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PRESENTING A PRODUCTIVITY ANALYSIS MODEL FOR IRAN OIL INDUSTRIES USING MALMQUIST NETWORK ANALYSIS

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Abstract: *Organizational performance evaluation is a crucial factor in making strategic decisions for the future. To plan for economic growth, it is important to measure the efficiency and productivity of organizations. Efficiency is a key indicator for evaluating the optimal performance of economic units. Petrochemical companies are vital components of a country's economy and their operations contribute to the growth and progress of different sectors. In countries where the economy relies heavily on this industry, such as ours, petroleum is of utmost importance. Data Envelopment Analysis is a widely used method for measuring productivity. This study aims to analyze the performance evaluation relates to the supply chain of petrochemical companies using network DEA and Malmquist index. Efficiency and performance indices are calculated for each stage of the process. The study determines the indices through literature review, expert consultation, analysis, and visits to petrochemical companies. The input- and output-oriented multiplier models are used to assess overall and stage efficiencies. Using the efficiency values, the Malmquist productivity index is determined. The study examines unit productivities for the years 1395 to 1398, and the results indicate that most of the units experienced productivity growth during this period.*

Key words*: Performance evaluation, network data envelopment analysis, Malmquist productivity index.*

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1. Introduction

Data envelopment analysis (DEA) is a widely used nonparametric method to evaluate the efficiency score and used for performance evaluation of production systems or activities (Charnes et al., 1978, 1989; Banker et al., 1984; Muniz et al., 2022; Arbabi et al., 2022). DEA uses mathematical planning models to evaluate the relative efficiency of DMUs (Charnes et al., 1978). Then, Golany and Roll (1989) presented an study for the application procedure of DEA. According to the model provided by (Charnes et al., 1978) one feature in DEA technique is that it considers multiple inputs and outputs (Färe et al., 1994). The Malmquist Productivity Index (MPI) was introduced by Malmquist (1953) as a measure for analyzing input consumption. Färe and Whittaker (1995) introduced an intermediate input model of dairy production using complex survey data. Later, Färe et al. (1994) developed a DEA-based Malmquist productivity index that measures productivity change over time. The MPI can estimate the performance change between two time periods by calculating the ratio of the distance of each point from a common technology. Bosetti et al. (2005) emphasized that Malmquist DEA methods applied to panel data enable the evaluation of DMUs' dynamic performance over time. Bagherzadeh Valami and Raeinojehdehi (2016) utilized DEA for ranking school teachers. Kumar et al. (2015) utilized DEA in Telecom sectors for efficiency evaluation. Kuo and Lin (2012) used DEA technique for supplier selection. This method is useful because regions may require more than one year to achieve the output levels given by the input factors. Like a moving average approach, regions in different years are treated as if they were different DMUs. This enables comparison of the efficiency of a DMU with its own efficiency in other years and with the efficiency of other DMUs. According to Färe et al. (1994) and Coelli (1996) it is evident that using Malmquist DEA methods to panel data allows for measuring changes in productivity over time and breaking them down into changes in efficiency and technology. Note that Färe et al. (1994) defined the distance function in this productivity index consists of two components. The first one which measures the technical efficiency change index, and the second one which measures the index of technical change. Murias et al. (2006) assumed a model in which inputs and outputs are partial indicators, characterized by the maxims ‘‘the fewer the better’’ and ‘‘the more the better’’. The DEA-Malmquist index method presents an effective solution and extends the DEA model, allowing it the measurement of total factor productivity change over time for decision-making units (DMUs) (Krishnasamy et al., 2004). The DEA-Malmquist index method estimates the change in TFP between two adjacent data points by determining the ratio of the distance of each data point to a common boundary of the production possibilities, which is referred to as the Malmquist-TFP index (Färe et al., 1994). In recent studies mathematical models for calculating Malmquist index are developed to better consider the situations of real-world problems. Zhou et al. (2023) analyzed efficiency of Chinese primary healthcare institutions using the Malmquist-DEA models. Carboni and Russu (2015) presented an application of Malmquist-DEA and Self-organizing Map Neural Clustering in evaluation of Regional Wellbeing in Italy. Chaubey et al. (2022) provided a Malmquist-DEA model for efficiency and productivity evaluation of the Indian agriculture sectors.

Multi-level decision-making units are prevalent in many organizations, and evaluating their performance using data envelopment analysis requires a careful consideration of their internal relationships (Kao & Hwang, 2008; Kao & Liu, 2014). Considering multi-level and network systems then became popular and several studies introduced in this field (Ebrahimnejad et al., 2014, Shermeh et al., 2016, Fukuyama & Weber, 2010; Holod & Lewis, 2011). Various methods have been proposed for the performance evaluation of such units, with data envelopment analysis being a common one (Shahbeyk & Banihashemi, 2023). However, one of the major challenges' organizations are assessing subsystems that have cause-and-effect relationships and are influenced by time (Hsieh & Lin, 2010; Huang et al., 2014). Therefore, a dynamic model that accounts for time is necessary for an accurate assessment. One such unit that requires a performance evaluation model is the supply chain, which often involves reversible factors and multiple levels. The evaluation of organizational efficiency and productivity is crucial for making strategic decisions in network systems (Chaubey et al., 2022; Lee & Worthington, 2016; Li et al., 2014a). Thus, this holds true for petrochemical companies, which are fundamental to a country's economy. In this investigation, we propose a performance evaluation model for the petroleum supply chain (Mahmoudi et al., 2021) and measure productivity using the network-based Malmquist index.

The proposed model incorporates reversible relations and is entirely unique. Furthermore, the use of the network-based Malmquist productivity relations with a network structure that contains reversible relations is a novel approach. Returned intermediate products are considered in DEA-network mathematical modelling.

This study also demonstrates the effectiveness of the model by applying it to a case study from the petroleum industry.

In the following sections, we provide an introduction to data envelopment analysis and supply chain, explain the modeling procedure of the data envelopment analysis for productivity evaluation using aggregate and total efficiencies in the petroleum industry's supply chain, present a practical example, and offer our conclusions and recommendations in Section 5.

2. Introduction to data envelopment analysis (DEA) and supply chain

In the following subsections DEA technique, network DEA models, and Malmquist index are briefly reviewed.

2.1. DEA Concept and Principles

Assume we have n DMUs, and each DMU_j (j=1,…,n) produces s outputs Y_{rj}(r=1,…,s) using m inputs X_{ij}(i=1,…,m). DEA calculates the performance evaluation for DMU_o as follows. Model (1) is known as the input-oriented CCR in the envelopment form.

$$\begin{aligned} & Min \quad \theta \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta\, x_{io} \ , \ i = 1,2,\cdots,m \\ & \sum_{j=1}^{n} \lambda_j x_{rj} \ge y_{ro} \, , \quad r = 1,2,\cdots,s \\ & \lambda_j \ge 0 \ , \ j = 1,2,\cdots,n \end{aligned} \tag{1}$$

Please note that the model is always feasible and $0 < \theta^* \le 1$. If $\theta^* = 1$, then DMU_o is efficient and otherwise inefficient. The dual envelopment form which is known as the multiplier form is as follows:

$$
\begin{aligned}
& Max \ \sum_{r=1}^{s} u_r y_{r\circ} \\
& s.t. \ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{s} v_i x_{ij} 0 \leq, j = 1, \cdots, n \\
& \quad \sum_{i=1}^{m} v_i x_{i\circ} = 1 \\
& u_r \geq \circ \quad , \quad r = 1, \cdots, s \\
& v_i \geq \circ \quad , \quad i = 1, \cdots, m
\end{aligned} \tag{2}
$$

2.2. Network data envelopment analysis

The network data envelopment analysis (DEA) approach in conventional DEA models treats decision-making units as a "black box" and disregards their internal structure (Li et al., 2014b). To address this issue and to accurately calculate efficiency, Färe (1991) and Färe and Grosskopf (2000) introduced the concept of network DEA. They argued that conventional DEA models overlook the organizational processes of decision-making units in their evaluations, treating them as a black box where inputs are transformed into outputs without considering the inner workings of these units. However, improving performance requires assessing different organizational processes at various levels and distinguishing successful and unsuccessful units (Färe & Groskopf, 2000).

In conventional DEA models, two common approaches are used to measure the efficiency of multi-stage organizations. The first approach is aggregation (black box), where various sections are combined and treated as a single company. This approach neglects the connection between internal activities and cannot calculate the effect of the efficiency of each unit on the efficiency of the entire organization. Additionally, there is a possibility of choosing inappropriate inputs and outputs and making unreasonable evaluations of decision-making units.

The second approach is separation, where each unit's efficiency is evaluated individually. This method enables the calculation of the efficiency of each unit within the company among different decision-making units. However, this approach does not maintain the connection between different stages, as shown in Figure 2.

Consider a two-stage system with inputs of DMUs, outputs of DMUs, and interconnection between subunits (Z). The first subunit's outputs are the second subunit's inputs. The second subunit does not consume any exogenous input, and the first subunit does not produce any exogenous output. In 2008, Kao and Hwang proposed a model to evaluate the efficiency of a two-stage decision-making unit, as shown in Figure 3.

$$
\begin{aligned}
& E_k^s = max \ {\sum_{r=1}^{s} u_r y_{rk}} \Big/ {\sum_{i=1}^{m} v_i x_{ik}} \\
& s.t{:} \ {\sum_{r=1}^{s} u_r y_{rk}} \Big/ {\sum_{i=1}^{m} v_i x_{ik}} \leq 1. \ j = 1. \dots . n \\
& \quad {\sum_{p=1}^{q} w_p z_{pj}} \Big/ {\sum_{i=1}^{m} v_i x_{ij}} \leq 1. \ j = 1. \dots . n
\end{aligned} \tag{3}
$$

$$\sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{p=1}^{q} w_p z_{pj} \leq 1.\ j = 1.\ldots.n$$

$$u_r.v_i.w_p \geq \varepsilon\ r = 1.\ldots.s.i = 1.\ldots.m.p = 1.\ldots.q$$

2.3. Malmquist productivity index

Malmquist, a Swedish economist, introduced the Malmquist living standard index in 1953 (Malmquist, 1953). Caves et al. (1982) included this index for the first time in the production theory. They proposed an extension of the production technology change in the cases of multiple inputs and outputs. Färe et al. (1994) utilized data envelopment analysis techniques to compute the Malmquist index. They divided the index into two factors: efficiency change and technology change. Using linear programming techniques and data envelopment analysis, Färe established a suitable method for evaluating the empirical production function for multiple inputs and outputs. In data envelopment analysis, the optimal efficient frontier is obtained using a set of decision-making units without prioritizing inputs and outputs. The decision-making units on the efficient frontier have the highest level of output or the lowest level of input. The Malmquist productivity index combines efficiency change and technology change. The Malmquist productivity index can be calculated by using the following distance functions, using the efficiency obtained by data envelopment analysis models.

3. Malmquist productivity index of the petroleum supply chain

The objective of this section is to introduce a network-DEA model based on a real case study that incorporates returned products in a network and considers intermediate products for evaluating the progress and regression of supply chains in the oil industry. Färe developed a suitable method for evaluating the empirical production function for multi-input and multi-output cases using linear programming techniques and data envelopment analysis. In data envelopment analysis, the optimal efficient frontier is obtained without prioritizing inputs and outputs, using a set of decision-making units. The decision-making units on the efficient frontier have the highest level of output or the lowest level of input. The Malmquist productivity index combines efficiency change and technology change. The Malmquist productivity index can be calculated using the following distance functions or other similar functions:

$$D(X_p, Y_p) = inf\{\theta/(\theta X_{p,} Y_p) \in PPS\} \tag{4}$$

In some exceptional cases, the above relations only indicate changes in the efficient frontier at time t+1 in comparison to time t, and hence, they cannot be an accurate measure for calculating technology changes. Moreover, this method overlooks the changes in efficiency. If $D^k(X^k, Y^k) = 1$, it is assumed that the kth unit is efficient; however, this function does not determine the inefficiency score's distance. Due to the inefficiency and nonlinearity of the technology frontier, Färe divided the productivity index into two factors. Using DEA techniques, the efficient frontier is determined for DMUs. The production function at times t and t+1 is given, and to calculate the Malmquist index, four linear programming problems must be solved:

$$
\begin{aligned}
& D^t(X_p^t, Y_p^t) = \min \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{ip}^t \quad i = 1 \dots m \\
& \sum_{j=1}^{n} \lambda_j \, y_{rj}^t \ge y_{rp}^t \quad r = 1 \dots s \\
& \lambda_j \ge 0 \quad j = 1 \dots n
\end{aligned}
\tag{5}
$$

where x_{ip}^t is the i^{th} input and y_{rp}^t is the r^{th} output of the DMU_p at time t. The efficiency score $D^t(X^t, Y^t) = \theta$ denotes how much the DMU_p input can be reduced to produce the same output. Instead of time t, the CCR problem is solved for time t+1 and $D^{t+1}(X^{t+1}, Y^{t+1})$, which is the technical efficiency of DMU_p at time t+1. $D^t(X^{t+1}, Y^{t+1})$ for DMU_p, which is the distance of DMU_p at time t+1 to frontier t, is obtained by the following linear programming problem:

$$
\begin{aligned}
& D^t(X_p^{t+1}, Y_p^{t+1}) = \min \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{ip}^{t+1} \quad i = 1 \dots m \\
& \sum_{j=1}^{n} \lambda_j \, y_{rj}^t \ge y_{rp}^{t+1} \quad r = 1 \dots s \\
& \lambda_j \ge 0 \quad j = 1 \dots n
\end{aligned}
\tag{6}
$$

$D^{t+1}(X^t, Y^t)$, DMU_p distance with t coordination to t+1 efficient frontier, is similarly calculated. To calculate the input-oriented Malmquist productivity index, this value needs to be the solution to the following linear programming problem:

$$
\begin{aligned}
& D^{t+1}(X_p^t, Y_p^t) = \min \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta x_{ip}^t \quad i = 1 \dots m \\
& \sum_{j=1}^{n} \lambda_j \, y_{rj}^{t+1} \ge y_{rp}^t \quad r = 1 \dots s \\
& \lambda_j \ge 0 \quad j = 1 \dots n
\end{aligned}
\tag{7}
$$

If one can assume $D^t(X^t, Y^t)$ and $D^{t+1}(X^{t+1}, Y^{t+1})$ must be one to be efficient, then the relative efficiency change can be defined as:

$$TEC_p = \frac{D_p^{t+1}(x_p^{t+1}, y_p^{t+1})}{D_p^t(x_p^t, y_p^t)}$$

The positive shift in a portion of the frontier occurs only if this portion, at time t+1 compared to the corresponding point at time t, expands the set of production capabilities. Conversely, a negative shift in a portion of the frontier occurs only if this portion, at time t+1 compared to the corresponding point at time t, reduces the production possibility set and moves inside. Färe proposed a geometrical combination to define the technology change between times t and t+1:

$$FS_p = \sqrt{\frac{D_p^t(x_p^{t+1}, y_p^{t+1})}{D_p^{t+1}(x_p^{t+1}, y_p^{t+1})} \cdot \frac{D_p^t(x_p^t, y_p^t)}{D_p^{t+1}(x_p^t y_p^t)}}$$

The following three situations will happen for the technology change index:

- $FS_p>1$, the frontier has a positive shift, or in other words, there is progression.
- $FS_p<1$, the frontier has a negative shift, or in other words, there is regression.
- $FS_p=1$ denotes there is no need for shift, or the frontier does not change.

The input-oriented Malmquist productivity index for each DMU_p at times t and t+1 is obtained by the product of efficiency change and technology change as the following relation:

$$M_p = \frac{D_p^{t+1}(x_p^{t+1}, y_p^{t+1})}{D_p^t(x_p^t, y_p^t)} * \sqrt{\frac{D_p^t(x_p^{t+1}, y_p^{t+1})}{D_p^{t+1}(x_p^{t+1}, y_p^{t+1})} \cdot \frac{D_p^t(x_p^t, y_p^t)}{D_p^{t+1}(x_p^t, y_p^t)}}$$

and if one simplifies the above relation, M_p will be

$$M_p = \sqrt{\frac{D_p^t(x_p^{t+1}, y_p^{t+1})}{D_p^t(x_p^t, y_p^t)} \cdot \frac{D_p^{t+1}(x_p^{t+1}, y_p^{t+1})}{D_p^{t+1}(x_p^t, y_p^t)}}$$

The Malmquist productivity index is defined as a convex geometrical combination because it exposes even the slightest inefficiencies and any change in each efficiency will have an impact on the index. There are three possible scenarios:

- $Mp>1$ denotes the increase in productivity, and there is progression.
- $Mp<1$ denotes the decrease in productivity, and there is regression.
- $Mp=1$ denotes no change in productivity for times t and t+1.

To calculate productivity using the Malmquist index, we will use the concepts and formulations presented in the introduction section, as well as those mentioned above. The following relations will be used for the evaluation:

$$M_o^{t1} = \left[\frac{D_o^{t1}(x_o^{t2}, y_o^{t2})}{D_o^{t1}(x_o^{t1}, y_o^{t1})}\right], M_o^{t2} = \left[\frac{D_o^{t2}(x_o^{t2}, y_o^{t2})}{D_o^{t2}(x_o^{t1}, y_o^{t1})}\right]$$

$$M_o = \left[\frac{D_o^{t1}(x_o^{t2}, y_o^{t2})}{D_o^{t1}(x_o^{t1}, y_o^{t1})} \times \frac{D_o^{t2}(x_o^{t2}, y_o^{t2})}{D_o^{t2}(x_o^{t1}, y_o^{t1})}\right]^{\frac{1}{2}}$$

$$M_o = \frac{D_o^{t2}(x_o^{t2}, y_o^{t2})}{D_o^{t1}(x_o^{t1}, y_o^{t1})}\left[\frac{D_o^{t1}(x_o^{t2}, y_o^{t2})}{D_o^{t2}(x_o^{t2}, y_o^{t2})} \times \frac{D_o^{t1}(x_o^{t1}, y_o^{t1})}{D_o^{t2}(x_o^{t1}, y_o^{t1})}\right]^{\frac{1}{2}}$$

M_0 represents the Malmquist productivity index that can be broken down into two factors: technology change and efficient frontier change.

$$TEC_o = \frac{D_o^{t2}(x_o^{t2}, y_o^{t2})}{D_o^{t1}(x_o^{t1}, y_o^{t1})}$$

$$FS_o = \left[\frac{D_o^{t1}(x_o^{t2}, y_o^{t2})}{D_o^{t2}(x_o^{t2}, y_o^{t2})} \times \frac{D_o^{t1}(x_o^{t1}, y_o^{t1})}{D_o^{t2}(x_o^{t1}, y_o^{t1})}\right]^{\frac{1}{2}}$$

Calculating the Malmquist productivity index for an output-oriented system with multiplier form:

Initially, we want to obtain the Malmquist productivity index in an output-oriented model for the whole system and the three levels of the above figure. Consider the following models for evaluating the efficiency of the D_o system at times t1 and t2. We have to obtain four efficiency scores proportional to the above discussion.

A: In the following model, both the unit under evaluation at time t1 and the efficient frontier at time t1 are considered:

$$\begin{aligned}
D_o^{t_1}(x_o^{t_1}, y_o^{t_1}) = Min \sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + k_1 y_{1o}^{t_1} + \sum_{f=1}^{2} w_f z_{fo}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} \\
+ \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} + k_2 y_{2o}^{t_1} \\
s.t.\, u_1 d_{1o}^{t_1} + u_2 d_{2o}^{t_1} = 1 \\
\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \ge 0, \quad j = 1,\dots,n \\
\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \ge 0, \quad j = 1,\dots,n \\
\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_2 d_{2j}^{t_1} + t_0^3 \ge 0, \quad j = 1,\dots,n \\
\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \ge 0, \quad j = 1,\dots,n \\
\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} + \sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_1 d_{1j}^{t_1} \\
- u_2 d_{2j}^{t_1} \ge 0, \quad j = 1,\dots,n \\
k_r \ge 0, v_i^1 \ge 0, v_l^2 \ge 0, v_p^3 \ge 0, w_f \ge 0, u_s \ge 0, \quad \forall r,i,l,p,f,s \\
t_0^1, t_0^2, t_0^3, t\,Free
\end{aligned} \tag{8}$$

The optimal solution to the above model denotes the aggregate efficiency of the unit under evaluation at time t1.

B: In the following model, both the unit under evaluation at time t2 and the efficient frontier at time t2 are considered.

$$\begin{aligned}
D_o^{t_2}(x_o^{t_2}, y_o^{t_2}) = Min \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + k_1 y_{1o}^{t_2} + \sum_{f=1}^{2} w_f z_{fo}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} \\
+ k_2 y_{2o}^{t_2} \\
s.t.\, u_1 d_{1o}^{t_2} + u_2 d_{2o}^{t_2} = 1 \\
\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} - \sum_{f=1}^{2} w_f z_{fj}^{t_2} - u_1 d_{1j}^{t_2} + t_0^1 \ge 0, \quad j = 1,\dots,n \\
\sum_{f=1}^{2} w_f z_{fj}^{t_2} - \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \ge 0, \quad j = 1,\dots,n \\
\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \ge 0, \quad j = 1,\dots,n
\end{aligned} \tag{9}$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \;\; j = 1,\dots,n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} + \sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_1 d_{1j}^{t_2} - u_2 d_{2j}^{t_2} \geq 0, \;\; j = 1,\dots,n$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

The optimal solution to the above model denotes the aggregate efficiency of the unit under evaluation at time t2.

C: In the above model, the unit under evaluation at time t2 and the efficient frontier at time t1 are considered.

$$D_o^{t_1}(x_o^{t_2}, y_o^{t_2}) = \text{Min} \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + k_1 y_{1o}^{t_2} + \sum_{f=1}^{2} w_f z_{fo}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} + k_2 y_{2o}^{t_2}$$

$$\text{s.t.}\; u_1 d_{1o}^{t_2} + u_2 d_{2o}^{t_2} = 1 \tag{10}$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1,\dots,n$$

$$\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1,\dots,n$$

$$\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_2 d_{2j}^{t_1} + t_0^3 \geq 0, \quad j = 1,\dots,n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \geq 0, \;\; j = 1,\dots,n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} + \sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_1 d_{1j}^{t_2} - u_2 d_{2j}^{t_2} \geq 0, \;\; j = 1,\dots,n$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

As the aggregate efficiency of the unit under evaluation at time t2 and the efficient frontier at time t1 are considered, the optimal solution to the objective function of the above model does not represent efficiency.

D: In the above model, the unit under evaluation at time t1 and the efficient frontier at time t2 are considered.

$$D_o^{t_2}(x_o^{t_1},y_o^{t_1}) = Min\sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + k_1 y_{1o}^{t_1} + \sum_{f=1}^{2} w_f z_{fo}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} + k_2 y_{2o}^{t_1}$$

$$\begin{aligned}
& s.t.\, u_1 d_{1o}^{t_1} + u_2 d_{2o}^{t_1} = 1 \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} - \sum_{f=1}^{2} w_f z_{fj}^{t_2} - u_1 d_{1j}^{t_2} + t_0^1 \geq 0, \quad j=1,\dots,n \\
& \sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} - \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \geq 0, \quad j=1,\dots,n \\
& \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \geq 0, \quad j=1,\dots,n \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \quad j=1,\dots,n \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} + \sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_1 d_{1j}^{t_1} - u_2 d_{2j}^{t_1} \geq 0, \quad j=1,\dots,n \\
& k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s \\
& t_0^1, t_0^2, t_0^3\, Free
\end{aligned} \tag{11}$$

As the aggregate efficiency of the unit under evaluation at time t1 and the efficient frontier at time t2 are considered, the optimal solution to the objective function of the above model does not represent efficiency.

Therefore, the Malmquist productivity index is obtained by the following relation using aggregate efficiency in a network:

$$M_o = \frac{D_o^{t2}(x_o^{t2},y_o^{t2})}{D_o^{t1}(x_o^{t1},y_o^{t1})}\left[\frac{D_o^{t1}(x_o^{t2},y_o^{t2})}{D_o^{t2}(x_o^{t2},y_o^{t2})} \times \frac{D_o^{t1}(x_o^{t1},y_o^{t1})}{D_o^{t2}(x_o^{t1},y_o^{t1})}\right]^{\frac{1}{2}}$$

Mo larger than, smaller than, or equal to one, respectively, denote growth, reduction, and no change in the efficiency of the unit, considering the aggregate efficiency. TEC larger than, smaller than, or equal to one denotes growth, reduction, and no change in the aggregate technical efficiency of the unit, respectively. Finally, FS greater than, smaller than, or equal to one denotes technological progress (approaching the frontier to the center or enlarging the production possibility set) and no change in the technology change or efficient frontier with time.

Now, considering different values for the two parts of FSo, the following cases can be examined:

Case A: $\frac{D_o^{t1}(x_o^{t1},y_o^{t1})}{D_o^{t2}(x_o^{t1},y_o^{t1})} > 1$ and $\frac{D_o^{t1}(x_o^{t2},y_o^{t2})}{D_o^{t2}(x_o^{t2},y_o^{t2})} > 1$

In this case, DMUo at both times is located in the part of the production possibility set where the highest technological progress occurred, and the PPS frontier has a positive shift, and FSo>1. This is the best case.

Case B: $\frac{D_o^{t1}(x_o^{t1},y_o^{t1})}{D_o^{t2}(x_o^{t1},y_o^{t1})} < 1$ and $\frac{D_o^{t1}(x_o^{t2},y_o^{t2})}{D_o^{t2}(x_o^{t2},y_o^{t2})} < 1$

In this case, DMUo at both times is located in the part of the production possibility set where the highest technological regress occurred, the PPS frontier has a negative shift, and FSo<1. This is the worst case.

Case C: $\frac{D_o^{t1}(x_o^{t1},y_o^{t1})}{D_o^{t2}(x_o^{t1},y_o^{t1})} < 1$ and $\frac{D_o^{t1}(x_o^{t2},y_o^{t2})}{D_o^{t2}(x_o^{t2},y_o^{t2})} > 1$

In this case, it is possible FSo<1 or FSo>1, and it can be concluded that DMUo moved from the part of PPS with technological regress to the part with technological progress, and the strategy change of DMUo in using different inputs was useful. Now, if the technological progress in time t2 compensates for the regress that occurred in time t1, FSo>1, otherwise FSo<1.

Case D: $\frac{D_o^{t1}(x_o^{t1},y_o^{t1})}{D_o^{t2}(x_o^{t1},y_o^{t1})} > 1$ and $\frac{D_o^{t1}(x_o^{t2},y_o^{t2})}{D_o^{t2}(x_o^{t2},y_o^{t2})} < 1$

In this case, it is possible FSo<1 or FSo>1, and it can be concluded that DMUo moved from the part of PPS with technological progress to the part with technological regress, and the strategy change of DMUo in using different inputs was not useful.

Now consider formulation for the Malmquist productivity index for an input-oriented system with multiplier form.

According to the above relations and similar to the output-oriented concept, the Malmquist productivity index can be defined with respect to the efficiency of the whole network. Therefore, the following four models are considered for different times:

A: In the following model, both the unit under evaluation and the efficient frontier is considered at time t_1.

$$\bar{D}_o^{t_1}\left(x_o^{t_1}, y_o^{t_1}\right) = Min \sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} \tag{12}$$

$$s.t. \sum_{r=1}^{2} u_r d_{ro}^{t_1} = 1$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1,\dots,n$$

$$\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1,\dots,n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \geq 0, \quad j = 1,\dots,n$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

The optimal solution to the above model represents the total efficiency of the unit under evaluation at time t_1.

B: In the following model, both the unit under evaluation and the efficient frontier is considered at time t_2.

$$\bar{D}_o^{t_2}(x_o^{t_2}, y_o^{t_2}) = Min \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2}$$
$$s.t. \sum_{r=1}^{2} u_r d_{ro}^{t_2} = 1$$
$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} - \sum_{f=1}^{2} w_f z_{fj}^{t_2} - u_1 d_{1j}^{t_2} + t_0^1 \geq 0, \quad j = 1, \dots, n$$
$$\sum_{f=1}^{2} w_f z_{fj}^{t_2} - \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \geq 0, \quad j = 1, \dots, n \tag{13}$$
$$\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \geq 0, \quad j = 1, \dots, n$$
$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \quad j = 1, \dots, n$$
$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$
$$t_0^1, t_0^2, t_0^3 Free$$

The optimal solution to the above model represents the total efficiency of the unit under evaluation at time t_2.

C: In the following model, the unit under evaluation and the efficient frontier are considered at times t_2 and t_1, respectively.

$$D_o^{t_1}(x_o^{t_2}, y_o^{t_2}) = Min \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2}$$
$$s.t. \sum_{r=1}^{2} u_r d_{ro}^{t_2} = 1$$
$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1, \dots, n$$
$$\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1, \dots, n \tag{14}$$
$$\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_2 d_{2j}^{t_1} + t_0^3 \geq 0, \quad j = 1, \dots, n$$
$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \geq 0, \quad j = 1, \dots, n$$
$$\sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} - \sum_{r=1}^{2} u_r d_{ro}^{t_2} \geq 0,$$
$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$
$$t_0^1, t_0^2, t_0^3 Free$$

As the total efficiency of the unit under evaluation and the efficient frontier are considered at times t_2 and t_1, respectively, the solution to the objective function of the above model does not represent efficiency.

D: In the following model, the unit under evaluation and the efficient frontier are considered at times t_1 and t_2, respectively.

$$D_o^{t_2}\left(x_o^{t_1}, y_o^{t_1}\right) = Min \sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1}$$

$$s.t. \sum_{r=1}^{2} u_r d_{ro}^{t_1} = 1 \tag{15}$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} - \sum_{f=1}^{2} w_f z_{fj}^{t_2} - u_1 d_{1j}^{t_2} + t_0^1 \geq 0, \qquad j = 1, \ldots, n$$

$$\sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} - \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \geq 0, \qquad j = 1, \ldots, n$$

$$\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \geq 0, \qquad j = 1, \ldots, n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \qquad j = 1, \ldots, n$$

$$\sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} - \sum_{r=1}^{2} u_r d_{ro}^{t_1} \geq 0,$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \qquad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

As the total efficiency of the unit under evaluation and the efficient frontier are considered at times t_1 and t_2, respectively, the solution to the objective function of the above model does not represent efficiency.

Consequently, the Malmquist productivity index in terms of total efficiency in a network is calculated by the following relation.

$$\bar{M}_o = \frac{\bar{D}_o^{t2}(x_o^{t2}, y_o^{t2})}{\bar{D}_o^{t1}(x_o^{t1}, y_o^{t1})} \left[\frac{\bar{D}_o^{t1}(x_o^{t2}, y_o^{t2})}{\bar{D}_o^{t2}(x_o^{t2}, y_o^{t2})} \times \frac{\bar{D}_o^{t1}(x_o^{t1}, y_o^{t1})}{\bar{D}_o^{t2}(x_o^{t1}, y_o^{t1})} \right]^{\frac{1}{2}}$$

M_o larger than, smaller than, or equal to one, respectively, denote growth, reduction, and no change in the efficiency of the unit, considering the total efficiency. TEC larger than, smaller than, or equal to one denotes growth, reduction, and no change in the total technical efficiency of the unit, respectively. Finally, FS greater than, smaller than, or equal to one denotes technological progress (approaching the frontier to the center or enlarging the production possibility set) and no change in the technology change or efficient frontier with time.

Now, considering different values for the two parts of FS_o, the following cases can be examined:

Case A: $\frac{\bar{D}_o^{t1}(x_o^{t1},y_o^{t1})}{\bar{D}_o^{t2}(x_o^{t1},y_o^{t1})} > 1$ and $\frac{\bar{D}_o^{t1}(x_o^{t2},y_o^{t2})}{\bar{D}_o^{t2}(x_o^{t2},y_o^{t2})} > 1$

In this case, DMU_o at both times is located in the part of the production possibility set where the highest technological progress occurred, the PPS frontier has a positive shift, and $FS_o>1$. This is the best case.

Case B: $\frac{\bar{D}_o^{t1}(x_o^{t1},y_o^{t1})}{\bar{D}_o^{t2}(x_o^{t1},y_o^{t1})} < 1$ and $\frac{\bar{D}_o^{t1}(x_o^{t2},y_o^{t2})}{\bar{D}_o^{t2}(x_o^{t2},y_o^{t2})} < 1$

In this case, DMU_o at both times is located in the part of the production possibility set where the highest technological regress occurred, the PPS frontier has a negative shift, and $FS_o<1$. This is the worst case.

Case C: $\frac{\bar{D}_o^{t1}(x_o^{t1},y_o^{t1})}{\bar{D}_o^{t2}(x_o^{t1},y_o^{t1})} < 1$ and $\frac{\bar{D}_o^{t1}(x_o^{t2},y_o^{t2})}{\bar{D}_o^{t2}(x_o^{t2},y_o^{t2})} > 1$

In this case, it is possible $FS_o<1$ or $FS_o>1$, and it can be concluded that DMU_o moved from the part of PPS with technological regress to the part with technological progress, and the strategy change of DMU_o in using different inputs was useful. Now, if the technological progress in time t^2 compensates for the regress that occurred in time t^1, $FS_o>1$, otherwise $FS_o<1$.

Case D: $\frac{\bar{D}_o^{t1}(x_o^{t1},y_o^{t1})}{\bar{D}_o^{t2}(x_o^{t1},y_o^{t1})} > 1$ and $\frac{\bar{D}_o^{t1}(x_o^{t2},y_o^{t2})}{\bar{D}_o^{t2}(x_o^{t2},y_o^{t2})} < 1$

In this case, it is possible $FS_o<1$ or $FS_o>1$, and it can be concluded that DMU_o moved from the part of PPS with technological progress to the part with technological regress, and the strategy change of DMU_o in using different inputs was not useful.

Consider the following models for obtaining Do at times t1 and t2. Please note that the evaluating model is output-oriented.

A: In the following model, both the unit under evaluation and the efficient frontier is considered at time t_1.

$$
\begin{aligned}
& G_o^{t_1}\left(x_o^{t_1}, y_o^{t_1}\right) = Max u_1 d_{1o}^{t_1} + u_2 d_{2o}^{t_1} \\
s.t. & \sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + k_1 y_{1o}^{t_1} + \sum_{f=1}^{2} w_f z_{fo}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} + k_2 y_{2o}^{t_1} = 1 \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1,\dots,n \\
& \sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1,\dots,n \\
& \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_2 d_{2j}^{t_1} + t_0^3 \geq 0, \quad j = 1,\dots,n \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} + \sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_1 d_{1j}^{t_1} \\
& \quad - u_2 d_{2j}^{t_1} \geq 0, \quad j = 1,\dots,n
\end{aligned}
\tag{16}
$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$
$$t_0^1, t_0^2, t_0^3, t Free$$

The optimal solution to the above model represents the aggregate efficiency of the unit under evaluation at time t_1.

$$\begin{aligned}
& G_o^{t_2}(x_o^{t_2}, y_o^{t_2}) = Max u_1 d_{1o}^{t_2} + u_2 d_{2o}^{t_2} \\
& s.t. \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + k_1 y_{1o}^{t_2} + \sum_{f=1}^{2} w_f z_{fo}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} + k_2 y_{2o}^{t_2} = 1 \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} - \sum_{f=1}^{2} w_f z_{fj}^{t_2} - u_1 d_{1j}^{t_2} + t_0^1 \geq 0, \quad j = 1, \ldots, n \\
& \sum_{f=1}^{2} w_f z_{fj}^{t_2} - \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \geq 0, \quad j = 1, \ldots, n \\
& \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \geq 0, \quad j = 1, \ldots, n \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \quad j = 1, \ldots, n \\
& \sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} + \sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_1 d_{1j}^{t_2} \\
& \quad - u_2 d_{2j}^{t_2} \geq 0, \quad j = 1, \ldots, n \\
& k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s \\
& t_0^1, t_0^2, t_0^3 Free
\end{aligned} \tag{17}$$

B: In the above model, both the unit under evaluation and the efficient frontier is considered at time t_2.

The optimal solution to the above model represents the aggregate efficiency of the unit under evaluation at time t_2.

C: In the above model, the unit under evaluation and the efficient frontier are considered at times t_2 and t_1, respectively.

$$
\begin{aligned}
&G_o^{t_1}(x_o^{t_2}, y_o^{t_2}) = Min\, u_1 d_{1o}^{t_2} + u_2 d_{2o}^{t_2} \\
&s.t. \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + k_1 y_{1o}^{t_2} + \sum_{f=1}^{2} w_f z_{fo}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} + k_2 y_{2o}^{t_2} = 1 \\
&\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1, \dots, n \\
&\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1, \dots, n \\
&\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_2 d_{2j}^{t_1} + t_0^3 \geq 0, \quad j = 1, \dots, n \\
&\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \geq 0, \quad j = 1, \dots, n \\
&\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} + \sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_1 d_{1j}^{t_2} \\
&\qquad - u_2 d_{2j}^{t_2} \geq 0, \quad j = 1, \dots, n \\
&k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s \\
&t_0^1, t_0^2, t_0^3\, Free
\end{aligned}
\tag{17}
$$

As the aggregate efficiency of the unit under evaluation and the efficient frontier are considered at times t_2 and t_1, respectively, the optimal solution to the objective function of the above model does not represent efficiency.

D: In the above model, the unit under evaluation and the efficient frontier are considered at times t_1 and t_2, respectively.

$$
\begin{aligned}
&G_o^{t_2}(x_o^{t_1}, y_o^{t_1}) = Max\, u_1 d_{1o}^{t_1} + u_2 d_{2o}^{t_1} \\
&s.t. \sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + k_1 y_{1o}^{t_1} + \sum_{f=1}^{2} w_f z_{fo}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} + k_2 y_{2o}^{t_1} = 1 \\
&\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + k_1 y_{1j}^{t_2} - \sum_{f=1}^{2} w_f z_{fj}^{t_2} - u_1 d_{1j}^{t_2} + t_0^1 \geq 0, \quad j = 1, \dots, n \\
&\sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} - \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \geq 0, \quad j = 1, \dots, n \\
&\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \geq 0, \quad j = 1, \dots, n \\
&\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \quad j = 1, \dots, n
\end{aligned}
\tag{18}
$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} + \sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_1 d_{1j}^{t_1}$$
$$- u_2 d_{2j}^{t_1} \geq 0, \; j = 1, \ldots, n$$
$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$
$$t_0^1, t_0^2, t_0^3 Free$$

As the aggregate efficiency of the unit under evaluation and the efficient frontier are considered at times t_1 and t_2, respectively, the optimal solution to the objective function of the above model does not represent efficiency.

Therefore, the Malmquist productivity index in terms of aggregate efficiency in a network is obtained by the following relation.

$$M_o = \frac{G_o^{t2}(x_o^{t2}, y_o^{t2})}{G_o^{t1}(x_o^{t1}, y_o^{t1})} \left[\frac{G_o^{t1}(x_o^{t2}, y_o^{t2})}{G_o^{t2}(x_o^{t2}, y_o^{t2})} \times \frac{G_o^{t1}(x_o^{t1}, y_o^{t1})}{G_o^{t2}(x_o^{t1}, y_o^{t1})} \right]^{\frac{1}{2}}$$

M_o larger than, smaller than, or equal to one, respectively, denote growth, reduction, and no change in the efficiency of the unit, considering the aggregate efficiency. TEC larger than, smaller than, or equal to one denotes growth, reduction, and no change in the aggregate technical efficiency of the unit, respectively. Finally, FS greater than, smaller than, or equal to one denotes technological progress (approaching the frontier to the center or enlarging the production possibility set) and no change in the technology change or efficient frontier with time.

Now, considering different values for the two parts of FS_o, the following cases can be examined:

Case A: $\frac{G_o^{t1}(x_o^{t1},y_o^{t1})}{G_o^{t2}(x_o^{t1},y_o^{t1})} > 1$ and $\frac{G_o^{t1}(x_o^{t2},y_o^{t2})}{G_o^{t2}(x_o^{t2},y_o^{t2})} > 1$

In this case, DMU_o at both times is located in the part of the production possibility set where the technological progress occurred, the PPS frontier has a positive shift, and $FS_o>1$. This is the best case.

Case B: $\frac{G_o^{t1}(x_o^{t1},y_o^{t1})}{G_o^{t2}(x_o^{t1},y_o^{t1})} < 1$ and $\frac{G_o^{t1}(x_o^{t2},y_o^{t2})}{G_o^{t2}(x_o^{t2},y_o^{t2})} < 1$

In this case, DMU_o at both times is located in the part of the production possibility set where the technological regress occurred, the PPS frontier has a negative shift, and $FS_o<1$. This is the worst case.

Case C: $\frac{G_o^{t1}(x_o^{t1},y_o^{t1})}{G_o^{t2}(x_o^{t1},y_o^{t1})} < 1$ and $\frac{G_o^{t1}(x_o^{t2},y_o^{t2})}{G_o^{t2}(x_o^{t2},y_o^{t2})} > 1$

In this case, it is possible $FS_o<1$ or $FS_o>1$, and it can be concluded that DMU_o moved from the part of PPS with technological regress to the part with technological progress, and the strategy change of DMU_o in using different inputs was useful. Now, if the technological progress in time t^2 compensates for the regress that occurred in time t^1, $FS_o>1$, otherwise $FS_o<1$.

Case D: $\frac{G_o^{t1}(x_o^{t1},y_o^{t1})}{G_o^{t2}(x_o^{t1},y_o^{t1})} > 1$ and $\frac{G_o^{t1}(x_o^{t2},y_o^{t2})}{G_o^{t2}(x_o^{t2},y_o^{t2})} < 1$

In this case, it is possible $FS_o<1$ or $FS_o>1$, and it can be concluded that DMU_o moved from the part of PPS with technological progress to the part with technological regress, and the strategy change of DMU_o in using different inputs was not useful.

Now, considering that the Malmquist productivity index can also be defined in terms of the total efficiency of the network, we consider the following models.

A: In the above model, both the unit under evaluation and the efficient frontier is considered at time t_1.

$$\bar{G}_o^{t_1}(x_o^{t_1}, y_o^{t_1}) = Max \sum_{r=1}^{2} u_r d_{ro}^{t_1}$$

$$\text{s.t.} \sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} = 1$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1,\ldots,n \tag{19}$$

$$\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1,\ldots,n$$

$$\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} + k_2 y_{2j}^{t_1} - u_2 d_{2j}^{t_1} + t_0^3 \geq 0, \quad j = 1,\ldots,n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \geq 0, \quad j = 1,\ldots,n$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

The optimal solution to the above model represents the total efficiency of the unit under evaluation at time t_1.

B: In the above model, both the unit under evaluation and the efficient frontier is considered at time t_2.

The optimal solution to the above model represents the total efficiency of the unit under evaluation at time t_2.

B: In the above model, the unit under evaluation and the efficient frontier are considered at times t_2 and t_1, respectively.

$$\bar{G}_o^{t_1}(x_o^{t_2}, y_o^{t_2}) = Max \sum_{r=1}^{2} u_r d_{ro}^{t_2}$$

$$s.t. \sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} = 1$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + k_1 y_{1j}^{t_1} - \sum_{f=1}^{2} w_f z_{fj}^{t_1} - u_1 d_{1j}^{t_1} + t_0^1 \geq 0, \quad j = 1, \dots, n \tag{20}$$

$$\sum_{f=1}^{2} w_f z_{fj}^{t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} - \sum_{r=1}^{2} k_r y_{rj}^{t_1} + t_0^2 \geq 0, \quad j = 1, \dots, n$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_1} - \sum_{r=1}^{2} u_r d_{rj}^{t_1} \geq 0, \quad j = 1, \dots, n$$

$$\sum_{i=1}^{3} v_i^1 x_{io}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_2} - \sum_{r=1}^{2} u_r d_{ro}^{t_2} \geq 0,$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

As the total efficiency of the unit under evaluation and the efficient frontier are considered at times t_2 and t_1, respectively, the optimal solution to the objective function of the above model does not represent efficiency.

D: In the above model, the unit under evaluation and the efficient frontier are considered at times t_1 and t_2, respectively.

$$\bar{G}_o^{t_2}(x_o^{t_1}, y_o^{t_1}) = Max \sum_{r=1}^{2} u_r d_{ro}^{t_1}$$

$$\sum_{f=1}^{2} w_f z_{fj}^{t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} - \sum_{r=1}^{2} k_r y_{rj}^{t_2} + t_0^2 \geq 0, \quad j = 1, \dots, n$$

$$\sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} + k_2 y_{2j}^{t_2} - u_2 d_{2j}^{t_2} + t_0^3 \geq 0, \quad j = 1, \dots, n \tag{21}$$

$$\sum_{i=1}^{3} v_i^1 x_{ij}^{1t_2} + \sum_{l=1}^{2} v_l^2 x_{lj}^{2t_2} + \sum_{p=1}^{3} v_p^3 x_{pj}^{3t_2} - \sum_{r=1}^{2} u_r d_{rj}^{t_2} \geq 0, \quad j = 1, \dots, n$$

$$\sum_{i=1}^{3} v_i^1 x_{io}^{1t_1} + \sum_{l=1}^{2} v_l^2 x_{lo}^{2t_1} + \sum_{p=1}^{3} v_p^3 x_{po}^{3t_1} - \sum_{r=1}^{2} u_r d_{ro}^{t_1} \geq 0,$$

$$k_r \geq 0, v_i^1 \geq 0, v_l^2 \geq 0, v_p^3 \geq 0, w_f \geq 0, u_s \geq 0, \quad \forall r,i,l,p,f,s$$

$$t_0^1, t_0^2, t_0^3 \, Free$$

As the total efficiency of the unit under evaluation and the efficient frontier are considered at times t_1 and t_2, respectively, the optimal solution to the objective function of the above model does not represent efficiency.

Therefore, the Malmquist productivity index in terms of total efficiency in a network can be obtained by the following relation.

$$\bar{M}_o = \frac{\bar{G}_o^{t2}(x_o^{t2}, y_o^{t2})}{\bar{G}_o^{t1}(x_o^{t1}, y_o^{t1})}\left[\frac{\bar{G}_o^{t1}(x_o^{t2}, y_o^{t2})}{\bar{G}_o^{t2}(x_o^{t2}, y_o^{t2})} \times \frac{\bar{G}_o^{t1}(x_o^{t1}, y_o^{t1})}{\bar{G}_o^{t2}(x_o^{t1}, y_o^{t1})}\right]^{\frac{1}{2}}$$

$\bar{M}_o$ larger than, smaller than, or equal to one, respectively, denote growth, reduction, and no change in the efficiency of the unit, considering the aggregate efficiency. TEC larger than, smaller than, or equal to one denotes growth, reduction, and no change in the aggregate technical efficiency of the unit, respectively. Finally, FS greater than, smaller than, or equal to one denotes technological progress (approaching the frontier to the center or enlarging the production possibility set) and no change in the technology change or efficient frontier with time.

Now, considering different values for the two parts of FS_o, the following cases can be examined:

Case A: $\frac{\bar{G}_o^{t1}(x_o^{t1}, y_o^{t1})}{\bar{G}_o^{t2}(x_o^{t1}, y_o^{t1})} > 1$ and $\frac{\bar{G}_o^{t1}(x_o^{t2}, y_o^{t2})}{\bar{G}_o^{t2}(x_o^{t2}, y_o^{t2})} > 1$

In this case, DMU_o at both times is located in the part of the production possibility set where the technological progress occurred, the PPS frontier has a positive shift, and $FS_o>1$. This is the best case.

Case B: $\frac{\bar{G}_o^{t1}(x_o^{t1}, y_o^{t1})}{\bar{G}_o^{t2}(x_o^{t1}, y_o^{t1})} < 1$ and $\frac{\bar{G}_o^{t1}(x_o^{t2}, y_o^{t2})}{\bar{G}_o^{t2}(x_o^{t2}, y_o^{t2})} < 1$

In this case, DMU_o at both times is located in the part of the production possibility set where the technological regress occurred, the PPS frontier has a negative shift, and $FS_o<1$. This is the worst case.

Case C: $\frac{\bar{G}_o^{t1}(x_o^{t1}, y_o^{t1})}{\bar{G}_o^{t2}(x_o^{t1}, y_o^{t1})} < 1$ and $\frac{\bar{G}_o^{t1}(x_o^{t2}, y_o^{t2})}{\bar{G}_o^{t2}(x_o^{t2}, y_o^{t2})} > 1$

In this case, it is possible $FS_o<1$ or $FS_o>1$, and it can be concluded that DMU_o moved from the part of PPS with technological regress to the part with technological progress, and the strategy change of DMU_o in using different inputs was useful. Now, if the technological progress in time t^2 compensates for the regress that occurred in time t^1, $FS_o>1$, otherwise $FS_o<1$.

Case D: $\frac{\bar{G}_o^{t1}(x_o^{t1}, y_o^{t1})}{\bar{G}_o^{t2}(x_o^{t1}, y_o^{t1})} > 1$ and $\frac{\bar{G}_o^{t1}(x_o^{t2}, y_o^{t2})}{\bar{G}_o^{t2}(x_o^{t2}, y_o^{t2})} < 1$

In this case, it is possible $FS_o<1$ or $FS_o>1$, and it can be concluded that DMU_o moved from the part of PPS with technological progress to the part with technological regress, and the strategy change of DMU_o in using different inputs was not useful.

4. Case study in the petroleum industry

Many decision-making units have more than one stage. To evaluate the performance of such units using data envelopment analysis, the whole unit cannot be considered a black box, but the inner relationships must be taken into account. Different methods have been proposed to evaluate the performance of multi-stage units.

As stated in Section 3, two different methods in multiplier form are introduced for evaluating a supply chain consisting of inner and reversible connections with system-independent inputs and outputs. One of these methods considered the aggregate efficiency evaluation of the supply chain, and the other considered the total efficiency evaluation of the system from input-oriented and output-oriented viewpoints. In both cases, the network stages efficiencies are also evaluated. Each of these methods has its own unique theoretical properties, and we will discuss them in measuring productivity using the Malmquist index.

In this section, a practical example in the petroleum industry is discussed. Both of the introduced approaches are implemented in this example, and the results will be examined. The data used for this case study are illustraded in Tables 1 and 2. The information and data are estimated by the experts from website eia.gov.ir. A part of the structure of the petroleum industry is extracted. Considering the discussed models for evaluating total and aggregate efficiencies, we will study the progress and regress of the units under evaluation in terms of the Malmquist productivity index by considering the total and aggregate efficiencies from the input-oriented and output-oriented viewpoints.

It should be noted that we do not use the hybrid data envelopment analysis model because, in this practical example, the purpose is to study and examine the results from input-oriented and output-oriented viewpoints. This approach is recommended as a reference for future studies to researchers considering the hybrid data envelopment analysis models.

Table 1. Data relate to the stage 1 in the network system

	Stage 1							
DMUs	Inputs					Intermediate		Output
	x11	x12	x13	x14	x15	Z11	Z12	d
	Ethane	Propane	Human Workforce	Energy consumption	Returned Propane	Propane	Propylene	Ethelene
1	620	325	64	4480	82	117	387	333650
2	540	280	73	5110	75	120	260	322500
3	720	460	123	8610	200	240	530	307500
4	120	1000	38	2660	122	147	650	81000
5	650	390	96	6720	135	170	290	43500
6	750	450	84	5880	180	260	370	427500
7	910	650	101	7070	215	370	440	562500
8	720	480	84	5880	190	320	280	450000
9	420	225	34	3480	132	217	387	433650
10	340	580	83	6110	95	320	460	622500
11	340	360	73	1610	100	140	130	207500
12	160	900	154	8660	202	127	750	61000
13	450	190	56	8720	235	270	390	83500
14	820	850	74	2880	120	160	870	327500
15	820	350	91	2070	195	470	940	862500
16	900	880	64	8880	290	120	580	750000
17	270	625	104	7480	172	470	790	733650
18	820	580	118	9110	175	460	270	522500
19	470	825	94	6480	182	570	940	807500
20	520	280	88	6110	145	120	880	765490

Table 2. Data relate to the stages 2 and 3 in the network system

DMUs	Stage 2				Stage 3			
	Input		Intermediate		Inputs			Output
	x2 Human Workforce	x2 Energy Consumption	y Tetrameter	y Benzene	x3 Human Workforce	x3 Decyl Benzene	x3 Energy Consumption	d Ethylene
1	47	3456	317	182	42	447	4536	143
2	53	3786	360	175	35	472	6543	120
3	68	4327	550	200	35	620	5674	135
4	34	5367	180	122	17	197	5413	137
5	57	4298	325	135	24	397	7654	105
6	48	3987	450	180	36	497	5672	142
7	65	4871	595	215	52	710	6437	187
8	75	5001	380	190	40	464	9254	175
9	37	5456	417	192	62	447	4536	153
10	73	2786	560	165	25	272	5543	160
11	88	8327	250	100	75	720	8674	175
12	74	2367	190	212	47	870	2413	157
13	27	8298	225	235	74	297	9654	125
14	88	9987	650	280	26	497	3672	132
15	16	6871	195	315	82	410	7437	127
16	95	2001	880	290	20	964	2254	195
17	26	9871	217	192	94	897	9654	173
18	95	8001	860	125	86	297	9672	190
19	56	2871	250	220	72	910	8437	185
20	75	4001	380	121	90	264	8254	172

4.1. Different forms of implementing decision-making units

A Decision-making unit is an entity that converts inputs to outputs. In DEA, decision-making units must be homogenous and have similar objectives and functions. This method measures productivity directly by considering the ratio between various inputs (or resources) to various produced outputs (or services). Therefore, the problem variables can be divided into the inputs and outputs groups. Determining input and output variables is so important in using DEA as the results of solving the DEA model depends on the types of chosen inputs and outputs in a way that changing an input or output variable will change the model results. Therefore, if the input or output variables are defined correctly, the performance evaluation of DMUs will be more realistic. Extracting evaluation inputs and outputs that are chosen among a set of indices is the most important part of this investigation. It should be noted that pursuing different goals in evaluation results in choosing different input and output indices. On the other hand, indices serve the role of warning decision-makers about possible or hidden problems in a set of specific fields or continuing the favorable process in other fields.

To identify the indices, the overall processes of different petrochemical companies are analyzed, and finally, the process portrayed in the following figure is confirmed by various experts in this field. Please see Figure 1. Independent inputs of the first stage are; Ethane, Propane, Human Workforce, and Energy Consumption. Propane is also returned back from the second stage to the first stage. The outputs of the first stage which are the inputs of the second stage are; Propane, and Propylene. Independent inputs of the second stage are Human Workforce, and Energy Consumption. Independent output of the first stage is Decyl Benzene. Tetrameter is the intermediate product of the second stage which is the input of the third stage. Independent inputs of the third stage are; Human Workforce, and Energy Consumption. The final output of the third stage is Ethylene.

In the Figure 1 the processes are depicted in three stages: the olefin unit, tetramer unit, and dodecylbenzene unit. Each of these units has specified functions, which in the following, we describe the most important ones.

According to the definition, petrochemical is referred to industries in which the hydrocarbons existing in crude oil or natural gas undergo a set of chemical processes to convert into new chemical products. Producing petrochemical products is such that in some cases, a main upstream unit produces raw material for other units, such as the olefin unit that produces ethylene and propylene to satisfy the need of the polyethylene and polypropylene units. Therefore, given the variety and difference in processes used in petrochemical complexes, the energy evaluation of each unit is done separately. On the other hand, in this industry, like refining industries, the fuel is consumed as feed in some units such as the olefin unit. The common energy carriers in petrochemical complexes are natural gas and fossil fuels.

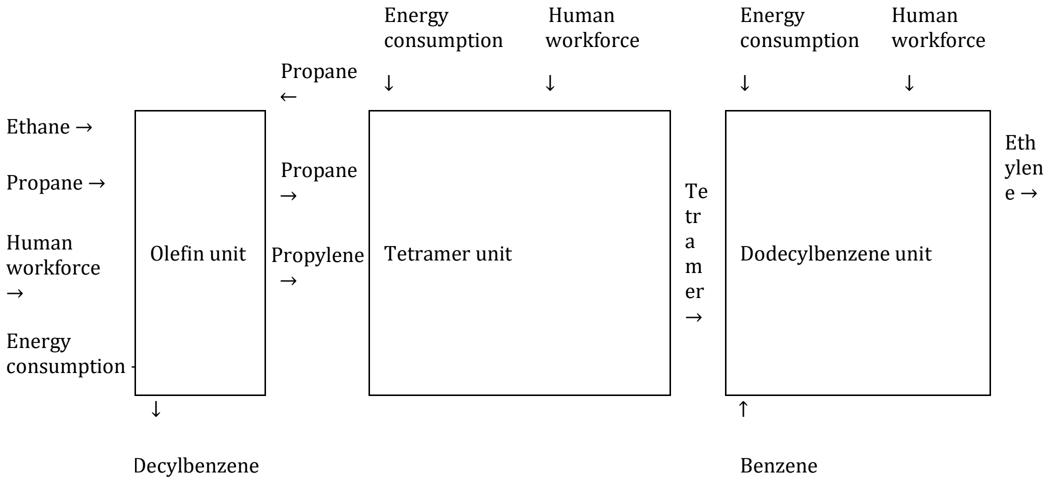

Figure 1. The petrochemical process

Producing petrochemical products is such that in some cases, a main upstream unit produces raw material for other units, such as the olefin unit that produces ethylene and propylene to satisfy the need of the polyethylene and polypropylene units.

After extensive evaluation, indices proportionate to the structure of the supply chain are found. We extracted data from twenty petrochemical units that have similar processes after consultation with several of them. These data are presented in the table below.

4.2. Malmquist productivity index

4.2.1. Output-oriented malmquist productivity index with aggregate efficiency model

In order to calculate the Malmquist productivity index, data from different petrochemical companies between 1395 to 1398 are extracted. By doing calculations at 95-96, 96-97, and 97-98 intervals, the trend lines are obtained for different units. Data analysis is performed based on these graphs. Please see Table 3.

Table 3. The output-oriented Malmquist productivity index for aggregate efficiency for 95-96

DMUs	D11 $D^t(t)$	D22 $D^{t+1}(t+1)$	D21 $D^{t+1}(t)$	D12 $D^t(t+1)$	MPI Malmquist Index
DMU1	1.26	1.08	1.47	1	1.12
DMU2	1.36	1.01	1.68	1	1.12
DMU3	2.18	1.22	2.25	1.01	1.12
DMU4	1.21	1.75	1.08	1.73	0.95
DMU5	2.57	1.63	2.4	1.33	1.07
DMU6	1.6	1.08	1.75	1.09	1.04
DMU7	1.55	1.36	1.83	1.06	1.23
DMU8	1.67	1.21	1.73	1	1.12
DMU9	1.01	1.46	1.11	1.25	1.13
DMU10	1	1.01	1	1	1
DMU11	1.75	1.42	1.91	1.33	1.08
DMU12	1.54	2.58	1.55	2.32	1.06
DMU13	2.03	1.4	2.17	1.14	1.14
DMU14	1.39	1.13	1.59	1	1.13
DMU15	1.01	2.01	1	1.04	1.38
DMU16	1.07	1.01	1.26	1	1.09
DMU17	1.02	1.33	1.07	1.22	1.07
DMU18	1.51	1	1.7	1	1.06
DMU19	1.01	2.52	1.14	2.3	1.11
DMU20	1.01	1.07	1	1	1.03

Figure 2 reveals that only petrochemical unit 4 has seen regress, and other units undergo progress. Unit 15 has the most progress, and some units showed insignificant progress.

Subsequently, given the data of 96-97 and 97-98, the indices are calculated proportionally to the previous interval. Please see Table 4 and Table 5.

Table 4. The output-oriented Malmquist productivity index for aggregate efficiency for 96-97

10	D11 $D^t(t)$	D22 $D^{t+1}(t+1)$	D21 $D^{t+1}(t)$	D12 $D^t(t+1)$	MPI Malmquist Index
DMU1	1.25	1.12	1.34	1	1.03
DMU2	1.26	1.01	1.38	1.12	1.16
DMU3	2.48	1.12	2.12	1.12	1.12
DMU4	1.31	1.44	1.43	1.53	0.93
DMU5	2.57	1.54	1.75	1.23	1.17
DMU6	1.43	1.08	1.45	1.16	1.12
DMU7	1.29	1.21	1.73	1.06	1.27
DMU8	1.76	1.12	1.45	1	1.07
DMU9	1.11	1.33	1.13	1.12	1.18
DMU10	1	1.01	1	1	1
DMU11	1.45	1.42	1.62	1.25	1.14
DMU12	1.43	1.68	1.59	2.74	1.14
DMU13	1.76	1.34	2.27	1.23	1.23
DMU14	1.39	1.13	1.387	1.43	1.11
DMU15	1	1.56	1	1.04	1.46
DMU16	1.23	1.23	1.76	1	1.13
DMU17	1.07	1.23	1.07	1.05	1.09
DMU18	1.54	1	1.23	1	1.03
DMU19	1.9	1.65	1.45	1.14	1.17
DMU20	1.11	1.18	1	1	1.21

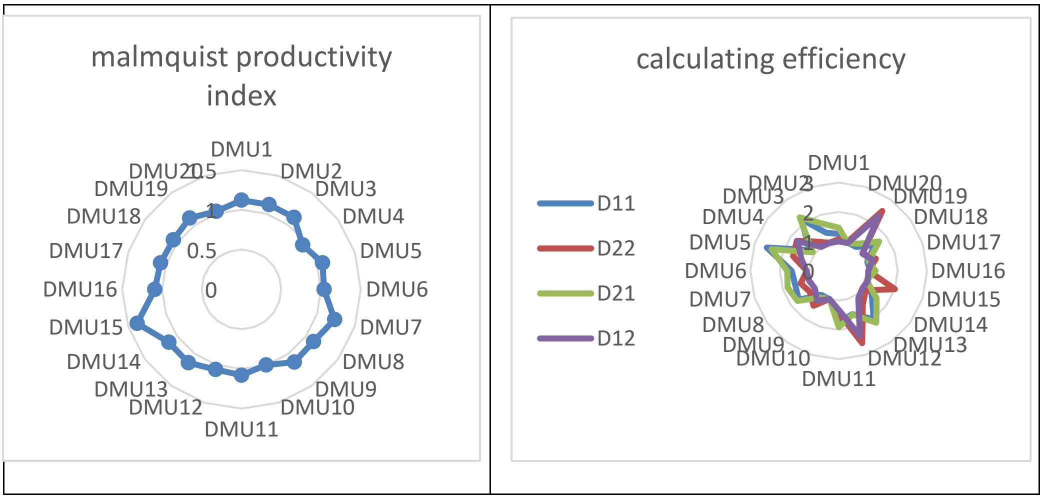

Figure 2. The Malmquist productivity index graphs

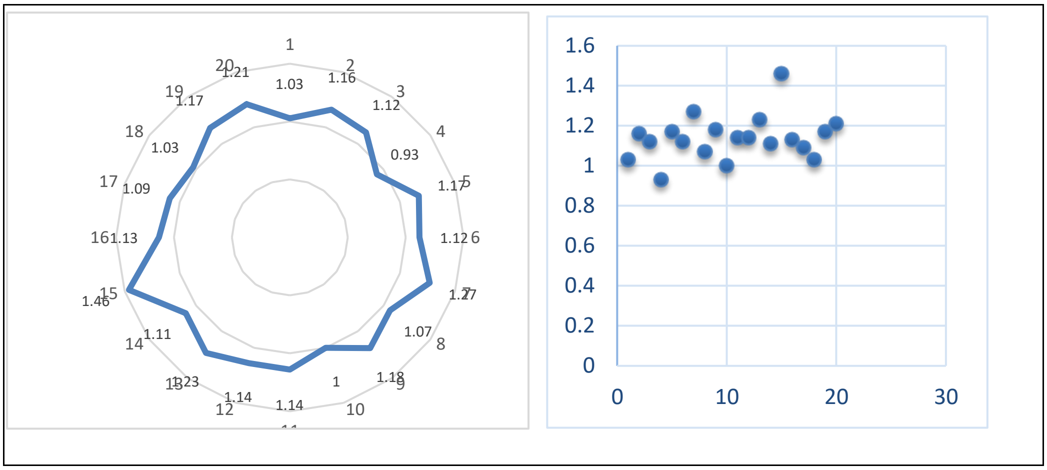

Figure 3. The Malmquist productivity index graphs for 96-97

Table 5. The output-oriented Malmquist productivity index for aggregate efficiency for 97-98

DMUs	D11 $D^t(t)$	D22 $D^{t+1}(t+1)$	D21 $D^{t+1}(t)$	D12 $D^t(t+1)$	MPI Malmquist Index
DMU1	1.36	1.12	1.32	1	1.23
DMU2	1.36	1.04	1.54	1	1.14
DMU3	1.68	1.12	1.65	1.01	1.16
DMU4	1.11	1.45	1.12	1.32	0.97
DMU5	1.87	153	2.12	1.24	1.17
DMU6	1.54	1.18	1.34	1.09	1.24
DMU7	1.45	1.26	1.56	1.06	1.43
DMU8	1.57	1.11	1.54	1	1.19
DMU9	1.13	1.35	1.11	1.12	1.03
DMU10	1.03	1.01	1	1	1.09
DMU11	1.45	1.32	1.61	1.23	1.18
DMU12	1.86	2.43	1.54	76	1.01
DMU13	1.78	1.33	1.32	1.14	1.04
DMU14	1.23	1.21	1.22	1	1.23
DMU15	1.12	1.65	1	1.04	1.57
DMU16	1	1.01	1.15	1	1.16
DMU17	1.12	1.18	1.07	1.12	1.11
DMU18	1.31	1	1.37	1	1.03
DMU19	1.08	2.09	1.09	1.76	1.19
DMU20	1.06	1.12	1	1	1.05

Similarly, unit 15 shows much more progress than the other units. The status of the units is shown graphically in Figure 3. As can be seen, only unit 4 shows regress.

Similar to the above discussion, the calculation is done for 97-98, and the results are presented in Tables 6-8.

Similar to the previous intervals, unit 15 shows much more progress than the other units. The status of the units is illustrated in the following figures. The only unit that shows regress is unit 4. Please see Figure 4.

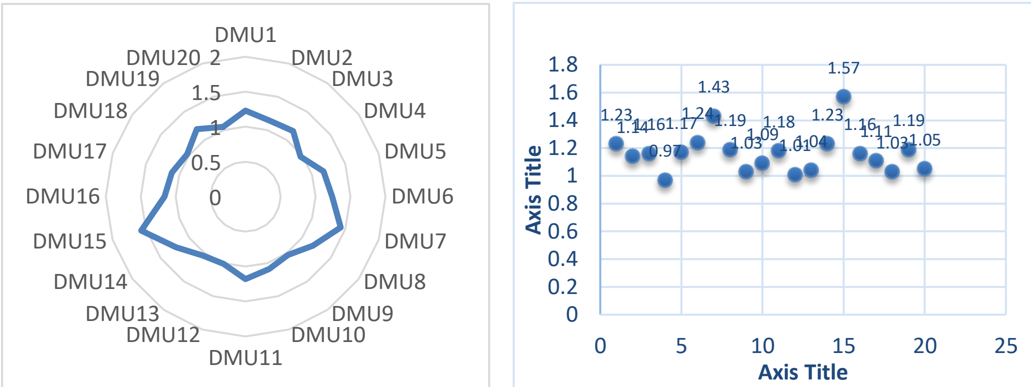

Figure 4. The Malmquist productivity index graphs for 97-98

Variations in the Malmquist index for all the units are presented in on graph in Figure 5.

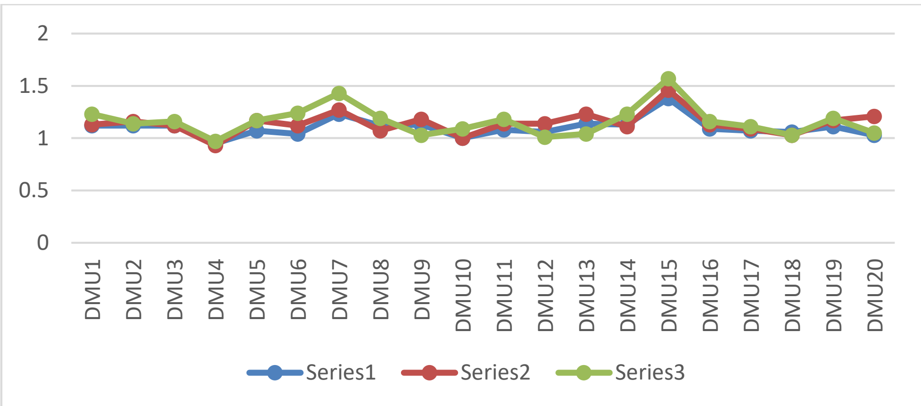

Figure 5. The Malmquist productivity index graphs for 96-98

To analyze the Malmquist index separately for each petrochemical unit, a graph for each unit must be created, and the following graphs show the variation of these indices.

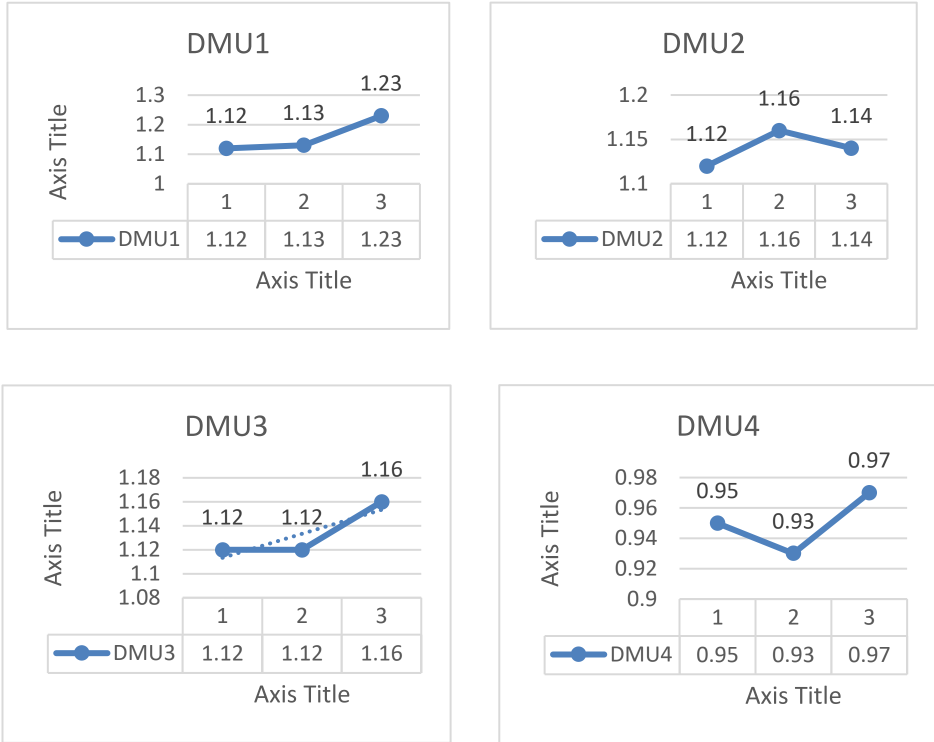

Figure 6. The Malmquist productivity index graphs of units one to four for 96-98

As can be seen in Figure 6, unit 1 shows progress during all these years. Unit 2 also underwent progress during 95-96, 96-97, and 97-98, but its progress during 97-98 was slightly lower than its progress in the previous interval. Unit 3 saw progress during all these years as well. However, unit 4 shows regress as its Malmquist index is lower than one. Please see Figure 7.

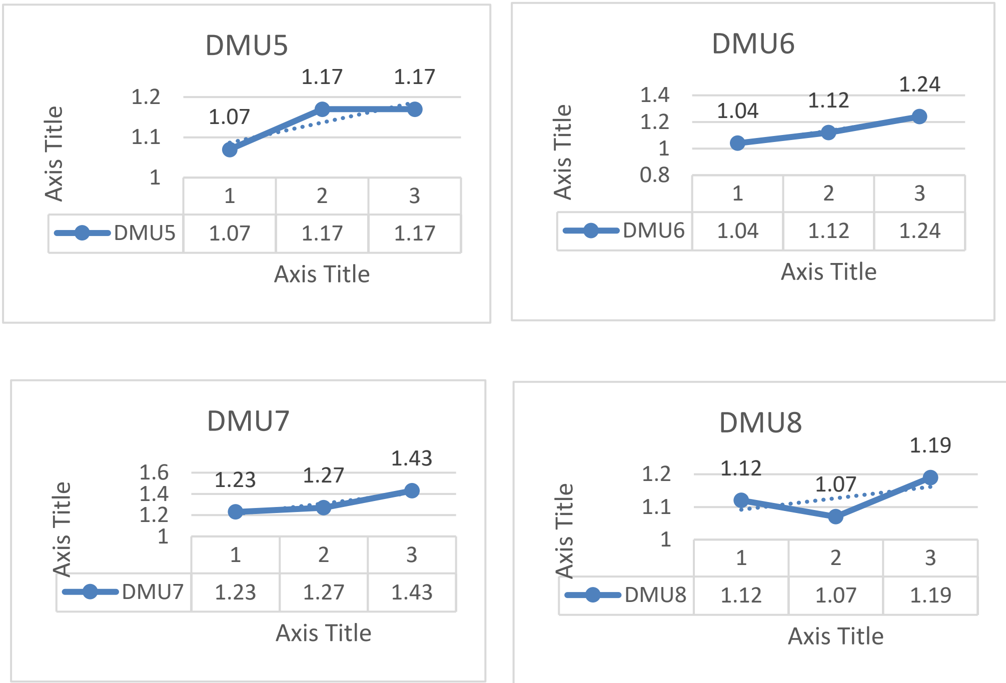

Figure 7. The Malmquist productivity index graphs of units four to eight for 96-98

Progress is clearly visible in units 5, 6, and 7. Although unit 8 has Malmquist indices greater than one, during 96-97, its progress was lower than the previous interval. Consider Figure 8.

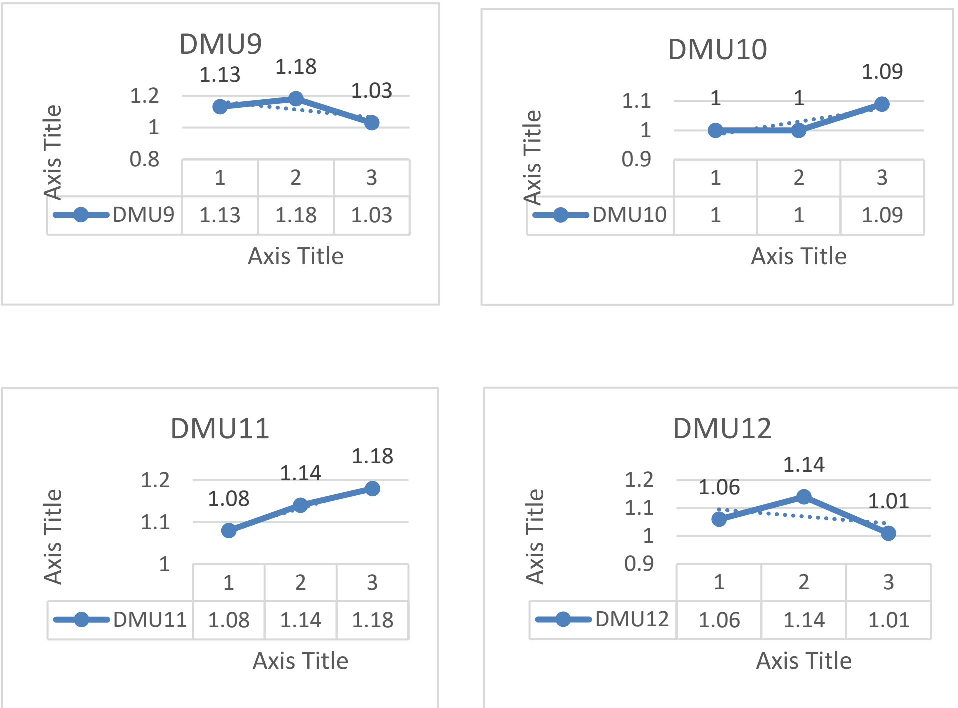

Figure 8. The Malmquist productivity index graphs of units 9 to 12 for 96-98

Unit 11 shows increasing progress during these years. Although the Malmquist indices for the other units are greater than one and they show progress, they show a decreasing trend with a negative rate. Consider Figure 9.

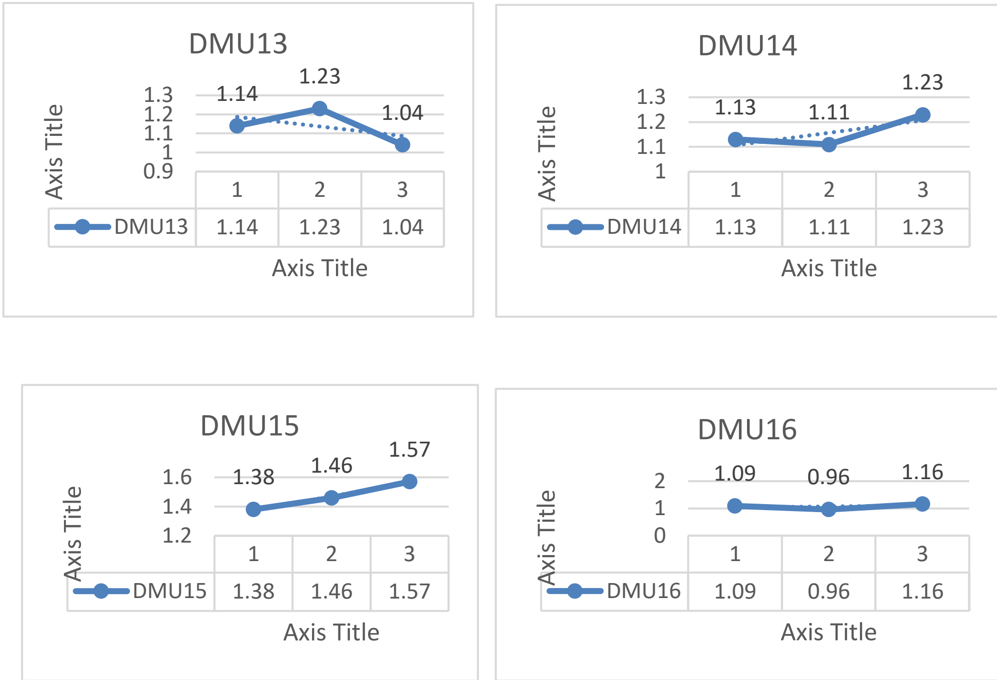

Figure 9. The Malmquist productivity index graphs of units 13 to 16 for 96-98

As can be seen, petrochemical unit 15 has the best conditions among other units. Consider Figure 10.

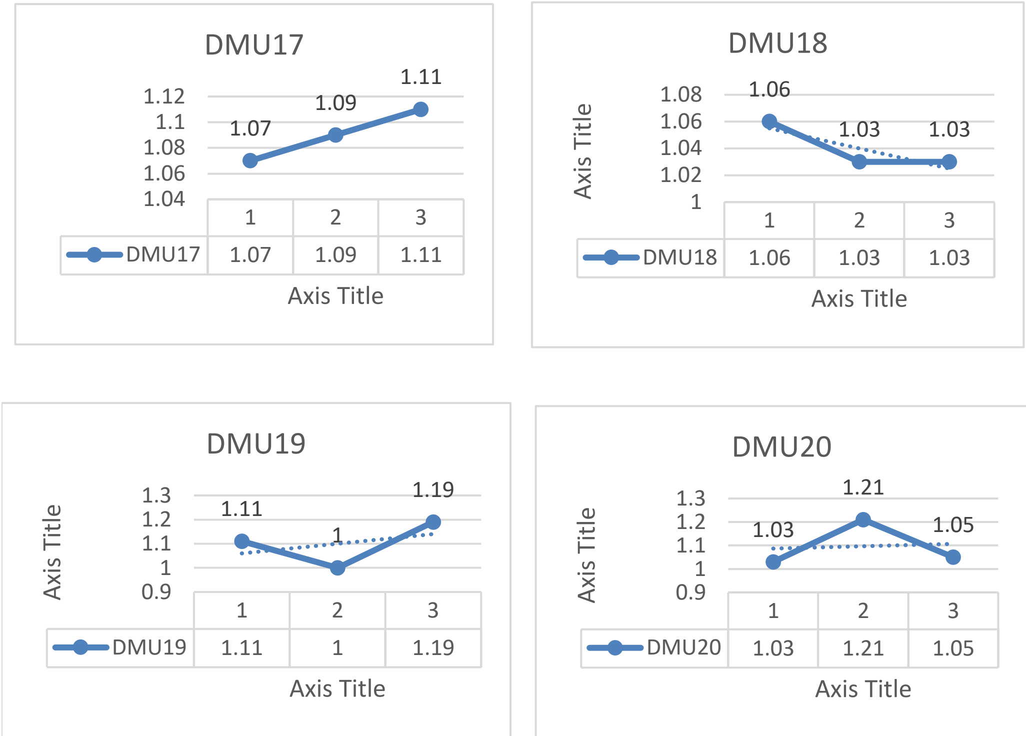

Figure 10. The Malmquist productivity index graphs of units 13 to 16 for 96-98

As shown in Figure 10, the graphs show progress for units 17 to 20 as well, albeit with many variations.

4.2.2. Input-oriented malmquist productivity index with aggregate efficiency model

Similar to the previous section, the input-oriented Malmquist productivity index can be calculated based on the aggregate efficiency evaluation model for different petrochemical units from 1395 to 1398. However, only the calculation for the 95-96 interval is presented below. Please see Table 6.

Table 6. The input-oriented Malmquist productivity index for aggregate efficiency

DMUs	t11 $D^{t}(t)$	t22 $D^{t+1}(t+1)$	t21 $D^{t+1}(t)$	t12 $D^{t}(t+1)$	MPI Malmquist Index
DMU1	0.78	0.94	0.78	1	0.97
DMU2	0.73	0.99	0.59	1	0.9
DMU3	0.45	0.96	0.6	1	0.74
DMU4	0.99	0.99	0.9	0.85	1.03
DMU5	0.52	0.79	0.6	0.69	1.15
DMU6	0.56	0.95	0.46	0.92	0.92
DMU7	0.58	0.64	0.69	0.93	0.9
DMU8	0.56	0.99	0.74	1	1.14
DMU9	0.99	0.95	0.9	1	0.93
DMU10	1	0.99	1	1	1
DMU11	0.55	0.63	0.5	0.72	0.89
DMU12	0.84	0.59	0.91	0.63	1.01
DMU13	0.38	0.99	0.56	0.83	1.33
DMU14	0.84	0.89	0.54	1	0.75
DMU15	0.99	0.44	1	0.96	0.69
DMU16	0.99	0.99	1	1	1
DMU17	0.98	0.92	0.93	0.87	1
DMU18	0.9	1	0.8	1	0.94
DMU19	0.99	0.65	0.87	0.62	0.97
DMU20	0.99	0.93	1	1	0.97

In the Figure 11 reveals that the petrochemical units 4, 5, 8, and 12 show progress, and the other units undergo a regress.

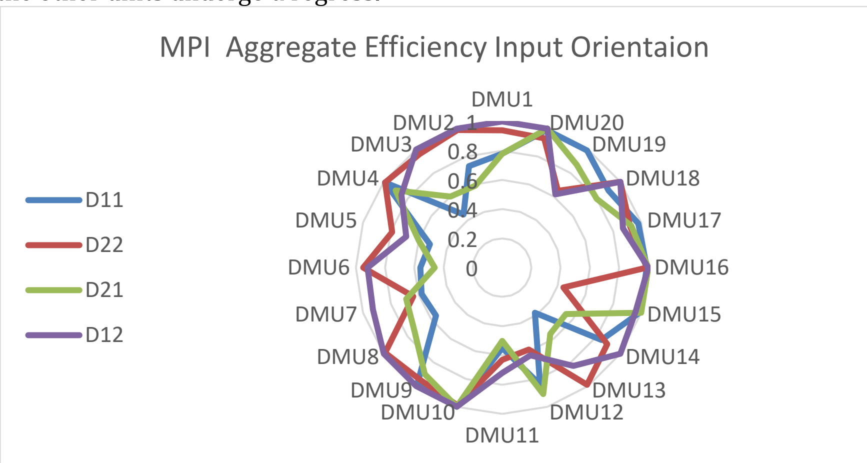

Figure 11. Malmquist index components

4.2.3. Output-oriented malmquist index with total efficiency model

Similar to the previous section, the output-oriented Malmquist productivity index can be calculated based on the total efficiency evaluation model for different petrochemical units from 1395 to 1398. However, only the calculation for the 95-96 interval is presented below. Consider Table 7.

Table 7. The output-oriented Malmquist productivity index for total efficiency

DMUs	t11	t22	t21	t12	MPI
DMU1	1.26	1.08	1.47	1	1.12
DMu2	1.36	1.01	1.68	1	1.11
DMU3	2.18	1.22	2.25	1.01	1.12
DMU4	1.21	1.75	1.08	1.73	0.95
DMU5	2.57	1.63	2.4	1.33	1.07
DMU6	1.6	1.08	1.75	1.09	1.04
DMU7	1.55	1.36	1.83	1.06	1.23
DMU8	1.67	1.21	1.73	1	1.12
DMU9	1.01	1.46	1.11	1.25	1.13
DMU10	1	1.01	1	1	1
DMU11	1.75	1.42	1.91	1.33	1.08
DMU12	1.54	2.58	1.55	2.32	1.06
DMU13	2.03	1.4	2.17	1.14	1.14
DMU14	1.39	1.13	1.59	1	1.13
DMU15	1.01	2.01	1	1.04	1.38
DMU16	1.07	1.01	1.26	1	1.09
DMU17	1.02	1.33	1.07	1.22	1.07
DMU18	1.51	1	1.7	1	1.06
DMU19	1.01	2.52	1.14	2.3	1.11
DMU20	1.01	1.07	1	1	1.03

Please pay attention to the Figure 12. This graph gives an overview of changes in efficiency and the efficient frontier.

4.3.4. Input-oriented malmquist productivity index with total efficiency model

Similar to the previous section, the input-oriented Malmquist productivity index can be calculated based on the total efficiency evaluation model for different petrochemical units from 1395 to 1398. However, only the calculation for the 95-96 interval is presented below in Table 8.

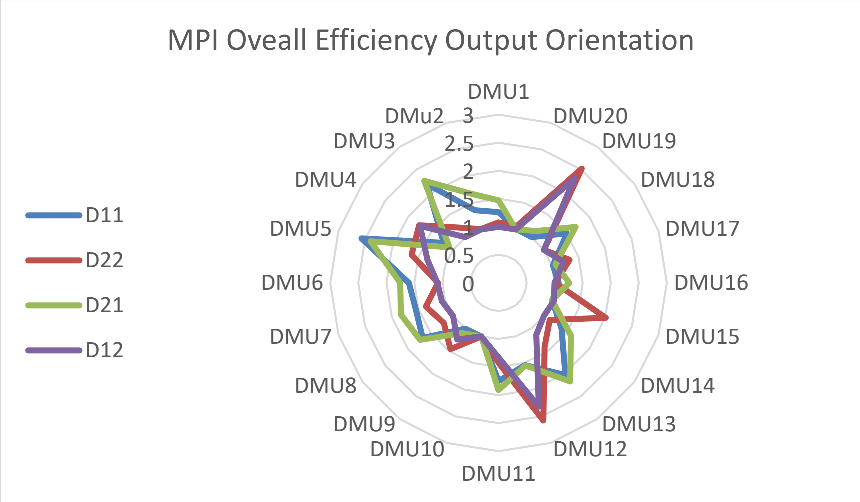

Figure 12. Malmquist index components

Table 8. The input-oriented Malmquist productivity index for total efficiency

DMUs	t11	t22	t21	t12	MPI
DMU1	0.86	0.94	0.78	1	0.92
DMU2	0.73	0.99	0.59	1	0.9
DMU3	0.13	0.96	0	1	0.16
DMU4	0.99	0.99	1	0.85	1.09
DMU5	0.52	0.79	0.14	0.69	0.55
DMU6	0.71	0.95	0.46	0.92	0.82
DMU7	0.58	0.64	0	1	0.06
DMU8	0.56	0.99	0.74	1	1.14
DMU9	0.99	0.95	0.9	1	0.93
DMU10	1	0.99	1	1	1
DMU11	0.55	0.63	0.5	0.72	0.89
DMU12	0.84	0.59	0.91	0.16	1.99
DMU13	0.38	0.99	0.56	0.83	1.33
DMU14	0.84	0.89	0.54	1	0.75
DMU15	0.99	0.44	1	0.96	0.68
DMU16	0.99	0.99	1	1	1
DMU17	0.98	0.92	0.93	0.87	1
DMU18	0.9	1	0.8	1	0.94
DMU19	0.99	0.65	0.87	0.62	0.97
DMU20	0.99	0.93	1	1	0.97

Please pay attention to the graph in Figure 13. This graph gives an overview of changes in efficiency and the efficient frontier.

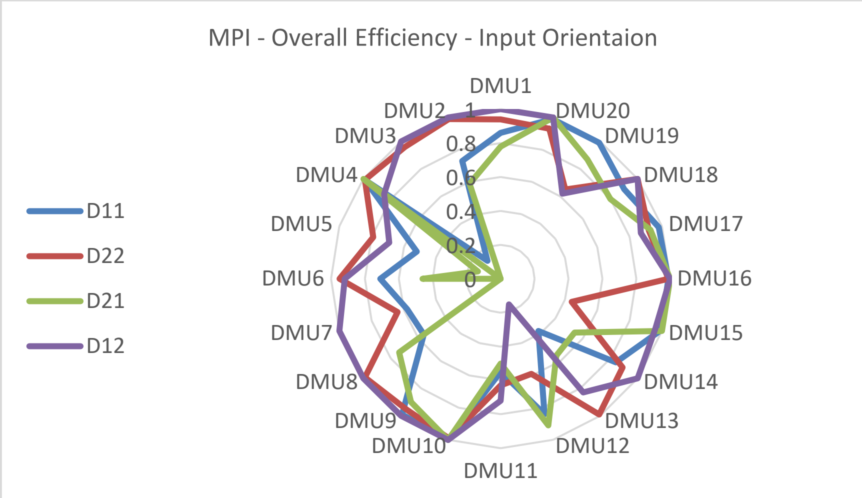

Figure 13. Malmquist index components

5. Conclusion

The investigation presented in this paragraph has two crucial features, which include the development of a performance evaluation model for the petrochemical supply chain and the measurement of the network-based Malmquist productivity index for calculating productivity. The aim of the study was to create a mathematical network model that can assess the performance of an organization, allowing productivity to be calculated based on aggregate and total efficiencies. The productivity evaluation was carried out across the petrochemical units in the country, and the information obtained was modeled in three stages. This information was then sent to the organization's managers to assist with decision-making.The mathematical model was formulated using the data envelopment analysis technique, and both input and output-oriented evaluations were performed based on aggregate and total efficiency evaluation. One significant advantage of the proposed method is that it was formulated for a case study in an oil company, and it allows for the consideration of every aspect of the production system, including intermediate products. However, the investigation was limited by various factors, such as the lack of access to data on petrochemical units for performance evaluation and ranking, which affected the naming of units. Additionally, the results of the investigation are only valid for the time of collecting the data and not forever. Despite the limitations, the investigation's results indicate that petrochemical units in the country have not been evaluated from the perspective of production process performance, and the methodology used can be applied to other oil and gas refineries and gas transmission districts. Furthermore, the study's methodology can be recommended for implementation in other organizations of the Oil Ministry, such as refineries, gas transmission districts, and gas refineries, among others. For future studies, it may be worthwhile to investigate the impact of inaccurate data on the mathematical model's output. Overall, the investigation provides a framework for evaluating the performance of organizations in the petrochemical industry, assisting managers with decision-making and identifying areas for improvement.

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Conflicts of Interest*:* The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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