

# **A STUDY ON POLLUTION SENSITIVE SPONGE IRON BASED PRODUCTION TRANSPORTATION MODEL UNDER FUZZY ENVIRONMENT**

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**Abstract:** *Over the last few years, sponge iron-based production transportation and pollution problems for major sponge iron producing countries are triggering a critical issue. The excess of marginal pollution from production industries and their disintegration takes drives towards the change of policymaking. The sustainable development of any country signifies the reduction of biohazards, which in turn improves the health index and livelihood status of people across the world. Keeping this in mind, a cost depreciation problem for the bi-layer integrated supply chain model has been built up. We consider the functional dependencies among all considerable decision variables like production rate, consumption rate which leads to the pollution rate of different countries exclusively. In this study, we have shown how production and rail freight transport relates to pollution. To draw several graphs and numerical computations we use MATLAB software and C programming via solution algorithm respectively. The comparative study has been presented using general fuzzy as well as cloudy fuzzy systems. Lastly, we have justified our proposed model using sensitivity analysis along with graphical interpretation.*

**Key words:** *Production, Pollution, Transportation, Cloudy Fuzzy, Modeling, Optimization.*

## **1. Introduction**

This section has been splitted into two subsections namely General overview and Motivation and specific study.

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### **1.1. General Overview**

Due to globalization, the manufacturing of iron as well as steel and other materials, such as aluminum and materials for chemical-based products are playing a substantial role in the competitive market. We know, the total energy use and different levels of pollution come from various stages in the production process. The ferroalloys production is minor in contrast with base materials such as steel and aluminum. The major portion of complete man-made pollution has been incorporated by the severe environmental effects of silicon and ferroalloys. Taking consideration of the by-products of the production of sponge iron, the daily increment in use of general assets like electrical as well as electronic products, chemical materials, iron, and steel items is alarming contents. Also, the large content of daily exhausted baby food products, lapsed drugs are the burning issues for our environment. The manufacturers have extensively marketed for updated items to remain in competition in the global game. With the inclusion of upgraded commodities, old stocks become useless for consumption and thus generate various liabilities causing a severe impact on our surroundings.

To realize, control as well as minimizing the surrounding pollution linked with the manufacturing process, their products and actions, the insight of life-cycle concept is booming method for any industry. It is important to consider the environmental issues in sponge iron production from a broad view. For the superior perception of the environmental problem the understanding of drawbacks with a production chain from "cradle to grave" using Life Cycle Assessment (LCA) is in dire necessity. The inclusion of the LCA study dishes out an integrated analysis of resources, substantial, and health effects on the system. Also, it paves the way for environmental advancements by carrying out significant opportunities. In a complete LCA, the total environmental cost consists of all material sources as well as energy resources inducted in the process from raw materials up to production and transportation.

### **1.2. Motivation and Specific Study**

In the literature, several research articles are available in which most of them are associated to cost benefits and controlling carbon emissions from the vehicles used in the transportation itself. Sarkar et al. (2015) investigated the outcome of an uneven lot size model for changeable establishment cost and carbon discharge cost in an SC problem. Madadi et al. (2010) came about a multi-tiered inventory management settlement with shipment cost emolument. Recently, the consequence of changing shipment and outpouring of carbon in the three-echelon SC model has been reported by Sarkar et al. (2016). Depending on electrical energy on railway shipment, Bryan et al. (2008) proposed a model. A detailed review with an introduction to controlling novel mechanization for carbon combustion had been presented by Sithole et al. (2018). In the case of Ferromanganese and steel, Sjoqvist et al. (2001) reported the outcome of carbon excretion during cleaning. It has to be mentioned, the correlation among manufacturing, shipment, carbon release, and environmental contamination is also being included in the Ferro industry-related conveying problem. Recent works suggest the severe knock-on our surrounding by the transport sector and this setback forces us to reconsider the environmental effect due to the transport organizing and operations. The main culprits from transportation are consist of different oxides of Carbon and Nitrogen, as well as different organic chemicals. The rising environmental consciousness among people, enormous competition as well as strict policies from the government enforce the manufacturer for minimization of this severe pollution for the sake of mankind (Nouira et al., 2016). The important model by Benjaafar et al. (2010)

describes the way of controlling carbon footprint in supply chains. Aarthi (2017), Chen et al. (2013), Mancini et al. (2016), Akten and Akyol (2018) suggest a different model (like the EOQ model) and methods to combat the rising carbon footprint. Grzywiński (2019) and Grzywiński et al. (2019), analysed various optimization techniques using metaheuristic algorithm and Jaya algorithm. Eirgash et al. (2019) described a multi-objective inventory model with trade credit. Bera et al. (2020) studied the impacts of air pollution in covid situation in the urban areas. A risk assessment of bankrupt cases in European Countries was done by Bărbuță-Mișu and Madaleno (2020). Abualigah et al. (2021) developed an arithmetic optimization algorithm to solve supply chain management problems. It has been observed that all earlier authors have focused to measure the carbon emission due to production, although along with it, transportation plays a major role.

The Fuzzy system is utilized when there is the existence of some non-random uncertain parameters in the system. After Zadeh (1965) developed the fuzzy set theory, there is multiple reports by renowned scientists across the globe (Kumar et al., 2012; De & Sana, 2013; De et al., 2014). Along with this, the production mechanism has been considerably investigated using cloudy fuzzy set (De & Mahata, 2017; Karmakar et al., 2017, 2018) and triangular dense fuzzy set (De & Beg, 2017). After the invention of triangular dense lock fuzzy sets by De (2017), De and Mahata (2020) developed a supply chain backordering model under triangular lock fuzzy environment Bhattacharya et al. (2020, 2021) developed pollution sensitive inventory models with the effect of corruption as well as global warming and solved these via fuzzy system. Giri et al. (2021) solved a price dependent multi-item inventory model using intuitionistic fuzzy number.

The above-reported literature suggests no one has investigated the industrial supply chain (SC) problem that includes the pollution due to production as well as transportation. Indeed, methodology over fuzzy learning theory was not popularly utilized yet. Hence in our study, we present out an article that includes cost minimization two-layer SC problem having two-way pollution channel under learning fuzzy environment. We solve the specific inventory management problem into three sub cases: one by crisp approach, another by general fuzzy approach and the other by cloudy fuzzy approach. We have also developed a solution algorithm to solve the problem in each case. We also include a sensitivity analysis table to show the stability of the parameters involving in the model.

The organization of this article is developed as follows: section one is introduction followed by motivation and specific study. Section 2 includes preliminaries that focuses definition of general and cloudy fuzzy sets and their defuzzification techniques. Section 3 describes notations, assumptions and a case study. Section 4 indicates formulation of crisp inventory model. Section 5 includes the general fuzzy mathematical model and its defuzzification method; section 6 develops cloudy fuzzy mathematical model and its defuzzification method with a solution algorithm, pseudo code of C programming; section 7 and 8 indicates numerical illustration and sensitivity analysis respectively. Sections 9 develops graphical illustrations; section 10 represents the merits and demerits of the article and finally section 11 keeps a conclusion followed by scope of future work.

## 2. Preliminaries

In this section, we shall give some definitions and basic formulae that are used to formulate and solve the proposed model.

**2.1. Pollution function**

From the report by Karmakar et al. (2017), we have taken the differential equation which governed the production-pollution rate is:

$$\begin{cases} \dot{X} = aX - rX^2 - \alpha XY, & a, r, \alpha > 0 \\ \dot{Y} = -cY + \gamma XY, & c, \gamma > 0 \end{cases} \tag{1}$$

From above, the pollution y (%) with production rate (p) is governed by

$$y = 0.45 + 0.01p - 0.25 \log(p) \tag{2}$$

**2.2. Normalized General Triangular Fuzzy Number (NGTFN)**

Let A be an NGTFN having the form  $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ . Then the membership function of the fuzzy set  $\tilde{A}$  is defined by

$$\mu(\tilde{A}) = \begin{cases} 0, & \text{if } a < a_1 \text{ and } a > a_3 \\ \frac{a-a_1}{a_2-a_1}, & \text{if } a_1 \leq a \leq a_2 \\ \frac{a_3-a}{a_3-a_2}, & \text{if } a_2 \leq a \leq a_3 \end{cases} \tag{3}$$

Now, the index value of  $\mu(\tilde{A})$  due to Yager (1981) is obtained as

$$I(\tilde{A}) = \frac{1}{2} \int_0^1 [L(\alpha) + R(\alpha)] d\alpha = \frac{(a_1+2a_2+a_3)}{4} \tag{4}$$

for the left and right  $\alpha$ -cuts  $L(\alpha) = a_1 + (a_2 - a_1)\alpha$  and  $R(\alpha) = a_3 - (a_3 - a_2)\alpha$  respectively.

**2.3. Cloudy Normalized Triangular Fuzzy Number (CNTFN) (De and Mahata, 2016)**

A fuzzy number  $\tilde{A} = \langle a_1, a_2, a_3 \rangle$  is called cloudy normalized fuzzy number if, after an infinite time, the set converges to a singleton crisp set. That is, if the time  $t \rightarrow \infty$ , the set  $\tilde{A}$  becomes  $A = \{a_2\}$ . For example, we consider the fuzzy number

$$\tilde{A} = \langle a_2 \left(1 - \frac{\rho}{1+t}\right), a_2, a_2 \left(1 + \frac{\sigma}{1+t}\right) \rangle, \text{ for } 0 < \rho, \sigma < 1 \tag{5}$$

Here we see that both  $\lim_{t \rightarrow \infty} a_2 \left(1 - \frac{\rho}{1+t}\right)$  and  $\lim_{t \rightarrow \infty} a_2 \left(1 + \frac{\sigma}{1+t}\right)$  converges to  $a_2$ . Then its membership function for  $t \geq 0$  is given by

$$\mu(x, t) = \begin{cases} 0 & \text{if } x < a_2 \left(1 - \frac{\rho}{1+t}\right) \text{ and } x > a_2 \left(1 + \frac{\sigma}{1+t}\right) \\ \frac{x - a_2 \left(1 - \frac{\rho}{1+t}\right)}{\frac{\rho a_2}{1+t}} & \text{if } a_2 \left(1 - \frac{\rho}{1+t}\right) \leq x \leq a_2 \\ \frac{a_2 \left(1 + \frac{\sigma}{1+t}\right) - x}{\frac{\sigma a_2}{1+t}} & \text{if } a_2 \leq x \leq a_2 \left(1 + \frac{\sigma}{1+t}\right) \end{cases} \tag{6}$$

Now the index value of  $\tilde{A}$  is given by

$$I(\tilde{A}) = \frac{1}{2T} \iint_{\alpha=0, t=0}^{\alpha=1, t=T} \{L^{-1}(\alpha, t) + R^{-1}(\alpha, t)\} dadt = a_2 \left[1 + \frac{\sigma - \rho \log(1+T)}{4T}\right] \tag{7}$$

For the left and right  $\alpha$ -cuts  $L^{-1}(\alpha, t) = a_2 \left(1 - \frac{\rho}{1+t} + \frac{\rho\alpha}{1+t}\right)$  and  $R^{-1}(\alpha, t) = a_2 \left(1 + \frac{\sigma}{1+t} - \frac{\sigma\alpha}{1+t}\right)$  respectively.

### 3. Notations and Assumptions

In this section we shall discuss the notations and assumptions that are used throughout the proposed model.

#### Notations

- $p$  : Production rate per cycle (MT/year) (Decision variable)  
 $y$  : Pollution index (%)  
 $\tau_1$  : Production run time (Decision variable) (year)  
 $\tau_2$  : Transportation time (year)  
 $\tau_3$  : Inventory exhaust time (year)  
 $d$  : Rate of demand for each cycle (MT/year)  
 $q$  : Total quantity in order (MT)  
 $\delta$  : Deterioration rate per unit time  
 $l$  : Transportation distance (Mile)  
 $C_p$  : Manufacturing expenditure for each item (\$)  
 $h_p$  : Carrying expenditure for each item for each interval of time at maker's plant(\$)  
 $h_r$  : Carrying expenditure for each item for each interval of time at dealer's shop (\$)  
 $C_{pol}$  : Pollution expenditure (\$) (per one item)  
 $C_t$  : Transportation cost (\$) (per unit MT per Mile)  
 $C_d$  : Deterioration price (\$) (for each item for each interval of time)  
 $C_c$  : Global social expenditure of carbon (\$)  
 $k_1$  : Setup cost at production plant (\$)  
 $k_2$  : Setup cost at retailer side (\$)

#### Assumptions

1. Replenishments are instantaneous.
2. Shortages are not allowed.
3. Lead time is zero.
4. A producer has the sole responsibility to transport the items to a retailer.
5. Deterioration occurs and deteriorated items cannot be recoverable.
6. A producer has a separate transportation facility.
7. Pollution during production is controlled by the inbuilt technology of the production process but 100% pollution reduction is not possible.
8. No deterioration is viewed in the final product during transportation.

#### 3.1. Case Study

Let us extend the case study performed by Karmakar et al. (2017, 2018). These studies were involved in the manufacturing and pollution of a sponge iron industry. Our focus of interest is to measure pollution due to the transportation of products by a freight train. Also, through managerial insights as well as learning experiences, we try to cut down the standard inventory expenditure. With a diameter of 1200 km (estimated), in this single managerial controlled industry, the different orders are put

down immediately using different shipment systems. The various expenditure information obtained from the industry is presented in Table 1.

**Table 1.** Data information for the concerned industry

Establishment expenditure per cycle \$20000	Carrying expenditure for each cycle in manufacturing plant per MT \$5	Degradation value in each cycle in manufacturing plant per each thing \$10	Degradation fraction 0.1
Contamination expenditure for a manufacturer for one MT \$43.89	Carrying expenditure for each cycle in dealer plant for one MT \$10	manufacturing price for each MT \$327.56	
Social expenditure for Carbon per MT \$417	Shipment expenditure for unit gallon fuel \$3.5	Distance crossed in a freight train 600 Miles	

The research problem is

- i) Is it possible to control the contamination and reach the least annual average expenditure in our proposed SI production?
- ii) What is the ideal quantity of order numbers which results in a minimum inventory cost?
- iii) Whether our cloudy fuzzy system is more effective to reduce the pollution of the supply chain as well as average inventory cost than the crisp and general fuzzy system.

#### 4. Formulation of crisp mathematical Model

We consider the above assumptions and notations for developing an imperfect production process by Bhattacharya et al. (2021). The proposed mathematical model for average inventory cost minimization is governed by

$$z = \frac{1}{\tau_1} [HC + PC + DC + TC + TPC + SC + PPC]$$

$$z = \frac{h_p p}{\delta} \left(1 + \frac{e^{-\delta \tau_1} - 1}{\delta \tau_1}\right) + C_d p \left(1 + \frac{e^{-\delta \tau_1} - 1}{\delta \tau_1}\right) + \frac{C_p p}{\delta} \left(1 + \frac{e^{-\delta \tau_1} - 1}{\delta \tau_1}\right) + C_{pol} p + \frac{K_1}{\tau_1} + C_t \times 0.00424628ld + C_c \times 0.0000431445ld + \frac{h_r d \tau_1}{2} + \frac{K_2}{\tau_1} \tag{8}$$

This SC model is represented by

$$\left\{ \begin{array}{l} \text{Minimize } z = \frac{h_p p}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + C_d p \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) + \frac{C_p p}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right) \\ + C_{pol} p + \frac{K_1}{\tau_1} + C_t \times 0.00424628ld + C_c \times 0.0000431445ld + \frac{h_r d \tau_1}{2} + \frac{K_2}{\tau_1} \\ \text{subject to, } \tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1 \\ q = \frac{p}{\delta}(1 - e^{-\delta\tau_1}) = d\tau_1 \\ y = 0.45 + 0.01p - 0.25 \log(p) \end{array} \right. \quad (9)$$

### 5. Construction of SC Model under general fuzzy system

All our expenditure parameters ( $\bar{C}$ ) with demand rate (d) in our prescribed SC model also obeys TFN is organized as  $\tilde{C}_i = \langle C_{i1}, C_{i2}, C_{i3} \rangle, i = 1, 2, \dots, 9, = (h_p, C_d, C_p, C_{pol}, K_1, K_2, h_r, C_t, C_c)$ , and  $\tilde{d} = \langle d_1, d_2, d_3 \rangle$ . Also, due to fuzzification of parameters like demand rate, the order quantity, production rate, and pollution level will assume values in the following:

$$\left\{ \begin{array}{l} \tilde{q} = \langle q_1, q_2, q_3 \rangle \geq \langle d_1\tau_1, d_2\tau_1, d_3\tau_1 \rangle \\ \tilde{p} = \langle p_1, p_2, p_3 \rangle \geq \langle d_1\tau_1\delta(1 - e^{-\delta\tau_1}), d_2\tau_1\delta(1 - e^{-\delta\tau_1}), d_3\tau_1\delta(1 - e^{-\delta\tau_1}) \rangle \\ \tilde{y} = \langle y_1, y_2, y_3 \rangle \\ = \langle 0.45 + 0.01p_1 - 0.25 \log p_3, 0.45 + 0.01p_1 - 0.25 \log p_3, 0.45 + 0.01p_1 - 0.25 \log p_3 \rangle \end{array} \right. \quad (10)$$

Then the corresponding fuzzy problem of the crisp problem (9) can be written as

$$\left\{ \begin{array}{l} \min \tilde{z} \cong \tilde{p} \sum_{i=1}^4 \tilde{C}_i f_i + \tilde{C}_5 f_5 + \tilde{C}_6 f_6 + \tilde{d} \sum_{i=7}^9 \tilde{C}_i f_i \\ \text{subject to, } \tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1, \\ \tilde{q} \cong \frac{\tilde{p}}{\delta}(1 - e^{-\delta\tau_1}) \cong \tilde{d}\tau_1 \\ \tilde{y} \cong 0.45 + 0.01\tilde{p} - 0.25 \log \tilde{p} \end{array} \right. \quad (11)$$

where

$$\left\{ \begin{array}{l} f_1 = \frac{1}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right), f_2 = \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right), \\ f_3 = \frac{1}{\delta} \left(1 + \frac{e^{-\delta\tau_1} - 1}{\delta\tau_1}\right), f_4 = 1, f_5 = \frac{1}{\tau_1}, f_6 = \frac{1}{\tau_1}, \\ f_7 = \frac{\tau_1}{2}, f_8 = 0.00424628l, f_9 = 0.0000431445l \end{array} \right. \quad (12)$$

#### 5.1. Defuzzification under general fuzzy system

Obeying TFN, all of our fuzzy objectives can be written as  $\tilde{z} = \langle z_1, z_2, z_3 \rangle$  and the components are represented as:

$$\left\{ \begin{array}{l} z_1 = p_1 \sum_{i=1}^4 C_{i1} f_i + C_{51} f_5 + C_{61} f_6 + d_1 \sum_{i=7}^9 C_{i1} f_i \\ z_2 = p_2 \sum_{i=1}^4 C_{i2} f_i + C_{52} f_5 + C_{62} f_6 + d_2 \sum_{i=7}^9 C_{i2} f_i \\ z_3 = p_3 \sum_{i=1}^4 C_{i3} f_i + C_{53} f_5 + C_{63} f_6 + d_3 \sum_{i=7}^9 C_{i3} f_i \end{array} \right. \quad (13)$$

Using equation (4), our fuzzy problem (11) is converted to Crisp cost minimization problem by replacing the respective index parameter with mentioned constraints are written as

$$\text{Minimize } I(\tilde{z}) = \frac{1}{4}(z_1 + 2z_2 + z_3) \quad (14)$$

$$\text{Subject to } \begin{cases} \tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1, I(\tilde{d}) = \frac{(d_1 + 2d_2 + d_3)}{4}, \\ I(\tilde{q}) = I(\tilde{d})\tau_1, I(\tilde{q}) = \frac{I(\tilde{p})}{\delta}(1 - e^{-\delta\tau_1}), \\ I(\tilde{y}) = 0.45 + 0.01 I(\tilde{p}) - 0.25 \log[I(\tilde{p})] \end{cases} \quad (15)$$

and the values of  $z_i, i = 1, 2, 3$  are found from (13)

### 6. Formulation of Cloudy Fuzzy Model

With cloud type flexibility, all our cost parameters ( $\bar{C}$ ) with indent rate ( $d$ ) connected with the model represented as:

$$\begin{cases} \bar{C}_i = \langle C_{i1}, C_{i2}, C_{i3} \rangle = \langle C_{i2} \left(1 - \frac{\rho_c}{1+t}\right), C_{i2}, C_{i2} \left(1 + \frac{\sigma_c}{1+t}\right) \rangle \\ \tilde{d} = \langle d_1, d_2, d_3 \rangle = \langle d_2 \left(1 - \frac{\rho_d}{1+t}\right), d_2, d_2 \left(1 + \frac{\sigma_d}{1+t}\right) \rangle \end{cases} \quad (16)$$

where  $\rho_c, \sigma_c, \rho_d, \sigma_d$  are fuzzy system deviation parameters for cost vector and demand rate respectively.

Then the cloudy fuzzy problem will be of the form (11) whose fuzzy cost parameters ( $\bar{C}$ ) and fuzzy demand rate ( $\tilde{d}$ ) follow the membership function as per subsection 2.3. Simultaneously, the fuzzy order quantity, fuzzy production rate, and fuzzy pollution level are of the form given in (17).

$$\begin{cases} \tilde{q} = \langle q_1, q_2, q_3 \rangle = \langle d_1\tau_1, d_2\tau_1, d_3\tau_1 \rangle \\ \tilde{p} = \langle p_1, p_2, p_3 \rangle = \langle d_1\tau_1\delta(1 - e^{-\delta\tau_1}), d_2\tau_1\delta(1 - e^{-\delta\tau_1}), d_3\tau_1\delta(1 - e^{-\delta\tau_1}) \rangle \\ \tilde{y} = \langle y_1, y_2, y_3 \rangle \\ = \langle 0.45 + 0.01p_1 - 0.25 \log p_3, 0.45 + 0.01p_1 - 0.25 \log p_3, 0.45 + 0.01p_1 - 0.25 \log p_3 \rangle \end{cases} \quad (17)$$

#### 6.1. Defuzzification of Cloudy Fuzzy Model

From equation (11), our fuzzy problem has been transformed into a similar Crisp problem using equation (7). All our fuzzy components are represented in (16-17). We have replaced the respective index parameter with mentioned constraints and might be presented as

$$\text{Minimize } I(\tilde{z}) = \frac{1}{2t_1} \int_0^{t_1} (z_1 + 2z_2 + z_3) \quad (18)$$

subject to the constraints



$$\left\{ \begin{array}{l} \tau_2 = \frac{3}{2}\tau_1, \tau_3 = \frac{5}{2}\tau_1, I(\tilde{d}) = d_2 + \frac{d_2}{4}(\sigma_d - \rho_d) \frac{\log(1+t_1)}{t_1}, \\ I(\tilde{q}) = d_2 t_1 + \frac{d_2}{4}(\sigma_d - \rho_d) \log(1+t_1), \\ I(\tilde{p}) = \frac{\delta}{1-e^{-\delta t_1}} \left[ d_2 t_1 + \frac{d_2}{4}(\sigma_d - \rho_d) \log(1+t_1) \right] \\ I(\tilde{y}) = 0.45 + 0.01 \left[ \frac{\delta}{1-e^{-\delta t_1}} \left\{ d_2 t_1 + \frac{d_2}{4}(\sigma_d - \rho_d) \log(1+t_1) \right\} \right] \\ -0.25 \log \left[ \frac{\delta}{1-e^{-\delta t_1}} \left[ d_2 t_1 + \frac{d_2}{4}(\sigma_d - \rho_d) \log(1+t_1) \right] \right] \end{array} \right. \quad (19)$$

(For details see Appendix 2-3)

and the values of new  $z_i, i = 1, 2, 3$  can be obtained with the replacement of fuzzy components given in (16-17) into the relations (13).

## 6.2. Solution Algorithm

Here, we shall develop a solution algorithm for solving the model under crisp, general fuzzy and cloudy fuzzy environment.

**Step 0:** START.

**Step 1:** Set the main nonlinear equality constrained crisp arithmetic optimization problem  $Z(X)$  stated in equation (9).

**Step 2:** Optimize the crisp problem and store the results at  $X_0 = (Z_0, Y_0)$ .

**Step 3:** Using the results of Step 2, formulate the non-linear problem via general fuzzy system in (11) and (12) and solve the defuzzified problem (14) and (15) via Yager'index method.

**Step 4:** Store the results obtained from step 3 at  $X_1 = (Z_1, Y_1)$ .

**Step 5:** Formulate the problem (9) in cloudy fuzzy system in (17) and solve the defuzzified cloudy system at (18) subject to the constraints (19).

**Step 6:** Store the results obtained from step 5 at  $X_2 = (Z_2, Y_2)$ .

**Step 7:** Compare the solutions by computing the inequalities  $X_0 < X_1 < X_2$  or  $X_0 > X_1 > X_2$  or  $X_0 > X_1 < X_2$  etc.

**Step 8:** Take optimum solution  $X_2$  when  $X_0 > X_1 > X_2$ .

**Step 9:** END.

The Pseudo Code of C programming is given below.

---

```
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
#include <time.h>
void main()
{
    int i,lower =100, upper=999, count=10;
    float t1[count],hp=5.0,d=600.0,lambda[count],p[count],z[count],del=0.01,
    cd=10.0, cp=327.56, cpol=43.89,k1=10000,ct=3.5,l=600.0,cc=417.0,k2=10000,
    hr=4,f1,f2,f3,f4,f5,f6,f7,f8,f9;
    for(i=0;i<count;i++)
    {
        t1[i]=(float)((rand()%(upper-lower+1))+lower)/1000;
    }
    for(i=0;i<count;i++)
    {
```

```

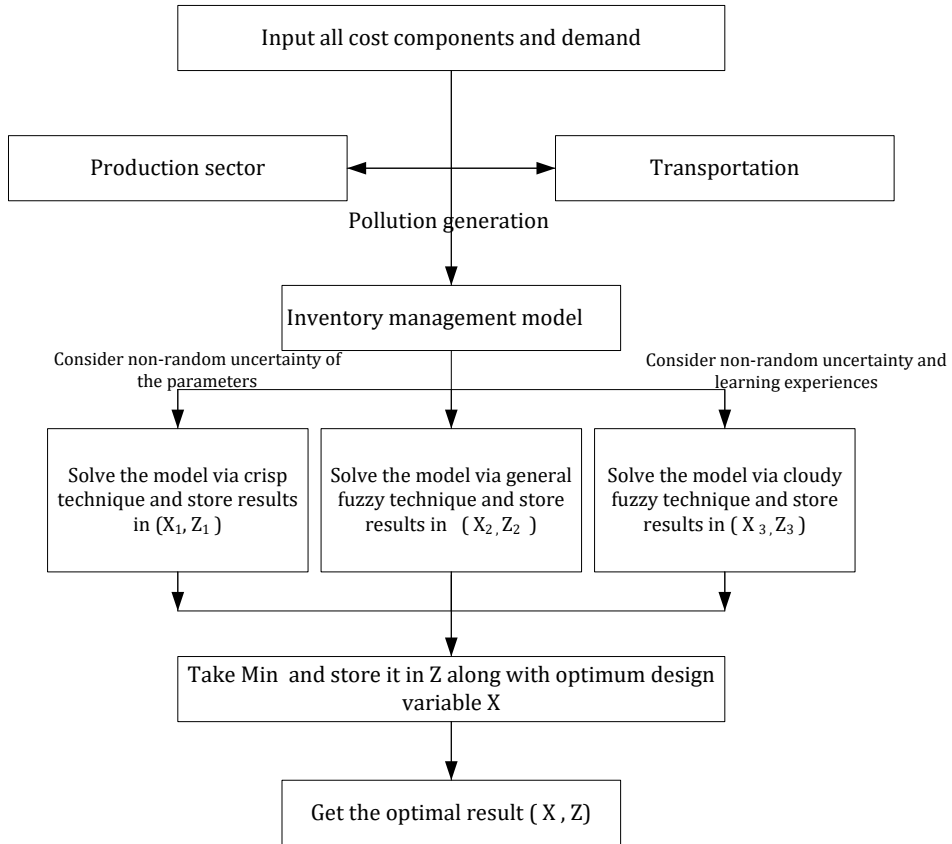
lambda[i]=del*t1[i]/(1-(exp(-del*t1[i])));
}
for(i=0;i<count;i++)
{
p[i]=d*lambda[i];
}
for(i=0;i<count;i++)
{
f1=(1+((exp(-del*t1[i]))-1)/(del*t1[i]))/del; f2=del*f1; f3 = f1; f4 =1; f5
=1/t1[i]; f6 = f5; f7 = 0.5*t1[i]; f8=0.00424628*f1; f9=0.0000431445*f1;
z[i]=p[i]*(hp*f1+cd*f2+cp*f3+cpol*f4)+k1*f5+k2*f6+d*(hr*f7+ct*f8+cc*f9);
}
float min_z = z[0]; int min_index = 0;
for(i=1;i<count;i++)
{
if(min_z>z[i])
{
min_z=z[i];
min_index=i;
}
}
float min_t1=t1[min_index]; float min_p = p[min_index]; float
min_lambda[min_index]; float t2=1.5*min_t1; float t3 = 2.5*min_t1;
float q = d*min_t1; float y = 0.45 + (0.01*min_p)-(0.25*log(min_p));
printf("Z\t\t%f\n",min_z); printf("T1\t\t%f\n",min_t1); printf("T2\t\t%f\n",t2);
printf("T3\t\t%f\n",t3); printf("P\t\t%f\n",min_p); printf("Q\t\t%f\n",q);
printf("Y\t\t%f\n",y); printf("D\t\t%f\n",d);
}

```

---

### 6.3. Schematic Diagram

Here we include a schematic diagram of the proposed study which shows the novelty of the article (Figure 1).



**Figure 1.** Schematic diagram of the study

## 7. Numerical Illustration

From the data set mentioned in Table 1 (subsection 3.1) and using the pollution function, the obtained minimized results have been displayed in Table 2. Also, the computed results using general fuzzy and cloudy fuzzy of the problem related to SC cost have been recorded in Table 2. We have considered fuzzy system parameters  $(\rho_c, \sigma_c, \rho_d, \sigma_d) = (0.3, 0.1, 0.2, 0.1)$  for our numerical computations.

**Table 2.** Minimized solutions of SC model under different environments

Model	$p^*$ (MT)	$y^*$ (%)	$\tau_1^*$ (Year)	$\tau_2^*$ (Year)	$\tau_3^*$ (Year)	$q^*$ (MT)	$z^*$ (\$)	$\frac{Z^* - Z_*}{Z_*}$ $\times 100\%$
Crisp	611.37	4.96	0.3768	0.5652	0.9620	226.07	117542.80	0
General Fuzzy	596.12	4.82	0.3779	0.5668	0.9447	221.06	114160.20	-2.88
Cloudy Fuzzy	598.34	4.83	0.3751	0.5626	0.9377	220.27	111893.40	-4.81

Table 2 represents the optimal SC expenditure for minimal order quantity, cycle time, contamination level, and manufacturing rate for three separate cases which are crisp, general as well as cloudy fuzzy systems. Cloudy fuzzy results minimum SC expenditure \$ 111893.40 for contamination share of 4.83% for the manufacturing time 0.3751 year with 220.27 MT of customer order. The SC expenditure grows to \$ 114160.20 while negligible minimization of the pollution level reaches to 4.82% for 221.06 MT customer order using a normal fuzzy system. The crisp model of SC is remarkably costly (\$ 117542.80) when air contamination level reaches to 4.96% for the customer order of 226.07 MT. Comparing with the crisp optimal solution, the SC cost-benefit for the cloudy fuzzy model becomes 4.81% which is superior to the 2.88% that was found for the general fuzzy system.

### 8. Sensitivity Analysis

After obtaining the best efficient cost in cloudy fuzzy, the sensitivity dependence for the same has been investigated. For observing the variation of SC cost with contamination level, customer order, manufacturing rate with several expenditure parts, all fuzzy variables ( $\rho_c, \sigma_c, \rho_d, \sigma_d$ ) have been changed with (+50%, +30%, -30%, -50%) accordingly and the outcomes have represented in Table 3.

**Table 3.** Sensitivity study with % variation of ( $\rho_c, \rho_d, \sigma_c, \sigma_d$ )

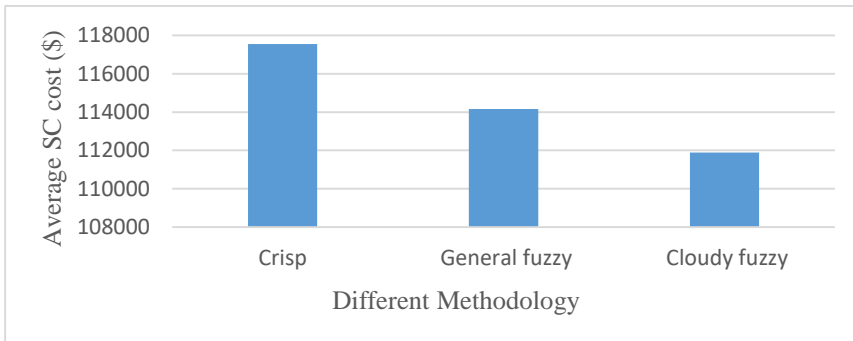
Fuzz y para mete rs	% chan ge	$\tau_1^*$ (Year)	$\tau_2^*$ (Year)	$\tau_3^*$ (Year)	$y^*$ (%)	$q^*$ (MT)	$z^*$ (\$)	$p^*$ (MT)	$\frac{Z^* - Z_0}{Z^*}$ $\times 100\%$	$\frac{Y^* - Y_0}{Y^*}$ $\times 100\%$
$\rho_c$ 0.3	+50	0.372	0.558	0.930	4.83	218.42	108570.4	598.24	-7.63	-2.56
	+30	0.373	0.560	0.933	4.83	219.13	109900.3	598.28	-6.5	-2.54
	-30	0.377	0.565	0.942	4.84	221.38	113884.5	598.41	-3.11	-2.5
	-50	0.378	0.567	0.946	4.84	222.10	115210.8	598.45	-1.98	-2.5
$\sigma_c$ 0.1	+50	0.376	0.564	0.940	4.84	220.75	113211.4	598.37	-3.68	-2.52
	+30	0.376	0.563	0.939	4.84	220.56	112684.2	598.36	-4.13	-2.52
	-30	0.375	0.562	0.937	4.84	219.98	111102.4	598.33	-5.48	-2.52
	-50	0.374	0.561	0.936	4.84	219.79	110575.0	598.32	-5.93	-2.52
$\rho_d$ 0.2	+50	0.561	0.842	1.403	4.78	323.30	116773.3	592.50	-0.65	-3.65
	+30	0.499	0.749	1.248	4.80	289.86	114235.8	595.15	-2.81	-3.15
	-30	0.374	0.560	0.934	4.91	222.26	112626.8	606.08	-4.18	-1.03
	-50	0.373	0.559	0.932	4.96	223.59	113115.2	611.25	-3.77	-0.02
$\sigma_d$ 0.1	+50	0.374	0.560	0.934	4.90	221.74	112786.4	604.78	-4.05	-1.27
	+30	0.374	0.561	0.935	4.87	221.15	112429.4	602.21	-4.35	-1.77
	-30	0.442	0.663	1.105	4.82	258.01	112359.9	596.85	-4.41	-2.8
	-50	0.481	0.722	1.203	4.81	279.99	113321.5	595.74	-3.59	-3.02

Table 3 shows that all the cloudy fuzzy system parameters are moderately sensitive relative to the crisp optimal solution. Our range of manufacturing run time, cycle time, air contamination share, the customer order, the manufacturing rate, and the average inventory cost assume value between (0.3719-0.5611) year, (0.9298-1.4028) year, (4.779-4.959) %, (218.42-323.30) MT, (592.50-611.25) MT and \$ (108570.40-116773.30) respectively. The overall cost-benefit lies within (0.65, 7.63) % and the contamination change within (0.02, 3.65) %. By this study, we also notice that the

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 maximum cost-benefit occurs from the 50% increment of all left fuzzy deviation parameters of all unit costs and the maximum contamination reduction occurs due to the 50% increment of left fuzzy deviation parameter of demand rate explicitly.

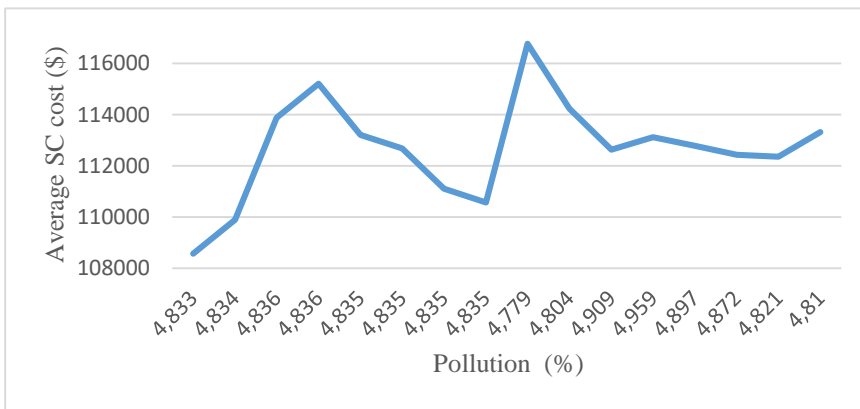
### 9. Graphical Illustrations

Different types of figures depend on the obtained outputs on several optimized solutions represented in table 2 and table 3 has been drawn.



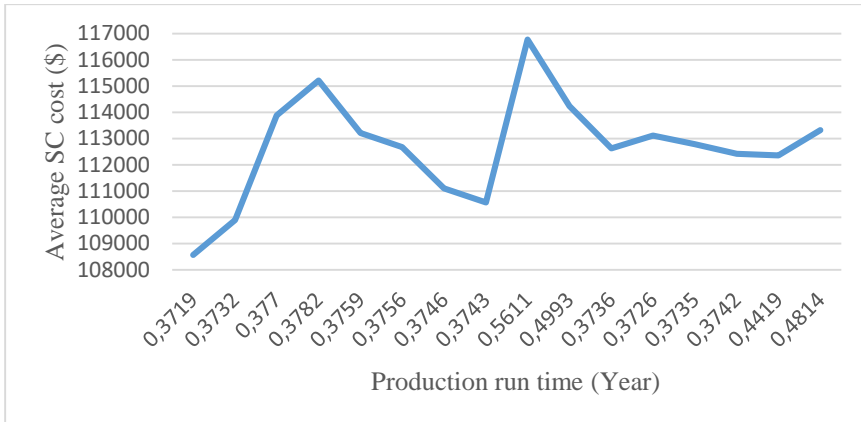
**Figure 2.** Different inventory cost for separate cases

Figure 2 shows minimum SC cost occurs due to cloudy fuzzy systems rather than crisp and general fuzzy systems. In the cloudy fuzzy system, the average SC cost takes a value around \$112000 whereas it takes jumps to around \$114000 and \$117000 corresponding to the general fuzzy and crisp model respectively.



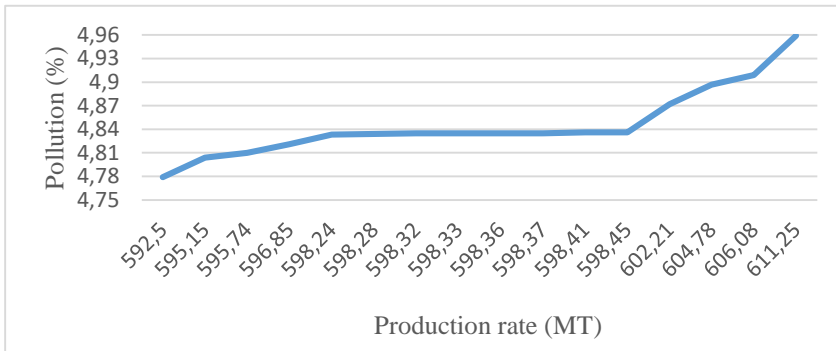
**Figure 3.** Average SC expenditure with different pollution

Figure 3 represents the dependence of average inventory expenditure with different contamination levels while other parameters remain independent. With contamination level reaches 4.833%, the expenditure goes to a minimum. But, at the smaller contamination level of 4.8 %, the average inventory expenditure reaches its maximum value. So, in place of achieving a cleaner environment, we are bound to pay more. The strong dependence of SC expenditure on contamination level is represented in the above graph.



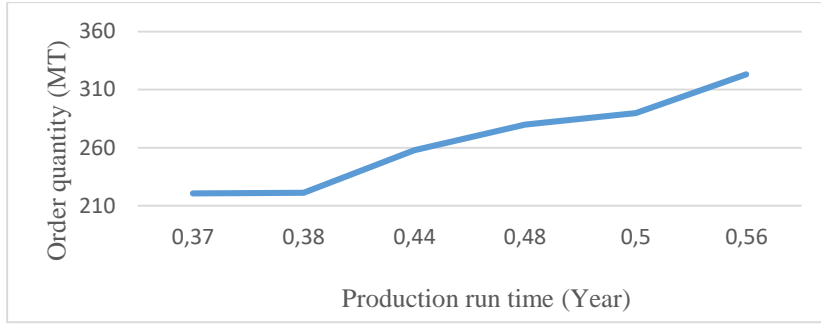
**Figure 4.** Variation of average SC cost due to production run time

Expenditure of inventory with manufacturing time has represented in Figure 4. The least value for SC cost has been obtained with 0.3719 years of manufacturing time. Similarly, with an increase in production time, the greatest average SC cost has been obtained.



**Figure 5.** Variation of pollution rate with respect to production rate

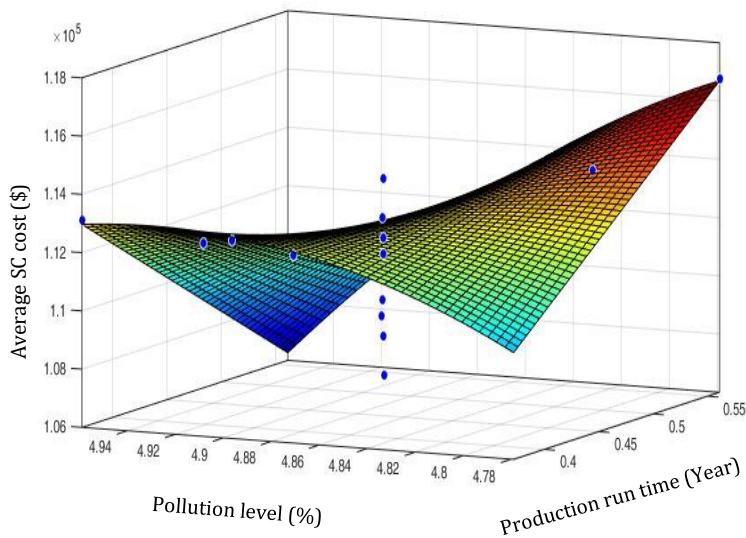
Naturally, the contamination level reaches its minimum with a minimum production rate. We see in Figure 5 that the pollution level remains almost stable within the manufacturing limit (598.24-598.45) MT. The pollution level increases slowly during the production range (592.5-598.24) MT. But when the production rate is increased (more than 598.45 MT) then the level of pollution is also increased almost exponentially.



**Figure 6.** Variation of order quantity with respect to production run time

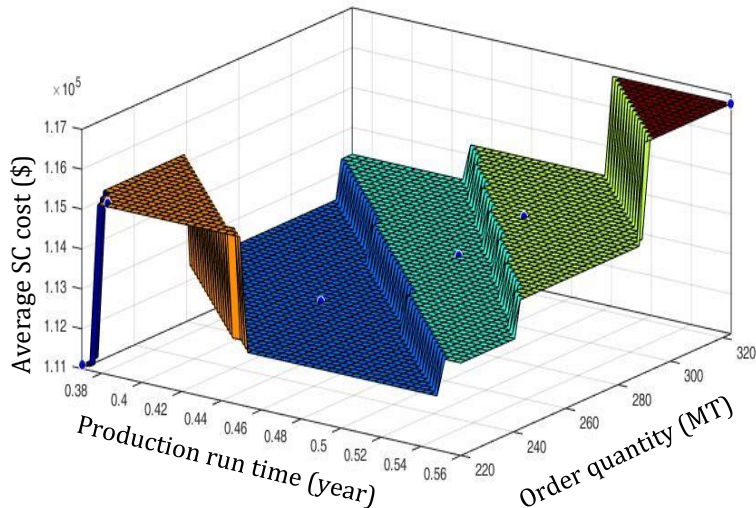
Figure 6 reveals the dependence of customer order (MT) with manufacturing time (year). We see that the order quantity curve has increased with the maximum slope with the rise of production run time of more than 0.54 years. When the production run time lies within 0.37-0.38 years then the order quantity curve takes a horizontal line by taking value near 220 MT. But interestingly, the order quantity curve gets an arc upwards (concave) having the range 220-290 MT with manufacturing time 0.38 ~ 0.5 year exclusively.

Figure 7 shows the surface-like structure of SC cost due to changes in production run time and the pollution level. We see that the cost is minimum at the pollution level 4.94% and production run time 0.55 years approximately. Also, the average SC cost reaches its maximum at the minimum pollution level 4.78% and the production run time 0.55 years approximately.



**Figure 7.** Dependence of manufacturing cost with production run time and pollution

The surface curve gets a bend at the middle of the graph the optimum of which parametric values are production run time near 0.45 years, pollution level near 4.84 %, and average SC cost near  $\$1.12 \times 10^5$ .



**Figure 8.** Variation of system cost with respect to production time and order quantity

Figure 8 represents the dependence of mean SC expenditure on customer order and manufacturing time. With approximately 0.98 years of manufacturing time and a customer order of 220 MT the average expenditure reaches a minimum, but it attains the greatest value for 0.56-year manufacturing time and 320 MT customer order.

## 10. Merits and Demerits

In this section we shall discuss the merits and demerits of the proposed approach.

### Merits:

- i) Pollution sensitive SC model has been analysed intelligently with the help of cloudy fuzzy number that gives the measure of learning experiences over time.
- ii) We incorporated a sensitivity analysis table to show the limitations and stability of the parameters involving in the model.
- iii) A comparative study with solution algorithm has been done to show the superiority of the optimum results in cloudy fuzzy system.
- iv) Our real case study data supports the learning model with new operational method.
- v) Any decision maker can easily use this method before going to final decision.

### Demerits:

Since, this model is solely devoted to learning theory so, lack of information gathering can harm the model. However, the numerical study is not checked by some other learning theory to draw an absolute global decision.



## 11. Conclusions

In this study, we have developed a two-channel pollution level on a two-layer supply chain deteriorated sponge iron manufacturing model under a cloudy fuzzy environment. For channel 1 of pollution corresponds to the number of items produced per time and that for the second channel it depends upon the distances travelled and the items transported. The production run time and the demand quantity may matter over the minimization of SC cost and pollution control. The basic novelty of the study is the incorporation of non-random uncertainty of the parameters of the model. We wish to know the average inventory cost, cycle time, pollution level (%) when the demand rate and all cost parameters are getting non- randomly uncertain. This problem has been solved by using general fuzzy set theory. Another important contribution of this article is to study the effect of learning experience in the inventory system. We want to study at what extent the decision maker could be able to reduce the average inventory cost and the pollution reduction for sustainable production within specific cycle time via learning experiences in the inventory process. To incorporate learning experience, we have introduced cloudy fuzzy number for the system parameters. We have also developed a solution algorithm and pseudo code of C programming to solve the mathematical model in different approaches like crisp, general fuzzy and cloudy fuzzy system. Moreover, we have incorporated a sensitivity analysis of the parameters to study the limitations and their stability for model validation. The table shows that we can control 3.65 % pollution by controlling the demand cut of 50 %. The restoration of SC cost may rise to 7.63 % by the control of all unit cost components cut by 50% each. We see that the minimum average inventory cost \$117542.80 occurs in cycle time 0.962 year due to production 611.37 MT (metric ton), order quantity 226.07 MT with pollution contribution 4.96% in crisp problem. In general, fuzzy situation, the minimum average inventory cost \$114160.20 occurs in cycle time 0.945 year due to production 596.12 MT (metric ton), order quantity 221.06 MT with pollution contribution 4.82% exclusively. In cloudy fuzzy approach, the minimum average inventory cost \$111893.40 occurs in cycle time 0.938 year due to production 598.34 MT (metric ton), order quantity 220.27 MT with pollution contribution 4.83% alone. So, the decision maker could be able to minimize the average inventory cost up to 2.88 % in general fuzzy approach and 4.81 % by applying cloudy fuzzy technique respectively. Also, for sustainability, a situation has come to balance production-demand-pollution-production run time altogether and this is only possible when the DM opts cloudy fuzzy system. However, some common managerial insights from this study can be drawn as follows:

- i) Cloudy fuzzy approach is better than crisp and general fuzzy approach.
- ii) The decision maker/manager could not ignore the issue of environmental pollution that are being deposited into the environment day by day from both the production process and transportation process. Rather, it should have to accept before going to furnish final decision over production-based inventory management problems.
- iii) Increase of production (order quantity) carries some pollution in environment. So, it can be reduced through sustainable production.

### Scope of future work

Various model can be studied using different fuzzy systems related to learning theory like Monsoon fuzzy theory, Fuzzy approximate reasoning, Doubt fuzzy approach etc. in near future. Taking different types of fuzzy set like Neutrosophic fuzzy sets, some

other complex and more realistic models can be developed in future. Also, several pollution function may be developed in the way of developing new research.

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**Appendix A**

**A.1:**  $\tau_3 - \tau_2 = 2(\tau_2 - \tau_1)$ ,  $\tau_1 = 2(\tau_2 - \tau_1)$  This implies  $\tau_2 = \frac{3}{2}\tau_1$ ,  $\tau_3 = \frac{5}{2}\tau_1$

**A.2:** The cloudy fuzzy objective value  $\tilde{z} = \langle z_1, z_2, z_3 \rangle$  can be reduced as follows:

$$\left\{ \begin{aligned} Z_1 &= \frac{d_2\tau_1\delta}{(1-e^{-\delta\tau_1})} \left\{ \left( \frac{h_p}{\delta} + C_d + \frac{C_p}{\delta} \right) \left( 1 + \frac{e^{-\delta\tau_1-1}}{\delta\tau_1} \right) + C_{pol} \right\} \left( 1 - \frac{\rho_d}{1+t} \right) \left( 1 - \frac{\rho_c}{1+t} \right) + \left( \frac{K_1}{\tau_1} + \frac{K_2}{\tau_1} \right) \left( 1 - \frac{\rho_c}{1+t} \right) \\ &\quad + d_2 \left\{ \frac{h_r\tau_1}{2} + C_t 0.00424628l + C_c 0.0000431445l \right\} \left( 1 - \frac{\rho_d}{1+t} \right) \left( 1 - \frac{\rho_c}{1+t} \right) \\ Z_2 &= \frac{d_2\tau_1\delta}{(1-e^{-\delta\tau_1})} \left\{ \left( \frac{h_p}{\delta} + C_d + \frac{C_p}{\delta} \right) \left( 1 + \frac{e^{-\delta\tau_1-1}}{\delta\tau_1} \right) + C_{pol} \right\} + \frac{K_1}{\tau_1} + \frac{K_2}{\tau_1} \\ &\quad + d_2 \left\{ \frac{h_r\tau_1}{2} + C_t 0.00424628l + C_c 0.0000431445l \right\} \\ Z_3 &= \frac{d_2\tau_1\delta}{(1-e^{-\delta\tau_1})} \left\{ \left( \frac{h_p}{\delta} + C_d + \frac{C_p}{\delta} \right) \left( 1 + \frac{e^{-\delta\tau_1-1}}{\delta\tau_1} \right) + C_{pol} \right\} \left( 1 + \frac{\sigma_d}{1+t} \right) \left( 1 + \frac{\sigma_c}{1+t} \right) + \left( \frac{K_1}{\tau_1} + \frac{K_2}{\tau_1} \right) \left( 1 + \frac{\sigma_c}{1+t} \right) \\ &\quad + d_2 \left\{ \frac{h_r\tau_1}{2} + C_t 0.00424628l + C_c 0.0000431445l \right\} \left( 1 + \frac{\sigma_d}{1+t} \right) \left( 1 + \frac{\sigma_c}{1+t} \right) \end{aligned} \right.$$

Now,

$$\begin{aligned} &\frac{1}{2} \int_0^{t_1} (Z_1 + 2Z_2 + Z_3) dt \\ &= \frac{1}{2} \int_0^{t_1} \left[ \frac{d_2\tau_1\delta}{(1-e^{-\delta\tau_1})} \left\{ \left( \frac{h_p}{\delta} + C_d + \frac{C_p}{\delta} \right) \left( 1 + \frac{e^{-\delta\tau_1-1}}{\delta\tau_1} \right) + C_{pol} \right\} \times \left\{ 2 + \left( 1 - \frac{\rho_d}{1+t} \right) \left( 1 - \frac{\rho_c}{1+t} \right) + \left( 1 + \frac{\sigma_d}{1+t} \right) \left( 1 + \frac{\sigma_c}{1+t} \right) \right\} + \left( \frac{K_1}{\tau_1} + \frac{K_2}{\tau_1} \right) \left( 4 + \frac{\sigma_c - \rho_c}{1+t} \right) + d_2 \left\{ \frac{h_r\tau_1}{2} + C_t 0.00424628l + C_c 0.0000431445l \right\} \times \left\{ 2 + \left( 1 - \frac{\rho_d}{1+t} \right) \left( 1 - \frac{\rho_c}{1+t} \right) + \left( 1 + \frac{\sigma_d}{1+t} \right) \left( 1 + \frac{\sigma_c}{1+t} \right) \right\} \right] dt \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \int_0^{t_1} \left[ \left[ \left( \frac{d_2 \tau_1 \delta}{(1-e^{-\delta \tau_1})} \right) \left\{ \left( \frac{h_p}{\delta} + C_d + \frac{C_p}{\delta} \right) \left( 1 + \frac{e^{-\delta \tau_1} - 1}{\delta \tau_1} \right) + C_{pol} \right\} + d_2 \left\{ \frac{h_r \tau_1}{2} + \right. \right. \right. \\
 &C_t 0.00424628l + C_c 0.0000431445l \left. \left. \left. \right] \right] \times \left\{ 4 + \frac{(\sigma_d + \sigma_c) - (\rho_d + \rho_c)}{1+t} + \frac{(\sigma_d \sigma_c + \rho_d \rho_c)}{(1+t)^2} \right\} + \left( \frac{K_1}{\tau_1} + \right. \right. \\
 &\left. \left. \frac{K_2}{\tau_1} \right) \left( 4 + \frac{\sigma_c - \rho_c}{1+t} \right) \right] dt = \left[ \left[ \left( \frac{d_2 \tau_1 \delta}{(1-e^{-\delta \tau_1})} \right) \left\{ \left( \frac{h_p}{\delta} + C_d + \frac{C_p}{\delta} \right) \left( 1 + \frac{e^{-\delta \tau_1} - 1}{\delta \tau_1} \right) + C_{pol} \right\} + d_2 \left\{ \frac{h_r \tau_1}{2} + \right. \right. \right. \\
 &C_t 0.00424628l + C_c 0.0000431445l \left. \left. \left. \right] \right] \times \frac{1}{2} \left\{ 4t_1 + ((\sigma_d + \sigma_c) - (\rho_d + \rho_c)) \log(1 + \right. \right. \\
 &t_1) + \frac{t_1}{1+t_1} (\sigma_d \sigma_c + \rho_d \rho_c) \left. \left. \right\} + \frac{1}{2} \left( \frac{K_1 + K_2}{\tau_1} \right) \left\{ 4t_1 + (\sigma_c - \rho_c) \log(1 + t_1) \right\} \right]
 \end{aligned}$$

**A.5** Adding left and right  $\alpha$ - cuts of membership function of cloudy fuzzy demand  $\tilde{d}$  we get,

$$L^{-1}(\alpha, t) + R^{-1}(\alpha, t) = d_1 + d_3 + \alpha(-d_1 + 2d_2 - d_3)$$

$$\begin{aligned}
 \text{Now, } I(\tilde{d}) &= \frac{1}{2t_1} \iint_{\alpha=0}^{\alpha=1} \{d_1 + d_3 + \alpha(-d_1 + 2d_2 - d_3)\} d\alpha dt \\
 &= \frac{1}{2t_1} \int_{t=0}^{t_1} \left[ \{(d_1 + d_3)\alpha\}_0^1 - \left\{ (d_1 - 2d_2 + d_3) \frac{\alpha^2}{2} \right\}_0^1 \right] dt \\
 &= \frac{1}{2t_1} \int_{t=0}^{t_1} \left[ (d_1 + d_3) - \frac{1}{2}(d_1 - 2d_2 + d_3) \right] dt \\
 &= \frac{1}{2t_1} \int_{t=0}^{t_1} \frac{1}{2} [(d_1 + d_3 + 2d_2)] dt = \frac{1}{2t_1} \int_{t=0}^{t_1} \frac{1}{2} \left[ d_2 \left\{ \left( 1 - \frac{\rho}{1+t} \right) + \left( 1 + \frac{\sigma}{1+t} \right) \right\} + 2d_2 \right] dt \\
 &= \frac{1}{2t_1} \int_{t=0}^{t_1} \frac{d_2}{2} \left[ 4 + \frac{\sigma - \rho}{1+t} \right] dt = \frac{1}{2t_1} \left[ \frac{d_2}{2} \{4t_1 + (\sigma - \rho) \log(1 + t_1)\} \right] \\
 &= \left( \frac{1}{2t_1} \times \frac{d_2}{2} \times 4t_1 \right) + \frac{1}{2t_1} \times \frac{d_2}{2} \times (\sigma - \rho) \log(1 + t_1) = d_2 + \frac{d_2}{4} (\sigma - \rho) \frac{\log(1+t_1)}{t_1} \\
 &= d_2 \left[ 1 + \frac{\sigma - \rho}{4} \frac{\log(1 + t_1)}{t_1} \right]
 \end{aligned}$$

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