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PROBABILISTIC LINGUISTIC Q-RUNG ORTHOPAIR FUZZY ARCHIMEDEAN AGGREGATION OPERATORS FOR GROUP DECISION-MAKING

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Abstract: *To express uncertain and imprecise information systematically, the concept of probabilistic linguistic q-rung ortho-pair fuzzy set (PLqROFS), which is an advanced version of linguistic intuitionistic fuzzy set and linguistic Pythagorean fuzzy set, considering the instantaneous occurrence of stochastic and non-stochastic-uncertainty. There isn't yet any literature on PLqROFSs that addresses the issue of the relative importance of experts and criteria. The evaluation's findings consequently become irrational. Additionally, the aggregation operators that are currently available on PLqROFSs are too rigid. The primary goal is to resolve these problems by creating a new methodology that makes use of a new flexible aggregation operator. In this paper, a novel integrated framework is suggested to address concerns with group decision-making in PLqROFSs settings by combining the strengths of the power average operator (PAO), the Archimedean operator, and the full consistency method (FUCOM). With the extended variance approach on PLqROFSs, the weight of decision experts is methodically determined in this line. Additionally, the FUCOM on PLqROFSs is used to determine the weight of the criterion. Some probabilistic linguistic q-rung ortho-pair fuzzy Archimedean weighted and power weighted aggregation operators are suggested to aggregate decision experts' preferences. To discuss the viability of the suggested technique, the challenge of choosing a*

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CO2 storage location is given. As alternatives, we have taken into account oilfields, gas fields, basalt formations, and coal resources. Basalt is the best choice, according to the outcome. The stability of our method is demonstrated by the sensitivity analysis of the criteria weights. The comparative analysis demonstrates that, in comparison to the ones already in use, our model is more significant and realistic.

Keywords: *Probabilistic linguistic q-rung ortho-pair fuzzy sets, PLqRFSs, Archimedean aggregation operators, Full consistency method, FUCOM, multi-criteria group decision-making, MCGDM.*

1. Introduction

Group decision-making (GDM) (Saha et al., 2021; Mishra et al., 2022; Ivanovic et al., 2022; Saha et al., 2022; Krishankumar et al., 2022; Senapati et al., 2023), is a complex and attractive decision problem that gets ratings/opinions from multiple experts to choose a suitable element from the set of elements based on diverse competing criteria (Riaz et al., 2021). In recent times, researchers widely adopted qualitative preferences in the GDM process to flexibly share her/his opinions on objects/criteria. Herrera & Martínez (2000) framed the idea of a "linguistic term set" (LTS) and promoted linguistic decision-making that considers qualitative terms directly as preference values and decision methods attempt to select suitable objects based on such rating information. Rodriguez et al. (2012) showed that LTS was unable to accept more than one qualitative term as a rating argument, which is unreasonable due to the practical uncertainty that exists in the decision process. To handle the issue, a "hesitant fuzzy linguistic term set" (HFLTS) was proposed that could flexibly accept more than one term as rating information thereby allowing experts to effectively share their opinions. To achieve this flexibility, HFLTS integrated the idea of LTS and "hesitant fuzzy set" (HFS) Torra (2010) Although HFLTS is attractive, it cannot assign weights to the diverse terms, which indicates that all the terms are of equal importance and that is unreasonable in practical decision problems. To resolve the issue, Pang et al. (2016) came up with a "probabilistic linguistic term set" (PLTS) that associates occurrence probability to the qualitative rating thereby assigning unequal weights to the terms. Attracted by the PLTS, scholars adopted it for GDM by proposing operators (Kobina et al., 2017; P. Liu & Li, 2019; P. Liu & Teng, 2018), ranking methods (Krishankumar et al., 2019; Ramadass et al., 2020; Sivagami et al., 2019), entropy/distance measures (Lin & Xu, 2018; Su et al., 2019), and others (Krishankumar et al., 2019; Liao et al., 2017, 2020; X. Zhang & Xing, 2017).

To cope with practical situations, equivocal human judgments were taken into consideration, which gave rise to the idea of fuzzy sets (FSs) (Zadeh, 1965). The FS theory, on the other hand, can control reality emerging from computational observation and comprehension, which includes ambiguity, partial belongingness, inaccuracy, sharpness limitations, and so forth. In the GDM model, "decision experts (DEs)" might assess the "belongingness grade (BG)" of an element to a set of diverse grades in various realistic settings due to their individual opinion, time constraints, and the lack of information. To evade the concern, an HFS was developed by (Torra & Narukawa, 2009), according to which doctrine a BG should comprise several distinct BGs. As an FS extension, HFS has attracted much researchers' interest in treating with vagueness in realistic problems. Recently, the HFS has powerfully been associated with the "intuitionistic fuzzy set (IFS)" (Atanassov, 1986) an extension of the FS. The relevance of the IFS, however, is restricted because the sum of the BG μ and the "non-belongingness grade (NBG)" ν cannot exceed 1, that is $\mu+\nu \leq 1$. However, it was later noticed that, depending on the preferences suggested by DEs for complex GDM issues, the given constraint was not satisfied. For example, if a DE favors BG 0.7 and NBG 0.5 while using IFSs, their sum exceeds 1 at that point. To obtain this circumstance type, Yager (2013) pioneered the notion of "Pythagorean fuzzy sets (PFSs)" with the BG μ and the NBG ν, complying with the condition $\mu^2+\nu^2 \leq 1$. The "q rung ortho-pair fuzzy sets (qROFSs)" pioneered by Yager (2017) hold the constraint that the q^{th} powers sum of the BG and the NBG lies between 0 and 1, i.e. $0 \leq \mu^q+\nu^q \leq 1$. When q=1, the qROFSs are reduced to IFSs, and when q=2, to PFSs, which means that qROFSs are the extended versions of IFSs and PFSs.

Inspired by the flexibility of q-ROFS, Liu & Liu (2019) presented a "linguistic q-rung ortho-pair fuzzy set (L-qROFS)" along with power operator to utilize the advantages of both qualitative terms and q-ROFS so that flexibility in rating improves. The "linguistic intuitionistic fuzzy sets (LIFSs)" (Zhang, 2014) and "linguistic Pythagorean fuzzy sets (LPFSs) (Garg, 2018) are particular cases of L-qROFS for q=1 and q=2 respectively. Lin et al. (2020) extended the Heronian operator along with weighted variants for GDM with L-qROFS. (Akram et al., 2021) put forward the Einstein operator with weighted variants under L-qROFS context for GDM. Liu et al. (2022) put forward the generalized point operator along with the weighted versions for aggregating L-qROFS information to perform GDM. Though the L-qROFS is attractive, the assignment of weights to the multiple terms is missing and that is considered to be crucial information in the decision process. Inspired by the claim, Liu & Huang (2020) proposed a "probabilistic linguistic q-rung ortho-pair fuzzy set" (PL-qROFS) which is a generalization of L-qROFS that includes probability for the terms that potentially adds support to the decision process. Earlier works on L-qROFS have primarily focused on preference aggregation, but other decision phases have to be still explored for rational decision-making.

1.1. Research gaps and our motivation

Our motivations are as follows:

1. The instantaneous occurrence of stochastic and non-stochastic ambiguity in genuine issues is not taken into account by the LIFSs (Garg, 2018; Zhang, 2014).
2. Works from Zhang (2014), Garg (2018), and Liu & Huang (2020) form some theoretical base by presenting operational laws, but it fails to provide a rational and flexible decision process.
3. Aggregation operators (AOs) are utilized to combine all the input data into a single entity. They are effectively used for information processing, specifically decision-making, pattern recognition, data mining, and machine learning for the last two decades. Aggregation operators in Zhang's method (2014), Garg's method (2018), Liu & Huang's method (2020) cannot effectively handle extreme values provided by some experts who tend to be biased or unwilling to participate in the decision process.
4. In practice, not all of the requirements are equally important. For instance, a teaching faculty member's qualifications and experience in the field are valued more highly than their age. As a result, priority must be supplied logically to determine the weights of the criteria. Weights of criteria are not methodically derived in the relevant methodologies now in use (Garg, 2018; Liu & Huang, 2020; Zhang, 2014), which could lead to subjectivity and mistakes in the

process. These methods (Garg, 2018; Liu & Huang, 2020; Zhang, 2014) are also unable to solve the issue that arises when applying the priority of a link among criteria for the evaluation of criteria weights.

5. Experts' weight assignment is a matter of significant concern for the process of aggregation. Experts' weight must be assessed systematically to mitigate subjective randomness from. This is completely missing in Zhang's method (2014), Liu & Huang's method (2020).
6. A completely aggregation-based method under hesitant and probabilistic information for ranking is still unexplored.
7. A sensitivity analysis of criteria weights is missing in existing works (Garg, 2018; Liu & Huang, 2020; Zhang, 2014).

1.2. Contribution of the paper

Motivated by these claims, a new integrated method for GDM is put forward, which consists of the following:

1. To cope with ambiguous data, we use PLqROFSs. In fact, by enabling stochastic and non-stochastic uncertainty to emerge immediately in real-world circumstances, PLqROFS restores the dependability of the GDM techniques (Garg, 2018; Zhang, 2014). Therefore, PLqROFSs outperform LIFSs and LPFSs (Garg, 2018; Zhang, 2014).
2. Archimedean t-norms (t-Nms) and t-conorms (t-Cnms) are the generalizations of a large number of other t-Nms and t-Cnms. So, some new operational laws are developed by taking the advantage of Archimedean t-norm and t-conorm (Klement & Mesiar, 2005; Klir & Yuan, 1996; Nguyen et al., 2018) for the theoretical superiorities.
3. To provide an aggregation operator that enables argument values to support one another during the aggregate process, a power average must be used. So combining Archimedean operators and power averaging operators, Archimedean power weighted average and geometric operators are developed with their properties for handling extreme value situations from experts.
4. Criteria weights determination tools are divided into two categories: subjective and objective. The subjective methods namely AHP, FUCOM, and BWM select weights based on the consideration or judgments of decision-makers. FUCOM (Pamučar et al., 2018a) technique is extended to PL-qROFS for criteria weight determination. so that consistent weights are obtained with a rational understanding of the views of experts.
5. Also, the variance approach is put forward for experts' weight assessment through methodical procedure. This aids in the reduction of subjectivity and biases in the process.
6. A new ranking algorithm is developed by utilizing the developed AOs.
7. Sensitivity analysis of weights reveals the robustness of the developing ranking technique with PL-qROFS information.

1.3. Arrangement of the paper

We summarize the remaining paper below. Some vital concepts related to PLqROFS, Archimedean operators, and PAO are presented in Section 2. Section 3 deals with the presented Archimedean operations for the PLqROFNs and the associated PLqROF-Archimedean AOs, such as PLqROFAWAA, PLqROFAWGA, PLqROFAPWAA, and PLqROFAPWGA. The MCGDM method is discussed in the PLqROFSs context in section 4. A case study of the choice of CO_2 storage location is taken in Section 5. . Section 6 deals with the results and discussions. Finally, we wrap up the entire research in section 7.

Table 1. List of abbreviations

Acronyms	Definition
LT	Linguistic term
LTS	Linguistic term set
LSF	Linguistic scale function
LIFS	Linguistic Intuitionistic fuzzy set
LPFS	Linguistic Pythagorean fuzzy set
PLqROF	Probabilistic linguistic *q*-rung ortho-pair fuzzy
PLqROFS	PLqROF set
PLqROFN	PLqROF number
MCDM	Multi-criteria decision making
MCGDM	Multi-criteria group decision making
DE	Decision expert
DM	Decision-making
PAO	Power average operator
FUCOM	Full consistency method
PLqROFAWAA	PLqROF Archimedean weighted averaging aggregation
PLqROFAWGA	PLqROF Archimedean weighted geometric aggregation
PLqROFAPWAA	PLqROF Archimedean power-weighted averaging aggregation
PLqROFAPWGA	PLqROF Archimedean power-weighted geometric aggregation
Λ_n	Set of all positive integers up to *n*
AHP	Analytic Hierarchy Process
BMW	Best worst method
FUCOM	Full consistency method
MEREC	Method based on the removal effects of criteria
CRITIC	CRiteria Importance Through Intercriteria Correlation

2. Preliminaries

Here, we concisely review the existing concepts. For this, we first listed all the abbreviations, in Table 1, used in the entire paper for the better readability of the paper.

2.1. Linguistic term set

Definition 1 (Zadeh, 1975)**:** A linguistic term set (LTS) $\Gamma = \{\ell_u : u = 0,1,\ldots,2z\}$ is a set (ℓ_u signifies a "linguistic value (LV)" and z being a non-negative integer), holding the constraints:

1. Negation$\left(\ell_u\right) = \ell_v$ where $u + v = 2z$.

2. $\ell_u \leq \ell_v$ if $u \leq v$.

Definition 2 (Xu, 2004; 2005)**:** Suppose β_u denotes a numerical value and $\beta_u \in [0,1]$. Then a "linguistic scale function (LSF)" is represented as a mapping $\xi : \ell_u \to \beta_u$ $(u = 0,1,2,\ldots,2z)$ where $0 \leq \beta_0 \leq \beta_1 \leq \ldots.. \leq \beta_{2z}$. The symbol β_u $(u = 0,1,2,\ldots,2z)$ is used to express the LTs $\ell_u (u = 0,1,\ldots,2z)$, which symbolize the semantics of LTs.

The most frequently utilized LSF is: $\xi(\ell_u) = \dfrac{u}{2z}(u = 0,1,2,\ldots.,2z)$ and its inverse is $\xi^{-1}(\beta_u) = 2z\ell_u \ (\beta_u \in [0,1])$.

2.2. Probabilistic linguistic q-rung orthopair fuzzy sets

Definition 3 (Liu & Huang, 2020)**:** For a given set U and a LST $\Gamma = \{\ell_u : u = 0,1,\ldots.,2z\}$, a probabilistic linguistic q-rung orthopair fuzzy set **(**PLqROFS) $Q_\ell(\zeta)$ on U is given by

$$Q_\ell(\zeta) = \{< y, \mu_\ell(\zeta)(y), \gamma_\ell(\zeta)(y) >: y \in U\}$$

where $\mu_\ell(\zeta)(y) = \{\ell_r(\zeta^{(r)}) : \ell_r \in \Gamma, 0 \leq \zeta^{(r)} \leq 1\}$ and $\gamma_\ell(\zeta)(y) = \{\ell_s(\zeta^{(s)}) : \ell_s \in \Gamma, 0 \leq \zeta^{(s)} \leq 1\}$ where the LTs ℓ_r and ℓ_s are associated with probabilities $\zeta^{(r)}$ and $\zeta^{(s)}$ respectively satisfying the condition $0 \leq (\xi(\max_r \ell_r))^q + \xi(\max_s \ell_s))^q \leq 1 \ (q \geq 1)$.

If $Q_\ell(\zeta)$ is singleton, we obtain a PLqROFN $\hbar =< \{\ell_r(\zeta^{(r)})\}, \{\ell_s(\zeta^{(s)})\} >$.

To handle the aggregation process in a simplistic way, Wu et al. (2018) introduced the concept of "adjustment of probabilities". In this paper, we extend it under the PLqROF setting. To understand the process of adjustment, example 1 is employed.

Example 1: (Wu et al., 2018) For a LST $\Gamma = \{\ell_u : u = 0,1,\ldots,6\}$ and two PLqROFNs $\hbar^{(1)} =< \{\ell_2(0.7), \ell_3(0.3)\}, \{\ell_4(0.5), \ell_6(0.5)\} >$ and $\hbar^{(2)} =< \{\ell_1(1)\}, \{\ell_2(0.4), \ell_3(0.6)\} >$, the adjusted PLqROFNs are: $\tilde{\hbar}^{(1)} =< \{\ell_2(0.7), \ell_3(0.3)\}, \{\ell_4(0.4), \ell_4(0.1), \ell_6(0.5)\} >$ and $\tilde{\hbar}^{(2)} =< \{\ell_1(0.7), \ell_1(0.3)\}, \{\ell_2(0.4), \ell_3(0.1), \ \ell_3(0.5)\} >$.

Definition 4 (Liu & Huang, 2020)**:** For a PLqROFN $\hbar =< \{\ell_r(\zeta^{(r)})\}, \{\ell_s(\zeta^{(s)})\} >$, the score value is given by

$$S(\hbar) = \sum_r (\xi(\ell_r)(\zeta^{(r)}))^q - \sum_s (\xi(\ell_s)(\zeta^{(s)}))^q \tag{1}$$

Sometimes score values become insufficient for the comparison of PLqROFNs. As an instance, take two PLqROFNs $\hbar^{(1)} =< \{\ell_2(1)\}, \{\ell_2(0.5)\}, \{\ell_3(0.5)\} >$ and $\hbar^{(2)} =< \{\ell_1(1)\}, \{\ell_0(0.5), \ell_1(0.5)\} >$. If q=2, then $S(\hbar^{(1)}) = S(\hbar^{(2)})$. Score values can't efficiently deal with this situation. To solve the concern, Liu and Huang (2020) defined the accuracy value.

Definition 5 (Liu & Huang, 2020)**:** For a PLqROFN $\hbar =< \{\ell_r(\zeta^{(r)})\}, \{\ell_s(\zeta^{(s)})\} >$, the accuracy value is presented as follows:

$$A(\hbar)=\sum_r(\xi(\ell_r)(\zeta^{(r)}))^q+\sum_s(\xi(\ell_s)(\zeta^{(s)}))^q \tag{2}$$

Definition 6 (Liu & Huang, 2020): Let $\hbar^{(1)}$ and $\hbar^{(2)}$ be two PLqROFNs. Then, an ordering structure for PLqROFNs can be represented as

(A) If $S(\hbar^{(1)})\succ S(\hbar^{(2)})$, then $\hbar^{(1)}\succ\hbar^{(2)}$

(B) If $S(\hbar^{(1)})\succ S(\hbar^{(2)})$, then

(a) If $A(\hbar^{(1)})\succ A(\hbar^{(2)})$, then $\hbar^{(1)}\succ\hbar^{(2)}$

(b) If $A(\hbar^{(1)})\prec A(\hbar^{(2)})$, then $\hbar^{(1)}\prec\hbar^{(2)}$

(c) If $A(\hbar^{(1)})=A(\hbar^{(2)})$, then $\hbar^{(1)}=\hbar^{(2)}$

3. Archimedean weighted and power-weighted aggregation operators

In this section, we deploy Archimedean operations (Klement & Mesiar, 2005; Klir & Yuan, 1995; Nguyen et al., 2018) between PLqROFNs.

3.1. Archimedean operations

Definition 7: For the adjusted PLqROFNs $\tilde{\hbar}^{(j)}=<\{\ell_{rj}(\zeta^{(r)})\},\{\ell_{sj}(\zeta^{(s)})\}>(j=1,2)$, we propose the Archimedean operations among PLqROFNs as

(i) $$\tilde{\hbar}^{(1)}\tilde{\oplus}\tilde{\hbar}^{(2)}=\left\langle\xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\sum_{j=1}^{2}\psi((\xi(\ell_{rj}))^q)\right)}\right)(\zeta^{(r)}),\xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\sum_{j=1}^{2}\delta((\xi(\ell_{sj}))^q)\right)}\right)(\zeta^{(s)})\right\rangle \tag{3}$$

(ii) $$\tilde{\hbar}^{(1)}\tilde{\otimes}\tilde{\hbar}^{(2)}=\left\langle\xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\sum_{j=1}^{2}\delta((\xi(\ell_{rj}))^q)\right)}\right)(\zeta^{(r)}),\xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\sum_{j=1}^{2}\psi((\xi(\ell_{sj}))^q)\right)}\right)(\zeta^{(s)})\right\rangle \tag{4}$$

(iii) $$\lambda\tilde{\hbar}^{(1)}=\left\langle\xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\lambda\psi((\xi(\ell_{r1}))^q)\right)}\right)(\zeta^{(r)}),\xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\lambda\delta((\xi(\ell_{s1}))^q)\right)}\right)(\zeta^{(s)})\right\rangle \tag{5}$$

(iv) $$(\tilde{\hbar}^{(1)})^\lambda=\left\langle\xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\lambda\delta((\xi(\ell_{r1}))^q)\right)}\right)(\zeta^{(r)}),\xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\lambda\psi((\xi(\ell_{s1}))^q)\right)}\right)(\zeta^{(s)})\right\rangle \tag{6}$$

Theorem 1: Let $\tilde{\hbar}^{(j)}=<\{\ell_{rj}(\zeta^{(r)})\},\{\ell_{sj}(\zeta^{(s)})\}>\left(j=1,2\right)$ be two adjusted PLqROFNs. Then, for any $\lambda,\lambda_1,\lambda_2>0$, we have,

(i) $\tilde{\hbar}^{(1)}\tilde{\oplus}\tilde{\hbar}^{(2)}=\tilde{\hbar}^{(2)}\tilde{\oplus}\tilde{\hbar}^{(1)}$

(ii) $\tilde{\hbar}^{(1)}\tilde{\otimes}\tilde{\hbar}^{(2)}=\tilde{\hbar}^{(2)}\tilde{\otimes}\tilde{\hbar}^{(1)}$

(iii) $\lambda(\tilde{\hbar}^{(1)}\tilde{\oplus}\tilde{\hbar}^{(2)})=(\lambda\tilde{\hbar}^{(1)})\tilde{\oplus}(\lambda\tilde{\hbar}^{(2)})$

(iv) $(\tilde{\hbar}^{(1)}\tilde{\otimes}\tilde{\hbar}^{(2)})^\lambda=(\tilde{\hbar}^{(1)})^\lambda\tilde{\otimes}(\tilde{\hbar}^{(2)})^\lambda$

(v) $(\lambda_1+\lambda_2)\tilde{\hbar}^{(1)}=(\lambda_1\tilde{\hbar}^{(1)})\tilde{\oplus}(\lambda_2\tilde{\hbar}^{(1)})$

(vi) $(\tilde{\hbar}^{(1)})^{\lambda_1+\lambda_2}=(\tilde{\hbar}^{(1)})^{\lambda_1}\tilde{\otimes}(\tilde{\hbar}^{(1)})^{\lambda_2}$

Proof: Straightforward.

3. 2. Proposed weighted operators

The PLqROFAWAA and PLqROFAWGA development was covered in this subsection.

Definition 8: Suppose $\tilde{\hbar}^{(j)} =< \{\ell_{rj}(\zeta^{(r)})\},\{\ell_{sj}(\zeta^{(s)})\} > \left(j \in \Lambda_n\right)$ be an assortment of PLqROFNs. Then we define:

$$PLqROFAWAA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)}) = \overset{n}{\underset{j=1}{\oplus}}(w_j\tilde{\hbar}^{(j)}) \quad (7)$$

where $w_j(>0)$ is the weight of $\tilde{\hbar}^{(j)}$ such that $\sum_{j=1}^{n} w_j = 1$.

Theorem 2: The aggregated value $PLqROFAWAA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)})$ is again a PLqROFN and

$$PLqROFAWAA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)}) = \left\langle \xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\sum_{j=1}^{n} w_j\psi((\xi(\ell_{rj}))^q)\right)}\right)(\zeta^{(r)}), \xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\sum_{j=1}^{n} w_j\delta((\xi(\ell_{sj}))^q)\right)}\right)(\zeta^{(s)}) \right\rangle \quad (8)$$

Proof: Straightforward.

Definition 9: Suppose $\tilde{\hbar}^{(j)} =< \{\ell_{rj}(\zeta^{(r)})\},\{\ell_{sj}(\zeta^{(s)})\} > (j \in \Lambda_n)$ be an assortment of adjusted PLqROFNs. Then we define:

$$PLqROFAWGA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)}) = \overset{n}{\underset{j=1}{\otimes}}(\tilde{\hbar}^{(j)})^{w_j} \quad (9)$$

where $w_j(>0)$ is the weight of $\tilde{\hbar}^{(j)}$ such that $\sum_{j=1}^{n} w_j = 1$.

Theorem 3: The aggregated value $PLqROFAWGA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)})$ is again a PLqROFN and

$$PLqROFAWGA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)}) = \left\langle \xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\sum_{j=1}^{n} w_j\delta((\xi(\ell_{rj}))^q)\right)}\right)(\zeta^{(r)}), \xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\sum_{j=1}^{n} w_j\psi((\xi(\ell_{sj}))^q)\right)}\right)(\zeta^{(s)}) \right\rangle \quad (10)$$

3. 3. Archimedean Power weighted operators

Here, based on the power operator (Yager, 2013), we show the development of the operators *PLqROFAPWAA* and *PLqROFAPWGA*.

Definition 10: Suppose $\tilde{\hbar}^{(j)} =< \{\ell_{rj}(\zeta^{(r)})\},\{\ell_{sj}(\zeta^{(s)})\} > (j \in \Lambda_n)$ be an assortment of adjusted PLqROFNs. Then the *PLqROFAPWAA* operator is given by

$$PLqROFAPWAA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)}) = \overset{n}{\underset{j=1}{\oplus}}(\Theta_j\tilde{\hbar}^{(j)}) \quad (11)$$

Here, $$\Theta_j = \left(\left(1+\sum_{i=1, j\neq i}^{n} Supp(\tilde{\hbar}^{(i)},\tilde{\hbar}^{(j)})\right)w_j\right) \Big/ \left(\sum_{j=1}^{n} w_j\left(1+\sum_{i=1, j\neq i}^{n} Supp(\tilde{\hbar}^{(i)},\tilde{\hbar}^{(j)})\right)\right) \quad (12)$$

where $Supp(\tilde{\hbar}^{(i)},\tilde{\hbar}^{(j)}) = 1 - \tilde{D}(\tilde{\hbar}^{(i)},\tilde{\hbar}^{(j)})$ and $\tilde{D}(\tilde{\hbar}^{(i)},\tilde{\hbar}^{(j)})$ symbolizes the distance between $\tilde{\hbar}^{(i)}$ and $\tilde{\hbar}^{(j)}$.

Theorem 4: Then the aggregation of $PLqROFAPWAA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)})$ is again a PLqROFN and

$$PLqROFAPWAA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)})$$

$$=\left\langle \xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\sum_{j=1}^{n}\Theta_j\psi((\xi(\ell_{rj}))^q)\right)}\right)(\zeta^{(r)}),\xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\sum_{j=1}^{n}\Theta_j\delta((\xi(\ell_{sj}))^q)\right)}\right)(\zeta^{(s)})\right\rangle \quad (13)$$

where Θ_j is defined by Eq. (12).

Definition 11: Suppose $\tilde{\hbar}^{(j)} = <\{\ell_{rj}(\zeta^{(r)})\},\{\ell_{sj}(\zeta^{(s)})\}> (j\in\Lambda_n)$ be an assortment of adjusted PLqROFNs. Then the *PLqROFAPWGA* given by

$$PLqROFAPWGA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)}) = \tilde{\otimes}_{j=1}^{n}(\tilde{\hbar}^{(j)})^{\Theta_j} \quad (14)$$

Theorem 5: The aggregated value $PLqROFAPWGA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)})$ is a PLqROFN and

$$PLqROFAPWGA(\tilde{\hbar}^{(1)},\tilde{\hbar}^{(2)},...,\tilde{\hbar}^{(n)})$$

$$=\left\langle \xi^{-1}\left(\sqrt[q]{\delta^{-1}\left(\sum_{j=1}^{n}\Theta_j\delta((\xi(\ell_{rj}))^q)\right)}\right)(\zeta^{(r)}),\ \xi^{-1}\left(\sqrt[q]{\psi^{-1}\left(\sum_{j=1}^{n}\Theta_j\psi((\xi(\ell_{sj}))^q)\right)}\right)(\zeta^{(s)})\right\rangle \quad (15)$$

4. Proposed MCGDM Methodology

Consider a group decision-making problem where *m* different alternatives $A_i\ (i\in\Lambda_m)$ are evaluated by DEs $D_k\ (k\in\Lambda_l)$ in PLqROFSs setting over the set of *n* distinct criteria $C_j\ (j\in\Lambda_n)$. Our proposed methodology is as follows:

4.1. Formation of the initial assessment matrices

Step 1: Prepare PLqROF-matrices representing the initial evaluations of DEs.
Consider $\Re_k = \left[\hbar^{(ijk)}\right]_{m\times n}\ (i\in\Lambda_m, j\in\Lambda_n, k\in\Lambda_l)$ as the initial assessment of the DE *D*$_k$.
For evaluation, we take the LST $\Gamma=\{\ell_u : u=0,1,...,2z\}$.
Step 2: Find the DEs' original assessment rating in the updated PLqROFNs forms $\tilde{\hbar}^{(ijk)} = \left[<\{\ell_{ijkr}(\zeta^{(r)})\},\{\ell_{ijks}(\zeta^{(s)})\}>\right] (i\in\Lambda_m, j\in\Lambda_n, k\in\Lambda_l)$.

4.2. Determination of Experts' weights

Here, we offer a new approach for DEs weight calculation under the PLqROFS context. Popular methods from the latter context are the "analytical hierarchy process (AHP)" (Saaty, 2002), "stepwise weight assessment ratio analysis (SWARA)" (Koksalmis & Kabak, 2019), and others. Statistical variance is a useful and straightforward tool for weighting value, which considers the doctrine of variation (Liu et al., 2016). Kao (2010) correctly mentioned the effectiveness of the variance tool and concluded the model deals the hesitancy/ambiguity efficiently. Koksalmis & Kabak (2019) discussed the significance of DEs' weight and its usage in mitigating biases from direct elicitation. Driven by these claims, we plan to lengthen the variance approach for DEs weight calculation under the PLqROFSs context. Steps for calculation are given by:

Step 1: Obtain *l* matrices of order *m*×*n* with adjusted PLqROF information.

Step 2: Transform the adjusted PLqROFNs into single values f_{ijk}^{net} using Eqs. (16)-(18) to obtain net effect/significance.

$$c_{ijk} = \sum_{r} \ell_{ijkr} \times \zeta^{(r)} \tag{16}$$

$$d_{ijk} = \sum_{s} \ell_{ijks} \times \zeta^{(s)} \tag{17}$$

$$f_{ijk}^{net} = c_{ijk} + d_{ijk} \tag{18}$$

where c_{ijk} is the single value from the membership side and d_{ijk} is the single value from the non-membership side for the k^{th} DE.

Step 3: Compute the net variance exhibited by each DE using Eq. (19).

$$\text{var}^{(k)} = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{\left(f_{ijk}^{net} - \overline{f_{ijk}^{net}}\right)^2}{n-1} \tag{19}$$

where $\overline{f_{ijk}^{net}}$ and $\text{var}^{(k)}$ denotes the mean and variance respectively for k^{th} DE.

Step 4: Obtain the confidence/non-hesitation factor for each DE by taking the complement of the normalized variance. Specifically, a DE with high hesitancy will produce a low confidence factor and ultimately, the weight is low. This concept is used to obtain the weights of DEs in Eqs. (20)-(22) as

$$NV^{(k)} = \frac{\text{var}^{(k)}}{\sum_{k} \text{var}^{(k)}} \tag{20}$$

$$CF^{(k)} = 1 - NV^{(k)} \tag{21}$$

$$\varpi_k = \frac{CF^{(k)}}{\sum_{k} CF^{(k)}} \tag{22}$$

where $CF^{(k)}$ is the confidence factor, $NV^{(k)}$ is the normalized variance value, and ϖ_k is the weight of k^{th} DE.

4.3. Computation of supports and power weights

Step 1: Estimate the supports $Supp(\tilde{\hbar}^{(ijk)}, \tilde{\hbar}^{(ijt)})\ (k, t \in \Lambda_l; k \neq t)$.

Step 2: Calculate the values Θ_{ijk} utilizing Eq. (12) assuming that $\varpi_k\ (k \in \Lambda_l)$ are weights of the decision experts $D_k\ (k \in \Lambda_l)$.

4.4. Formation of aggregated and normalized decision-matrices

Step 1: Create the "aggregated-PLqROF-matrix (A-PLqROF-M)".
The *PLqROFAPWAA* or *PLqROFAPWGA* operator is employed to obtain the A-PLqROF-M $\left[\tilde{\hbar}^{(ij)}\right]_{m\times n}$ as follows:

$$\tilde{\hbar}^{(ij)} = PLqROFAPWAA(\tilde{\hbar}^{(ij1)}, \tilde{\hbar}^{(ij2)}, \ldots, \tilde{\hbar}^{(ijl)}) \tag{23}$$

$$\tilde{\hbar}^{(ij)} = PLqROFAPWGA(\tilde{\hbar}^{(ij1)}, \tilde{\hbar}^{(ij2)}, \ldots, \tilde{\hbar}^{(ijl)}) \tag{24}$$

Suppose the aggregated PLqROF matrix is $\Re = \left[\tilde{\hbar}^{(ij)}\right]_{m\times n} = \left[<\{\ell'_{ijr}(\zeta^{(r)})\},\{\ell'_{ijs}(\zeta^{(s)})\}>\right]_{m\times n}$.

Step 2: Obtain the normalized A-PLqROF-M $\Re^N = \left[\tilde{\hbar}'^{(ij)}\right]_{m\times n}$. Here,

$$\tilde{\hbar}'^{(ij)} = \begin{cases} <\{\ell_{ijr}(\zeta^{(r)})\},\{\ell_{ijs}(\zeta^{(s)})\}>, & \text{if } C_j \in Q_B \\ <\ell_{ijs}(\zeta^{(s)})\},\{\ell_{ijr}(\zeta^{(r)})\}>, & \text{if } C_j \in Q_C \end{cases} \quad (25)$$

where Q_B, Q_C denote the beneficial and cost criteria, respectively.

4.5. Determination of criteria weights

The FUCOM is defined following the concepts of comparisons in pairs of characteristics and the validation of the outcomes by defining the "deviation from the maximum consistency (DMC)" (Pamučar et al., 2018a). In recent times, there are various disciplines in which the FUCOM has been implemented successfully such as, evaluation of the airline traffic (Badi & Abdulshahed, 2019), evaluation of the period of installation of security procedure (Pamučar et al., 2018b), road traffic route evaluation for hazardous products (Noureddine & Ristic, 2019), selection of equipment for storage schemes in the logistics (Fazlollahtabar et al., 2019), evaluation urban mobility scheme (Pamucar et al., 2020), evaluation of a suitable territory in Spain's autonomous societies (Yazdani et al., 2020) and others. Saha et al. (2022a) solved the "healthcare waste treatment method (HCWTM) assessment problem using q-ROFSs, FUCOM, and "double normalization based multi-aggregation (DNMA)" methods. Mishra et al. (2022a) developed a DEA-FUCOM-MABAC methodology on HFSs for "sustainable supplier selection (SSS) in the automotive industry.

In this paper, for estimating the criteria weights, we apply the FUC0M method (Pamučar et al., 2018a).

4.6. Determination of ranking order

Step 1: Compute the A-PLqROF-M $\tilde{\hbar}^{(i)}$ $(i \in \Lambda_m)$ corresponding to A_i $(i \in \Lambda_m)$ as follows:

$$\tilde{\hbar}^{(i)} = PLqROFAWAA(\tilde{\hbar}'^{(i1)}(\zeta), \tilde{\hbar}'^{(i2)}(\zeta), \ldots, \tilde{\hbar}'^{(in)}(\zeta)) \quad (26)$$

or

$$\tilde{\hbar}^{(i)} = PLqROFAWGA(\tilde{\hbar}'^{(i1)}(\zeta), \tilde{\hbar}'^{(i2)}(\zeta), \ldots, \tilde{\hbar}'^{(in)}(\zeta)) \quad (27)$$

Here, $\tilde{\zeta}$ denotes normalized probability.

Step 2: Estimate the scores values of the A-PLqROF-M $\tilde{\hbar}^{(i)}$ $(i \in \Lambda_m)$ corresponding to A_i $(i \in \Lambda_m)$ using Definition 4.

Step 3: Prioritize the alternatives A_i $(i \in \Lambda_m)$ with the use of Definition 6.

5. Case study: CO_2 storage location selection

The commencement of investigation on the option of CO_2 neutralization by receiving and its storage in suitably chosen geological surroundings took place at the initiating in the 1990s (Bachu, 2000; Koteras et al., 2020). Sequestration of CO_2 is a significant system to obtain CO_2 emission reduction. CO_2 storage locations can be categorized into the following types: geological, biological, and oceanic sequestration, respectively (Hsu et al., 2012). From the safe CO_2 storage location, CO_2 can be injected into deep geological storage (GS) in the supercritical state. The GS of CO_2 is the most appropriate location selection. Based on the research, three kinds of GS can be applied in the procedure of CO_2 GS as deep saline structures, oil and gas reservoirs, and unmixable coal sheets (Guo et al., 2020). Following these, the deep saline water sheet has the leading storage capacity. The structure in which CO_2 is stored is known as a reservoir, and the upper portion is known as a cap rock layer. Based on the numerous parameters namely geological circumstances, engineering approaches, and force majeure of storage location, CO_2 may escape from the GS reservoir and harm the environment and human beings, so the assessment of CO_2 storage location is a significant portion of whole Carbon capture, utilization and storage (CCUS) project management. In India, a huge stable CO_2 sink has been utilized in the Deccan volcanic region, which comprises the drainage of Kutch, Deccan, and Saurashtra. The potentials of CO_2 storage in geological structures are deliberated in the characteristic of a suitable storage location of the geological structure, along with the tightness and veracity of contiguous layers, which may establish natural insulation of suitable location. The further features are associated with natural storage settings, which have an impact on sustaining the integrity of the location. To decide on GS of CO_2, it is essential to assess various attributes that assess the procedure employment from sustainability perspective. The suitability of any specific location, is consequently, based on various concerns, containing the nearness to CO_2 sources and other reservoirs with definite assets namely porosity, permeability, and leakage capacity. For CCUS to flourish, it is expected that each storage variety would always store massive amounts of injected CO_2, keeping the gas sequestered from the environment in perpetuity (Folger, 2021).

For ecological storage, CO_2 is saturated underground in a variety of topographical situations in muddy bowls. In the bowls, oil and gas basins and vacant areas, un-mineable coal layers, and saline designs are possible regions. Additional likely storing terminuses for the sequestration of CO_2 contain assimilated sinkholes, basalt rocks, and natural shale. These kinds of land provisions exist on land as well as seaward in several regions all over the world. Nevertheless, to appropriately release the injurious natural effects on environment from CO_2 accretion, the capacity must be tenacious. A storage space with lifelong worth shows that CO_2, it comprises will not stumble over the climate at a huge rate for several years. The development of CO_2 storage is stirring progressively in India (Kumar et al., 2019). Based on the literature review and DEs opinions, five possible option storage locations in the Indian context, named Coal deposits (A_1), Gas-field (A_2), Basalt (A_3), Aquifer (A_4), and Oilfield (A_5) are selected. Assume that a committee of three DEs D_1, D_2, D_3 to choose the suitable storage location over 12 criteria, and depicted in Table 2.

Table 2. The details description of considered criteria

Criteria	Description	Type
Cost (C_1)	Considers the overall cost including initial cost, transportation costs, maintenance costs, and others.	*Cost*
Storage capacity (C_2)	Considers the capacity of underground geological structures.	Benefit
Regional risks (C_3)	Considers the risks namely earthquake risk, natural risk, and others in the region.	*Cost*
Reservoir area and net thickness (C_4)	The capacity of the geological structure specified that the reservoir has a high net thickness	Benefit
Caprock permeability and thickness (C_5)	CO2 storage in the long term must demand cap rocks with satisfactory thickness for safe storage	Benefit
Transportation availability (C_6)	Quality of transportation and distribution systems.	Benefit
Porosity (C_7)	Necessary to assess the potential volume accessible for CO2 sequestration in depleted oil and gas reservoirs	Benefit
Availability of infrastructure (C_8)	Technological features and accessibility of fundamental infrastructure, pressure, and flow structures	Benefit
Distances to suppliers & Resources (C_9)	Distances to roads and power line, respectively and, availability of raw materials.	Benefit
Sustainability (C_{10})	Sustainability in the long term signifies the environmental, social and, economic feasibility of the storage.	Benefit
Legal constraints (C_{11})	Government instructions, ecological legislation, bureaucracy, work and, health safety.	Benefit
Environment and public (C_{12})	Social accountability, community behaviors, environmental protocols.	Benefit

6. Results and discussion

6.1. Problem solution

We have applied the *PLqROFAWAA* operator (taking *q*=3) to solve the case study defined in Section 5.1. Here, we consider the LST $\Gamma = \{\ell_0$ =extremely bad, ℓ_1 =bad, ℓ_2 =moderately bad, ℓ_3 =moderate, ℓ_4 =good, ℓ_5 =very good, and ℓ_6 =very very good}. Table 3 presents the initial assessment matrices. The initial evaluation ratings of the DEs in the form of adjusted PLqROFNs are depicted in Table S1 of the Supplementary material. The variance, normalized variance, confidence factor and weight of DEs are estimated by Eqs. (19), (20), (21), and (22) respectively. These are given in Table 4. We compute the supports and denote them as $S^{(kt)}\,(k, r \in \Lambda_3; k \neq t)$ in Table S2 of the supplementary material. From Eq. (12), values of $\Theta_{ijk}\ (i \in \Lambda_5; j \in \Lambda_{12}; k \in \Lambda_3)$ are evaluated (Table S3 of the supplementary file) by taking $\varpi_1 = 0.32834,\ \varpi_2 = 0.33417, \varpi_3 = 0.33749$. From Eq. (23) and taking $\delta(t) = -\ln t$ (where $t \in (0,1]$), we obtain A-PLqROF-M. The entries are then normalized using Eq. (25). The final normalized A-PLqROF-M is given in Table S4 of the supplementary material.

Next, to determine criteria weights, we assume that the ranking of criteria: $C_1>C_2>C_3>C_4>C_5>C_6>C_7>C_8>C_9>C_{10}>C_{11}>C_{12}$. The comparison is prepared with the first-ranked C_1 criterion and using the scale [1, 9]. Hence, the preferences of criteria ($\hbar_{C_{j(k)}}$) for each attribute ranked in Step 1 are achieved (Table 5). The final model for predicting the weight values uses the comparative preferences of the attributes, which are computed based on the attained preferences of the attributes as described below.

$$\min \chi$$
$$s.t.\begin{cases} \left|\frac{w_1}{w_2}-1.2\right| \le \chi,\ \left|\frac{w_2}{w_3}-1.17\right| \le \chi,\ \left|\frac{w_3}{w_4}-1.29\right| \le \chi,\ \left|\frac{w_4}{w_5}-1.15\right| \le \chi,\ \left|\frac{w_5}{w_6}-1.14\right| \le \chi,\ \left|\frac{w_6}{w_7}-1.17\right| \le \chi, \\ \left|\frac{w_7}{w_8}-1.07\right| \le \chi,\ \left|\frac{w_8}{w_9}-1.17\right| \le \chi,\ \left|\frac{w_9}{w_{10}}-1.09\right| \le \chi,\ \left|\frac{w_{10}}{w_{11}}-1.05\right| \le \chi,\ \left|\frac{w_{11}}{w_{12}}-1.13\right| \le \chi,\ \left|\frac{w_1}{w_3}-1.40\right| \le \chi, \\ \ldots \\ \left|\frac{w_5}{w_7}-1.33\right| \le \chi,\ \left|\frac{w_6}{w_8}-1.25\right| \le \chi,\ \left|\frac{w_7}{w_9}-1.25\right| \le \chi,\ \left|\frac{w_8}{w_{10}}-1.27\right| \le \chi,\ \left|\frac{w_9}{w_{11}}-1.14\right| \le \chi,\ \left|\frac{w_{10}}{w_{12}}-1.18\right| \le \chi, \\ \sum_{j=1}^{12} w_j = 1,\ w_j \ge 0, \forall j \end{cases}$$

For solving the model with Lingo 17.0 tool, the weight values of attributes and DFC $\chi = 0.00$ are computed (Table 6).

The aggregated PLqROFNs $\tilde{\hbar}^{(i)}\ (i \in \Lambda_5)$ (with normalized probabilities) corresponding to $A_i\ (i \in \Lambda_5)$ are computed using the PLqROFAWAA operator Eq. (26) (taking $\delta(t) = -\ln t, t \in (0,1]$). The score value of the aggregated PLqROFNs $S(\tilde{\hbar}^{(i)})\ (i \in \Lambda_5)$ is calculated by Eq. (1) and is given as: $S(\tilde{\hbar}^{(1)})$ =0.000064, $S(\tilde{\hbar}^{(2)})$ =0.002744, $S(\tilde{\hbar}^{(3)})$ =0.000226, $S(\tilde{\hbar}^{(4)})$ =-0.001166, $S(\tilde{\hbar}^{(5)})$ =0.001493.

Rank the alternatives $A_i\ (i \in \Lambda_5)$ according to the score values $S(\tilde{\hbar}^{(i)})\ (i \in \Lambda_5)$. Thus the priority order is: $A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$ where “f ” signifies “better than”. So, the best option is A_2.

Table 3. Initial decision matrix

		C1	C2	C3	C4	C5	C6
D1	A1	<{ℓ_3(.5), ℓ_4(.2), ℓ_5(.3)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}, {ℓ_2(.5), ℓ_3(.5)}>	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_2(1)}>	<{ℓ_5(1)}, {ℓ_3(.5), ℓ_4(.5)}>	<{ℓ_4(1)}, {ℓ_4(.7), ℓ_5(.3)}>
	A2	<{ℓ_3(.5), ℓ_4(.5)}, {ℓ_5(.3), ℓ_6(.7)}>	<{ℓ_5(1)}, {ℓ_2(.5), ℓ_4(.5)}>	<{ℓ_2(.6), ℓ_3(.4)}, {ℓ_2(1)}>	<{ℓ_4(.7), ℓ_6(.3)}, {ℓ_2(.5), ℓ_4(.5)}>	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_3(.5), ℓ_4(.5)}>	<{ℓ_6(1)}, {ℓ_5(1)}>

	A$_3$	<{ℓ_5(1)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_4(1)}, {ℓ_5(1)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_3(.5), ℓ_4(.5)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_2(.5), ℓ_3(.5)}, {ℓ_5(1)}>	<{ℓ_2(1)}, {ℓ_4(1)}>
	A$_4$	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}, {ℓ_2(.5), ℓ_3(.5)}>	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_2(1)}>	<{ℓ_5(1)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_4(1)}, {ℓ_5(1)}>	<{ℓ_4(.7), ℓ_5(.3), {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_2(.4), ℓ_3(.6)}>
	A$_5$	<{ℓ_2(.6), ℓ_3(.4)}, {ℓ_2(1)}>	<{ℓ_4(.7), ℓ_6(.3)}, {ℓ_2(.5), ℓ_4(.5)}>	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_4(.7), ℓ_6(.3)}>	<{ℓ_6(1)}, {ℓ_1(.4), ℓ_2(.6)}>	<{ℓ_5(.3), ℓ_6(.7)}, {ℓ_2(.6), ℓ_3(.4)}>	<{ℓ_2(.5) ℓ_4(.5)}, {ℓ_4(.7), ℓ_6(.3)}>
D$_2$	A$_1$	<{ℓ_5(1)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_4(1)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_4(1)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_2(1)}, {ℓ_5(1)}>	<{ℓ_3(.3), ℓ_4(.7)}, {ℓ_2(.4), ℓ_3(.6)}>
	A$_2$	<{ℓ_4(.7), ℓ_6(.3)}, {ℓ_4(.7), ℓ_6(.3)}>	<{ℓ_3(.5), ℓ_4(.5)}, {ℓ_5(.3), ℓ_6(.7)}>	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_2(.5), ℓ_4(.5)}>	<{ℓ_2(1)}, {ℓ_2(.6), ℓ_3(.4)}>	<{ℓ_2(.6), ℓ_3(.4)}, {ℓ_4(.7), ℓ_6(.3)}>	<{ℓ_5(1)}, {ℓ_3(.5), ℓ_4(.5)}>
	A$_3$	<{ℓ_2(1)}, {ℓ_2(.5), ℓ_3(.5)}>	<{ℓ_3(.3), ℓ_4(.7)}, {ℓ_4(1)}>	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.4)}, {ℓ_5(1)}>	<{ℓ_4(1)}, {ℓ_2(.4), ℓ_3(.6)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_2(1)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_3(.3), ℓ_4(.7)}>
	A$_4$	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_4(1)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_2(1)}, {ℓ_2(.5), ℓ_3(.5)}>	<{ℓ_3(.3), ℓ_4(.7)}, {ℓ_4(1)}>	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_4(1)}, {ℓ_4(1)}>
	A$_5$	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_2(.5), ℓ_4(.5)}>	<{ℓ_2(1)}, {ℓ_2(.6), ℓ_3(.4)}>	<{ℓ_2(.6), ℓ_3(.4)}, {ℓ_6(1)}>	<{ℓ_5(1)}, {ℓ_2(.5), ℓ_4(.5)}>	<{ℓ_4(.7), ℓ_6(.3)}, {ℓ_1(.4), ℓ_2(.6)}>	<{ℓ_5(.1), ℓ_6(.9)}, {ℓ_2(1)}>
D$_3$	A$_1$	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}, {ℓ_4(1)}>	<{ℓ_2(1)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_5(1)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_2(1)}>	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_2(1)}, {ℓ_2(1)}>
	A$_2$	<{ℓ_2(.6), ℓ_3(.4)}, {ℓ_3(.5),	<{ℓ_2(.5), ℓ_4(.5)}, {ℓ_5(.3),	<{ℓ_5(.3), ℓ_6(.7)}, {ℓ_4(.7),	<{ℓ_6(1)}, {ℓ_2(1)}>	<{ℓ_5(1)}, {ℓ_2(.6), ℓ_3(.4)}>	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_2(.5),

		ℓ_4(.5)}>	ℓ_6(.7)}>	ℓ_6(.3)}>			ℓ_4(.5)}>
	A$_3$	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_2(1)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_4(1)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_2(1)}>	<{ℓ_5(1)}, {ℓ_2(.4), ℓ_3(.6)}>	<{ℓ_2(1)}, {ℓ_2(1)}>
	A$_4$	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_5(1)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_2(1)}>	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_2(.3), ℓ_3(.7)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_4(1)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_5(.5), ℓ_6(.5)}>
	A$_5$	<{ℓ_5(.3), ℓ_6(.7)}, {ℓ_4(.7), ℓ_6(.3)}>	<{ℓ_6(1)}, {ℓ_2(1)}>	<{ℓ_5(1)}, {ℓ_5(.3), ℓ_6(.7)}>	<{ℓ_1(1)}, {ℓ_1(.4), ℓ_2(.6)}>	<{ℓ_3(.5), ℓ_4(.5)}, {ℓ_5(.3), ℓ_6(.7)}>	<{ℓ_5(.3), ℓ_6(.7)}, {ℓ_6(1)}>

		C$_7$	C$_8$	C$_9$	C$_{10}$	C$_{11}$	C$_{12}$
D$_1$	A$_1$	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_2(.4), ℓ_3(.6)}>	<{ℓ_2(.5), ℓ_3(.5)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_2(1)}, {ℓ_5(1)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_5(1)}>	<{ℓ_5(1)}, {ℓ_4(1)}>
	A$_2$	<{ℓ_5(.3), ℓ_6(.7)}, {ℓ_2(.6), ℓ_3(.4)}>	<{ℓ_2(.5), ℓ_4(.5)}, {ℓ_4(.7), ℓ_6(.3)}>	<{ℓ_2(1)}, {ℓ_4(.7), ℓ_6(.3)}>	<{ℓ_2(.5), ℓ_4(.5)}, {ℓ_1(.4), ℓ_2(.6)}>	<{ℓ_4(.7), ℓ_6(.3)}, {ℓ_1(.4), ℓ_2(.6)}>	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_6(1)}>
	A$_3$	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_5(1)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_3(.5), ℓ_4(.5)}, {ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_2(.4), ℓ_3(.6)}>	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}, {ℓ_2(.5), ℓ_3(.5)}>	<{ℓ_2(.4), ℓ_3(.6)}, {ℓ_2(1)}>
	A$_4$	<{ℓ_2(.5), ℓ_3(.5)}, {ℓ_5(1)}>	<{ℓ_2(1)}, {ℓ_4(1)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_3(.5), ℓ_4(.5)}>	<{ℓ_5(1)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_3(.5), ℓ_4(.5)}, {ℓ_4(.7), ℓ_5(.3)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_5(.5), ℓ_6(.5)}>
	A$_5$	<{ℓ_2(1)}, {ℓ_1(.4), ℓ_2(.6)}>	<{ℓ_2(.5), ℓ_4(.5)}, {ℓ_6(1)}>	<{ℓ_4(.7), ℓ_6(.3)}, {ℓ_3(.5), ℓ_4(.5)}>	<{ℓ_1(.4), ℓ_2(.6)}, {ℓ_5(1)}>	<{ℓ_3(.5), ℓ_4(.5)}, {ℓ_5(.3), ℓ_6(0.7)}>	<{ℓ_5(1)}, {ℓ_2(.5), ℓ_4(.5)}>
D$_2$	A$_1$	<{ℓ_1(.1), ℓ_2(.6), ℓ_3(.3)}, {ℓ_5(.5), ℓ_6(.5)}>	<{ℓ_4(1)}, {ℓ_4(.5), ℓ_5(.5)}>	<{ℓ_4(.7), ℓ_5(.3)}, {ℓ_2(.5), ℓ_3(.5)}>	<{ℓ_5(.5), ℓ_6(.5)}, {ℓ_4(1)}>	<{ℓ_2(.5), ℓ_3(.5)}, {ℓ_2(1)}>	<{ℓ_4(1)}, {ℓ_3(.3), ℓ_4(.7)}>

	A_2	$<\{\ell_4(.7), \ell_6(.3)\}, \{\ell_1(.4), \ell_2(.6)\}>$	$<\{\ell_5(.3), \ell_6(.7)\}, \{\ell_2(1)\}>$	$<\{\ell_2(.5), \ell_4(.5)\}, \{\ell_6(1)\}>$	$<\{\ell_2(.6), \ell_3(.4)\}, \{\ell_2(.5), \ell_4(.5)\}>$	$<\{\ell_6(1)\}, \{\ell_2(.6), \ell_3(.4)\}>$	$<\{\ell_2(.5), \ell_4(.5)\}, \{\ell_5(1)\}>$
	A_3	$<\{\ell_2(.5), \ell_3(.5)\}, \{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}>$	$<\{\ell_4(1)\}, \{\ell_4(1)\}>$	$<\{\ell_5(1)\}, \{\ell_5(.5), \ell_6(.5)\}>$	$<\{\ell_2(.4), \ell_3(.6)\}, \{\ell_4(1)\}>$	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_4(.7), \ell_5(.3)\}>$	$<\{\ell_4(1)\}, \{\ell_5(.5), \ell_6(.5)\}>$
	A_4	$<\{\ell_4(.7), \ell_5(.3)\}, \{\ell_2(1)\}>$	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_3(.3), \ell_4(.7)\}>$	$<\{\ell_2(.5), \ell_3(.5)\}, \{\ell_5(1)\}>$	$<\{\ell_4(1)\}, \{\ell_2(.4), \ell_3(.6)\}>$	$<\{\ell_5(1)\}, \{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}>$	$<\{\ell_2(.4), \ell_3(.6)\}, \{\ell_4(1)\}>$
	A_5	$<\{\ell_2(.5), \ell_4(.5)\}, \{\ell_2(.6), \ell_3(.4)\}>$	$<\{\ell_2(.6), \ell_3(.4)\}, \{\ell_5(1)\}>$	$<\{\ell_6(1)\}, \{\ell_4(.7), \ell_6(.3)\}>$	$<\{\ell_2(.5), \ell_4(.5)\}, \{\ell_3(.5), \ell_4(.5)\}>$	$<\{\ell_4(.7), \ell_6(.3)\}, \{\ell_4(.7), \ell_6(.3)\}>$	$<\{\ell_3(.5), \ell_4(.5)\}, \{\ell_5(.3), \ell_6(.7)\}>$
D_3	A_1	$<\{\ell_4(1)\}, \{\ell_4(.7), \ell_5(.3)\}>$	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_5(.5), \ell_6(.5)\}>$	$<\{\ell_5(1)\}, \{\ell_5(.5), \ell_6(.5)\}>$	$<\{\ell_2(1)\}, \{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}>$	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_2(.4), \ell_3(.6)\}>$	$<\{\ell_1(.1), \ell_2(.6), \ell_3(.3)\},\{\ell_2(1)\}>$
	A_2	$<\{\ell_3(.5), \ell_4(.5)\}, \{\ell_5(.3), \ell_6(.7)\}>$	$<\{\ell_5(.3), \ell_6(.7)\}, \{\ell_6(1)\}>$	$<\{\ell_4(.7), \ell_6(.3)\}, \{\ell_5(.3), \ell_6(.7)\}>$	$<\{\ell_2(1)\}, \{\ell_1(.4), \ell_2(.6)\}>$	$<\{\ell_5(.3), \ell_6(.7)\}, \{\ell_5(1)\}>$	$<\{\ell_1(.4), \ell_2(.6)\}, \{\ell_1(.4), \ell_2(.6)\}>$
	A_3	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_4(1)\}>$	$<\{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}, \{\ell_5(.5), \ell_6(.5)\}>$	$<\{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}, \{\ell_4(.7), \ell_5(.3)\}>$	$<\{\ell_2(1)\},\{\ell_5(.5), \ell_6(.5)\}>$	$<\{\ell_4(.7), \ell_5(.3)\}, \{\ell_5(1)\}>$	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_2(1)\}>$
	A_4	$<\{\ell_5(1)\}, \{\ell_2(.4), \ell_3(.6)\}>$	$<\{\ell_2(1)\}, \{\ell_2(1)\}>$	$<\{\ell_5(.5), \ell_6(.5)\}, \{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}>$	$<\{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}, \{\ell_2(1)\}>$	$<\{\ell_1(.1), \ell_2(.6), \ell_3(.3)\}, \{\ell_4(1)\}>$	$<\{\ell_2(1)\}, \{\ell_5(.5), \ell_6(.5)\}>$
	A_5	$<\{\ell_4(.7), \ell_6(.3)\}, \{\ell_5(1)\}>$	$<\{\ell_2(1)\}, \{\ell_1(.4), \ell_2(.6)\}>$	$<\{\ell_5(.3), \ell_6(.7)\}, \{\ell_2(.6), \ell_3(.4)\}>$	$<\{\ell_1(.4), \ell_2(.6)\}, \{\ell_2(.5), \ell_4(.5)\}>$	$<\{\ell_2(.6), \ell_3(.4)\}, \{\ell_3(.5), \ell_4(.5)\}>$	$<\{\ell_2(.5), \ell_4(.5)\}, \{\ell_5(.3), \ell_6(.7)\}>$

Table 4. DE's criteria weight determination

	Net variance	normalized variance	confidence factor	Weight
D_1	82.79482	0.343319	0.656681	0.32834
D_2	79.98237	0.331657	0.668343	0.33417
D_3	78.38245	0.325023	0.674977	0.33749

Table 5. Preferences of attributes

Criteria	C1	C2	C3	C4	C5	C6
$\hbar_{C_{j(k)}}$	1	1.2	1.4	1.8	2.1	2.4
Criteria	C7	C8	C9	C10	C11	C12
$\hbar_{C_{j(k)}}$	2.8	3	3.5	3.8	4	4.5

Table 6. Criteria weights

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
w_j	0.1750	0.1463	0.1249	0.0974	0.0832	0.0732
Criteria	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
w_j	0.0625	0.0585	0.0500	0.0461	0.0439	0.0391

6.2. Sensitivity investigation (SI)

Here, we utilize "sensitivity investigation (SI)" to assess the influence of an appropriate attribute on the results of the introduced model. An attribute is chosen as the "most significant attribute" if it has the highest weight value. It was suggested by several authors (Saha et al., 2021a)] that Eq. (28) can be applied to assess the weights proportionality through the assessment.

$$w_c = (1 - w_s) \times \frac{w_c^{\,0}}{W_c^{\,0}} = w_c^{\,0} - \alpha_c \times \Delta x \qquad (28)$$

where w_c = Variation in attribute weights in the SA,

w_s = Weight of the most prominent attribute,

$w_c^{\,0}$ =Original values of the attribute weights,

$W_c^{\,0}$ = Sum of actual values of modified attribute weights,

α_c = Weight coefficient of elasticity.

The relative significance for different values of weights is articulated by α_c, when we relate the variations made in the most important weight. α_c is computed using the expression as

$$\alpha_c = \frac{w_c^{\,0}}{W_c^{\,0}} \qquad (29)$$

The assumptions during the SA are as follows:

(1) α_s (Weight coefficient of elasticity of an appropriate attribute) is given;

(2) The ratio of weight values remains unchanged in the process of SA.

From Eq. (28), we observe that the variation amount applied to a weight set is signified by Δx based on weight elasticity values. We can compute the limit values of Δx as:

$$-w_s^{\,0} \le \Delta x \le \min\left\{\frac{w_c^{\,0}}{\alpha_c}\right\} \qquad (30)$$

The original weights of the characteristics are then approximated based on the pre-set parameters after we have expressed the limitations for Δx. A set of attribute weights is computed by Eq. (31) and Eq. (32).

$$w_s = w_s^{\,0} + \alpha_s \times \Delta x \tag{31}$$

$$w_c = w_c^{\,0} - \alpha_c \times \Delta x \tag{32}$$

where $w_s^{\,0}$ = Original weight of the most significant criterion to SA,

$w_c^{\,0}$ = Original value of criteria weights.

It is observed that $\sum w_s + \sum w_c = 1$ is deliberated as the general form of proportionality of weights, which holds the expression. The priority of the options is estimated by consideration of new attributes weights.

Here, the highest weight coefficient degree w_1 = 0.1750 and C_1 is considered as the most influential attribute. Subsequently, the weight elasticity coefficients (Table 7) are assessed and, the variation of weight coefficient (Δx) is found to lie in the range -0.1750≤ Δx ≤0.8251. According to the given limits for the variation of weight coefficient values of an attribute, various attribute weight sets (CWS1, CWS2, CWS3, … , CWS12) for SA are calculated. The interval -0.1750≤ Δx ≤0.8251 is separated into twelve sets. For each set, original values of criteria weights are computed by Eq. (31) and Eq. (32), and are depicted in Table 8. Thus, the scores of options are calculated for various attribute weights and are represented in Figure 1. A_1 is rated fourth in all cases, A_2 is ranked first in all cases, A_3 is ranked third in all cases, A_4 is ranked fifth in all situations, and A_5 is ranked second in all cases after evaluating the ranking positions of the alternatives for various attribute weight sets. This analysis demonstrates that, in comparison to all other options, alternative A_2 is more palatable. We can see from Figure 1 that the ranking order does not change, and as a result, the average of the SRCC (r_A) values is "1," demonstrating a "very high correlation" between alternative rankings. As a consequence, the findings demonstrate the validity and dependability of the priority of alternatives determined using the established methodology.

Table 7. weight coefficient of elasticity of attributes

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
α_c	1	0.1773	0.1514	0.1180	0.1008	0.0887
Criteria	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
α_c	0.0757	0.0709	0.0606	0.0559	0.0532	0.0474

Table 8. Twelve sets of attribute weights for SA

	CWS1	CWS2	CWS3	CWS4	CWS5	CWS6
C1	0	0.405	0.46	0.515	0.57	0.625
C2	0.1773	0.1055	0.0957	0.086	0.0762	0.0664
C3	0.1513	0.0900	0.0817	0.0734	0.0650	0.0567
C4	0.1180	0.0702	0.0637	0.0572	0.0507	0.0442
C5	0.1008	0.06	0.0544	0.0489	0.0433	0.0378
C6	0.0887	0.0527	0.0479	0.0430	0.0381	0.0332
C7	0.0757	0.0450	0.0409	0.0367	0.0325	0.0284

C8	0.0709	0.0421	0.0382	0.0343	0.0304	0.0265
C9	0.0606	0.0360	0.0327	0.0293	0.0260	0.0227
C10	0.0558	0.0332	0.0301	0.0271	0.0240	0.0209
C11	0.0532	0.0316	0.0287	0.0258	0.0228	0.0199
C12"	0.0473	0.0282	0.0255	0.0229	0.0203	0.0177
	CWS7	CWS8	CWS9	CWS10	CWS11	CWS12
C1	0.68	0.735	0.79	0.845	0.9	0.955
C2	0.0567	0.0469	0.0372	0.0274	0.0177	0.0079
C3	0.0484	0.0401	0.0317	0.0234	0.0151	0.0068
C4	0.0377	0.0312	0.0247	0.018	0.0118	0.0053
C5	0.0322	0.0267	0.0211	0.0156	0.0100	0.0045
C6	0.0283	0.0235	0.0186	0.0137	0.0088	0.0039
C7	0.0242	0.0200	0.0159	0.0117	0.0075	0.0034
C8	0.0226	0.0187	0.0148	0.0109	0.0070	0.0031
C9	0.0193	0.0160	0.0127	0.0093	0.0060	0.0027
C10	0.0178	0.0148	0.0117	0.0086	0.0055	0.0025
C11	0.0170	0.0141	0.0111	0.0082	0.0053	0.0023
C12"	0.0151	0.0125	0.0099	0.0073	0.0047	0.0021

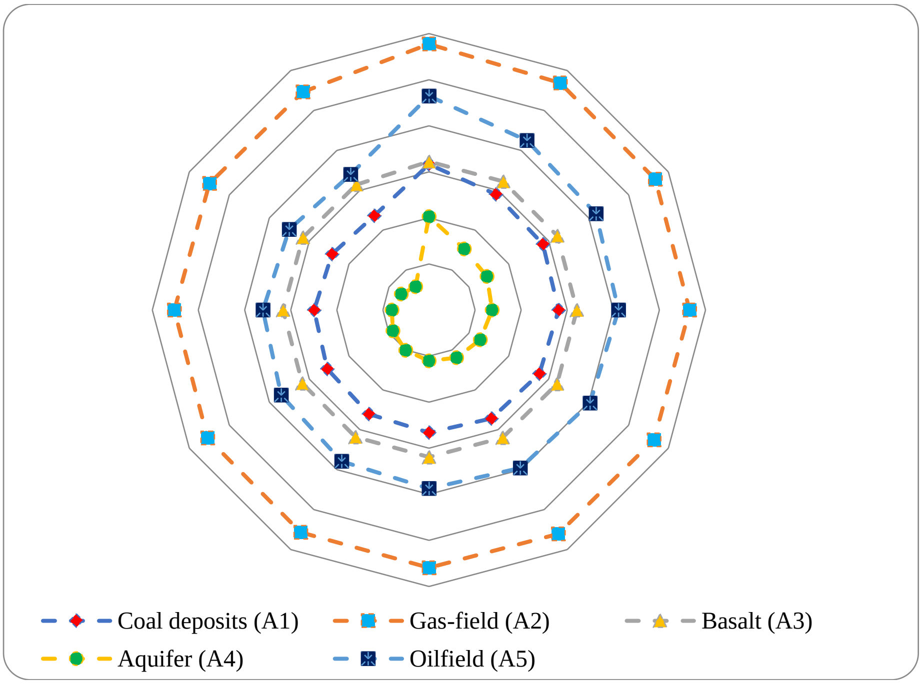

Figure 1. Scores of options over diverse attribute weight sets

6.3. Comparative investigation

This section compares results from theoretical and numerical angles. To compare the presented method 5 with some extant techniques on PLqROF, LPF and, LIF settings, respectively, we assess different extant methods, such as Liu and Huang's method (2020), Garg's method (2018) and, Zhang's method (2014). To elucidate the usefulness of the introduced model, we apply these methods to the aforementioned case study. Table 9 provides a summary of the results.

Table 9. Comparative study: Proposed with extant methods

	Methods			
	Developed approach	Liu and Huang's method (2020)	Garg's method (2018)	Zhang's method (2014)
Information type	PLqROF	PLqROF	Linguistic Pythagorean fuzzy	Linguistic intuitionistic fuzzy
Criteria weights assessment	FUCOM	Assumed	Assumed	Assumed
Prioritization between criteria	Considered	Not considered	Not considered	Not considered
Calculation of DEs weight	Variance Approach	Assumed	Normal distribution	Assumed
Generality and flexibility of the AOs	Very high	Very low	Very low	Very low
Whether captures hesitancy in prioritizations	Yes	Yes	No	No
Whether tackles with probabilistic information	Yes	Yes	No	No
Whether reduces the influences of outrageous measuring information from biased DEs	Yes	No	No	No
Ranking of alternatives	$A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$	$A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$	Can't be determined	Can't be determined

If we apply Huang and Liu's method (2020) to the same numerical example with DEs weights (initial weights) 0.32834, 0.33417, 0.33749 and criteria weights 0.1750, 0.1463, 0.1249, 0.0974, 0.0832, 0.0732, 0.0625, 0.0585, 0.0500, 0.0461, 0.0439, 0.0391, then we get a slightly different preference order $A_2 \succ A_5 \succ A_4 \succ A_1 \succ A_3$, but the optimal option coincides with our result. Next, If we apply Garg's method (2018) and Zhang's method (2014) to the same case study described earlier with DEs weights (initial weights) 0.32834, 0.33417, 0.33749 and criteria weights 0.1750, 0.1463, 0.1249, 0.0974, 0.0832, 0.0732, 0.0625, 0.0585, 0.0500, 0.0461, 0.0439, 0.0391, we do not obtain any ranking order for the reason that Garg's method (2018) and Zhang's method (2014) do not deal with the probabilistic hesitant uncertainty.
The major disadvantages of the existing methods are:

1. Liu and Huang's method (2020) is based on the behavioral TOPSIS method, which is an extension of TOPSIS methods under the PLqROF environment. However, Opricovic & Tzeng (2004) have proposed a guarantee of the non-exactness of the solution obtained using TOPSIS with the perfect solution. It means that the method developed by Liu and Huang's method (2020) is not that useful.
2. In Garg's technique (2018) and Zhang's method (2014), DEs represent information using only one LT as a membership value and only one LT as a non-membership value. However, in practice, DEs occasionally struggle to explain the outcomes of their assessments using a single LT format and are reluctant to use anyone in particular.
3. In Garg's method (2018) and, Zhang's method (2014), all assessment values are assumed to have equal importance. However, in practice, DEs can have varying levels of liking for several potential LTs.
4. In real situations, all criteria don't have equal importance. The weight of the criteria needs to be assessed very logically. In the decision-making methodologies developed by Liu and Huang (2020), Liu and Huang's method (2020), criteria weights were given arbitrarily during the aggregation of criteria values. Consequently, the final ranking gets influenced.
5. Experts' weight assignment, which is a matter of significant concern for the process of aggregation, is completely missing in Liu and Huang (2020), and, Zhang's method (2014).

Table 9 demonstrates the advantages of the introduced method. From the assessment, we can deduce the following:

1. We used PLqROFS-based information, a dependable technique, to manage ambiguous data. The dependability and adaptability of conventional DM approaches are significantly improved by PLqROFSs by accounting for the simultaneous occurrence of stochastic and non-stochastic uncertainty in real issues (Zhang, 2014; Garg, 2018). PLqROFSs are superior to LIFSs and LPFSs as a result.
2. The PLqROF-Archimedean weighted average and geometric AOs can efficiently aggregate the PLqROF information with greater generality and flexibility because PLqROF weighted average and geometric AOs (2020), PLqROF Einstein weighted average and geometric AOs, and PLqROF Hamachar weighted average and geometric AOs are specific examples of introduced AOs.
3. DEs' weights are calculated by extending the variance approach in PLqROF environment. The advantages of the variance approach are (i) it is straightforward; (ii) it can effectively reflect the DEs' hesitation during preference expression; and (iii) it assumes all preferences (data points) before

assessing the variability in the distribution, unlike other statistical measures such as minimum/maximum.

4. Our method for determining the weights of the criterion makes use of the FUCOM technique. The FUCOM indicated fewer variances to obtain the criteria weights from the most favorable ratings as compared to the BWM, AHP, and others. Thus, the method that is presented lessens MCGDM process errors.
5. The two-way comparative approach (stochastic approach like Liu and Huang's method (2020), and non-stochastic approaches like Garg's method (2018) and, Zhang's method (2014)) establishes our model as a superior and most effective one in tackling DEs' judgments in MCGDM problems.

7. Conclusions

In this paper, we have used PLqROFSs, which are generalizations of qROFSs, LIFSs and, LPFSs to handle uncertain and inaccurate information in decision-making. The existing tools, proposed so far for aggregating PLqROF data, are classified into algebraic operations, and even we observe both lacking flexibility and generality during the aggregation process. Due to this reason, we have suggested new operations amongst PLqROFNs in this study using Archimedean operations. The evolved operations' refined characteristics are examined. Additionally, we have spoken with the developed operations about several PLqROF AOs, including PLqROFWAA, PLqROFWGA, PLqROFPWAA, and PLqROFPWGA operators. In the suggested methodology, DEs weights are estimated using the variance approach, whereas criteria weights are derived using FUCOM. Here, one case study addressing the choice of a CO2 storage location is taken into consideration to better understand the created method we previously exhibited. The sensitivity assessment reveals the suggested operator's robustness. Furthermore, by drawing comparisons, we are better able to state categorically that the developed approach can be applied to resolve MCGDM issues in a PLqROF environment. It is pertinent to state here that many existing operators in connection with the PLqROF -information can be considered special cases of the developed AOs.

The managerial implications are discussed related to the study as follows:

(i) The developed model utilizes the PLqROF doctrine to offer a decision structure that integrates inaccurate information inherent in the CO_2 storage site selection.

(ii) It improves the theoretical perception of PLqROFSs by offering a new structural base for MCGDM where DEs gain flexibility from the generic structure and allows ease of decision-making.

The main limitation of the developed model is that it can't deal with any consensus-reaching process when expert(s) opinion(s) is(are) biased. Moreover, the proposed model doesn't consider dependency among multiple numbers of criteria. To overcome all these things, in the future, one can develop consensus-based decision-making models with Bonferroni mean or Hamy Mean or Heronian mean or Maclaurin Symmetric mean operators. Besides, for the determination of criteria weights, objective methods like MEREC, CRITIC, entropy measure, maximum deviation method, optimization models can be utilized. Although, we have used a case study related to sustainable material selection, other case studies (Deb et al., 2022; Hezam et al., 2023; Krishankumar et al., 2021; Liu et al., 2022; Saha et al., 2023) can also be considered.

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