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Inverse Data Envelopment Analysis Model to Improve Efficiency by Increasing Outputs

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ABSTRACT

Inverse Data Envelopment Analysis (IDEA) has gained significant attention among researchers as an analytical tool for assessing efficiency. Estimating input values while ensuring cost efficiency through changes in output quantities is complex and sensitive. In contrast to conventional data envelopment analysis methods, IDEA enables the quantification of input/output variations resulting from output/input reductions or expansions while preserving the measurement efficiency level. This paper aims to introduce a novel approach for estimating input values by incrementally increasing the value of each output of decision-making units (DMUs) during the evaluation process, thereby maintaining or improving cost efficiency. By utilizing IDEA and manipulating the output values of the DMU under evaluation, the input values are estimated while ensuring constant or enhanced cost efficiency. A simple numerical example and a case study from a Turkish automotive company are presented to validate the proposed method. The obtained results demonstrate significant improvements and hold promising prospects, indicating the potential applicability of this approach to other similar problems and research areas.

1. Introduction

In recent years, many studies have been conducted on Data Envelopment Analysis (DEA) in various fields. Data envelopment analysis as mathematical programming is a non-parametric method for evaluating the relative efficiency of a DMU, compared to other DMUs, which use homogeneous inputs to produce homogeneous outputs [1]. Charnes introduced DEA by proposing the CCR model [2]. Then the following question was raised: "If inputs (outputs) are changed in a particular DMU (increase/decrease) among a group of DMUs, and it is assumed that DMU maintains and improves its performance compared to other DMUs, to what extent will the outputs (inputs) of DMU under evaluation be changed?" IDEA, as an essential subject, can be used in practical and theoretical forms and provides answers to this question. Inverse DEA was first studied and evaluated by Zhang [3]. Wei proposed a model to respond to the following question: "If several inputs are increased in a particular

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unit among a group of DMU, and it is assumed that DMU under evaluation maintains its effectiveness in comparison with other units, to what extent has the output of the DMU under evaluation increased?"[4].

Yan discussed the IDEA with preference cone constraints by discussing related multi-objective and weighted programming problems [5]. Vencheh proposed a sufficient condition to estimate a given DMU's input/output values when some or all output/input vector values increase while maintaining the efficiency index [6]. Lertworasirikul considered the inverse BCC model for a resource allocation problem, where increasing some outputs and decreasing other outputs of the considered DMU can be considered simultaneously [7]. Behavioral objectives and input or output prices maximize revenue, minimize costs, and maximize profits. The ratio of the minimum cost to the current cost is called cost efficiency. Farrell introduced cost-effectiveness and played an essential role in developing the concept of data envelopment analysis [8]. Cost efficiency (CE), as a model of DEA, is a tool for measuring the ability of a DMU to reach the current output with minimal cost. Fukuyama used direct and indirect quasi-distance functions input to measure the output allocation efficiency [9]. Lin examined China's local economic governments by integrating DEA and the analytic hierarchy process [10]. Banihashem assessed multistage supply chains' profit efficiency, cost, and revenue in three stages [11]. Khodabakhshi investigated the equitable allocation of joint fixed-income costs through the DEA [12]. Mozafari presented cost-revenue efficiency models in DEA-R [13]. Sahoo measured cost-revenue efficiency and profit in DEA by the directional distance function method [14]. A focused approach for the reallocation of resources based on revenue performance between a set of DMUs in a centralized environment was presented by Fang [15]. Corporate cost-revenue efficiency with the revised Russell measure was estimated and analyzed by Aparicio [16].

Ghiyasi investigated the inverse data envelopment analysis based on cost and revenue efficiency with a multi-objective linear programming structure [17]. Amin introduced a new IDEA model to identify the levels of input and output inherited from the merged units to adjust the merger target in the presence of negative data [18]. Amin proposed a new goal-setting method in integration using Goal Programming (GP) and IDEA [18]. Hassanzadeh proposed a new two-model IDEA to address the problems of resource allocation and investment analysis concerning aspects of sustainable development in the presence of negative data [19]. Khoshfetrat has addressed the issue of maintaining relative efficiency values with inverse DEA with imprecise data [20]. Guijarro developed a regenerative model where all DMUs meet a predefined global efficiency level [21]. Soleimani presented a model based on the IDEA approach to maintain cost and revenue efficiency [22]. Their proposed model becomes single-objective linear programming, an advantage and a way to differentiate their method from Ghiyasi's [17] method.

Khoshfetrat used IDEA to maintain relative efficiency values with imprecise data [20] . Khoshfetrat used multi-objective linear programming (MOLP) and DEA models to measure non-discretionary performance [23]. Gatimbu analyzed the environmental performance of the tea industry in Kenya using the IDEA and regression approach [24]. Zhang proposed an integrated framework of inverse DEA called non-radial DEA based on non-radial DEA by multi-objective programming [25]. In the proposed model, Monzeli determined the efficiency of DMUs that some of their input and output components may be undesirable [26]. Moghaddas developed a network IDEA model to evaluate the performance of SSCs according to the nature of network systems [27]. Oukil assessed the potential impact of integration on optimizing energy consumption using IDEA for application in agricultural production [28]. Boubaker investigated a new approach to bank management with the help of IDEA under COVID-19 [29]. Orisaremi proposed a new application of the IDEA model to implement lean manufacturing practices in the petroleum industry [30]. Younesi presented an IDEA model based on the non-radial slacks model in the presence of uncertainty using continuous and integer interval data

[31]. Khoshandam presented an IDEA model for a two-stage network structure production system in the presence of undesirable factors [32]. Moreover, in recent years, various scholars have proposed several approaches to DEA models. For more details see [33-41].

However, very few studies have been related to inverse data coverage analysis and cost efficiency. Therefore, as a research gap in this article, a new approach is presented using inverse data envelopment analysis in calculating input values by changing output values and improving cost efficiency. This method can be used to estimate the inputs so that the cost efficiency value of the DMU under evaluation remains constant or improves. The inputs are estimated by increasing the desired outputs to maintain or improve cost efficiency using IDEA.

This article is organized as follows: In Section 2, DEA, IDEA, and cost efficiency will be introduced. Section 3 presents the proposed model (improving cost efficiency by increasing outputs), and section 4 presents numerical and practical examples. In section 5, the paper ends with a conclusion.

2. DEA, IDEA, and Cost Efficiency

2.1. DEA

Productivity is among the concepts of human interests because every wise and prudent person prefers to do their best to get the best product. Productivity can be expressed as the ratio between return (output) and input. The two main categories for measuring productivity are parametric and non-parametric methods. In parametric methods, there must be a default function, and DMUs with an output are used.

However, there is no predetermined function in the nonparametric method, and the attempt is to obtain a test function through observations. In the most critical nonparametric methods for evaluating DMUs, their performance is estimated based on linear programming. Charnes proposed a model for evaluating the efficiency of DMUs [2]. They generalized their model based on the nonparametric method Farrell raised, which could compare and evaluate DMUs with multiple homogeneous inputs and outputs [8]. The basic model of DEA posed by Charnes is as follows [2]:

$$\begin{aligned} &\min \theta \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \qquad , \qquad i = 1, \dots, m \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \qquad , \qquad r = 1, \dots, s \\ &\lambda_j \ge 0, \qquad j = 1, \dots, n \quad , \quad \theta \ free \end{aligned} \tag{1}$$

Eq. (1) is known as an input-oriented CCR model with constant returns to scale where n is the number of DMUs ($\mathrm{DMU_j}$, $\mathrm{j} = 1, \dots, \mathrm{n}$) and m different inputs $\mathrm{x_{ij}}$, $\mathrm{i} = 1, \dots, \mathrm{m}$ are used to produce s outputs $\mathrm{y_{rj}}$, $\mathrm{r} = 1, \dots, \mathrm{s}$. Eq. (1) is always feasible and has a finite optimal solution $0 < \theta^* \le 1$. If $\theta^*_o = 1$, $\mathrm{DMU_o}$ is called efficient; otherwise, it is non-efficient.

2.1. IDEA

The inverse DEA approach answers the question: "If the efficiency index θ^* remains unchanged while the outputs increase, by how much must the inputs to the DMU_o change?" To answer this question, suppose the outputs of DMU_o are increased from Y_o to $\beta_o = \mathrm{y_o} + \Delta \mathrm{y_o}$, $(\mathrm{y_o} + \Delta \mathrm{y_o} > 0)$ where the vectors $Y_o > 0$ and $Y_o \neq 0$ as a result, the input vector $\propto_o = \mathrm{x_o} + \Delta \mathrm{x_o} > 0$ is estimated, provided that the efficiency value θ^* remains. For this estimation, the following MOLP model is applied [4]:

$$\begin{aligned} &Min\ (\alpha_{1o}, \alpha_{2o}, \dots, \alpha_{mo}) \\ &s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta^* \alpha_{io} \qquad , \qquad i = 1, \dots, m \end{aligned} \tag{2}$$

$$\sum_{j=1}^{n} \lambda_j \, y_{rj} \geq \beta_{ro} \quad , \quad r = 1, \dots, s$$
$$\alpha_{io} \geq x_{io} \quad , \quad i = 1, \dots, m$$
$$\lambda_j \geq 0, \quad , \quad j = 1, \dots, n$$

2.3. Cost Efficiency

The concept of efficiency is a lack of resource wastage, optimal utilization of resources, and maximizing desired outputs. In addition to using quantitative values of inputs and outputs, cost efficiency takes advantage of the prices and cost of inputs in calculating efficiency.

Let $X_{m \times n}$, $Y_{s \times n}$ and $C_{m \times n}$ be input, output, and input cost matrices, respectively. The cost model to calculate the cost efficiency of DMU_o seeks to find a unit that consumes the least cost for purchasing inputs less than or equal to the units under evaluation to produce equal outputs to the output of the unit under evaluation. Therefore, taking into account c_{io} as the costs correspond to ith input of DMU_o, the cost model is as follows [42].

$$\begin{aligned} & min \;\; z = \sum_{i=1}^{m} c_{io} x_i \\ & s.t. \;\; \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i \quad , \quad i = 1, \dots, m \\ & \sum_{j=1}^{n} \lambda_j \, y_{rj} \geq y_{ro} \quad , \quad r = 1, \dots, s \\ & \lambda_j \geq 0 \quad , \quad j = 1, \dots, n \end{aligned} \tag{3}$$

Provided that λ^*, x^* and z^* are the optimal solution of Eq. (3), the total cost efficiency of oth unit under evaluation is defined by dividing the minimum of total costs by the observed costs:

$$E_{co} = \frac{\sum_{i=1}^{m} c_{io} \, x_i^*}{\sum_{i=1}^{m} c_{io} \;\; x_{io}} \tag{4}$$

where $0 < E_{co} \leq 1$ for $(o = 1, \dots, n)$.

Definition 1: DMU_o is total cost efficiency, if and only if $E_{co} = 1$.

3. The Proposed Model (Improving Cost Efficiency by Increasing Outputs)

The main problem of IDEA for cost efficiency is as follows:

How much is the amount of input changes of the DMU under evaluation by increasing outputs so that the cost efficiency of the DMU under evaluation is fixed or improved?

In other words, we must calculate the perturbations of input vectors by perturbing the output vectors and maintaining or improving the cost efficiency. Therefore, The main question is: what is its input value if the output value of the DMU_o , changes from y_o to $y_o + \Delta y_o$ $(y_o + \Delta y_o > 0)$ so that its cost efficiency is fixed or improves?

To answer the above problem, it is assumed that for DMU_o output changes from y_o to $y_o + \Delta y_o$ and the input changes from x_o to $x_o + \Delta x_o$ so that $\begin{pmatrix} x_o + \Delta x_o \\ y_o + \Delta y_o \end{pmatrix} > 0$. We recalculate the cost efficiency by replacing $\begin{pmatrix} x_o \\ y_o \end{pmatrix}$ in Model (3) with the vector $\begin{pmatrix} x_o + \Delta x_o \\ y_o + \Delta y_o \end{pmatrix}$. In this regard, DMU_o is ignored from the set of observations, and a new DMU_o is replaced in which the amount of the input vector $x_o + \Delta x_o > 0$ is unknown, and the output vector $y_o + \Delta y_o > 0$ is known. So, we have Eq. (5).

$$\begin{aligned} & min \quad z = \textstyle\sum_{i=1}^{m} c_{io}(x_{i0} + \Delta x_{i0}) \\ & s.t. \quad \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j \, x_{ij} + \lambda_o(x_{io} + \Delta x_{io}) \leq x_{io} + \Delta x_{io} \quad , \quad i = 1, \dots, m \end{aligned} \tag{5}$$

$$\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j\, y_{rj} + \lambda_o(y_{ro} + \Delta y_{ro}) \geq y_{ro} + \Delta y_{ro} \qquad , \qquad r = 1, \dots, s$$
$$\sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \geq z^*$$
$$x_{i0} + \Delta x_{i0} \geq 0, \qquad , \qquad i = 1, \dots, m$$
$$\lambda_j \geq 0, \quad j = 1, \dots, n$$

where z^* is the optimal solution of Eq. (3), c_{io} is the price or cost of the i_th input from the o_th DMU under evaluation and the new DMU_o' with new inputs and outputs $\begin{pmatrix} x_o+\Delta x_o \\ y_o+\Delta y_o \end{pmatrix}$ with the known input and output vector $\begin{pmatrix} x_o \\ y_o \end{pmatrix}$ Eq. (3) is replaced.

It should be noted that this model is always solvable because by choosing $\lambda_j = 0$ for $j = 1,2, \dots, n\,, j \neq o$ and $\lambda_o = 1$, this solution is always satisfactory in Eq. (5). Model (5) has an inequality constraint $\sum_{i=1}^{m} c_{io}(x_{i0} + \Delta x_{i0}) \geq z^*$ more than Eq. (3), which improves the cost efficiency value of DMU_o' is relative to DMU_o. $\lambda_j = 0$ for $j = 1,2, \dots, n\,, j \neq o$ and $\lambda_o = 1$ will be a feasible solution for Eq. (5). Therefore, in optimality $(1 - \lambda_o) \neq 0$. Eq. (5) is nonlinear, so we can reformulate Eq. (5) as follows:

$$\begin{aligned} \min \quad & z = \sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \\ s.t. \quad & \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j\, x_{ij} \leq (1 - \lambda_o)(x_{io} + \Delta x_{io}) \qquad , \qquad i = 1, \dots, m \;(1) \\ & \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j\, y_{rj} \geq (1 - \lambda_o)(y_{ro} + \Delta y_{ro}) \qquad , \qquad r = 1, \dots, s \;\;(2) \\ & \sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \geq z^* \\ & x_{io} + \Delta x_{io} \geq 0 \qquad , \qquad i = 1, \dots, m \\ & \lambda_j \geq 0 \;\;, \; j = 1, \dots, n \end{aligned} \tag{6}$$

Lemma 1 proves that the value of $(1 - \lambda_o)$ is non-negative:

Lemma 1- In constraints (6-1) and (6-2), the value of $1 - \lambda_o$ is non-negative ($(1 - \lambda_o) \geq 0$)

Proof: If $(1 - \lambda_o) < 0$, then condition (6-1) is untenable because $\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j\, x_{ij}$ the sum of the product of $\lambda_j \geq 0$ and $x_{ij} \geq 0$, which will be $\sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j\, x_{ij} \geq 0$. Now if $(1 - \lambda_o) < 0$ it will be $(1 - \lambda_o)(x_{io} + \Delta x_{io}) \leq 0$, means that condition (6-1) is impossible. Therefore, $(1 - \lambda_o) \geq 0$. The proof is completed.

Therefore, $(1 - \lambda_o) > 0$, then divide the sides of inequalities (6-1) and (6-2) by $(1 - \lambda_o)$. Then Eq. (6) changes into the following model:

$$\begin{aligned} \min \quad & z = \sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \\ s.t. \quad & \sum_{\substack{j=1\\ j\neq o}}^{n} \frac{\lambda_j}{1 - \lambda_o}\, x_{ij} \leq x_{io} + \Delta x_{io} \qquad , \qquad i = 1, \dots, m \\ & \sum_{\substack{j=1\\ j\neq o}}^{n} \frac{\lambda_j}{1 - \lambda_o}\, y_{rj} \geq y_{ro} + \Delta y_{ro} \qquad , \qquad r = 1, \dots, s \end{aligned} \tag{7}$$

$\sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \geq z^*$
$x_{io} + \Delta x_{io} \geq 0 \quad , \quad i = 1, \dots, m$
$\lambda_j \geq 0 \quad , \quad j = 1, \dots, n$

Now, by changing variable $\lambda'_j = \frac{\lambda_j}{1-\lambda_o}$ for $j = 1, \dots, n$ and $j \neq o$. For this purpose, the following model is applied.

$$
\begin{aligned}
& min \;\; z = \textstyle\sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \\
& s.t. \;\; \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda'_j \, x_{ij} \leq x_{io} + \Delta x_{io} \quad , \quad i = 1, \dots, m \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda'_j \, y_{rj} \geq y_{ro} + \Delta y_{ro} \quad , \quad r = 1, \dots, s \qquad (8) \\
& \textstyle\sum_{i=1}^{m} c_{io}(x_{io} + \Delta x_{io}) \geq z^* \\
& x_{io} + \Delta x_{io} \geq 0 \quad , \quad i = 1, \dots, m \\
& \lambda'_j \geq 0 \quad , \quad j = 1, \dots, n \;,\; j \neq o
\end{aligned}
$$

The following linear programming model is obtained by simplifying and rewriting Eq. (8). For simplicity, we apply the notation λ_j instead of λ'_j for $j = 1, \dots, n, j \neq o$ and by applying the variable change $x_i = x_{i0} + \Delta x_{i0}$ for $i = 1, \dots, m$, we have our model as:

$$
\begin{aligned}
& min \;\; z = \sum_{i=1}^{m} c_{io} \, x_i \\
& s.t. \;\; \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j \, x_{ij} - x_i \leq 0 \quad , \quad i = 1, \dots, m \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j \, y_{rj} \geq y_{ro} + \Delta y_{ro} \quad , \quad r = 1, \dots, s \qquad (9) \\
& \textstyle\sum_{i=1}^{m} c_{io} \, x_i \geq z^* \\
& x_i \geq 0 \quad , \quad i = 1, \dots, m \\
& \lambda_j \geq 0 \quad , \quad j = 1, \dots, n \;,\; j \neq o
\end{aligned}
$$

4. Numerical Experiments

This section shows the proposed method using a numerical example and a case study in a Turkish automotive company.

4.1. A Simple Numerical Example

Consider ten DMUs with two inputs ($X_{1_{old}}$ and $X_{2_{old}}$), prices (costs) of inputs (C_1 and C_2) and one output (Y), whose information is given in Table 1. The cost efficiency of each DMU is calculated using Eq. (4) and shown in the last column of Table 1.

Table 1
The data set of examples 4.1

	Inputs		The cost of each unit of input		Output	Cost efficiency
DMU_i	$X_{1_{old}}$	$X_{2_{old}}$	C_1	C_2	Y	
DMU1	6	3	8	9	10000	0.2743
DMU2	9	8	3	5	70000	0.9055
DMU3	4	7	6	9	30000	0.5862
DMU4	4	3	7	3	20000	0.6718
DMU5	5	6	9	10	10000	0.2190
DMU6	7	1	10	4	30000	0.7066
DMU7	9	4	6	2	20000	03226
DMU8	1	9	2	3	20000	0.3908
DMU9	8	2	2	7	10000	0.2333
DMU10	10	3	3	5	28000	0.5393

Table 2
Results

	New inputs		New outputs	
DMU_i	$X^*_{1_{new}}$	$X^*_{2_{new}}$	Y_{new}	New cost efficiency
DMU1	2.5714	2.2857	20000	0.5486
DMU2	17.500	2.5000	75000	0.9701
DMU3	9.3333	1.3333	40000	0.7816
DMU4	3.8571	3.4286	30000	1.0000
DMU5	2.5714	2.2857	20000	0.4381
DMU6	6.4286	5.7143	50000	1.0000
DMU7	5.1429	4.5714	40000	0.6452
DMU8	7.0000	1.0000	30000	0.5862
DMU9	3.9667	0.5667	17000	0.3967
DMU10	7.2333	1.0333	31000	0.5970

In Table 2, taking into account the new outputs Y_{new} for each DMU and applying Eq. (9), the new inputs are estimated, and then the cost efficiency of each DMU with new inputs and outputs again with Eq. (4) is calculated. The cost efficiency value of the DMU under evaluation does not deteriorate, i.e., it improves or remains constant.

This example shows that if DMU1 seeks to increase the output by 100% from 10000 to 20000, it can change the first input from 6 to 2.5714 and the second from 3 to 2.2857, improving its cost efficiency from 0.2743 to 0.5786. Also, if DMU6 wants to increase the output from 30000 to 50000, it can increase x1 from 7 to 6.4286 and x2 from 1 to 5.7143 and become a cost-effective DMU. For the rest of the DMUs, the process will be similar.

Table 1 calculates the cost efficiency of 10 DMUs with two inputs and one output. Then, for each DMU in the set of DMUs, we replace the new output (Y_{new}) with the output (Y) so that the inputs ($X^*_{1_{new}}$ and $X^*_{2_{new}}$) are estimated with the proposed model (model (9)) so that the cost efficiency does not deteriorate.

Table 2 shows the cost efficiency with estimated inputs and new outputs. In this example, the cost efficiency of all DMUs is improved after increasing the outputs and calculating the inputs with the proposed method.

4.2. An Application in the Automotive Industry

Here, real data from a Turkish automotive company that manufactures both passenger cars and light commercial vehicles is considered an application of the proposed models [43]. The performance of the new proposed model to improve cost efficiency by increasing the outputs is investigated. In 2010, the company produced 28.5% of all vehicles with 312,000 units and was recognized as the largest producer of commercial vehicles in Turkey. This Turkish company is one of the pioneers of the country's automobile industry, which exported its products to 80 countries in 2011, and the share of the total export of the automobile industry was 22.8%, with 180,698 units. The company has realized that long-term success is impossible without stakeholder collaboration. The company regularly measures customer, distributor, and supplier satisfaction and is always interested in increasing satisfaction and cooperation. In this research, DMUs are sales agents in this automobile company. (n=73) As shown in Table 3, this set of information includes seven inputs with a specific cost or price and five outputs, and the input and output factors are introduced as follows:

Input 1 (x_1): Number of branches.

Input 2 (x_2): Total number of vehicles for the fair (including branch).

Input 3 (x_3): Total number of test vehicles.

Input 4 (x_4): Total Sales Consultant.

Input 5: (x_5) Total number of sales staff.

Input 6 (x_6): Number of exhibits and activities provided by vendors.

Input 7 (x_7): The number of broadcast advertisements by multimedia.

Output 1 (y_1): The number of vehicles sold.

Output 2 (y_2): Average amount of satisfaction for all vendors determined by total customers.

Output 3 (y_3): Verification of after-sales service providers (in Turkish Lira TL).

Output 4 (y_4): Credit validation through borrowing is initiated for a client. (TL)

Output 5 (y_5): Sales Seller Confirmation on Spare Parts (TL).

Table 3 contains the data set for 73 Turkish automotive dealer vendors, including 7 inputs ($x_1, \ldots, x_7$) and 5 outputs ($y_1, \ldots, y_5$). P_i is the price or cost of inputs x_i ($i = 1, \ldots, 7$) that is shown in Table 4. We want to estimate the amount of inputs of DMUs so that the outputs of y_2, y_4 and y_5 increase by 10%, and the cost efficiency of DMUs does not deteriorate. Eq. (9) is calculated, and the results are presented in Table 5.

Table 3

The statistical description of 73 sales representatives of Turkish automotive companies

Vendors	Inputs							Outputs				
	x1	x2	x3	x4	x5	x 6	x7	y1	y2	y3	y4	y5
Minimum	1	3	1	3	13	10	21	37	65	18798	280500	63096
maximum	6	31	25	21	107	80	340	5923	97	8308527	13186358	34259621
Average	2	13	6	8	42	34	90	1133	91	2515892	3074610	5093856
Standard deviation	0.953644	5.439266	3.947862	4.584651	17.65326	15.29618	59.55592	1016.201	4.218236	1693429	2575697.4	7050819.2

Table 4
The statistical description of the price or cost of inputs (x_1, I, x_7) of the sales representative of the Turkish automotive company

Vendors	P1	p2	p3	p4	p5	p6	p7
Minimum	1,859,486	334,293	383,178	128,674	105,642	253,570	301,621
Maximum	27,206,006	7,671,397	5,590,642	1,087,982	1,099,072	537,924	648,648
Average	15,224,198	3,738,610	2,894,304	607,421	582,936	395,288	455,740
Standard deviation	7861664.9	2199490	1488718	278706.8	297002.8	84463.59	107085.6

Table 5
Results

Vendor	$x1_{new}$	$x2_{new}$	$x3_{new}$	$x4_{new}$	$x5_{new}$	$x7_{new}$	$x7_{new}$	Z_{old}	Z_{new}
Vendor1	1.51	4.36	6.91	6.40	22.82	30.93	57.26	0.85	0.90
Vendor2	1.07	4.59	3.37	4.28	29.32	19.59	49.65	0.74	0.81
Vendor3	1.09	4.96	5.81	5.11	17.89	22.97	43.71	0.80	0.87
Vendor4	1.05	8.37	8.19	10.52	49.81	36.04	95.77	0.70	0.76
Vendor5	1.10	4.92	4.31	5.51	19.99	28.60	56.56	0.76	0.82
Vendor6	1.37	7.05	5.39	6.78	27.59	22.34	88.90	0.57	0.59
Vendor7	1.13	12.94	4.95	13.10	49.18	46.62	149.93	0.72	0.78
Vendor8	1.15	9.97	3.90	10.59	46.99	25.09	62.83	0.83	0.91
Vendor9	1.34	11.13	7.06	12.54	46.86	34.21	97.89	1.00	1.00
Vendor10	1.14	11.50	4.85	11.36	44.87	43.40	141.50	1.00	1.00
Vendor11	1.44	7.27	3.06	6.39	32.28	31.30	87.29	0.52	0.55
Vendor12	3.33	4.17	3.69	8.06	36.35	25.64	56.42	0.62	0.66
Vendor13	1.77	7.98	4.29	8.01	41.81	23.94	66.68	0.75	0.79
Vendor14	1.06	5.90	5.10	5.19	19.16	15.77	45.21	1.00	1.00
Vendor15	1.14	4.11	6.16	5.54	16.52	22.15	43.20	0.66	0.70
Vendor16	1.06	8.16	4.41	4.76	26.27	30.67	65.07	0.95	1.00
Vendor17	1.11	6.62	5.55	4.47	24.19	11.13	31.33	1.00	1.00
Vendor18	1.20	15.41	6.84	18.14	66.92	52.61	167.10	0.69	0.73
Vendor19	1.29	20.58	7.61	18.40	65.72	78.93	218.54	1.00	1.00
Vendor20	1.41	11.20	6.22	13.24	54.72	44.47	131.31	0.64	0.67
Vendor21	1.47	14.13	5.52	13.41	54.71	54.55	158.68	0.92	0.97
Vendor22	1.17	6.54	1.57	4.78	23.16	28.06	60.51	0.58	0.62
Vendor23	1.45	9.60	3.35	8.28	39.11	42.15	132.24	0.83	0.87
Vendor24	1.50	8.20	2.77	7.01	36.06	36.84	75.16	0.73	0.77
Vendor25	6.04	26.33	6.08	20.36	117.51	68.92	107.11	1.00	1.00
Vendor26	0.79	3.37	2.27	3.46	14.26	19.00	27.10	0.39	0.43
Vendor27	1.39	6.63	3.56	5.38	28.10	28.58	63.80	0.67	0.73
Vendor28	1.62	7.16	5.42	6.83	32.34	33.41	79.11	0.69	0.74
Vendor29	1.09	8.53	4.51	4.01	28.35	28.79	46.29	0.65	0.69
Vendor30	2.67	10.50	4.86	11.75	66.66	47.73	146.34	0.61	0.62
Vendor31	1.10	4.89	5.16	5.07	24.36	24.65	57.76	0.66	0.73
Vendor32	1.02	6.20	2.54	4.69	22.24	27.86	55.45	0.74	0.80
Vendor33	1.09	5.61	1.34	4.43	22.92	26.60	38.18	0.86	0.94
Vendor34	1.37	6.53	11.54	4.77	37.57	28.20	77.33	0.39	0.41
Vendor35	1.24	13.58	7.57	13.28	55.10	49.72	138.13	1.00	1.00

Vendor	$x1_{new}$	$x2_{new}$	$x3_{new}$	$x4_{new}$	$x5_{new}$	$x7_{new}$	$x7_{new}$	Z_{old}	Z_{new}
Vendor36	1.68	8.26	3.00	8.63	46.05	30.33	70.19	0.75	0.78
Vendor37	1.03	8.00	4.10	6.60	27.42	32.32	86.59	0.70	0.77
Vendor38	1.05	5.33	1.14	4.25	22.17	25.48	34.90	0.93	1.00
Vendor39	1.15	3.45	6.89	5.74	14.93	27.57	43.65	0.63	0.69
Vendor40	1.15	6.25	5.37	5.68	20.24	18.14	51.64	1.00	1.00
Vendor41	1.68	5.65	5.50	6.62	32.03	32.10	80.31	0.63	0.67
Vendor42	1.23	7.02	5.59	6.52	25.04	34.56	76.40	0.87	0.93
Vendor43	1.39	8.30	4.62	7.16	37.55	34.92	96.84	0.65	0.69
Vendor44	1.02	6.51	2.57	5.01	23.27	28.94	56.76	0.98	1.00
Vendor45	1.16	6.01	3.49	5.25	22.28	22.07	44.77	0.95	1.00
Vendor46	1.09	6.73	4.29	3.74	23.18	22.55	38.09	0.67	0.72
Vendor47	1.27	8.29	4.96	9.09	43.85	36.98	88.44	0.80	0.85
Vendor48	1.17	11.77	6.18	10.64	37.31	40.31	113.26	0.78	0.84
Vendor49	1.30	18.94	6.75	25.43	89.48	63.75	232.46	1.00	1.00
Vendor50	1.11	13.26	4.64	11.44	43.65	53.48	143.40	0.88	0.91
Vendor51	1.15	8.03	2.29	6.70	30.32	35.55	81.56	0.52	0.55
Vendor52	1.43	17.21	6.26	15.07	57.79	62.05	171.05	0.87	0.94
Vendor53	1.49	19.71	6.68	26.12	91.16	76.82	248.72	1.00	1.00
Vendor54	1.45	6.72	6.09	7.23	28.83	35.99	95.02	0.74	0.78
Vendor55	1.59	22.10	7.40	18.60	66.40	73.20	208.79	1.00	1.00
Vendor56	1.39	20.91	11.00	15.81	78.70	110.39	361.47	1.00	1.00
Vendor57	1.09	7.60	4.77	4.91	24.52	26.13	65.31	0.58	0.62
Vendor58	1.84	5.46	5.67	6.06	26.52	27.60	48.35	0.93	1.00
Vendor59	2.87	6.68	4.28	8.69	41.25	33.67	80.59	0.73	0.78
Vendor60	1.56	6.22	5.63	6.22	26.87	17.74	53.32	0.87	0.93
Vendor61	1.90	16.15	21.21	12.67	76.46	76.60	264.10	1.00	1.00
Vendor62	1.45	5.63	6.54	6.96	25.83	33.78	67.42	0.77	0.82
Vendor63	1.08	8.16	6.53	9.72	36.05	23.49	63.35	1.00	1.00
Vendor64	1.08	7.29	2.02	7.14	33.98	23.67	42.16	1.00	1.00
Vendor65	1.13	9.41	4.72	4.41	31.83	30.74	55.28	0.69	0.73
Vendor66	1.40	7.74	3.96	6.04	30.80	34.19	80.06	0.61	0.65
Vendor67	1.09	6.41	2.06	5.30	25.94	27.23	47.23	0.78	0.85
Vendor68	1.66	6.70	6.21	7.72	33.09	36.83	83.61	0.63	0.66
Vendor69	1.12	6.74	2.14	5.41	26.03	30.42	55.08	0.74	0.80
Vendor70	1.46	6.35	3.93	6.19	29.15	23.41	55.08	0.62	0.66
Vendor71	1.26	3.88	6.36	5.71	19.29	27.59	48.92	0.69	0.74
Vendor72	1.14	6.46	11.10	6.31	31.84	37.11	108.95	1.00	1.00
Vendor73	1.30	4.85	5.90	5.93	21.43	24.09	58.19	0.54	0.58
Minimum	0.79	3.37	1.14	3.46	14.26	11.13	27.1	0.39	0.41
maximum	6.04	26.33	21.21	26.12	117.51	110.39	361.47	1.00	1.00
Average	1.4134247	8.89274	5.309178	8.529041	37.59233	35.93493	92.0637	0.781507	0.881507
Standard Deviation	0.6821193	4.82705	2.80383	4.884687	19.72429	17.33637	61.7816	0.164761	0.272415

When changing outputs and calculating new inputs, maintaining the cost efficiency of the unit under evaluation is very important. This is done by increasing some outputs by ten percent (output 2 (y_2): average value of satisfaction for all sellers, output 4 (y_4): loan credit confirmation and output

5 (y_5): seller sales confirmation) and calculating the values of the new inputs. It was evaluated for each of the DMUs.

With the help of solving the single-objective linear programming model of Eq. (9), the cost efficiency of DMUs under evaluation has been maintained and even improved. In Table 5, Z_{old} shows the cost efficiency by considering the inputs and outputs of Table 3, which is obtained from Eq. (4) and has a minimum of 0.39, an average of 0.781507, and a standard deviation of 0.164761, and new inputs after a 10% increase In this study, if Vendor1 seeks a 10% increase in the outputs of y_2, y_4, and y_5, it can change the vector of inputs from $X = (1,9,6,4,30,24,52)^t$ to $X = (1.51,4.36,6.91,6.40,22.82,30.93,57.26)^t$ so as to improve its cost efficiency from 0.85 to 0.90. Also, if Vendor 9 wants to increase the outputs y2, y4, and y5 by ten percent, it can change the inputs from $X = (1,8,8,10,49,36,95)^t$ to $X = (1.34,11.13,7.06,12.54,46.86,34.21,97.89)^t$ and still remain a cost-effective DMU. For the rest of the DMUs, it is processed similarly. Inputs x_i $(i = 1, \ldots, 7)$ are estimated by Eq. (9), and the cost efficiency of new data (Z_{new}) is calculated using Eq. (4) and has a minimum of 0.41, a mean of 0.881507, and a standard deviation of 0.272415. It can be clearly seen that the average cost efficiency has increased from 0.781507 to 0.881507. This shows an increase in the cost efficiency of DMUs after applying changes in some outputs and calculating new inputs. In this example, the cost efficiency of all DMUs has improved. This means that the decision-maker can predict the value of inputs when the value of some outputs increases and the unit is still cost-efficient.

5. Conclusions and Suggestions

While the cost efficiency is improved, estimating the amount of inputs is very complex and sensitive as output changes. The main contribution of this paper is presenting an alternative approach to calculating input values by changing outputs and improving cost efficiency. According to what is mentioned above, by changing the output value of the DMU under assessment, in any case, inputs can be estimated in such a way that its cost efficiency remains constant or improves. An alternative IDEA approach has been developed to achieve this goal. A numerical example and a Turkish automotive company case study have been studied to verify the model's accuracy. In particular, we have shown how to estimate the value of inputs of DMUs by increasing the value of some outputs to maintain or improve the cost efficiency of the DMU under evaluation. The results of the proposed model in the Turkish automotive company show that the cost efficiency of all DMUs has improved. In future studies, using two perspectives of fuzzy logic and network modeling in cost efficiency is suggested. In other words, what changes will be made in the original model if we use fuzzy data? Also, the primary model of this article is CCR. If we consider this model a series or parallel network, how can we generalize a model for cost efficiency? Maintaining profit efficiency with the IDEA approach will also be an exciting topic for future study. And these are interesting topics that can be addressed in the future.

Author Contributions

Research problem, M.T.Y.Sh. and A.A.; Conceptualization, M.T.Y.Sh.and A.A.; Methodology, M.T.Y.Sh.; Software, M.T.Y.Sh.; Validation, A.A, S.K., S.A.E. and M.T.Y.Sh.; Formal analysis, M.T.Y.Sh., S.A.E. and A.A.; Investigation, M.T.Y.Sh.; Resources, M.T.Y.Sh and A.A and S.K.; Data curation, M.T.Y.Sh. and A.A.; Writing, M.T.Y.Sh., A.A., S.K. and S.A.E.; Reviewing and editing, M.T.Y.Sh.; Visualization, M.T.Y.Sh., A.A. and S.A.E.; Supervision, A.A. and S.K.; All authors have read and approved the published version of the manuscript.

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Data availability statement
The datasets generated during and/or analysed during the current study are not publicly available due to the privacy-preserving nature of the data. However, they can be obtained from the corresponding author upon reasonable request.

Conflict of Interest
The authors declare no conflict of interest.

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