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A ROUGH SET THEORY APPLICATION IN FORECASTING MODELS

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Abstract. *This paper introduces the performance of different forecasting methods for tourism demand, which can be employed as one of the statistical tools for time series forecasting. The Holt-Winters (HW), Seasonal Autoregressive Integrated Moving Average (SARIMA) and Grey model (GM (1, 1)) are three important statistical models in time-series forecasting. This paper analyzes and compare the performance of forecasting models using rough set methods, Total Roughness (TR), Min-Min Roughness (MMR) and Maximum Dependency of attributes (MDA). Current research identifies the best time series forecasting model among the three studied time series forecasting models. Comparative study shows that HW and SARIMA are superior models than GM (1, 1) for forecasting seasonal time series under TR, MMR and MDA criteria. In addition, the authors of this study showed that GM (1, 1) grey model is unqualified for seasonal time series data.*

Key words: Forecasting, Mean Absolute Percent Error (MAPE), Rough Set, Total Roughness, Maximum Dependency Degree

1. Introduction

The future planning has emerged as a key component for the success of a large number of entrepreneurs round the corners of the world. It certainly makes it easy for all the countries to formulate economic policies as well as act upon them efficiently. The perfection and accuracy are quite important in the process of highly accurate and reliable prediction. It helps the government in formulating development policies concerned with economics, infrastructure and many other sectors of the global as well as the domestic economy. It even improves the decision-making process. Time series modeling and forecasting play a key role in accurate prediction. The current trend is

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incorporated for the future prediction; it thus becomes necessary to use highly consistent and precise forecasting tools.

Since the last couple of decades, a wide variety of forecasting models is available for the study of tourist arrivals and demand forecasting (Chu, 1998, Lim & McAleer, 2002; Wang, 2004). Suggesting Autoregressive Integrated Moving Average (ARIMA) model as a most suitable model for tourism demand forecast. ARIMA was first brought out by Box-Jenkins (Box & Jenkins, 1976) and presently it is the most accepted model for forecasting univariate time series data. ARIMA model is the combined result of autoregressive (AR) and Moving Average (MA) model. ARIMA model develops an optimal univariate future prediction. Moreover, the ARIMA model has received worldwide confidence due to its ability to handle stationary and non-stationary series with seasonal and non-seasonal elements (Pankratz, 1983). But, SARIMA is particularly designed for the time series data with trends and seasonal patterns. Holt-Winters has also gained more popularity to capture trend and seasonality (Winters, 1960). The HW seasonal method consists of the forecast equation and smoothing equations for level, trend, and seasonality. Later on, grey system theory developed by Deng states that a system whose internal sources such as system characteristics operation mechanisms and architecture are completely clear, called a white system (Deng, 1982). The added advantage of this system is that the theory cannot only estimate an uncertain system but sometimes it produces ideal results. For example, Tseng et al. (2001) was reported the application of the grey model to forecast Taiwan Machinery Industry and soft drinks time series data. However, Nguyen et al. (2013) studied the forecasting of tourist arrivals in Vietnam using GM (1, 1) grey model.

From the last many years variety of forecasting criteria has been used to select the best time series models (Lim & McAleer, 2002; Wang, 2004). Modeling and forecasting consist of a large number of criteria. For instance, Chu (1998) employed MAPE and U-statistics criteria to compare the Holt winters and SARIMA models. However, Chen et al, (2009) applied MAPE criterion to evaluate the forecasting accuracy of Holt-Winters, SARIMA, and Grey models. In earlier research accuracy of forecasting models have been evaluated using error based criteria (Goh & Law, 2002; Law & Au, 1999; Law, 2000). Sometimes it may be possible that one model may become a good one due to some set of criteria but at the same time some other model may turn out to be the best one due to some other set of criteria. Moreover, these indicators have very much been exploited and only marginal improvements might be expected from their continued use. This research proposed a new approach that applies rough set theory to select the highly accurate models in time series forecasting. The rough set theory has been introduced to deal with vagueness, imprecision, and uncertainty. The original rough set theory depends on the equivalence relation (indiscernibility relation). This approach is taken into consideration the attributes in accordance with the normalized values (Goh & Law, 2003). Rough set theory has been able to overcome one of its advantages in association with statistical analysis during the process of attribute selection using rough set indicators (Hassanein & Elmelegy, 2013; Herawan et al., 2010a, 2010b).

Current research is an extended effort of Chen et al. (2009) work where they have used the Holt-Winters (HW) model, Seasonal Autoregressive Integrated Moving Average (SARIMA) model and Grey Model (GM (1, 1)). GM (1, 1) has been used to define monthly inbound air travel arrivals to Taiwan and to distinguish the models based on their respective performance. Mean Absolute Percent Error (MAPE) has been used as an indicator to measure forecast accuracy. Based on the results derived, they concluded that the HW and SARIMA models are better reliable models than GM (1, 1).

The objective of this paper is to obtain the best forecasting models using TR, MMR, and MDA rough set indicators. Based on rough set information table, those techniques are used to calculate roughness of models. Then, compare these three models in accordance with the roughness. The authors of this study showed that the GM (1, 1) is an inadequate model for forecasting with seasonality as compared to HW and SARIMA models.

The rest of the research paper is organized as follows; Section 2 contains literature review, Section 3 briefly introduces the basic concepts rough set theory and some related properties. Section 4, presents an algorithmic approach for the evaluation of rough data using MAPE indicator. In Section 5, the experimental design and experimental results have been discussed. Finally, the paper is concluded in section 6.

2. Literature review

In recent years, rough set theory have been employed in various literature to select the clustering attributes. For example, Mazlack et al. (2000) proposed Bi-Clustering (BC) technique depend on balanced/unbalanced bi-valued attributes and Total Roughness (TR) technique based on the average accuracy of roughness (Pawlak, 1982; Pawlak & Skowron, 2007). The TR technique is useful for selecting the clustering attributes in the data set, where the maximum TR is the maximum accuracy for selecting clustering attributes. Three indicators i.e. TR, MMR, and MDA of the rough set theory have been successfully used. For instance, Parmar et al. (2007) developed a new method called Min-Min Roughness (MMR) to develop BC technique for the information system with many valued attributes. In this technique, attributes for approximation are calculated using well-known corporate to the lower and upper approximations of a subset of the universe in the information system. Herawan et al. (2010a, 2010b) developed a new technique known as Maximum Dependency of Attributes (MDA) to select clustering attributes. MDA technique is based on the dependency of attributes using rough set theory in an information system. These three techniques TR, MMR and MDA provide the same outcome in selecting the attributes. This makes the rough set criteria a very useful to select the different attributes. However, in previous literature, there is no any link of rough set theory with relationship time series modeling to select the best forecasting models. In time series analysis and forecasting, the selection of highly accurate model is very important to evaluate the best time series model. Hence, this research proposed a rough set criterion for strong evidence in the selection of best suitable time series models that is different another traditional statistical indicator.

Rough set theory has been consistently employed in a variety of research areas for the extraction of decision rules (Law & Au 1998, 2000, Goh & Law, 2003; Liou et al., 2016). Celotto et al. (2012) applied rough set theory based forecasting model in data of tourist service demand.. Moreover, Li et al. (2011) predicted tourism in Tangshan city of China using rough set model. Golmohammadi and Ghareneh (2011) analyze the importance of travel attributes by rough set approach. Celotto et al. (2012) applied rough set theory to summarize tourist evaluations of a destination. Faustino et al. (2011) present a rough set analysis of electrical charge demand in the United States and the level of the Sapucal river in Brazil. Liou et al. (2016) used the rough set theory to study the airline service quality to Taiwan. Sharma et al. (2019) proposed hybrid rough set based forecasting model and applied on tourism demand of air transportation passenger data set in Australia tourism demand.

Rough set theory use to alter the roughness of a data, which has been successfully applied to various real life decision making problems (Karavidić & Projović, 2018; Roy et al., 2018; Vasiljević et al., 2018). Moreover, the rough set concept can definitely be implemented to sets categorized by means of immaterial facts wherein statistical tools fail to provide fruitful outcomes (Pawlak, 1991). Pamučar et al. (2018) proposed interval rough number enabled AHP-MABAC model for web pages evaluation. Sharma et al. (2018) applied Modified Rough AHP-Mabac Method for Prioritizing Indian Railway Stations.

3. Rough Set Theory

The rough set theory was first introduced by Pawlak (1982). The rough set concept is a new mathematical technique to tackle vagueness, imprecision, and uncertainty (Pawlak, 1982; Pawlak & Skowron, 2007). It is a vital tool to examine the degree of dependencies and minimize the number of attributes within the dataset. Its success is partly owed to the following properties: (1) Analysis is performed on the hidden fact of the data; (2) Supplementary information on data is not required like specialist awareness or thresholds; (3) Equivalent relation is a basic idea of classical rough set theory. Whereas, the attribute might be assign with both the values symbolic or real.

Pawlak proposed that the rough set theory is established on the assumption that with every member of the universe of discourse we relate some information. For example, symptoms of the disease develop a crucial part of information where objects are the patients suffering from the certain disease. The objects become indiscernible (similar) when characterized by the same information in view of the available information about them. The indiscernibility relation created in this way is the mathematical foundation of the rough set theory.

The original concept of the rough set theory is the induction of approximation. The main aim of the rough set theory is the approximation of a set by a pair of two crisp sets called the lower and upper approximations of the sets.

3.1 Indiscernibility Relation

Let U be the non-empty finite set of all objects known as the universe and A is the finite set of all attributes, then the couple $S = (U, A)$ is known as an information system. For any non-empty subset B of A is associated with an equivalence relation INDS(B) relation,

$$INDS(B) = \{ (y_i, y_j) \in U \times U \mid \forall\, b \in B, b(\mathrm{y_i}) = \mathrm{b(y_j)} \} \quad (1)$$

where $b(y_i)$ represents the value of attribute b for the element y_i. $INDS(B)$ is called the Indiscernibility relation on U. The notion $[y_i]_{INDS(B)}$ represent the equivalence class of the indiscernibility relation. $[y_i]_{INDS(B)}$ is also called as elementary set with respect to the attribute B.

3.2. Lower and Upper Approximation

Lower approximation and upper approximation (Pawlak, 1982, 1991) of any set can be defined as follows:

For an information system S = (U, A) Given the set of attribute B ⊆ A, Y ⊆U, the lower and upper approximation of Y are defined as follows respectively,

$$\underline{Y_B} = \cup \{y_i \mid [y_i]_{INDS(B)} \subseteq Y\} \quad (2)$$

$$\overline{Y_B} = \cup \{y_i \mid [y_i]_{INDS(B)} \cap Y \neq \emptyset\} \quad (3)$$

Clearly, lower approximation contains all members which certain objects of Y and upper approximation consists all members which possible objects of Y.
The boundary region is the set of members that can possible member, but not surely, defined as follow:

$$BND_B(Y) = \overline{Y_B} - \underline{Y_B} \tag{4}$$

The boundary region of an exact (crisp) set is an empty set like the lower approximation and upper approximation of exact set are similar. If the boundary region of a set is non-empty i.e. $BND_B(Y) \neq \emptyset$, then the set Y has been referred to as rough (vague).

3.3. Roughness (R)

Inexactness of a category (set) is one of the reasons behind the existence of boundary line region. As the boundary line region of a category increases, the accuracy of the category decreases. To model such kind of imprecision the concept of accuracy of approximation (Pawlak, 1991) is very much required. Accuracy measure represented as follow:

$$\alpha_B(Y) = \frac{card\ \underline{\underline{Y_B}}}{card\ \overline{\overline{Y_B}}}$$

The accuracy is intended to compute the degree of satisfaction of our knowledge about the category (set). Obviously $0 \leq \alpha_B(Y) \leq 1$. If $\alpha_B(Y)$ =1, Y is exact with respect to B, if $\alpha_B(Y) < 1$, Y is rough with respect to B.

Assume that an attribute $a_i \in A$ having k-distinct values, say $\alpha_k, k = 1,2, \ldots, m$. Suppose $Y(a_i = \alpha_k), where\ k = 1,2, \ldots, m\ i$ a subset of the objects consists k-distinct values of attribute a_i. The roughness of TR (Mazlack et al., 2000) of the set$(a_i = \alpha_k)$, $k = 1, 2, \ldots, m$, with respect to aj, where $i \neq j$, represented by $R_{a_j}(Y \mid a_i = \alpha_k)$ as is defined by

$$R_{a_j}(Y \mid a_i = \alpha_k) = \frac{\left|\underline{\underline{\mathrm{Y}_{\mathrm{a_j}}(a_i=\alpha_k)}}\right|}{\left|\overline{\overline{\mathrm{Y}_{\mathrm{a_j}}(a_i=\alpha_k)}}\right|}, k = 1, 2, \ldots, m \tag{5}$$

3.3.1. Mean roughness (MR)

The values of Mean roughness of an attribute $a_i \in A$ with respect to another attribute $a_j \in A$, where, $i \neq j$, represented by the following formula

$$Rough_{a_j}(a_i) = \frac{\sum_{k=1}^{|V(a_i)|} R_{a_j}(Y|a_i=\alpha_k)}{|V(a_i)|} \tag{6}$$

where $V(a_i)$ is the set of all values of attribute $a_i \in$ A.

3.3.2. Total Roughness (TR)

The total roughness of the attribute $a_i \in$ A with respect to the attribute $a_j \in A$, where, $i \neq j$, represented by $TR\ (a_i)$, is defined by

$$TR(a_i) = \frac{\sum_{j=1}^{|A|} Rough_{a_j}(a_i)}{|A|-1} \tag{7}$$

The maximum value of TR, the finest selection choice of clustering attributes.

3.3.3. Minimum – Minimum Roughness (MMR)

From the TR system, the mean roughness of attribute a_i with respect to attribute a_j, where, $i \neq j$ is define by

$$MMRough_{a_j}(a_i) = 1 - \frac{\sum_{k=1}^{|V(a_i)|} R_{a_j}(Y|a_i=\alpha_k)}{|V(a_i)|}$$

$$MMRough_{a_j}(a_i) = 1 - Rough_{a_j}(a_i) \quad (8)$$

3.4. Maximum Dependency Attribute (MDA) [Herawan et al. (2010a, 2010b)]

Suppose S = (U, A) is information system and let a_i and a_j be any subsets of A. Dependency attribute a_i on a_j in a degree k (0<k<1), is denoted by $a_j \Rightarrow ka_i$. The degree k is derived:

$$k = \frac{\sum_{Y \in U/a_i} \left|\underline{a_j}(Y)\right|}{|U|} \quad (9)$$

If degree k =1, a_i is fully depends on a_j. Otherwise a_i partially depends on a_j.

For an assortment of clustering attributes a maximum degree of dependency of attributes is most appropriate. Because as greater the degree of dependence of attribute as much accurate for the assortment of clustering attribute.

3.5. Accuracy of TR, MMR and MDA Techniques

Section 4 involves the applications of TR, MMR and MDA techniques, used for the selection of better forecasting models. The calculation of the accuracy consists of (i) TR which makes the use of the total average of mean roughness (ii) MMR which relies upon the minimum of mean roughness and (iii) MDA depends on the maximum degree of dependency to select the forecasting attribute model. After determining the roughness and mean roughness of the attributes, we select the best model based on maximum TR, maximum MDA values and lower values of minimum roughness.

Example 1. To demonstrate the degree of dependency of attributes, we consider the information system as shown in Table 1.

Table 1. Representation of Food Junction database with ten objects and five attributes

Customer	Food quality (F_1)	Menu variety (F_2)	Service (F_3)	Atmosphere (F_4)	Cleanliness (F_5)
y_1	Good	Good	Poor	Good	Average
y_2	Poor	Poor	Poor	Average	Good
y_3	Poor	Good	Average	Good	Average
y_4	Good	Average	Poor	Average	Average
y_5	Good	Average	Poor	Good	Poor
y_6	Average	Good	Average	Good	Poor
y_7	Average	Good	Good	Good	Good
y_8	Poor	Poor	Poor	Average	Good
y_9	Good	Average	Poor	Average	Average
y_{10}	Good	Good	Average	Average	Good

Table 1 shows the food junction data set in which $U = \{y_1, y_2, .., y_{10}\}$ (where y_j represents a customer) and $A = (F_1, F_2, F_3, F_4, F_5)$, being $F_1, F_2, F_3, F_4,$ and F_5 attribute set.

- In dataset 1, indiscernibility relations can described as follows: $U/F_1 = \{(y_2, y_3, y_8), (y_6, y_7), (y_1, y_4, y_5, y_9, y_{10})\}$
  Since $F_1(y_1) = F_1(y_4) = F_1(y_5) = F_1(y_9) = F_1(y_{10}) = good$
  $F_1(y_2) = F_1(y_3) = F_1(y_8) = poor$

$F_1(y_6) = F_1(y_7) = average$

- $U/F_2 = \{(y_2, y_8), (y_4, y_5, y_9), (y_1, y_3, y_6, y_7, y_{10}),$
- $U/F_3 = \{(y_1, y_2, y_4, y_5, y_8, y_9), (y_3, y_6, y_{10}), (y_7),$
- $U/F_4 = \{(y_2, y_4, y_8, y_9, y_{10}), (y_1, y_3, y_5, y_6, y_7)\},$
- $U/F_5 = \{(y_5, y_6), (y_1, y_3, y_4, y_9,), (y_2, y_7, y_8, y_{10})\},$

Lower and upper approximation of subsets of U of attribute F_1 with respect to attribute F_2, F_3, F_4 and F_5 are given below:

(i) F_1 with respect to F_2

$\underline{X(F_1 = poor)} = \{y_2, y_8\}$, $\overline{X(F_1 = poor)} = \{y_1, y_2, y_3, y_6, y_7, y_8, y_{10}\}$

$\underline{X(F_1 = average)} = \{\emptyset\}$, $\overline{X(F_1 = average)} = \{y_1, y_3, y_6, y_7, y_{10}\}$

$\underline{X(F_1 = good)} = \{y_4, y_5, y_9\}$, $\overline{X(F_1 = good)} = \{y_1, y_3, y_4, y_5, y_6, y_7, y_9, y_{10}\}$.

(ii) F_1 with respect to F_3

$\underline{X(F_1 = poor)} = \{\emptyset\}$, $\overline{X(F_1 = poor)} = \{y_1, y_2, y_3, y_4, y_5, y_6, y_8, y_9, y_{10}\}$

$\underline{X(F_1 = average)} = \{y_7\}$, $\overline{X(F_1 = average)} = \{y_3, y_6, y_7, y_{10}\}$

$\underline{X(F_1 = good)} = \{\emptyset\}$, $\overline{X(F_1 = good)} = \{y_1, y_2, y_3, y_4, y_5, y_6, y_8, y_9, y_{10}\}$.

(iii) F_1 with respect to F_4

$\underline{X(F_1 = poor)} = \{\emptyset\}$, $\overline{X(F_1 = poor)} = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}\}$

$\underline{X(F_1 = average)} = \{\emptyset\}$, $\overline{X(F_1 = average)} = \{y_1, y_3, y_5, y_6, y_7\}$

$\underline{X(F_1 = good)} = \{\emptyset\}$, $\overline{X(F_1 = good)} = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}\}$.

(iv) F_1 with respect to F_5

$\underline{X(F_1 = poor)} = \{\emptyset\}$, $\overline{X(F_1 = poor)} = \{y_1, y_2, y_3, y_4, y_7, y_8, y_9, y_{10}\}$

$\underline{X(F_1 = average)} = \{\emptyset\}$, $\overline{X(F_1 = average)} = \{y_2, y_5, y_6, y_7, y_8, y_{10}\}$

$\underline{X(F_1 = good)} = \{\emptyset\}$, $\overline{X(F_1 = good)} = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}\}$

To explain in finding the degree of dependency of the attributes, we examine the information system as shown in Table1. From Table1, depend on each attribute there are five classes of U induced by indiscernibility relation on each attribute. Using (9), the degree of dependency of attribute F_2 on attribute F_1, denoted by, $F_1 \Rightarrow_k F_2$ can be computed as follows:

$$k = \frac{\sum_{Y \in U/F_2} \left| \underline{F_1(Y)} \right|}{|U|} = \frac{|\{y_6, y_7\}|}{|\{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}\}|} = 0.2$$

Similarly, we can compute $F_2 \Longrightarrow_k F_3$ as

$$k = \frac{\sum_{Y \in U/F_3} \left| F_2(Y) \right|}{|U|} = \frac{|\{y_2,\ y_4,\ y_5,\ y_8,\ y_9\}|}{|\{y_1, y_2,\ y_3,\ y_4,\ y_5,\ y_6, y_7, y_8, y_9, y_{10}\}|} = 0.5.$$

The degree of dependency of all the attributes in Table 1 can be outlined as in Table 2. In MDA performance, if the maximum degree of an attribute is equal with other attributes, then we check the next maximum value of the attribute. From the Table 2 the first highest degree of the attribute, i.e., 0.5 comes out in attributes F_1 and F_3. Then the second maximum degree of attribute F_1 is 0.1 even as in attribute F_3 is 0. Hence, attribute F_1 has been elected as a good attribute as compared with F_2, F_3, F_4 and F_5 attributes. Therefore, Food quality (F_1) is the most appropriate attribute for Food Junction data among rest of the attributes.

Table 2. Degree of Dependency using MDA criteria

	F_1	F_2	F_3	F_4	F_5	MDA
F_1	.....	0.5	0.1	0	0	0.5 0.1
F_2	0.2	.....	0.4	0	0	0.4 0.2
F_3	0	0.5	.....	0	0	0.5 0
F_4	0.2	0.2	0.1	.....	0.2	0.2
F_5	0	0.2	0.1	0	.....	0.2 0.1

4. Data description

4.1. Mean absolute percentage error (MAPE)

The mean absolute percentage error (MAPE), as well renowned as mean absolute percentage deviation (MAPD), is used as a measure of accuracy for constructing actual and fitted time series data. It usually denoted by the formula:

$$MAPE = \frac{\sum_{t=1}^{n} (|Ac_t - \widehat{Pr}_t|) / Ac_t}{n} \tag{10}$$

Where Ac_t $(t = 1, 2, \ldots, n)$ is the actual value, $\widehat{\text{Prt}}$ $(t = 1, 2, \ldots, n)$ represents the predicted values and n is the total number of observations.

4.2. Evaluation of rough set data

According to Lewis (1982), the range of MAPE is fixed for time series models. But there is no any strong evidence for others criteria. Therefore MAPE is suitable criteria in the rough set analysis as compared to different indicators. For the measurement of the accuracy of roughness, we normalize the MAPE values in the following two ways.

Case1. MAPE range according to Lewis (1982), the value of MAPE being less than 10% denotes a high degree of accuracy. Moreover, when it lies between 10-20% predictions is good, 20-50% is reasonable and more than 50% depicts inaccuracy in prediction.

Case 2. The accuracy in the level of prediction relies on the minimum value of MAPE. Henceforth in the paper, we have proposed our assumptions which offer us a valid and highly reliable result. In this sense value of MAPE, less than 5% provides highly accurate results. However, when it lies in the range of 5% to 10% the prediction is appreciably good and more than 10% shows inaccuracy in the prediction.

The proposed procedures of modeling are: first normalized decision table is constructed in the range (0 to 1). Then TR, MMR and MDA are calculated to select best criteria for forecasting. Also, the best criteria are selected on the basis of minimum MMR value and maximum TR and MDA. All the steps also described in Figure 1.

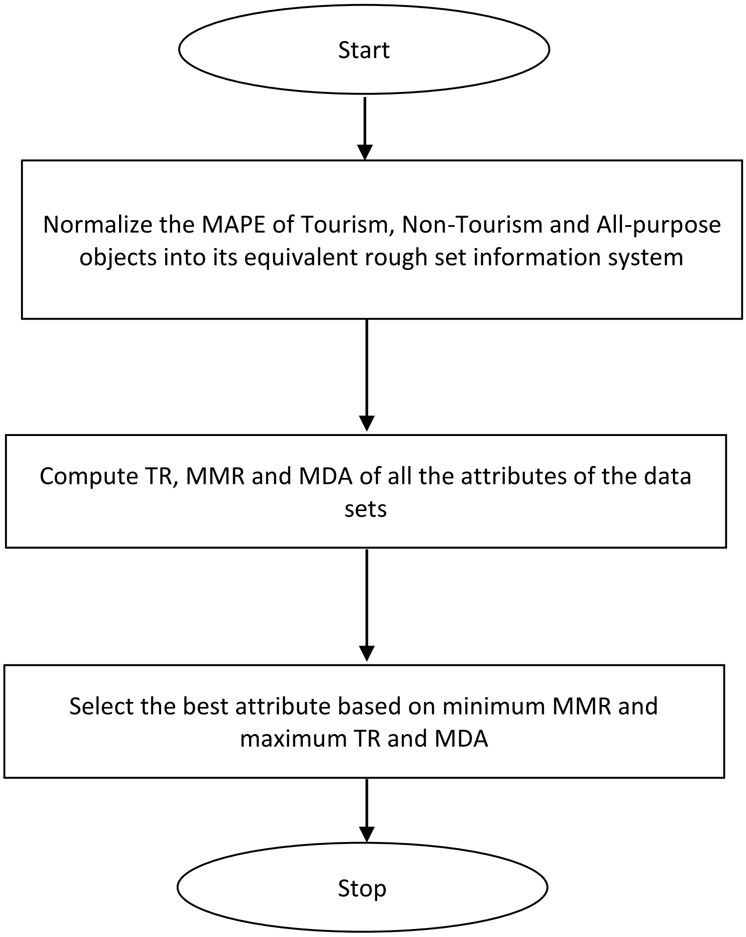

Figure 1. The Stages of Building best forecasting models.

4.3. Data

The study of Chen et al. (2009) has been used to evaluate MAPE value for this research as shown Table 3. In this research, the author analysed total arrivals of 12 countries objects, of Japan, Hong Kong, and the US during the period 1996: 1-2007:12 and three attributes SARIMA, Holt-Winters and Grey time series models. The entirety data series are separated into two groups, Tourism, and Non-Tourism.

5. Result analysis and discussions

In this section, we begin with the analysis of the data and make a comparison of their precise results. The comparison of the performance is based on the accuracy of the models. We have considered two test cases to bring out the comparison and evaluation of the accuracy of the three models using three different rough set criteria TR, MMR, and MDA.

5.1. Case 1: The rough set information system

We have incorporated three models which are Holt-Winters, SARIMA, and Grey model. All these three models have three normalized values, i.e. reasonable, good and high accuracy. These values are mention in Table 4 for considered twelve objectives.

Table 3. MAPE of fitted models (%) (Chen et al., 2009)

Objects	Holt-Winters	SARIMA Model	Grey Model
Tourism purpose			
Japan	17.90	10.24	29.80
Hong Kong	19.27	15.97	24.61
US	5.23	5.01	16.37
Total	16.56	7.03	31.16
Non-tourism purpose			
Japan	4.75	4.02	8.85
Hong Kong	4.03	4.87	19.57
US	3.54	3.24	7.55
Total	2.69	2.94	4.25
All-purposes			
Japan	10.54	8.50	23.12
Hong Kong	11.52	13.80	22.58
US	2.81	2.72	8.54
Total	5.55	8.95	16.04

Table 4. Rough set information system of three models (Case1)

Objects	Holt-Winters	SARIMA Model	Grey Model
Tourism purpose			
Japan	Reasonable	Reasonable	Reasonable
Hong Kong	Reasonable	Reasonable	Reasonable
Us	Good	Good	Reasonable
Total	Reasonable	Good	Reasonable
Non-tourism purpose			
Japan	High accuracy	High accuracy	Good
Hong Kong	High accuracy	High accuracy	Reasonable
Us	High accuracy	High accuracy	Good
Total	High accuracy	High accuracy	High accuracy
All-purposes			Reasonable
Japan	Reasonable	Good	Reasonable
Hong Kong	Reasonable	Reasonable	Good
US	High accuracy	High accuracy	Reasonable
Total	Good	Good	Reasonable

To serve this purpose we obtain the equivalence classes determined by indiscernibility relation from equation (1).

(i) $Y(\ Holt\ Winters\ =\ reasonable\ ) = \{\ 1,2\ 4,9,10\}, Y(Holt\ Winters\ = good) = \{3,12\},\ Y(Holt\ Winters\ =\ High\ accuracy) = \{5,6,7,8,11\},$ $U\ /\ Holt\ Winters\ = \{\{1,2,4,9,10\},\ \{3,12\},\ \{5,6,7,8,11\}\}$

(ii) $Y\ (SARIMA\ =\ reasonable) = \{1,2,10\}, Y(SARIMA\ =\ good) = \{3,4,9,12\}, (SARIMA\ =\ High\ accuracy) = \{5,6,7,8,11\}, U/SARIMA = \{\{1,2,10\},\{3,4,9,12\},\{5,6,7,8,11\}\}$

(iii) $Y\ (GM(1,1)\ =\ reasonable) = \{1,2,3,4,6,9,10,12\}, Y(GM(1,1) = good) = \{5,7,11\}, Y(GM(1,1)\ =\ High\ accuracy) = \{8\}, U/ GM(1,1) = \{\{1,2,3,4,6,9,10,12\},\{5,7,11\},\{8\}\}$.

In the next step, we finalize the lower and upper approximations of subset Y of U based on Holt-Winters with respect to SARIMA and Grey model using the formula in Equation (2) and (3).

(i) Holt-Winters with respect to SARIMA

$\underline{Y\ (Holt\ Winters\ =\ reasonable)} = \{1, 2, 10\}$,

$\overline{Y\ (Holt\ Winters\ =\ reasonable)} = \{1, 2, 3, 4, 9, 10, 12\}$,

$\underline{Y\ (Holt\ Winters\ =\ good)} = \{\emptyset\}$,

$\overline{Y\ (Holt\ Winters\ =\ good)} = \{3, 4, 9, 12\}$,

$\underline{Y\ (Holt\ Winters\ =\ High\ accuracy)} = \{5, 6, 7, 8, 11\}$,

$\overline{Y\ (Holt\ Winters\ =\ High\ accuracy\ )} = \{5, 6, 7, 8, 11\}$.

(ii) Holt-Winters with respect to grey

$\underline{Y\ (Holt\ Winters\ =\ reasonable)} = \{\emptyset\}$,

$\overline{Y\ (Holt\ Winters\ =\ reasonable)} = \{1, 2, 3, 4, 6, 9, 10, 12\}$,

$\underline{Y\ (Holt\ Winters\ =\ good)} = \{\emptyset\}$,

$\overline{Y\ (Holt\ Winters\ =\ good)} = \{1, 2, 3, 4, 6, 9, 10\}$,

$\underline{Y\ (Holt\ Winters\ =\ High\ accuracy)} = \{5, 6, 7, 8, 11\}$,

$\overline{Y\ (Holt\ Winters\ =\ High\ accuracy)} = \{5, 6, 7, 8, 11\}$.

Then, we obtain the roughness of all models, where TR makes the use of formula as in the Equation (5). The roughness of subsets of U consist distinct value of Holt-Winters with respect to SARIMA and Grey models are given below.

(i) Roughness(R) of Holt Winters with respect to SARIMA model
$R_{reasonable} = 0.42857$, $R_{good} = 0$ and $R_{high\ accuracy} = 1$

(ii) Roughness of Holt-Winters with respect to Grey model
$R_{reasonable} = 0$, $R_{good} = 0$ and $R_{high\ accuracy} = 1$.

Further, we obtain the mean roughness of all attributes model. Where TR uses the formula in Equation (6).

(i) Mean roughness(MR) of Holt-Winters with respect to SARIMA model
$MR_{HW} = 0.47619$.

(ii) Mean roughness of Holt-Winters with respect to grey model
$MR_{HW} = 0.11$.

Now finally, we obtain the total roughness of Holt-Winters with respect to the SARIMA and Grey model.

$TR(a_i)$ = (0.47 + 0.11) /2= 0.2931, where a_i is the numbers of attributes model, $i = 1, 2$.

Now, from Table 5, we observe that the TR value 0.2931 of Holt-Winters is much higher than TR value 0.1944 of Grey model. Thus, the Holt-Winters consider as a better model than the Grey model. Similarly, TR value of SARIMA is greater than that of the Grey model. Hence, the SARIMA model serves us as a better model in view of the Grey model.

Table 5. The total roughness of three model using TR criteria

Attribute	Holt-Winters	SARIMA Model	Grey Model	Total Roughness
Holt Winters	………	0.47619	0.11	0.2931
SARIMA Model	0.42857	………...	0.11	0.26929
Grey Model	0.19444	0.19444	…………...	0.19444

Fourthly, we find the mean roughness of the three attributes using the principle of the MMR technique as in the equation (8) which differs from that in the TR technique. Mean roughness of the model Holt-Winters with respect to SARIMA and Grey model is as below.

(i) MMR of Holt Winters with respect to SARIMA model
$MMR_{HW} = 1 - MR_{HW} = 1 - 0.47619 = 0.52381$

(ii) MMR of Holt Winters with respect to Grey model
$MMR_{HW} = 1 - MR_{HW} = 1 - 0.11 = 0.89$

Consequently, the MMR of the entire models is depicted in Table 6. We perceive that the MMR of the Holt-Winters and SARIMA have attained their minimum values respectively i.e., 0.52381 and 0.57143. These values are lower than that of the MMR of Grey model i.e., 0.80556. Thus, Holt-Winters and SARIMA are preferred as the good models. Also, the results of the experiment were summarized in Figure 2 and Figure 3 shows the accuracy of three models with TR, MMR, and MDA values. However, as far as MDA techniques are considered, Holt-Winters and SARIMA have the maximum degree of dependency in Table 7 in contrast with the Grey model. As a result, the Holt-Winters and SARIMA are preferred as the good models.

Table 6. The minimum-minimum roughness of three models using MMR criteria

Models	Holt-Winters	SARIMA Model	Grey Model	MMR
Holt Winters	……...	0.52381	0.89	0.52381
SARIMA Model	0.57143	………...	0.89	0.57143
Grey Model	0.80556	0.80556	………...	0.80556

Table 7. Degree of Dependency using MDA criteria of three models

Models	Holt-Winters	SARIMA Model	Grey Model	MDA
Holt Winters	.....	0.6667	0.333	0.6667
SARIMA Model	0.583	.....	0.333	0.583 0.333
Grey Model	0.583	0.583	.....	0.583

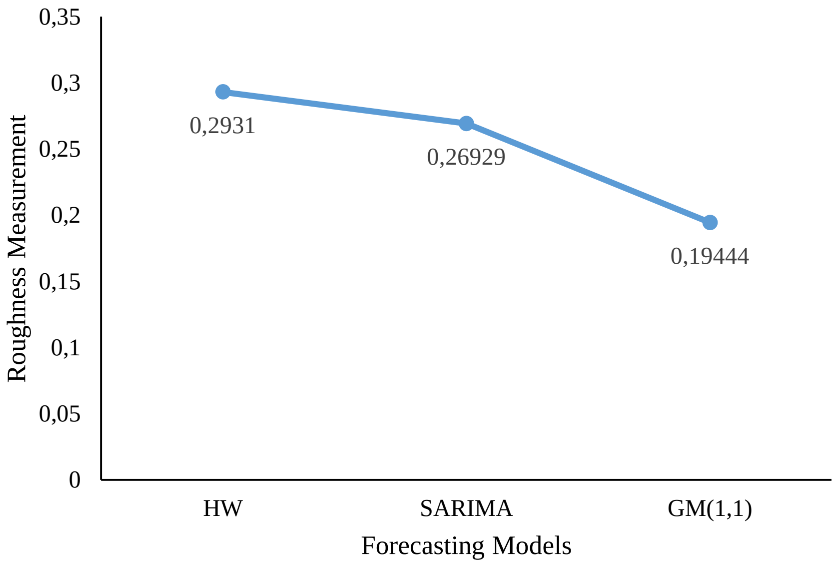

Figure 2. The roughness of HW, SARIMA and GM (1, 1) models using TR criterion for case 1

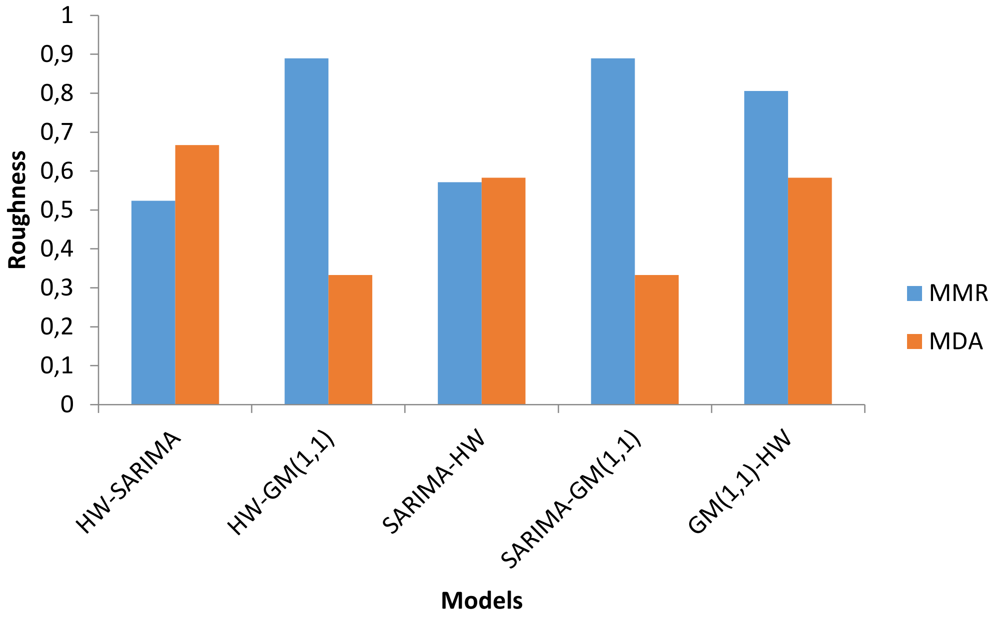

Figure 3. The roughness of HW, SARIMA and GM (1, 1) models using MMR and MDA criteria for Case 1

5.3. Case 2: The rough set information system

Table 8 shows in the information system of rough set data of case 2. In this case, the analysis to evaluate TR, MMR, and MDA of each method is similar as in case1. With TR techniques Holt-Winters model is considered an as good model because this attribute has the highest total roughness in Table 9 i.e. 0.5625 as compared to SARIMA and Grey model i.e. 0.29165 and 0.2065. Thus, we select Holt-Winters model is good as compare to SARIMA and Grey model. The results of MMR and MDA are given in Table 10 and Table 11. According to MMR and MDA reports, Holt-Winters models are chosen to select good model as compared to SARIMA and Grey model. Also, Figure 4 and Figure 5 illustrate the accuracy of three models with TR, MMR and MDA values which are involving the datasets of case 2. From the above considerations, we can see that Case 1 gives better results of three models in terms of rough accuracy compare to Case 2 results. Therefore, case 1 is an appropriate range of MAPE for applying the rough set approach in the field of time series modeling.

Table 8. Rough set information system of three models (Case 2)

Objects	Holt-Winters	SARIMA Model	Grey Model
Tourism purpose			
Japan	Good	Good	Reasonable
Hong Kong	Good	Good	Reasonable
US	High accuracy	High accuracy	Good
Total	Good	High accuracy	Reasonable
Non-tourism purpose			
Japan	High accuracy	High accuracy	High accuracy
Hong Kong	High accuracy	High accuracy	Good
US	High accuracy	High accuracy	High accuracy
Total	High accuracy	High accuracy	High accuracy
All-purposes			
Japan	Good	High accuracy	Reasonable
Hong Kong	Good	Good	Reasonable
US	High accuracy	High accuracy	High accuracy
Total	High accuracy	High accuracy	Good

Table 9. The total roughness of three models using TR criteria

Attribute	Holt-Winters	SARIMA Model	Grey Model	Total Roughness
Holt Winters	………	0.125	1	0.5625
SARIMA Model	0.29165	………..	0.29165	0.29165
Grey Model	0.33	0.083	………….	0.2065

Table 10. The minimum-minimum roughness of three models using MMR criteria

Attribute	Holt-Winters	SARIMA Model	Grey Model	MMR
Holt Winters	……..	0.875	0	0 0.875
SARIMA Model	0.70835	………..	0.70835	0.70835
Grey Model	0.67	0.917	………..	0.67 0.917

Table 11. Degree of Dependency using MDA criteria

Attribute	Holt-Winters	SARIMA Model	Grey Model	MDA
Holt Winters	.....	0.25	1	1
SARIMA Model	0.5833	.....	0.5833	0.5833
Grey Model	0.4167	0.25	.....	0.4167

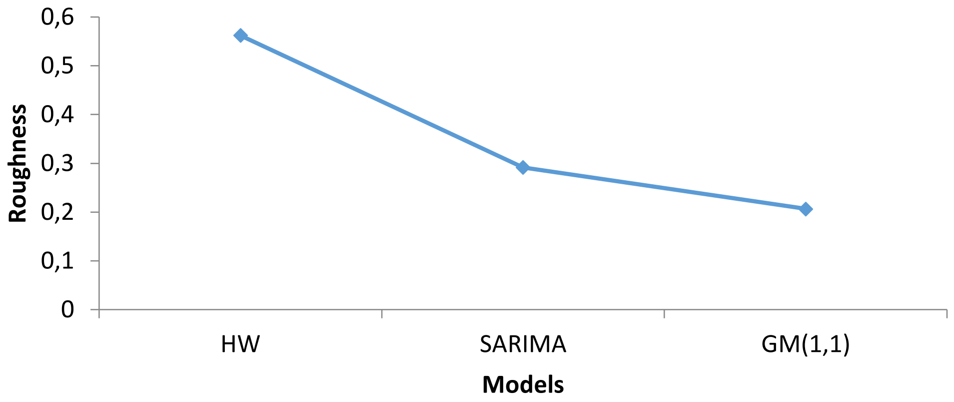

Figure 4. The roughness of HW, SARIMA and GM (1, 1) models based on TR criterion for Case 2

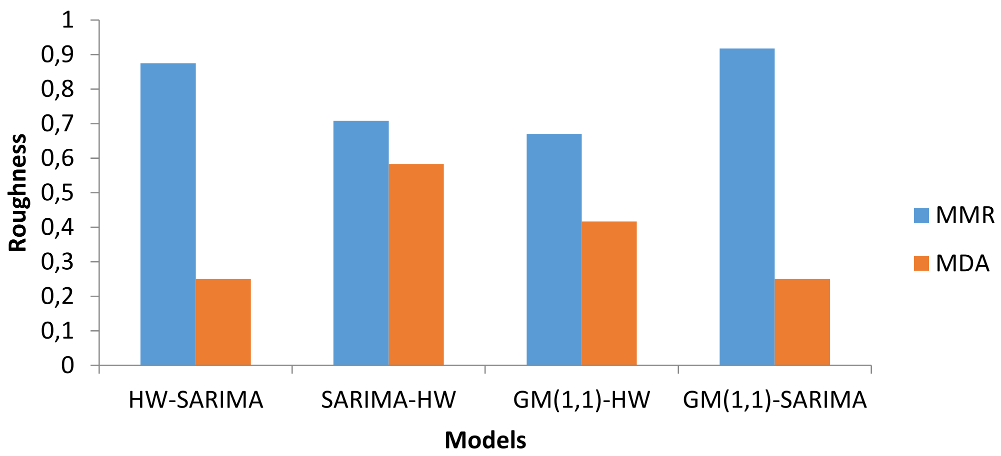

Figure 5. The accuracy of HW, SARIMA and GM (1,1) models based on MMR and MDA criteria for Case 2

6. Conclusions

Current research proposes a new technique to select the best forecasting model using rough set approach. This technique is based on the rough set criteria i.e., TR, MMR and MDA. In the present study, the problems concerning the selection of good forecasting model using rough set methods, such as normalized MAPE values. Two test cases are considered to implement rough set and three models i.e. Holt winters, SARIMA, and Grey models are employed. At first, we normalize MAPE values into the rough set information system and then we calculate the accuracy of each model. A close analysis of all the results furnished by the respective models, it reports that Holt-Winters and SARIMA models are far better models when compared to Grey model.

According to the analysis reports, Holt-Winters and SARIMA are good models in consideration of seasonal time series. Also, Grey model is an unqualified model to forecast seasonal time series under TR, MMR, and MDA rough set criteria. In this research, we would recommend that the proposed research is feasible and it offers a powerful statistical evidence for rough set methods. . Herewith these concepts, we suppose that various applications through rough set will be relevant in time series forecasting. The projected approach could also be useful in large real-life datasets. In future, one can apply these techniques in other time series forecasting models.

Our newly proposed technique differs from the traditional statistical method in the sense, that it provides an alternative way to selection best forecasting model for time series forecasting.

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