM approaches can be categorized into three groups:

(1) Cost measurement methods, exemplified by WSM (Weighted Sum Model) [21] and WPM (Weighted Product Model) [22];

(2) Target or reference level models, such as TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) [23] and VIKOR (VIse Kriterijumska Optimizacija kompromisno Resenje, in Serbian) [24];

(3) Excellence methods, including PROMETHEE (Preference Ranking Organization METHod for Enrichment of Evaluations) [25] and ORESTE (Organísation, Rangement Et SynThèse de donnéEs relarionnelles, in French) [26].

The principal challenges associated with decision-making utilizing the 3M approach, which currently lack definitive resolutions, are as follows:

i. Determining the qualitative and quantitative composition of MCDM methods to be included in the list for solving a specific problem.
ii. Establishing a methodology for comparing results obtained from different methods.
iii. Assessing the significance (weight) of the employed methods.
iv. Addressing the question of whether methods should be grouped and, if so, how to form these groups.
v. Defining a method for synthesizing the solution.

The ongoing research into these issues predominantly relies on the success of specific methods in addressing particular problems.

In the literature, one area of decision-making based on a plethora of methods is referred to as Multi-Objective Optimization on the basis of a Ratio Analysis (MOORA) [27]. MOORA involves the integration of standard methods such as WSM, WPM, Evaluation based on Distance from Average Solution (EDAS), Multi-Objective Optimization on the basis of Simple Ratio Analysis (MOOSRA), TOPSIS, etc., with subsequent expansion to MultiMOORA through the incorporation of the full multiplicative form Weighted Aggregated Sum Product Assessment (WASPAS) [13, 28]. The current outcome of MultiMOORA, regarding the selection of MCDM methods, is a ranking obtained by aggregating the results of ternary ranking methods, namely the relation system, control point approach, and full multiplicative form [11]. It is noteworthy that, apart from general comments, there is no explanation regarding the suitability of specific groups of methods for a given selection task. While the MultiMOORA concept, as a variant of the 3M approach, aims to enhance the reliability of the solution, it is currently non-constructive and lacks adequate justification in addressing the previously identified problems.

Given the abundance of MCDM methods, there is a need for studies that compare them with one another. In the research presented, this comparison is conducted using two lists: a ranked list and a rating list. The rating list represents an integral score for an alternative, formed by aggregating individual characteristics using the aggregation method.

In numerous instances, the ranks of alternatives obtained through different methods exhibit consistency, thereby enhancing the reliability of solutions—a fact easily corroborated through rank correlation. Conversely, assessment scores for the same set of methods are less correlated. Importantly, the rating list, in contrast to the ranking list, reveals the "fine" structure of relationships. In specific cases, it is conceivable that the ranks and ratings of alternatives for two or more methods follow the same order, as elaborated upon in the subsequent generalization of the MCDM method to the MCDM model. However, the disparity in the rating values of two or more alternatives may be negligible. Given the heightened sensitivity of ratings to the parameters of the MCDM model, such alternatives should effectively hold the same preference status, i.e., identical ranks [20].

The primary parameters that determine rating variations include:

- the method for determining the weight of criteria, with more than 30 options available in the absence of a preference criterion [29-32],
- the method of normalizing the decision matrix, involving more than 10 main methods [33-36], the selection of which is based on established principles
- the selection of a distance metric in the n-dimensional feature space for target level models (e.g., TOPSIS) [24].

Notably, the VIKOR method [24] appears to be the only MCDM method that adjusts rankings based on ratings. This method relies on the difference between the rating values of two or more alternatives. In the VIKOR method, the critical difference value is determined under the assumption that the ratings of alternatives (Q_i) are uniformly distributed on [0, 1] with DQ=1/(m-1) representing the length of the interval for m values of the Q_i rating of alternatives.

In the presented study, the 3M multi-criteria selection approach is adopted. The concept of the MCDM model is defined, representing the synthesis and unity of the aggregation method, weighing method, decision matrix normalization method, and choice of distance metrics. This approach significantly broadens the list of acceptable MCDM methods for problem-solving and specifies its tools.

This paper is dedicated to addressing fundamental questions within the context of MCDM. Specifically, it aims to elucidate the process of establishing a meaningful difference in the ratings of alternatives obtained in a given MCDM model, leveraging the relative performance indicator (RPI) of alternatives. A method designed to adjust the ranks of alternatives while taking into account the distinguishability of ratings is introduced. In the pursuit of a comprehensive MCDM framework, the RPI is emplyed to establish four distinct methods for aggregating the individual characteristics of alternatives that yield identical results. These methods include WSM, MABAC, TOPSIS(L_1) , and the Ratio System approach (RS), strategically avoiding redundancy within the 3M approach.

The article further explores how to compare ratings of alternatives obtained from different MCDM models using stepwise linear transformation. This enables the aggregation of ratings that account for the "thin" structure of relations between alternatives, enhancing result reliability, in contrast to the aggregation of a ranking list, which represents the "coarse" structure of relations. The fourth section provides a numerical example of the 3M approach, fully implementing the methodology presented in Section 3.

2. Preliminaries: MABAC, TOPSIS(L_1) and VIKOR MCDM methods

2.1 Nomenclature

MCDM methods:

CODAS : COmbinative Distance-based ASsessment
COPRAS : COmplex PRoportional Assessment
CRITIC : CRiteria Importance Through Intercriteria Correlation
GRA : Grey Relational Analysis
MABAC : Multi-Attributive Border Approximation area Comparison
MOORA : MultiObjective Optimization on the basis of a Ratio Analysis
3M : Multi-Method Model
ORESTE : Organization, Rangement Et Synthese De Donnes Relationnelles (fr.)
PROMETHEE : Preference Ranking Organization METHod for Enrichment Evaluation
ReS : Reverse Sorting algorithm
RS : the Ratio System approach
TOPSIS : Technique for Order Preference by Similarity to Ideal Solutions
TOPSIS(L_1) : TOPSIS with city block metric L_1
WASPAS : Weighted Aggregated Sum Product Assessment
WPM : Weighted Product Method
WSM : Weighted Sum Method
AHP : Analytic Hierarchy Process
EV : EigenVector

Designation:

A_i alternatives (objects) (i=1,…, m)
C_j^+, C_j^- criteria or objects properties (j=1,…, n), (+) benefit, (-) cost
a_{ij} elements of decision matrix D
r_{ij} normalized elements of decision matrix
$\overline{r}_j$ average value of jth criterion
$a_j^{\max}$ maximum element in criteria j
$a_j^{\min}$ minimum element in criteria j
w_j weight or importance of criteria (j=1,…, n)
Q_i the performance indicator of alternatives (objects) (i=1,…, m)
dQ_i relative performance indicator of alternatives (RPI)

Normalization methods:

Max $r_{ij} = \dfrac{a_{ij}}{a_j^{\max}}$, Sum $r_{ij} = \dfrac{a_{ij}}{\sum_{i=1}^{m} |a_{ij}|}$, Vec $r_{ij} = \dfrac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}}$, Max-Min $r_{ij} = \dfrac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}$

2.2 Initializing the MCDM problem

The process can be conceptualized as selecting the best (most preferred) alternative from a finite set of alternatives. Each alternative is characterized by a specific finite set of attributes within the framework of selected criteria. Alternatively, it can be understood as the grouping of alternatives into multiple preference groups, followed by the selection of a small set from them. Moreover, these problems aim to identify alternatives that are neither dominant nor effective. It is impossible to transition from a non-dominated solution to another solution without sacrificing at least one of the criteria [21].

In mathematical terms, the MCDM problem is defined as follows: the MCDM rank model for each alternative A_i calculates the performance indicator (Q) of each alternative (or key performance indicator (KPI), preference score, assessment score).

$$Q_i = f_k(A, C, D, '\omega', \ 'norm', 'dm') \quad (1)$$

Here, alternatives (A_i, i=1,…, m) represent different possible courses of action, and the solution space is presented as "possible alternatives." Decision variables are defined as components of a vector of alternatives.

Criteria (C_j, j=1,…, n) serve as tools for evaluating and comparing alternatives in terms of the consequences of their choice.

Measures are elements used to quantify an attribute of an alternative, with an attribute defined as a measurable characteristic. The set of all attributes forms the decision-making matrix $D=(a_{ij})$.

Each alternative attribute aggregation method (f_k) includes a method for estimating the weight of criteria ('w'), normalization method ('*norm*') of the decision matrices, selection of a metric for calculating distances in the n-dimensional space of criteria ('*dm*').

For example, WSM has the simple form

$$Q_i = \sum_{j=1}^{n} w_j r_{ij} \quad (2)$$

Here, r_{ij} represents the normalized values of the decision matrix.

The ranking list Q is arranged in descending (or ascending) order, based on which a ranking list of alternatives is formed. Without loss of generality, it is assumed that connected sorting and renumbering of these two lists has been performed in the form:

$$Q_1 \geq Q_2 \geq \ldots \geq Q_m \quad (3)$$

$$A_1 \succ A_2 \succ \ldots \succ A_m \quad (4)$$

Here, the symbol "$\succ$" indicates preference.

2.3 Multi-Attributive Border Approximation area Comparison (MABAC)

The MABAC [37] refers to cost estimation methods. The performance indicator of alternatives is defined as:

$$Q_i = \sum_{j=1}^{n} (v_{ij} - g_j) \quad (5)$$

$$v_{ij} = (r_{ij} + 1) \cdot \omega_j, \ i = 1, \ldots, m; \ j = 1, \ldots, n \quad (6)$$

$$r_{ij} = Max - Min(a) = \begin{cases} \dfrac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}, \ for \ Benefit \ type \ criteria \\ \dfrac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}, \ for \ Cost \ type \ criteria \end{cases} \quad (7)$$

$$g_j = \left(\prod_{i=1}^{m} v_{ij} \right)^{1/m} \quad (8)$$

The best alternative corresponds to the highest value of the performance indicator Q.

2.4 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

To determine the performance indicator of the *i*th alternative Q_i, a homogeneous function was used [21]:

$$Q_i = \frac{S_i^-}{S_i^+ + S_i^-} \tag{9}$$

where, r_{ij} represent normalized values of the decision matrix in accordance with (7),

$$v_{ij} = r_{ij} \cdot w_j,\ S_i^+ = d(v_{ij}, v_j^+),\ S_i^- = d(v_{ij}, v_j^-) \tag{10}$$

$$v_j^+ = \{\max_i v_{ij} \left| if\ j \in C_j^+;\ \min_i v_{ij} \right| if\ j \in C_j^-\} \tag{11}$$

$$v_j^- = \{\min_i v_{ij} \left| if\ j \in C_j^+;\ \max_i v_{ij} \right| if\ j \in C_j^-\} \tag{12}$$

where, S_i^+ and S_i^- represent the distances *d* between the ideal and anti-ideal objects respectively. Additionally, the alternative A_i in the *n*-dimension attributes space, which are defined in one of the L_p-metrics. The TOPSIS ranking result depends on the choice of distance metric (*dm*):

$$L_p(X,Y) = \left[\sum_{i=1}^{m} (x_i - y_i)^p\right]^{1/p},\ 1 \le p \le \infty; L_\infty(X,Y) = \max_i |x_i - y_i| \tag{13}$$

Let us denote the TOPSIS method with the CityBlock metric L_1 as TOPSIS(L_1) .

The best alternative corresponds to the highest value of the performance indicator *Q*.

2.5 VIsekriterijumsko KOmpromisno Rangiranje (VIKOR)

Similar in structure to the TOPSIS method, the VIKOR method is outlined [24]. In the initial step of the VIKOR method, a matrix of deviations of the natural values of alternative attributes from the ideal and anti-ideal objects is formulated.

To determine the performance indicator of the *i*th alternative Q_i, a homogeneous function is applied based on the strategies of maximal *R* and group utility *S*:

$$Q_i = \beta \cdot \frac{S_i - S^*}{S^- - S^*} + (1-\beta)\frac{R_i - R^*}{R^- - R^*} \tag{14}$$

$$v_{ij} = r_{ij} \cdot w_j \tag{15}$$

$$R_i = \max_j v_{ij};\ R^* = \min_i R_i;\ R^- = \max_i R \tag{16}$$

$$S_i = \sum_{j=1}^{n} v_{ij};\quad S^* = \min_i S_i;\ S^- = \max_i S_i \tag{17}$$

where, r_{ij} represent normalized values of the decision matrix in accordance with (7),

The parameter *β* serves as a balancing factor between total benefit (*S*) and the maximum individual deviation (*R*). Smaller values of *β* emphasize the strengthening of the group, while larger values increase the weight given by individual deviations.

The result of the VIKOR methodology yields three rating lists: S, *R* and *Q*. Alternatives are evaluated by sorting the values of *S*, *R* and *Q* according to the criterion of the minimum value.

As a compromise solution, alternative A_1 is proposed, the efficiency indicator *Q* of which has the lowest value and if the following two conditions are met:

1) "acceptable advantage": $Q(A_2) - Q(A_1) \ge 1/(m-1)$, where A_2 is an alternative to the second position in the *Q*-rating list,

2) "acceptable decision stability": alternative A_1 should also be best scored on *S* or/and *R*.

If one of the conditions 1) or 2) is not satisfied, then a set of compromise solutions is proposed, which consists of:

- the alternatives A_1 and A_2 if condition 1 is true and condition 2 is false, or

- the set of alternatives $\{A_1, A_2, \ldots, A_k\}$ if condition 1 is false; being k the position in the ranking of the alternative A_k verifying $Q(A_k)–Q(A_1)<1/(m–1)$ and $Q(A_{k+1})–Q(A_1)\geq 1/(m–1)$.

The peculiarity of the VIKOR method lies not only in maintaining a balance of utilities but also in containing an implicit procedure for evaluating the distinguishability of ranking results. Alternatives are considered indistinguishable if the performance indicators of the alternatives differ by less than $1/(m–1)$. The latter, however, is not substantiated in any way. The significant difference between the alternatives is detailed in Section 3.4.

2.6 The Ratio System approach (RS)

To determine the performance indicator of the ith alternative Q_i, in Ratio System, the non-beneficial sum is subtracted from the beneficial sum [27]:

$$Q_i = \sum_{j=1}^{n} sign(C_j) \cdot w_j \cdot r_{ij} \tag{18}$$

$$sign(C_j) = \begin{cases} 1, & if\ j \in C_j^{+} - \text{beneficial criteria} \\ -1, & if\ j \in C_j^{-} - \text{non beneficial criteria} \end{cases} \tag{19}$$

The best alternative corresponds to the highest value of the performance indicator Q.

3. Methodology

3.1 MCDM rank model

Historically, the MCDM method is understood primarily as a procedure for aggregating individual features of alternatives according to Eq. 1. Numerous studies, for example [1-20], demonstrate that the ranks (especially ratings) of alternatives can change with variations of any of the arguments in Eq. 1. It is also well known that the three main components of MCDM ranking methods, i.e. the method for assessing the significance of attributes (weight of criteria), the method for normalizing the decision matrix and the distance metric for target or reference level methods (TOPSIS, etc.) are essential and in many cases (tasks) determine the result ranking. If the choice of a set of alternatives A_i and criteria C_j are the prerogative of the decision maker (expert) and constitute an unformalized part of the solution to the problem, then the three parameters {‘*w*’, ’*norm*’, ‘*dm*’} are multivariate and their choice requires analysis of both the situation and results. As highlighted in the introduction section, a diverse set of tools is utilized, including:

- more than 10 basic methods for determining the weight of criteria in the absence of a preference criterion, providing a variety of options to choose from,

- over 5 basic methods for normalizing the decision matrix, with the choice guided by established principles

- three main distance metrics in n-dimensional feature space for MCDM methods based on distance from the “ideal” (L_1 - ‘CityBlock’, L_2 - 'Eucludean', L_∞ -‘Chebyshev’).

This implies that the range of options for solving MCDM problems is contingent on the number of possible (admissible) combinations of model arguments [2]. For instance, utilizing the WSM aggregation, over 150=10×5×3 different options can be implemented solely within the main methods-arguments of the model. Consequently, the ratings of alternatives will naturally differ across various models, with variations in ranks also observed.

Given these considerations, it is recommended to employ the term "MCDM rank model", specifying the arguments and methods utilized in problem-solving. For instance, the model:
Q_i=TOPSIS(*'w'*= AHP-EV, *'Norm'*=Max, *'dm'*= L_1)
employs the TOPSIS-method for attribute aggregation, the Max-method for normalization, the CityBlock-metric for distance measurement, and integrates a weight estimate derived from a matrix of paired criteria comparisons in the Analytic Hierarchy Process (AHP) and eigenvector method (EV).

It is crucial to emphasize that none of the method arguments used in the model holds priority over the others, rendering all models equal. In many instances, combining a triple of model parameters {'*w*', '*norm*', '*dm*'} does not significantly alter the ratings, nor does it affects the ranking of alternatives, characterizing the stability of the solution to variations in model parameters. This situation characterizes the stability of the solution to variations in model parameters. However, there are situations where the solution is unstable, and variations in model parameters lead to changes in ranking, resulting in an uncertain final choice. For example, works [34, 35] present cases where decision matrices yield different rank 1 alternatives for five distinct normalization methods.

Structurally, each valid triple {normalization, weighting, and aggregation method} is construed as one of the MCDM models. To utilize the MCDM model approach effectively, researchers must determine which normalization, weighting, and aggregation methods to employ and subsequently analyze the independence of models to ensure equal "voting" conditions for each model in the final candidate selection.

3.2 "Thin" structure of relationships in MCDM rank model

The concept of the MCDM rank model expands the array of decision options, necessitating comparative studies to aid decision-makers in selecting the most suitable method. It is crucial to underscore the distinctions between these methods, as the utilization of different approaches may yield discrepancies in the resultant ratings.

In contrast to the ranking list, the rating list reveals the "thin" structure of relationships. A scenario may arise where the difference between the rating values of two or more alternatives is insignificant, leading to variations in the order (ranks of alternatives) across different MCDM models. Due to the heightened sensitivity of rankings to the parameters of the MCDM model, such alternatives should ideally share the same preference status (i.e., possess identical ranks). Therefore, to ascertain the priority of alternatives, a mere comparison of ratings – absolute values of the efficiency indicator Q_i is insufficient. The identification of situations with high decision sensitivity is achievable through the use of the RPI of alternatives [34, 35]:

$$\mathrm{d}Q_p = \frac{(Q_p - Q_{p+1})}{rng(Q)} \cdot 100\%, \;\; p = 1,...,m-1 \tag{20}$$

where, Q_p is the value of the performance indicator corresponding to the *p*-rank alternative (ordered list), $rng(Q)=\max Q_i-\min Q_i=Q_1-Q_m$.

The dQ score represents the relative (given in the Q scale) increase or decrease in the performance indicator for an ordered list of alternatives. It is believed that two alternatives, the relative increase in dQ of which differ less than the value of a given a priori error, should be considered indistinguishable.

In order to demonstrate the above reasoning with an example, the ordered scale of results is Q_1=0.712>Q_2=0.711>…>Q_m=0.137. In fact, alternatives A_1 and A_2 are indistinguishable, since dQ_1=(0.712−0.711)/(0,712−0.137)·100≈0.17%.

Thus, the 0.17% difference in the ratings of the first and second alternatives is considered small, implying that the alternatives are indistinguishable and should be assigned the same rank. The process of determining the critical error value dQ_{cr} is explained in paragraph 3.4.

3.3 Comments on adjusting ranks in the VIKOR method

The VIKOR method stands out as the only MCDM method that performs ranking adjustments based on ratings. Figure 1 illustrates the rank adjustment process in the VIKOR method.

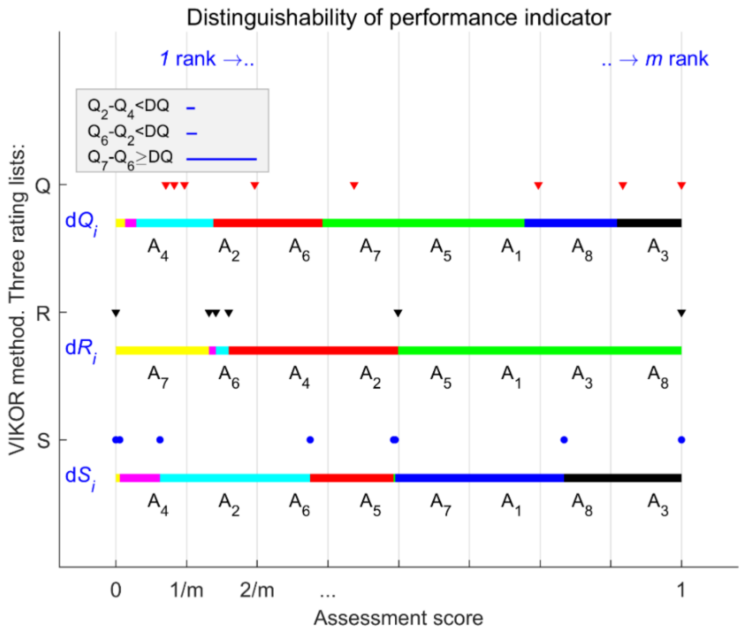

Fig. 1. Illustration of rank adjustment in the VIKOR method

Although $Q_6 > Q_2 > Q_4$ where a smaller value is considered better, alternatives A_4, A_2, A_6 according to VIKOR, are deemed indistinguishable and are assigned the same rank of 1.

Formula (13) represents the performance indicator Q of an alternative, striking a balance between the sum of particular features (normalized), considering significance (weight), and the largest normalized value of the feature, also taking into account weight. Two rating lists, S and R are pre-converted to [0, 1]:

$$\overline{S}_i = \frac{S_i - S^*}{S^- - S^*},\ \overline{R}_i = \frac{R_i - R^*}{R^- - R^*} \tag{21}$$

$$\overline{S}_i \in [0,1],\ \overline{R}_i \in [0,1] \tag{22}$$

The Max-Min transformation method, a linear normalization preserving the proportions of values [38], is then applied:

$$\frac{\overline{S}_p - \overline{S}_q}{\overline{S}_{\max} - \overline{S}_{\min}} = \frac{S_p - S_q}{S_{\max} - S_{\min}},\ \frac{\overline{R}_p - \overline{R}_q}{\overline{R}_{\max} - \overline{R}_{\min}} = \frac{R_p - R_q}{R_{\max} - R_{\min}} \tag{23}$$

Thus, the converted data still reflects the actual proportions of these ranking lists. This method ensures that the converted data retains the actual proportions of the original ranking lists.

VIKOR's ranking relies on the significant difference between the rating values of two or more alternatives. In the fundamental VIKOR method, the critical value of the difference DQ is determined based on the assumption that the normalized, dimensionless values of the ratings Q_i are uniformly distributed on [0, 1] resulting in DQ=1/(m−1) — the length of the interval for m values Q_i from [0, 1]. However, the assumption of a uniform distribution of the Q rating indicator may not always be valid, prompting the first remark. The critical value of DQ in the case of a non-uniform distribution of Q_i ratings should be chosen in accordance with the distribution law.

The second remark pertains to the balance formula (13), which, considering notation (21), takes the form:

$$Q_i = \beta \cdot \overline{S}_i + (1-\beta) \cdot \overline{R}_i \quad (24)$$

It is evident that $Q_i \in [0, 1]$ if and only if the values 0 and 1 for $\overline{S}_i$ and $\overline{R}_i$ are achieved simultaneously for two alternatives. In many cases, $Q_i \in (0, 1)$ indicating that the range of the set Q_i is less than 1. However, the predetermined critical value DQ is a fraction of 1, revealing an inherent inaccuracy.

The impact of these inaccuracies on the results should be considered in light of the subjectivity inherent in MCDM. The choice of alternatives, criteria, and methods of aggregation, normalization, weighting, and distance metrics is subjective. A minor deviation in the selection of DQ likely does not compromise the solution, the main error of which is influenced by the subjectivity stemming from the uncertainty in the choice of the MCDM model.

It's noteworthy that if, in formula (20), the range of the efficiency indicator is equal to 1, then the relative increase in the efficiency indicator dQ coincides with the absolute increase (without conversion to a percentage), as determined in the VIKOR method.

The relative increase in the relative performance indicator dQ, as determined by (20), serves as a universal metric for all MCDM ranking methods, encompassing WSM, CODAS, COPRAS, TOPSIS, PROMETHEE, etc., with variations in the arguments and methods of the model (1). The purpose of dQ lies in assessing the distinguishability of alternatives based on rating values and adjusting the ranks accordingly.

3.4 Distinction of alternatives

Addressing the question of how to determine the critical value dQ_{cr} of the relative increase in the efficiency indicator to assess the distinguishability of alternatives, the following approaches emerge:

I. *Subjective approach*

The subjective characteristic is established by the decision maker. It is reasonable to require that two alternatives be considered meaningfully different if the relative change in the performance measure exceeds a given value. For instance, a manufacturer may task the designer with improving the overall rating by at least 15% when releasing new models. Alternatively, a consumer might seek an alternative superior to analogues by at least dQ_{cr} percent; otherwise, the alternatives are deemed indistinguishable, resulting in the same rating. In this case, the relative error is set "a priori."

II. *Statistical approach*

The statistical approach relies on variations in the ranking of alternatives due to changes in the decision matrix D or variations caused by the design of the MCDM model.

In the first case, attributes may be imprecisely measured, data sources may be unreliable, errors in measurements may occur, measurements for different alternatives may be conducted using different methods, and some attributes may be random variables or defined by interval values. All these factors can alter the decision matrix D. By varying the corresponding elements of matrix D within a possible interval, one can estimate the magnitude of the rating dispersion for alternatives of ranks 1 and 2 [20].

In the second case, the choice of argument method also induces a change in the ranking of alternatives. In some cases, this change may cause a shift in the ranking (high sensitivity). It is often impossible to determine which method-argument is suitable for the problem at hand, and all available methods should be considered equal.

Let us consider the situation of variation of matrix D.

To statistically assess the distinguishability of the rating dQ_{cr}, it is necessary to obtain a representative statistical sample of size N for the performance indicators of alternatives Q_i with variations of the decision matrix a_{ij} in the range of acceptable values. The true value of an attribute is taken to be either the average of many observations or, in the absence of statistics, the available value, denoted as a_{ij}^0. It is assumed that the error estimate for the attribute a_{ij} is known (given): δ_{ij}=

$\delta(a_{ij})$. A special case is the situation when, for a fixed criterion, under the conditions of the same data source, the error in assessing alternatives does not depend on the alternatives, i.e. $\delta_{ij}= \delta_j$. Condition: $\delta_{ij}= \delta_j$ for all i=1,…, m, excludes the priority of alternatives before making decisions. This requirement is mandatory when evaluating alternatives, as an overestimated (underestimated) assessment of any alternative for any of the attributes entails its prioritization.

The variation of the decision matrix is performed using a random number generator uniformly distributed on the interval [−1, 1]. The variation algorithm has the following simple formula:

$$a_{ij}^{(k)} = a_{ij}^{o} \cdot (1+\delta_{ij} \cdot rnd_k()), \quad k=1,\ldots,N \tag{25}$$

where, $rnd_k()$ — a function that returns a random number uniformly distributed on the interval [−1, 1] for each variation of a matrix element D, δ_{ij} — relative error in estimating the attribute a_{ij}, for example at 5%, δ_{ij} =0.05.

For each variation of the decision matrix $a_{ij}^{(k)}$, the value of the performance indicator for each alternative $Q_i^{(k)}$, i=1,…, m is determined. Subsequently, based on the statistics $Q_i^{(k)}$, a statistical assessment of the standard error σ_Q of the performance indicator of alternatives of the 1st and 2nd ratings is carried out. In the situation of "method-argument" variation, there is also a similar statistical sample $Q_i^{(k)}$, only of a fixed volume, since the number of variations is finite.

The resulting indistinguishability estimate dQ_{cr} takes the form:

$$\mathrm{d}Q_{cr} = \frac{1.96\cdot(\sigma_{Q_1}+\sigma_{Q_2})}{rng(Q)}\cdot 100\% \tag{26}$$

where, σ_{Q1}, σ_{Q2} are, respectively, the standard deviation of the performance indicators Q_1 and Q_2 of alternatives of the 1st and 2nd ratings according to N statistical tests; $rng(\bar{Q})= \bar{Q}_1-\bar{Q}_m$ the average range of the rating.

Formula (26) corresponds to the statistical deviation between the values of the ratings Q_1 and Q_2 by the value of the 95% confidence interval, normalized to the Q value scale. The 1st and 2nd ranked alternatives are evidently crucial for decision making. If $dQ_2 > dQ_{cr}$ (or $dQ_3 > dQ_{cr}$) alternatives of the 1st and 2nd ranks (or the 2nd and 3rd ranks) are significantly distinguishable; otherwise, they are not distinguishable.

3.5 Rank adjustments. Ranking algorithm using difference criterion

For compromise solutions that include three alternatives of the first three ranks based on the ordering of Q_i values, 5 groups of solutions are possible according to the indistinguishability criterion:

1) I ≠ II, II ≠ III — alternatives of the first three ranks are significantly distinguishable,
2) I ≈ II, II ≠ III — alternatives of rank I and II are indistinguishable,
3) I ≠ II, II ≈ III — alternatives of rank II and III are indistinguishable,
4) I ≈ II, II ≈ III, I ≈III — alternatives of rank I, II and III are indistinguishable,
5) I ≈ II, II ≈ III, I ≠ III — alternatives of rank I, II and II, III are indistinguishable.

Let us divide the alternatives of the first three ratings ($Q_1 > Q_2 > Q_3$) into classes in accordance with the rating distinctiveness criterion (dQ_{cr}) in an ordered list as follows:

Group 1: $dQ_2 > dQ_{cr}$ and $dQ_3 > dQ_{cr}$
Group 2: $dQ_2 \leq dQ_{cr}$ and $dQ_3 > dQ_{cr}$
Group 3: $dQ_2 > dQ_{cr}$ and $dQ_3 \leq dQ_c$
Group 4: $dQ_2 \leq dQ_{cr}$ and $dQ_2 + dQ_3 \leq dQ_{cr}$
Group 5: elseif

If it is necessary to use more than three alternatives to analyze distinguishability, the number of classes will increase. Nevertheless, the algorithm for assigning a result to a specific class remains consistent — the combinatorial method. The following presents a function procedure implementing the technique for adjusting ranks based on the d*Q* indicator.

```
function R=Rank_dQ(Q, dQ, DQ)
  %-- DQ - Critical difference,%
  %-- dQ - RPI of alternatives
  %       for the sorted list of Q
  m=size(Q,1);
  R=[1:m];                %-- normal rank list
  s=0;
  for i=2:m
     if dQ(i)<=DQ & s<=DQ
        s=s+dQ(i);
        R(i)=R(i-1);       %-- corrected rank list
     end
     if dQ(i)<=DQ & s>DQ
        s=dQ(i);
        R(i)=i-1;          %-- corrected rank list
     end
     if dQ(i)>DQ
        s=0;
        R(i)=R(i);
     end
  end
end
```

An illustration of the rank adjustment for 8 alternatives in 12 different calculation options, with a random variation of the decision matrix *D* within 5% of the scale of changes in the values of each attribute, is presented in Figure 2.

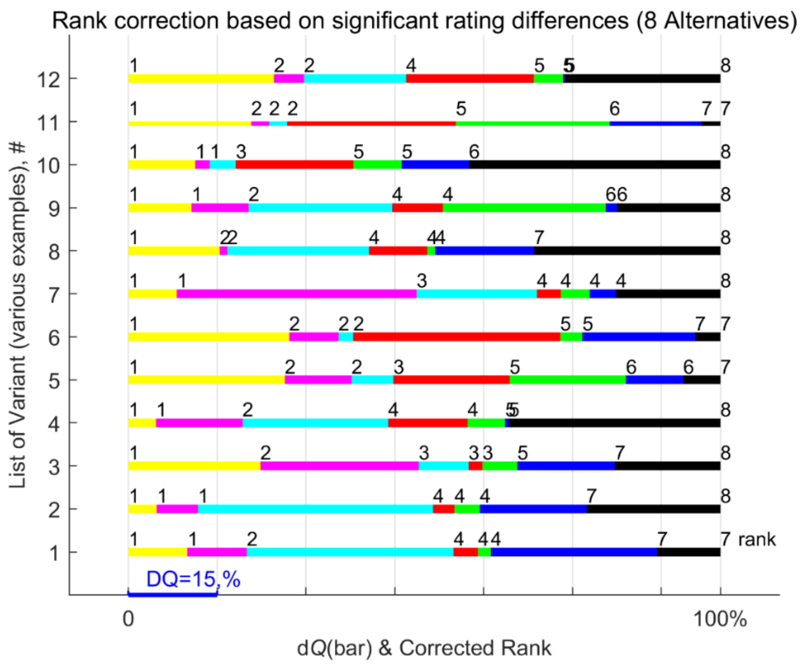

Fig. 2. Corrected rank based on significant difference

In the illustration provided, the critical value dQ_{cr} "a priori" is set to a relative error of 15%. The 1st rank is achieved in 20 cases, the 2nd rank is achieved in 16 cases (instead of 12), and so on. This implies that the competition between alternatives competing for leadership is intensifying.

3.6 Equivalence of the MABAC, TOPSIS(L_1) and RS methods to the WSM

Despite the notable difference in the aggregation formulas for the WSM, MABAC, TOPSIS(L_1) and RS methods, these approaches exhibit identical RPI values. Disparities in the aggregation formulas (2), (5)−(8), (9)−(13), (18)−(19) naturally result in distinct values for the performance indicator of alternatives Q_i. Nevertheless, the coincidence of dQ_i values stems from the scaling effect: transformations of the decision matrix D (formulas for aggregation of private features) do not alter the relative distances, demonstrating the invariance of dispositions of Q_i values.

In both instances, the authors encountered challenges in obtaining definitive proof of this effect due to the presence of nonlinearities in transformation formulas (8) and (9), complicating rigorous calculations. Nonetheless, the authors conducted numerous computational experiments, varying the problem dimension, decision matrix, normalization methods (performed exclusively for linear transformations), and weighting coefficients across a broad spectrum. In all cases, the dQ_i values remained consistent.

Figure 3 illustrates the results of one of several tests affirming the equivalence of the WSM, MABAC, TOPSIS(L_1) and RS methods (grey highlighted area).

The equivalence of methods becomes evident as dQ_i remains consistent for equivalent methods, while Q_i exhibits proportionality in terms of displacement and tension-compression.

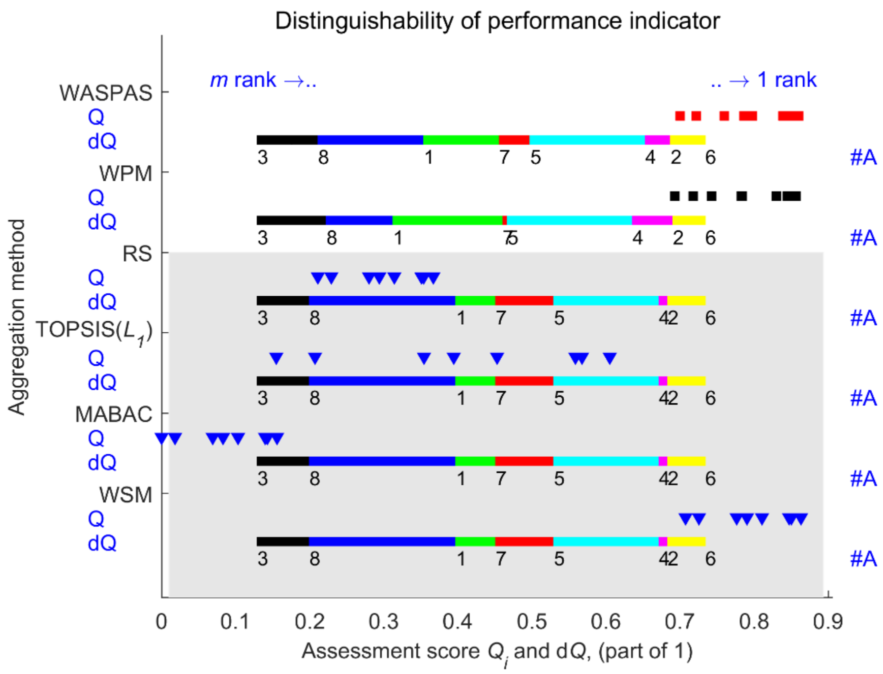

Fig. 3. Equivalence of WSM, MABAC, TOPSIS(L_1) and RS methods — numerical illustration

This observation leads to several consequential considerations:

- the MABAC, TOPSIS(L_1) and RS methods may be excluded from the collection of MCDM methods, as the simpler WSM method produces identical results,

- the identity between RS and WSM suggests that the ReS inversion [38], employed in WSM to transform cost criteria values, incorporates the inversion $-r$ and ensures the continuity of the aggregation method,

- the significance of the additive WSM method is heightened, given that its results are corroborated by TOPSIS(L_1), utilizing the longest distance from the anti-ideal approach (target level model), and the nonlinear aggregation method MABAC, employing a multiplicative component.

Equivalence tests for RPI (dQ) were also conducted to assess the aggregation methods WPM, WASPAS, CODAS, COPRAS, VIKOR, TOPSIS(L_2), PROMETHEE, and ORESTE. No evidence of RPI equality was found. Figure 3 illustrates the results for the WPM and WASPAS aggregation methods, serving as a comparative reference.

3.7 Synthesis of a solution within the list of acceptable MCDM ranking models

The simplest and most commonly employed synthesis approach involves the aggregation of rank lists, commonly known as Borda voting. However, due to the inherent nature of ranks not accurately reflecting the actual relationship structure among alternatives, there is a loss of information within the ranking list.

Option 1. Tournament-style counting

Tournament-style counting assigns each alternative in the 3M list ($m-i$) points for the i-rank, where m represents the number of alternatives. Thus, an alternative ranked 1st receives $m-1$ points, a 2nd rank gets $m-2$ points, ..., and so forth, with m-rank garnering 0 points, where m is the total number of alternatives.

Option 2. The Dowdall system

The Dowdall system awards each alternative in the 3M list $1/i$ point for i-rank. Consequently, a 1st rank receives 1 point, a 2nd rank obtains 1/2 points, and so forth, with m-rank acquiring $1/m$ point, where m is the number of alternatives. Notably, this method is more favorable to candidates with many first preferences compared to the traditional Borda count.

Subsequently, the scores for each alternative are aggregated across all models, potentially incorporating the weight of the model. When employing the weighted sum method for solution synthesis, there are inherent risks of inaccurately determining the weight of the model or group, which can significantly impact the resulting ranking.

In accordance with (20), the proportions between the points assigned to each alternative in Borda's approach are detailed as follows:

Option 1: dQ_i=$1/(m-1)\cdot 100\%$, meaning points are evenly distributed.

Option 2. dQ_p=$m/(m-1)\cdot 100/p\%$, indicating points are distributed inversely to the rating, with points for 1st rank being significantly higher than those for subsequent ranks. For m=8, this difference is respectively [57.1 19.1 9.5 5.7 3.8 2.7 2.0] %. This implies that alternatives are not on equal footing, necessitating justification. As previously noted, this method favors candidates with a greater number of first preferences.

3.8 Synthesis of a solution based on the transformation of the rating scale

Utilizing the solution to the MCDM problem through the 3M approach — the set of admissible MCDM rank models, let k models be selected.

Synthesizing a solution based on a rating scale is deemed preferable to synthesizing a solution based on a ranking scale. The latter scenario entails a loss of information inherent in the rating list, specifically the degree of proximity-removal of the integral indicator Q_i for various alternatives. Although the adjustment of the ranking list was previously demonstrated based on dQ_i, it still results in a loss of the "fine structure of relationships."

Various MCDM models possess distinct scales, with some having different directionalities, as exemplified by VIKOR where a lower Q_i value is considered better.

Hereinafter, the ease of transforming different scales into a common scale is demonstrated through Steps 1-4:

Step 1. Selecting an alternative (rank), wherein a larger value of the performance indicator Q_i is considered better, is taken as the base.

Step 2. For methods included in 3M, where a smaller Q_i value is preferred, an inversion of values is performed using the ReS algorithm [38]:

$$Q_i = -Q_i + Q_i^{\max} + Q_i^{\min} \tag{27}$$

This transformation preserves the proportions between the Q_i values.

Step 3. Shifting the values to the zero reference point:

$$Q_i = Q_i - Q_i^{\min} \tag{28}$$

This is relevant if the rating list contains negative values. In this case $Q_i^{\min}=0$.

Step 4. Transforming the Q_i values for all methods included in 3M to [0, 1] using the linear Max-Min transformation:

$$\overline{Q}_i = \frac{Q_i - Q_i^{\min}}{Q_i^{\max} - Q_i^{\min}} = \frac{Q_i}{Q_i^{\max}} \tag{29}$$

The normalized values are interpreted as a fraction of the largest value or, given $Q_i^{\min}=0$, as a fraction of the range of values, specific to each MCDM model. In accordance with [36, 38], for linear transformations the proportions between the Q_i values are preserved:

$$\frac{\overline{Q}_p - \overline{Q}_q}{\overline{Q}_i^{\max} - \overline{Q}_i^{\min}} = \frac{Q_p - Q_q}{Q_i^{\max} - Q_i^{\min}} \tag{30}$$

The best value of $\overline{Q}_i$ for all models is equal to 1 and is indifferent to the rating scale of each model.

Thus, for all methods included in 3M, the rating values of the alternatives are $\overline{Q}_i \in [0, 1]$, while maintaining the ordering and proportions of the original values.

Now, the synthesis of the solution is expressed as:

$$Qs_i = \sum_{j=1}^{k} \theta_j \overline{Q}_i^{(j)} \tag{31}$$

where, ϑ_j is the weight of the jth MCDM model in the selected 3M structure, j=1,..., k; Qs_i is the integral indicator of the ith alternative over k MCDM models.

The best alternative corresponds to the highest Qs_i value.

This resulting rating surpasses the Borda rating, reflecting the real proportions of ratings or the "thin" structure of relations. In contrast, the Borda ranking assigns scores in proportion to the ranking list under the same ordering, but these scores do not correspond to the actual proportions of the performance indicator of the alternatives.

4. Numerical example

4.1 Background data and methods

A numerical example is performed for a multi-criteria selection problem with a decision matrix D presented in the Table 1.

Within the framework of the 3M approach to decision-making, seven distinct aggregation

Table 1
Decision matrix D [8×5]

	Criteria: benefit(+)/cost(–)	C_1^+	C_2^+	C_3^-	C_4^+	C_5^-
Alternatives	A_1	71	4500	150	1056	478
	A_2	85	5800	145	2680	564
	A_3	76	5600	135	1230	620
	A_4	74	4200	160	1480	448
	A_5	82	6200	183	1350	615
	A_6	81	6000	173	1565	580
	A_7	80	5900	160	1650	610
	A_8	85	4700	140	1650	667

methods are employed, i.e., WSM, WPM, WASPAS, CODAS, COPRAS, TOPSIS(L2), GRA.

These methods are coupled with four normalization techniques [34]: IZ(Max,4), MS(Max,4), mIQR and Sgm(Z). Additionally, six methods for evaluating the weight of criteria, falling within the category of objective methods [32], are integrated, i.e., Entropy, CRITIC, SD using normalized decision matrix D for 4 different normalization methods.

The amalgamation of the aforementioned methods results in 7×4×6=168 MCDM models. Expanding the 3M list, an additional 6 models are derived by incorporating the VIKOR aggregation method along with the Max-Min normalization method for the decision matrix, accompanied by the same six methods for assessing criteria weights. Furthermore, the 3M list incorporates the PROMETHEE-II aggregation method paired with three distinct preference functions $H(d)$: V-Shape, Linear, Gauss. This combination, along with the same six methods for estimating criteria weights, yields an additional 18 models.

Similarly, the 3M list encompasses the ORESTE aggregation method, characterized by parameters α=0.5, β=0.05, γ=1.4 in conjunction with three different distance metrics L_1, L_2, L_∞. This configuration, along with the same six methods for estimating criteria weights, contributes 18 more models.

Consequently, the 3M approach in the presented example involves the utilization of a total of 210 models. The sequential transformations applied in normalizing the decision matrix for these methods are detailed in the Table 2.

A reader may understandably question the rationale behind such intricate transformations. Several reasons underlie this approach. Firstly, the selected normalization adheres to fundamental principles governing the normalization of multidimensional data [34-36]:

– preservation of the proportions of natural values,
– absence of prioritization among the normalized values of any criterion.

IZ(Max,4) constitutes a linear transformation that upholds the dispositions of natural attribute values while eliminating lower bound bias in the domains of normalized attribute values. This normalization proves more effective than standard Max normalization and aligns with Max-Min normalization, yet surpasses it by avoiding zero values for each attribute. The presence of null values renders some aggregation methods (WPM, WASPAS, COPRAS) impractical.

Additionally, Sum and Vec normalizations, causing shifts in both upper and lower bounds within the domains of normalized attribute values, are not recommended.

MS(Max,4) serves as a linear normalization method that preserves the dispositions of natural attribute values, as well as the equality of means and variances for all attributes. It is effective for data processing in population analysis, with the absence of criterion prioritization in the "average." Similar to Z-score but more effective for aggregation, it ensures that the range of values (0, 1] does not include negative values and zeros.

Table 2
Normalization procedure of decision matrix *D*

	method	*formulas*
1	IZ(Max,4) – IZ transform for Max where the smaller value is the median	$D=(a_{ij})$, $r_{ij}=\text{Max}(a_{ij})= a_{ij}/a_j^{max}$, $I=\text{median}(r_j^{min})$, $Z=1$ $u_{ij}=\frac{r_{ij}-r_j^{min}}{r_j^{max}-r_j^{min}}\cdot(Z-I)+I,\ \forall i=\overline{1,m};\ \forall j=\overline{1,n}$.
2	MS(Max,4) - MS transform for Max where the smaller value is the median	$D=(a_{ij})$, $u_{ij}=\frac{a_{ij}-\bar{a}_j}{s_j}$, $u1_{ij}=u_{ij}-\min_i \min_j(u_{ij})$, $u^*=\max_i \max_j(u1_{ij})$, $r_{ij}=\text{Max}(a_{ij})= a_{ij}/a_j^{max}$, $I=\text{median}(r_j^{min})$, $Z=\text{median}(r_j^{max})$, $v_{ij}=u1_{ij}\cdot(Z-I)/u^*$, $v^{out}_{ij}=v_{ij}+1-\max_i \max_j(v_{ij})$
3	mIQR[0,1] – standard linear transform with median & Interquartile Range and transform to [0,1]	$D=(a_{ij})$, $md_j=median_i(a_{ij})$, $r_{ij}=\frac{a_{ij}-md_j}{IQR_j}$ transform r_{ij} to $z_{ij}\in[0,1]$: $v_{ij}=r_{ij}-\min_i \min_j(r_{ij})$, $u_{ij}=v_{ij}/\max_i \max_j(v_{ij})$, $u_{ij}^{out}=u_{ij}+1-\max_i \max_j(u_{ij})$
4	Sgm(Z) – transform for Z-score with sigmoid function, Non-linear	$D=(a_{ij})$, $z_{ij}=\frac{a_{ij}-a_j^*}{s_j}$, $s_j=\left(\frac{1}{m}\cdot\sum_{i=1}^{m}(a_{ij}-\bar{a}_j)^2\right)^{0.5}$, $r_{ij}==\frac{1}{1+e^{-3z_{ij}}}$,

mIQR[0, 1] s akin to Z-score but utilizes a robust estimate for the bias parameter, incorporating the median and interquartile range for the scale compression parameter. The range of values (0, 1] does not include negative values and zero. For normalized values, the median values and interquartile range for all attributes are equal.

Sgm(Z) involves the conversion of Z-score values to (0, 1) using the sigmoid function (a "soft" transformation), reinforcing alternatives with a greater number of high feature values, although not necessarily the highest. The range does not encompass negative values and zero.

The domains of normalized values are presented in Figure 4.

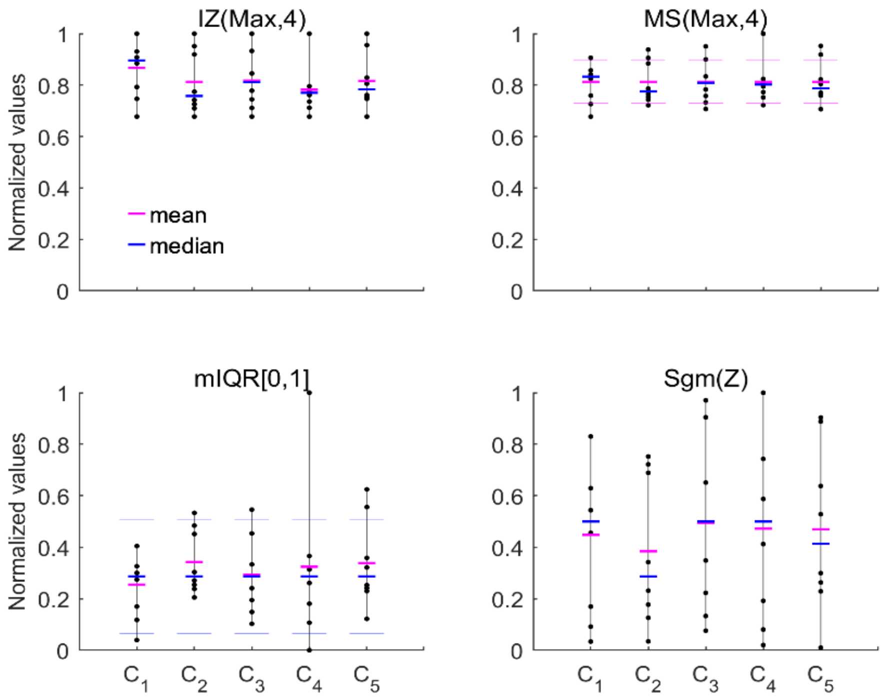

Fig. 4. Domains of normalized values of matrix *D* according to the Table 1.

For aggregation methods requiring the inversion of values for cost criteria, the universal ReS algorithm [38] is employed to invert normalized values.

Table 3 displays the values of the weighting coefficients and the methods utilized for normalizing the decision matrix.

Table 3
Weight of criteria

	method	w_1	w_2	w_3	w_4	w_5
1	SD, IZ(Max,4) -norm	0.210	0.222	0.198	0.174	0.196
2	SD, MS(Max,4) -norm	0.200	0.200	0.200	0.200	0.200
3	SD, mIQR[0, 1]-norm	0.151	0.143	0.172	0.341	0.193
4	SD, sgm(Z)-norm	0.195	0.179	0.212	0.216	0.198
5	CRITIC	0.142	0.252	0.199	0.221	0.186
6	Entropy	0.166	0.195	0.256	0.190	0.193
	Mean:	0.177	0.199	0.206	0.223	0.194
	Std:	0.0282	0.0373	0.0279	0.0599	0.0050

All six methods fall within the class of objective weighing methods [33]. It's important to note that this classification is not a reflection of the authors' preference. The presented example lacks real-world application support, precluding the use of subjective methods for comparison within AHP without the possibility of pairwise comparisons.

Of particular interest is row 2 of Table 3, where all weights are identical. This uniformity arises because, for this normalization method, the standard deviations for all attributes are equal. This example illustrates how data can be manipulated to achieve equal weights through straightforward linear transformations during normalization.

The most substantial weight variation in SD weighting methods occurs with mIQR[0,1] normalization. Any attempt to assign greater significance to a specific method is untenable. Weighting statistics demonstrate a criterion weight variation within the range of 10-15%, which is deemed acceptable.

4.2 Estimation of rating indistinguishability with variation of the decision matrix

The variation of the decision matrix *D*, including all its elements, is executed in accordance with formula (25), utilizing a given (expert) value δ_{ij}, representing the relative error in estimating the attribute a_{ij}.

For each variation of the decision matrix $a_{ij}.^{(k)}$, the efficiency indicator value $Q_i^{(k)}$, for each alternative i=1,…,m is determined. Subsequently, based on the $Q_i^{(k)}$, statistics, a statistical assessment of the standard error σ_{Q1} is conducted for the efficiency indicator of alternatives within the 1st and 2nd ratings, 2nd and 3rd ratings, and so forth. The resulting indistinguishability estimate dQ_{cr} is determined using formula (26). Table 4 provides an example of the results of assessing the critical value of indistinguishability.

With a 3% variation in the elements of the decision matrix, the average discriminability for the WSM(IZ(Max,4) model was approximately 4%. The distribution of the performance indicator *Q* for various models is illustrated in Figure 5.

Each fragment in Figure 5 illustrates the distribution of a performance measure for all alternatives within one MCDM rank model. The partial distributions for the rank 1 alternative are positioned at the bottom right of each fragment. The numerical values presented in the table inside each fragment indicate the proportion (%) of indistinguishable alternatives within the identified critical value dQ_{cr}.

Table 4
Example of critical value assessment. WSM(IZ(Max,4) model with equal weights, 1024 variations, δ_{ij}=δ=3%

rank-Ai	mean *Q*	std *Q*	d*Q*, %	dQ_{cr}, %
1-A4	0.873	0.011	-	-
2-A2	0.863	0.009	8.1	4.0
3-A6	0.829	0.013	26.6	4.3
4-A5	0.821	0.010	5.8	4.5
5-A8	0.821	0.011	0.0	4.1
6-A1	0.813	0.012	6.7	4.5
7-A7	0.805	0.012	6.1	4.7
8-A3	0.745	0.011	46.7	4.5

The results exhibit a normal distribution of the efficiency indicator *Q*, aligning with the central limit theorem in probability theory: the distribution of a sum of independent random variables subject to a uniform distribution is normal.

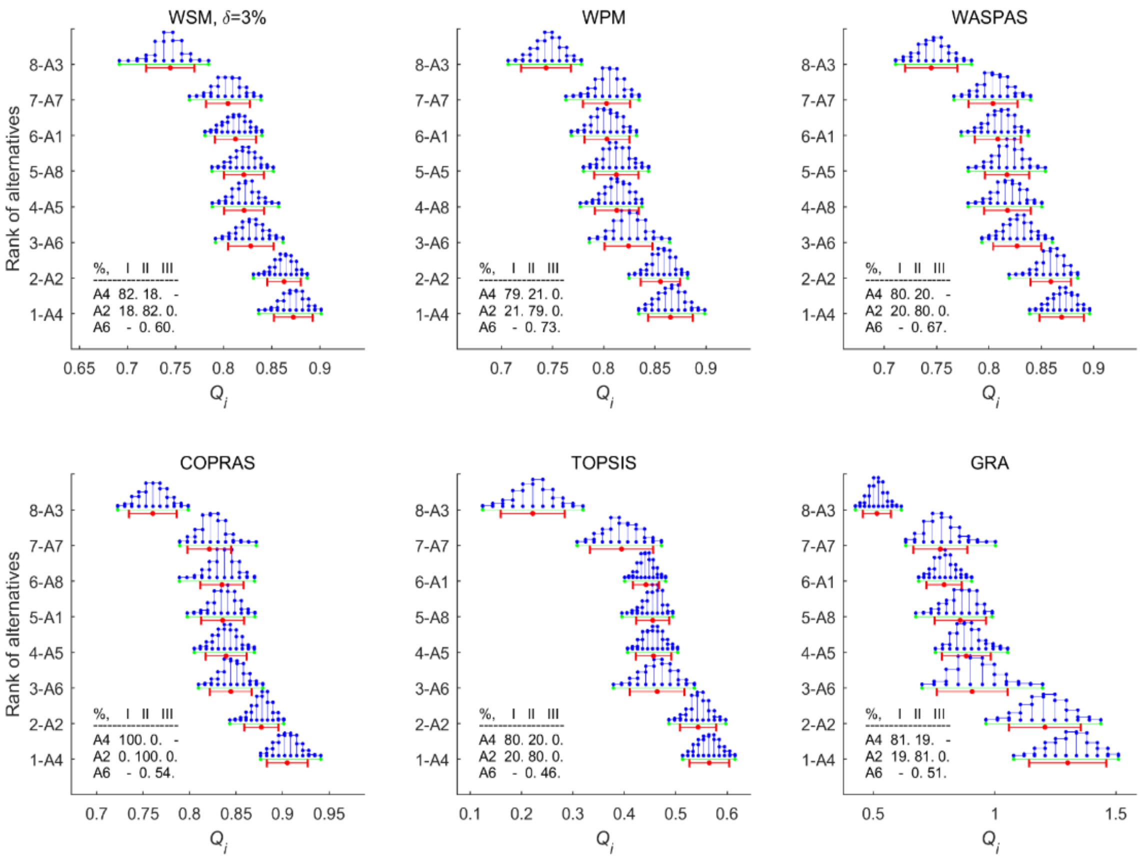

Fig. 5. Distribution of performance indicator *Q* for various models. 1024 variations, δ_{ij}=3%

4.3 Calculation results

The numerical values in Table 5 represent the number of occupied alternatives A_i (rows) for one of the 8 ranks (columns) in a series of 210 MCDM models.

Preferences, based on the criterion of the number of I, II, III, etc. ranks (left block of Table 4), are distributed as follows: A_4, A_2, A_6, A_5, and so forth Accounting for the distinctiveness of the rating (RPI) leads to increased competition among alternatives of lower ranks (right block of Table 5). For instance, Alternative A_2 occupies rank 1 in 119 models out of 210, compared to 122 for Alternative A_4. In terms of the sum of ranks 1 and 2, Alternative A_2 actually surpasses Alternative A_4.

Table 5
Ranking according to the highest score (210 models)

Rank	I	II	III	IV	V	VI	VII	VIII	∑
Without taking into account distinguishability in ratings									
A_1	0	16	13	5	22	43	98	13	210
A_2	89	92	10	1	0	2	5	11	210
A_3	18	0	0	0	0	0	14	178	210
A_4	102	77	10	2	4	0	8	7	210
A_5	0	7	20	57	52	60	13	1	210
A_6	1	16	141	22	9	17	4	0	210
A_7	0	2	2	52	51	47	56	0	210
A_8	0	0	14	71	72	41	12	0	210
∑	210	210	210	210	210	210	210	210	168

Table 5 (continued)

Rank	I	II	III	IV	V	VI	VII	VIII	∑
Distinctiveness in ratings dQ_{cr}=7%									
A_1	0	19	27	9	41	52	50	12	210
A_2	119	72	1	0	2	1	8	7	210
A_3	18	0	0	0	0	0	16	176	210
A_4	122	66	3	1	3	1	11	3	210
A_5	0	15	73	53	34	28	6	1	210
A_6	9	34	140	7	10	8	2	0	210
A_7	0	9	32	70	43	31	25	0	210
A_8	0	5	66	77	48	12	2	0	210
∑	268	220	342	217	181	133	120	199	1680

Histograms and the dynamics of the distribution of places for different values of the distinguishability criterion dQ_{cr} (set a priori) are presented in Figure 6.

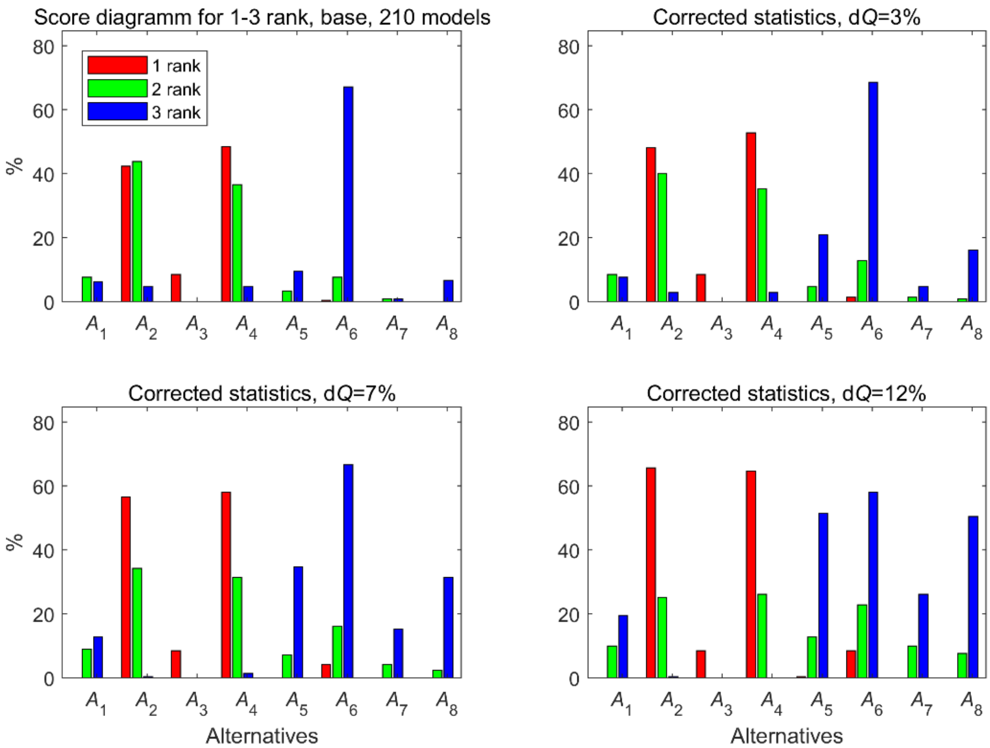

Fig. 6. Histogram of the distribution of alternative ranks in 210 models

For the problem under consideration, employing 210 ranking MCDM models, it is calculated that in 10% of cases, the relative difference in the ranking of alternatives between 1st and 2nd ranks does not exceed 3%, for 27% of models, this difference does not exceed 7%, and for 48% of models, this difference does not exceed 12%. This underscores the importance of considering errors as a necessary component when solving decision-making problems, especially in cases of sensitivity of ratings to the initial data or the choice of model components (methods).

Applying Borda's count may result in a change in rankings, as shown in Table 6, presented in two versions: Tournament counting and the Dowdall system. Unlike the results of simple statistics (Table 6), the Borda count, when accounting for distinctiveness, alters the 1st and 2nd rank alternatives (highlighted).

Table 6
Scoring without and taking into account the distinctiveness of the rating of alternatives (210 models)

	Borda count: tournament-style counting								the Dowdall system							
Rank	base		3%		7%		12%		base		3%		7%		12%	
#1	A_4	1255	A_4	1276	A_4	1294	A_2	1315	A_4	146.2	A_4	152.7	A_2	160.9	A_2	170.4
#2	A_2	1244	A_2	1263	A_2	1292	A_4	1308	A_2	142.9	A_2	150.1	A_4	159.8	A_4	165.0
#3	A_6	958	A_6	991	A_6	1037	A_6	1087	A_6	66.0	A_6	70.0	A_6	77.0	A_6	85.9
#4	A_8	679	A_8	774	A_8	848	A_8	931	A_5	46.5	A_5	51.2	A_5	56.5	A_5	63.9
#5	A_5	649	A_5	730	A_5	811	A_5	910	A_8	45.8	A_8	51.0	A_8	55.8	A_8	62.5
#6	A_7	528	A_7	593	A_7	690	A_7	779	A_3	42.0	A_7	43.2	A_7	48.1	A_7	53.8
#7	A_1	440	A_1	492	A_1	570	A_1	667	A_7	40.5	A_1	42.4	A_1	45.9	A_1	50.3
#8	A_3	127	A_3	128	A_3	128	A_3	131	A_1	40.3	A_3	42.0	A_3	42.0	A_3	42.0

The range of alternative rating values varies when using different aggregation methods, weighting coefficients, and normalization methods. Within the framework of the 210 models considered, the range of rating values is presented in Table 7.

Table 7
Value range of performance indicator of the alternatives Q_i for various aggregation methods (210 models)

Number of methods				Q_i	
Aggregation	Weight	Norm	∑	min	max
WSM	6	4	24	0.2	0.9
WPM	6	4	24	0.0	0.9
WASPAS	6	4	24	0.1	0.9
CODAS	6	4	24	0.0	6.5
COPRAS	6	4	24	0.1	1.1
TOPSIS	6	4	24	0.1	0.8
GRA	6	4	24	0.5	1.5
PROMETHEE	6	3	18	-3.8	3.0
ORESTE	6	3	18	69.5	153.0
VIKOR	6	1	6	0.0	1.0
Total models:			210		

To synthesize a solution based on ratings, the different ranges of values necessitate transforming the rating scale of each model into a single scale on the interval [0, 1] while maintaining the proportions of the original ratings. This is achieved through linear transformations (3.8-3.12). The degree of proximity, removal of the performance of various alternatives, is preserved under linear transformations, thereby maintaining the "thin" structure of relationships in the ratings of alternatives. All calculations were executed following the step-by-step algorithm described in section 3.8 and are presented in Table 8.

It is suggested that the indicated calculation of the integral rating is more effective than the Borda count, which distributes points in an even, inversely proportional, or otherwise voluntaristic manner in accordance with occupied places. Maintaining the proportions of the rating scale is considered a natural procedure.

Table 8
Rank based on rating score of alternatives (210 models)

Rank	A_i	Score	dQ_i, %	Rank	A_i	Score	dQ_i, %
#1	A_2	179.90	-	#5	A_5	110.27	0.8
#2	A_4	173.95	3.7	#6	A_7	103.78	4.0
#3	A_6	129.72	27.4	#7	A_1	95.79	5.0
#4	A_8	111.61	11.2	#8	A_3	18.72	47.8

5. Conclusions

Through the research presented in this study, the 3M multi-criteria selection approach was employed, embodying the synthesis and integration of the aggregation method, the weighing method, the normalization method of the decision matrix, and the selection of distance metrics. The acceptability of MCDM methods for resolving problems was significantly broadened, enhancing the instrumental toolkit employed in this context.

Within the investigation, the introduction and utilization of the RPI of alternatives allowed for the identification of significant differences in the ranking of alternatives obtained through a specific MCDM model. The RPI facilitated adjustments to the ranking of alternatives, exerting influence on subsequent rankings. Leveraging the RPI led to the identification of three methods for aggregating individual characteristics of alternatives with identical results: WSM, MABAC, TOPSIS(L_1) and RS eliminating the need for duplicating these methods within the 3M approach.

Given the profusion of MCDM methods, a necessity for comparative analyses emerged. This comparison was conducted utilizing two lists: ranked and rating. A method for the step-by-step linear transformation of ratings of alternatives obtained in various MCDM models was delineated, enabling the comparison and aggregation of ratings while preserving the degree of proximity. This preservation ensured the retention of the integral indicator Q_i for various alternatives during linear transformations, thereby preserving the “fine structure of relationships” of the ratings of alternatives, thereby enhancing the reliability of the results.

Directions for future research based on the 3M approach, currently lacking definitive solutions, encompass:

- Determining the MCDM methods to be incorporated into the list of methods used to address specific problems, considering both qualitative and quantitative composition;
- Establishing methodologies for comparing results obtained from different methods;
- Developing approaches for assessing the significance (weight) of methods;
- Exploring methodologies for grouping methods and formulating criteria for such grouping;
- Investigating methodologies for synthesizing solutions within the 3M framework.

Author Contributions

Conceptualization, methodology, validation, I.Z.M. and D.P.; software, visualization, writing—original draft preparation, I.Z.M.; formal analysis, writing—review and editing, D.P. All authors have read and agreed to the published version of the manuscript.

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Data availability statement

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declare that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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