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STOCK PORTFOLIO SELECTION USING A NEW DECISION-MAKING APPROACH BASED ON THE INTEGRATION OF FUZZY COCOSO WITH HERONIAN MEAN OPERATOR

Monika Narang[1*], Mahesh Chandra Joshi[1], Kiran Bisht[2] and Arun Pal[2]

[1] Department of Mathematics, Kumaun University, India
[2] Department of Mathematics, Statistics and Computer science, CBSH, GBPUA&T, Pantnagar, India

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Abstract: *The main objective of stock portfolio selection is to distribute capital to selected stocks to get the most profitable returns at a lower risk. The performance of a stock depends on a number of criteria based on the risk-return measures. Therefore, the selection of shares is subject to fulfilling a number of criteria. In this paper, we have adopted an integrated approach based on the two-stage framework. First, the heronian mean operator (improved generalized weighted heronian mean and improved generalized geometric weighted heronian mean) is combined with the traditional Combined compromise solution (CoCoSo) method to present a new decision-making model for dealing with stock selectionf problem. Second, Base-criterion method is used to calculate the relative optimal weights of the specified decision criteria. Despite the uncertainties, the advanced CoCoSo-H model eliminates the efficacy of anomalous data and make complex-decisions more flexible. A case study of stock selection for portfolio under National stock exchange (NSE) is discussed to validate the applicability of the proposed model. Different portfolio (P_1, P_2 & P_3) have been constructed using Particle swarm optimization (PSO). The outcome shows the prominence and stability of the proposed model when compare to previous studies.*

Key words: *Multi-criteria decision-making (MCDM), Heronian mean (HM), Combined compromise solution (CoCoSo), PSO, Portfolio analysis.*

* Corresponding author.
E-mail addresses: monikanarang5@gamil.com (Monika Narang), mcjoshi69@gmail.com (Mahesh Chandra Joshi), kiranbisht96@gmail.com (Kiran Bisht), arun_pal1969@yahoo.co.in (Arun Pal).

1. Introduction

Investing in the stock market over the past few decades has procreated increasing interest among investors as it offers an opportunity for flexible and transparent options of money to diversify risk with the potential for returns. The stock market is influenced by many direct and indirect factors and is full of opacity. Therefore, an investor has to examine a stock scrupulously before investing in it. In today's global economy, it is very difficult to analyse large amounts of information to arrive at an investment decision. Stock selection process include complex decision-making with many and often conflicting objectives. In the process of stock portfolio selection, there are broadly two stages: (a) some suitable shares are chosen; (b) the percentage of total investment for each share is obtained through different weighing schemes or through optimization techniques. The central problem is how to rank a group of stocks by evaluating them in terms of several criteria. Multi-criteria decision making (MCDM) is a controlled decision tool for calculating the weight of the evaluation criterion as well as ranking the alternatives present in problems with quantitative and qualitative criteria (Zeleny & Cochrane, 1973). MCDM methods have recently been getting phenomenal popularity and widespread applications (Durmic et al., 2020; Pamucar & Savin, 2020; Gorcun et al., 2021; Narang et al., 2021; Božanić et al., 2021). MCDM problems could be categorized into two classes: MODM and MADM.

MODM (Multiple-Objective Decision-making methods) refer to handling continuous problems with infinite number of options. On the other hand, MADM (Multiple-Attribute Decision-making methods) refer to discrete representations of a problem with many conflicting criteria and a limited number of alternatives. There are two main goals for solving practical problems by MCDM methods: calculating the optimum weight of the criterion and setting the rank of the alternatives. Scientists and researchers gave a new insight on how to mend the quality of decision making over the past decades and cited several methods for weight processing of criteria and alternatives as well as how to rank alternatives (Fontela & Gabus, 1972; Saaty, 1980, 1996; Rezaei, 2015; Haseli et al., 2019; Benayoun et al., 1996; Hwang & Yoon, 1981; Zavadskas, 1994; Pamucar et al., 2021; Zavadskas et al., 2001; Yazdani et al., 2018).

Base-Criterion method (Haseli et al., 2019) is the latest MCDM method that calculate the weights of criteria. In this method, one criterion is chosen as the base-criterion out of all the specified decision criteria. Then the relative importance of base-criterion to other criteria is determined on the numerical scale of 1/9 to 9. The concept of fuzziness which includes uncertainty and inaccuracy is first formalized by Zadeh (1965). So, the fuzzy extension of BCM is proposed (Haseli et al., 2020). The CoCoSo method (Yazdani et al., 2018) is new as well as a unique structure among several MCDM methods. It is highly capable of working with incomplete and uncertain data. Currently, the utility of CoCoSo method is increasing in many fields (Pamucar et al., 2018; Yazdani et al., 2019; Wen et al., 2019; Peng et al., 2020, Deveci et al., 2021).

In the proposed study, the CoCoSo method is modified by integrating it with the heronian mean operator (CoCoSo-H) under fuzzy environment to rank the stocks. A fuzzy Base-Criterion Method (F-BCM) is used to find the relative importance of criteria in a stock selection process. The derived weights of criteria are then used in

CoCoSo-H to obtain the most suitable stock. Illustration is done using stocks under NSE. Historical data is used to apply F-CoCoSo-H- BCM model to rank the stocks. Various portfolio has been then constructed by using particle swarm optimization based on the ranking obtained by proposed model. The return of the portfolio has been found to be satisfactory as compared to the previous studies. This hybrid approach (F-

CoCoSo-H-BCM) for solving stock selection problem has been discussed for the first time.

The aggregation of information provided by decision makers is a fundamental need of an information processing system such as decision making. Aggregation functions play a key role in MCDMs to slacken the dimensions of the criterion (Detyniekie, 2001). According to the conventional aggregation operators, criteria are independent, and the efficacy of the criterion is additive. Whereas, in real-world decision-making problems there are always different types of interrelationships between decision criteria, so this independent hypothesis cannot ordinarily be satisfied. The most fundamental and plausible aggregation operators are the Choquet integral (CI) & Sugeno integral (SI), Power average (PA), Bonferroni mean (BM) and Heronian Mean.

The HM operator (Beliakov et al., 2007; Sykora, 2009a, 2009b) has the peculiarity of catching the correlations of the aggregated arguments. The operators based on HM describes the interrelationships between different variables, explain the interrelationship between a decision criterion and itself and also differ the interrelationship between C_j and C_i from the interrelationship between criteria C_i and C_j.

In the CoCoSo method, criteria values procured by applying the weighted product method (WPM) and the weighted sum method (WSM) employ a significant impact on the final ranking of alternatives. The character of the WSM function is a simple linear one. WPM and WSM ignore the mutual impact of criteria. The traditional method can distort the result of aggregation and leads to incorrect prioritization of alternatives. Hence the application of the heronian mean operator (IGWHM- improved generalized weighted heronian mean and IGGWHM- improved generalized geometric weighted heronian mean) is introduced to overcome this drawback in the traditional CoCoSo method.

The reasons for the combination of F-BCM and CoCoSo-H are as follows:

- Despite uncertainties it can make complex stock selection processes much easier and efficient in multi-dimensional decision analysis systems.
- In the course of the selection process, F-BCM make use of subjective information that reflects the judgment and conduct of humans. Unlike AHP and F-BWM, the F-BCM method permits fully consistent pairwise comparison of evaluation criteria.
- Reforming the traditional CoCoSo method using new aggregation operators which has the ability of capturing the correlations of the aggregated arguments and removes the effect of the anomalous data.

The remaining paper is organized as follows: Section 2 focuses on the existing literature of stock portfolio selection. Section 3 decodes the proposed model. Section 4 gives a case study, results and discussions. Section 5 concludes the paper.

2. Related work

In the literature, there are number of approaches to handle and construct a portfolio. Portfolio optimization is based on the Modern Portfolio Theory (MPT) (Markowitz, 1952) established fifty years ago. The MPT is based on the principle that investors want the highest return with the lowest risk. Markowitz introduced the mean-variance method for the stock portfolio decision problem. This method works on the concept that "when the risk of stock portfolio is constant, we should try to maximize the return rate of stock portfolio and when the return rate of stock portfolio is constant, we should try to minimize the risk of stock portfolio". This theory was widely accepted and adopted by various researchers. But market efficiency is considered a core assumption in MPT, getting information about the markets every time is expensive and time-consuming (Grossman & Stiglitz, 1980). The capital asset pricing model (CAPM) was developed in the early 1960s (Sharpe, 1964; Treynor, 2015; Lintner, 1965a, 1965b). The CAPM is based on the concept that all risks should not affect asset prices. It establishes the relationship between expected return and systematic risk. Despite some shortcomings the CAPM formula is still widely used and is capable of easy comparison of investment options. However, it is difficult to evaluate the ability of firms with different inputs and outputs. The data envelopment analysis (DEA) models (Charnes et al., 1978; Banker et al., 1984) can purposefully combine multiple inputs and outputs of a unit into a single measure of overall organizational capacity. The DEA methodology is very efficient in a financial application such as measuring managerial efficiency using a company's financial statements.

Analytical hierarchy process (AHP) (Saaty, 1980) has been proposed to cope with the stock portfolio decision problem by evaluating the performance of each company in various levels of criteria. DEA, a nonparametric method, has been used (Edirisinghe & Zhang, 2008) for selecting and screening of stocks. Huang (2008) gave a new definition of risk and employed a genetic algorithm to deal with the stock portfolio decision making problem. Generally, in a portfolio selection problem the decision maker simultaneously considers conflicting objectives like rate of return, liquidity, and risk. So, the multi-objective programming (such as Goal programming, compromise programming) are used to select the portfolio (Abdelaziz et al., 2007; Ballestero, 2001; Aouni et al., 2005). The need to model the portfolio selection within the MCDM frame has been proposed (Hurson & Zopounidis, 1995) in 1995. After analysing the relevance of multi-criteria decision systems for financial decisions, a detailed discussion and review on portfolio selection (Zopounidis & Doumpos, 2013) is provided. An integrated and innovative method for building and selecting equity portfolios have been developed (Xinodas et al., 2010), which takes into account the inherent multidimensional nature of the problem while allowing DMs to incorporate their priorities in the decision process.

Due to accurate information and subjective opinions of experts that often appear in the stock portfolio decision making process, crisp values are insufficient to solve problems. The use of linguistic assessment in place of numerical values may be a more appropriate approach. A new decision-making method (Chen & Hung, 2009) is introduced for stock portfolio selection using linguistic valuation along with computing. In the proposed approach, they have used linguistic assessment to express the opinion of the experts and combined the linguistic TOPSIS (Technique for order preference by similarity to an ideal solution) and linguistic ELECTRE (Elimination and Choice Expressing Reality) methods for dealing with stock selection problem. Recently a fuzzy-ANP (Analytical network process) approach (Galankashi et al., 2020) is developed to rank various Tehran stock exchange (TSE) portfolios. Keeping in mind the uncertain nature of the portfolio selection problems, Ammar and Khalifa (2003). moved a formulation of fuzzy portfolio optimization. A method of group decision making has been developed (Tiryaki & Ahlatcioglu, 2005) in an ambiguous environment. In this method, Tirayaki shifted the linguistic value of the experts to a triangular fuzzy number and used a new fuzzy ranking and weighted algorithm to derive the investment ratio of each stock.

Ranking is one of the strategies to derive the concept of converting raw information into relevant information for decision making. A hybrid multi-criteria model (Fazli & Jafari, 2012) for investment in stock exchange is developed. In this methodology, DEMETAL (Decision-Making Trial and Evaluation Laboratory) method is used to build a relations-structure between criteria while VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method is used to select the most preferrable alternative for investment. A fuzzy cross-entropy-mean–variance–skewness models for portfolio optimization under several constraints under Bombay stock exchange (BSE) has been proposed (Bhattacharya et al., 2014). A new hybrid MCDM approach is introduced (Poklepović & Babić, 2014) to rank the stocks based on Spearman's rank correlation coefficient. An investor wants to create a balanced portfolio with shares representing different sectors. An attempt has been made to provide a DEA-TOPSIS based framework (Mansouri et al., 2014) in the context of TSE. A new ranking methodology (Dedania et al., 2015) that is based on the principle of comparing a company with companies in the same field have been proposed because the attributes that affect the growth of the company vary for different sectors. A popular MCDM method, AHP, is applied to achieve the rank of portfolio (Hota et al., 2015) for further decision-making process. Dincer (2015) has proposed a profit-based stock selection approach in the banking sector using fuzzy AHP and MOORA (The Multi-Objective Optimization Ratio Analysis) method.

A new technique has been proposed (Boonjing & Boongasame, 2017) for portfolio selection with two significant financial ratios (dividend yield and net profit margin) using the ELECTRE III method to enable small investors to make trading decisions easily. A portfolio selection model (Thakur et al., 2018) have been developed that prioritizes high-ranked stocks. In this approach, they have identified critical factors using the fuzzy Delphi method and used the Dempster – Shaffer evidence theory to rank the stocks under NSE. Ant colony optimization (ACO) is used to optimize (or construct) the portfolio. To predict the best performing company in the IT industry, a new Best-worst method (BWM) has been implemented (Krishna et al., 2018). A hybrid approach DEA-COPRAS (Complex Proportional Assessment) (Gupta et al., 2019) has been applied for portfolio selection at risk-return interface based on NSE. In which, DEA is applied to calculate the efficiency of the stock and COPRAS is used to rank the stocks.

Recently, a hybrid multi-criteria decision-making approach have been presented (Mills et al., 2020) under grey environment incorporating an integrated ANP and DEMETAL that provides both ranking and weighting information for optimal portfolio selection. AHP-TOPSIS (Gupta et al., 2021), a hybrid multi-criteria decision-making technique, has been developed. using which they have ranked the financial performance of selected Indian private banks. A hybrid fuzzy COPRAS base criterion method (Narang et al., 2021) has been implemented to rank the stocks under NSE. In this methodology, ranked asset algorithm has been used for the capital distribution among the stocks according to their rank.

3. Data and Methodology

To tackle the hesitant nature of the information of human mind, fuzzy sets are used. It is necessary to use linguistic variables for modeling the decisions of the decision maker's that can be expressed by trapezoidal fuzzy numbers.

3.1. Preliminaries

Definition (Trapezoidal fuzzy number) (Nourianfar & Montazer, 2013; Savitha & George, 2017) A Trapezoidal fuzzy number (TrFN) represented by T is defined as (a, b, c, d) where the membership function is expressed by

$$\mu_T(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & x \geq d \end{cases} \quad (1)$$

Definition (Statistical beta distribution) (Rahmani et al., 2016). The crisp value μ_T corresponding to the trapezoidal fuzzy number (a, b, c, d) based on Statistical Beta Distribution method (SBDM) can be obtained as follows

$$\mu_T = \frac{2a+7b+7c+2d}{18} \quad (2)$$

The readers can refer (Nourianfar & Montazer, 2013) to learn about basic operations of TrFNs.

Definition (Heronian mean) (Liu & Zhang, 2017). A HM operator of a set of non-negative values $X = \{x_1, x_2, \dots x_n\}$ is:

$$HM(x_1, x_2, \dots x_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \sqrt{x_i x_j} \quad (3)$$

3.2. The proposed F-CoCoSo-H-BCM method

3.2.1. Deriving weight of criteria through F-BCM

Fuzzy Base-Criterion Method has been moved (Haseli et al., 2020) in which the decision maker's opinions are expressed linguistically as human decisions are fraught with uncertainty and ambiguity. F-BCM is capable to obtain fully consistent results and calculate crisp weights using less pairwise comparisons than the existing MCDM methods such as AHP (Saaty, 1980), and BWM (Rezaei, 2015). This method is more accurate and at less time consuming because the execution of secondary comparisons is not necessary. Although, BCM and FUCOM (Full consistency method) (Pamucar et al., 2018) both the method performs n-1 pairwise comparisons to calculate optimal weights of criteria. But in BCM, complexity is low in terms of selecting a particular criterion as a base criterion as compared to FUCOM. So, the Fuzzy BCM method is preferred to calculate the weights of the criteria in this paper.

The fuzzy pairwise comparison matrix is as follows:

$$\tilde{T} = \begin{bmatrix} (1,1,1,1) & \tilde{T}_{12} & \tilde{T}_{13} & \dots & \tilde{T}_{1n} \\ \tilde{T}_{21} & (1,1,1,1) & \tilde{T}_{23} & \dots & \tilde{T}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{T}_{n1} & \tilde{T}_{n2} & \tilde{T}_{n3} & \dots & (1,1,1,1) \end{bmatrix} \quad (4)$$

where $\tilde{T}_{ij}$ represent the relative fuzzy importance of criteria i to j. Similarly, T_{ji} represent the relative fuzzy importance of criteria j to i. T_{ij} is a trapezoidal fuzzy number and when $i = j$, $\tilde{T}_{ij} = (1,1,1,1)$. Table 1 represents the corresponding TrFNs for pairwise comparison of criteria.

Table 1. TrFNs of F-BCM for pairwise comparison

Linguistic Set	TrFNs
Equally Important	(1,1,1,1)
Moderately Important	(1,1,2,3)
Strongly Important	(2,3,4,5)
Very Strongly Important	(4,5,6,7)
Extremely Important	(6,7,8,9)
Absolutely Important	(8,9,9,9)

The step-wise procedure of F-BCM is as follows.

Step-1 Determine and evaluates a set of criteria $\{C_1, C_2, C_{3,} \dots, C_n\}$ accordance with the opinion of decision maker.

Step-2 Identify one of the criteria as a base-criteria from a set of criteria.

Step-3 Pairwise comparisons are performed in this step. The relative fuzzy preference of the base criteria over the remaining criteria is derived using Table 1. The resulting vector of fuzzy base comparisons as follows.

$$\tilde{T}_B = (\tilde{T}_{B1}, \tilde{T}_{B2}, \tilde{T}_{B3}, \dots, \tilde{T}_{Bn})$$

$\check{T}_B$ represents a fuzzy base-criteria over the rest of the criteria vector. $\check{T}_{Bj}$ represents the fuzzy importance of the base-criteria over the rest of the criteria and it is obvious that $\check{T}_{BB} = (1,1,1,1)$.

Step-4 For identification of optimal fuzzy weights, a non-linear programming model on the basis of components derived from $\check{T}_B$ vector is as follows.

$\min \tilde{\xi}$

Such that

$$\begin{cases} \left|\frac{\widetilde{w}_B}{\widetilde{w}_j} - \tilde{T}_{Bj}\right| \leq \tilde{\xi} \\ \sum_{j=1}^{n} R(\widetilde{w}_j) = 1 \\ a_j^w \leq b_j^w \leq c_j^w \leq d_j^w \\ a_j^w \geq 0 \; for \; all \; j \end{cases} \tag{5}$$

Where $\widetilde{w}_B = (a_B^w, b_B^w, c_B^w, d_B^w)$, $\widetilde{w}_j = (a_j^w, b_j^w, c_j^w, d_j^w)$, $\tilde{\xi} = (a^{\xi}, b^{\xi}, c^{\xi}, d^{\xi})$.

The Equation 5 can be rewritten as

$\min \tilde{\xi}$

Such that

$$\begin{cases} \left|\frac{(a_B^w, b_B^w, c_B^w, d_B^w)}{(a_j^w, b_j^w, c_j^w, d_j^w)} - (a_{Bj}, b_{Bj}, c_{Bj}, d_{Bj})\right| \leq (k^*, k^*, k^*, k^*) \\ \sum_{j=1}^{n} R(\widetilde{w}_j) = 1 \\ a_j^w \leq b_j^w \leq c_j^w \leq d_j^w \\ a_j^w \geq 0 \; for \; all \; j \end{cases} \tag{6}$$

After solving the Equation 6, the optimal fuzzy weights can be transformed to crisp values by make use of SBDM presented in Equation 2.

The pairwise comparison of elements in Base-Criterion method done under the principle that $a_{Base,i} * a_{i,j} = a_{Base,j}$ and TrFNs satisfy the constraint $\left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{8}\right) \leq (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \leq (8,9,9,9)$ or $\left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}, \frac{1}{8}\right) \leq \frac{(a_{Bj}, b_{Bj}, c_{Bj}, d_{Bj})}{(a_{Bi}, b_{Bi}, c_{Bi}, d_{Bi})} \leq (8,9,9,9)$. This ensures the fully consistent solution of the optimization problem with $\xi = 0$.

3.2.2. Ranking of alternatives through Fuzzy CoCoSo-H

The traditional CoCoSo (Combined Compromise Solution) method for MCDM is primarily developed by Yazdani et al. (2018). Foundation of the CoCoSo method is based on the idea of SAW (Simple additive weighting), MEW (Multiplicative exponential weighting) and WASPAS (The Weighted Aggregates Sum Product Assessment) methods.

After defining a set of alternatives and criteria, the stepwise procedure to solve the CoCoSo-H MCDM model (Figure 1) is as follows.

Step-1 Decision matrix: The assessment of m alternatives $A = \{A_1, A_2, .., A_m\}$ with respect to n criteria $C = \{C_1, C_2, .., C_n\}$ is performed in the matrix $\widetilde{T}$. Decision maker give their opinions from a linguistic point of view. Table 2 unfolds the linguistic terms and their corresponding TrFNs.

$$\widetilde{T} = \begin{bmatrix} \left(T_{11}^{(a)}, T_{11}^{(b)}, T_{11}^{(c)}, T_{11}^{(d)}\right) & \left(T_{12}^{(a)}, T_{12}^{(b)}, T_{12}^{(c)}, T_{12}^{(d)}\right) & \cdots & \left(T_{1n}^{(a)}, T_{1n}^{(b)}, T_{1n}^{(c)}, T_{1n}^{(d)}\right) \\ \left(T_{21}^{(a)}, T_{21}^{(b)}, T_{21}^{(c)}, T_{21}^{(d)}\right) & \left(T_{22}^{(a)}, T_{22}^{(b)}, T_{22}^{(c)}, T_{22}^{(d)}\right) & \cdots & \left(T_{2n}^{(a)}, T_{2n}^{(b)}, T_{2n}^{(c)}, T_{2n}^{(d)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(T_{m1}^{(a)}, T_{m1}^{(b)}, T_{m1}^{(c)}, T_{m1}^{(d)}\right) & \left(T_{m2}^{(a)}, T_{m2}^{(b)}, T_{m2}^{(c)}, T_{m2}^{(d)}\right) & \cdots & \left(T_{mn}^{(a)}, T_{mn}^{(b)}, T_{mn}^{(c)}, T_{mn}^{(d)}\right) \end{bmatrix} \quad (7)$$

Table 2. TrFNs for CoCoSo-H

Linguistic Sets	TrFNs
Very Low	(1,1,2,3)
Low	(1,2,3,4)
Medium Low	(2,3,4,5)
Medium	(3,4,5,6)
Medium High	(4,5,6,7)
High	(5,6,7,8)
Very High	(6,7,8,9)
Very Very High	(7,8,9,9)
Extremely High	(8,9,9,9)

Step-2 Normalized decision matrix:

$$\widetilde{N} = \begin{bmatrix} \left(\gamma_{11}^{(a)}, \gamma_{11}^{(b)}, \gamma_{11}^{(c)}, \gamma_{11}^{(d)}\right) & \left(\gamma_{12}^{(a)}, \gamma_{12}^{(b)}, \gamma_{12}^{(c)}, \gamma_{12}^{(d)}\right) & \cdots & \left(\gamma_{1n}^{(a)}, \gamma_{1n}^{(b)}, \gamma_{1n}^{(c)}, \gamma_{1n}^{(d)}\right) \\ \left(\gamma_{21}^{(a)}, \gamma_{21}^{(b)}, \gamma_{21}^{(c)}, \gamma_{21}^{(d)}\right) & \left(\gamma_{22}^{(a)}, \gamma_{22}^{(b)}, \gamma_{22}^{(c)}, \gamma_{22}^{(d)}\right) & \cdots & \left(\gamma_{2n}^{(a)}, \gamma_{2n}^{(b)}, \gamma_{2n}^{(c)}, \gamma_{2n}^{(d)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\gamma_{m1}^{(a)}, \gamma_{m1}^{(b)}, \gamma_{m1}^{(c)}, \gamma_{m1}^{(d)}\right) & \left(\gamma_{m2}^{(a)}, \gamma_{m2}^{(b)}, \gamma_{m2}^{(c)}, \gamma_{m2}^{(d)}\right) & \cdots & \left(\gamma_{mn}^{(a)}, \gamma_{mn}^{(b)}, \gamma_{mn}^{(c)}, \gamma_{mn}^{(d)}\right) \end{bmatrix} \quad (8)$$

where normalized values of decision matrix

$$\gamma_{ij} = \left\{\gamma_{ij}^{(a)}, \gamma_{ij}^{(b)}, \gamma_{ij}^{(c)}, \gamma_{ij}^{(d)}\right\}$$

$$= \left\{\gamma_{ij}^{(a)} = \frac{\gamma_{ij}^{(a)}}{\gamma_j^{+}}; \gamma_{ij}^{(b)} = \frac{\gamma_{ij}^{(b)}}{\gamma_j^{+}}; \gamma_{ij}^{(c)} = \frac{\gamma_{ij}^{(c)}}{\gamma_j^{+}}; \gamma_{ij}^{(d)} = \frac{\gamma_{ij}^{(d)}}{\gamma_j^{+}};\right. \quad (9)$$

where $\gamma_j^{+} = \max_i\left(T_{ij}^{(d)}\right)$

Step-3 Weighted sequences of alternatives: Weighted sequences are enumerated by make use of fuzzy IGWHM function and fuzzy IGGWHM function (Liu & Zhang, 2017).

IWGHM is defined as

$$SH_i^{p,q} = \frac{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i\breve{\gamma}_i)^p(w_j\breve{\gamma}_j)^q\right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i)^p(w_j)^q\right)^{\frac{1}{p+q}}}$$

$$=\left(\left(\frac{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}\left(w_i\breve{\gamma}_i^{(a)}\right)^p\left(w_j\breve{\gamma}_j^{(a)}\right)^q\right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i)^p(w_j)^q\right)^{\frac{1}{p+q}}}\right),\left(\frac{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}\left(w_i\breve{\gamma}_i^{(b)}\right)^p\left(w_j\breve{\gamma}_j^{(b)}\right)^q\right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i)^p(w_j)^q\right)^{\frac{1}{p+q}}}\right),\right.$$

$$\left.\left(\frac{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}\left(w_i\breve{\gamma}_i^{(c)}\right)^p\left(w_j\breve{\gamma}_j^{(c)}\right)^q\right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i)^p(w_j)^q\right)^{\frac{1}{p+q}}}\right),\left(\frac{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}\left(w_i\breve{\gamma}_i^{(d)}\right)^p\left(w_j\breve{\gamma}_j^{(d)}\right)^q\right)^{\frac{1}{p+q}}}{\left(\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i)^p(w_j)^q\right)^{\frac{1}{p+q}}}\right)\right) \quad (10)$$

where $p,q \geq 0$, w_j unfold the relative weights of criteria, $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.
IGGWHM is defined as

$$PH_i^{p,q} = \frac{1}{p+q}\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\breve{\gamma}_i + q\breve{\gamma}_j\right)^{\frac{2(n+1-i)w_j}{n(n+1)\sum_{k=i}^{n}w_k}}$$

$$=\left(\left(\frac{1}{p+q}\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\breve{\gamma}_i^{(a)} + q\breve{\gamma}_j^{(a)}\right)^{\frac{2(n+1-i)w_j}{n(n+1)\sum_{k=i}^{n}w_k}}\right),\left(\frac{1}{p+q}\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\breve{\gamma}_i^{(b)} + q\breve{\gamma}_j^{(b)}\right)^{\frac{2(n+1-i)w_j}{n(n+1)\sum_{k=i}^{n}w_k}}\right),\left(\frac{1}{p+q}\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\breve{\gamma}_i^{(c)} + q\breve{\gamma}_j^{(c)}\right)^{\frac{2(n+1-i)w_j}{n(n+1)\sum_{k=i}^{n}w_k}}\right),\left(\frac{1}{p+q}\prod_{i=1}^{n}\prod_{j=i}^{n}\left(p\breve{\gamma}_i^{(d)} + q\breve{\gamma}_j^{(d)}\right)^{\frac{2(n+1-i)w_j}{n(n+1)\sum_{k=i}^{n}w_k}}\right)\right) \quad (11)$$

where $p,q \geq 0$, w_j unfold the relative weights of criteria, $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.
Parameters p and q represent the stability parameters.

Step-4 Relative significance: Three pooling strategies are enumerated for each option. the concernment of alternatives within the strategies could be procreated by make use of the following equations.

$$K_{iHa} = \frac{SH_i + PH_i}{\sum_{i=1}^{m}(SH_i + PH_i)} \quad (12)$$

$$K_{iHb} = \frac{SH_i}{\min_i(SH_i)} + \frac{PH_i}{\min_i(PH_i)} \quad (13)$$

$$K_{iHc} = \frac{\lambda SH_i + (1-\lambda)PH_i}{\lambda \max_i(SH_i) + (1-\lambda)\max_i(PH_i)}, \quad 0 \leq \lambda \leq 1 \quad (14)$$

The coefficient λ unfolds the stagnation and ductility of the proposed fuzzy CoCoSo-H model.

Step-5 Final table for the ranking of alternatives: The rank of an alternative is defined on the basis of the value of K_{ih}. The higher the value of K_{ih}, the higher the priority of the alternative.

$$K_{iH} = \frac{(K_{iHa} + K_{iHb} + K_{iHc})}{3} + (K_{iHa}.K_{iHb}.K_{iHc})^{1/3} \quad (15)$$

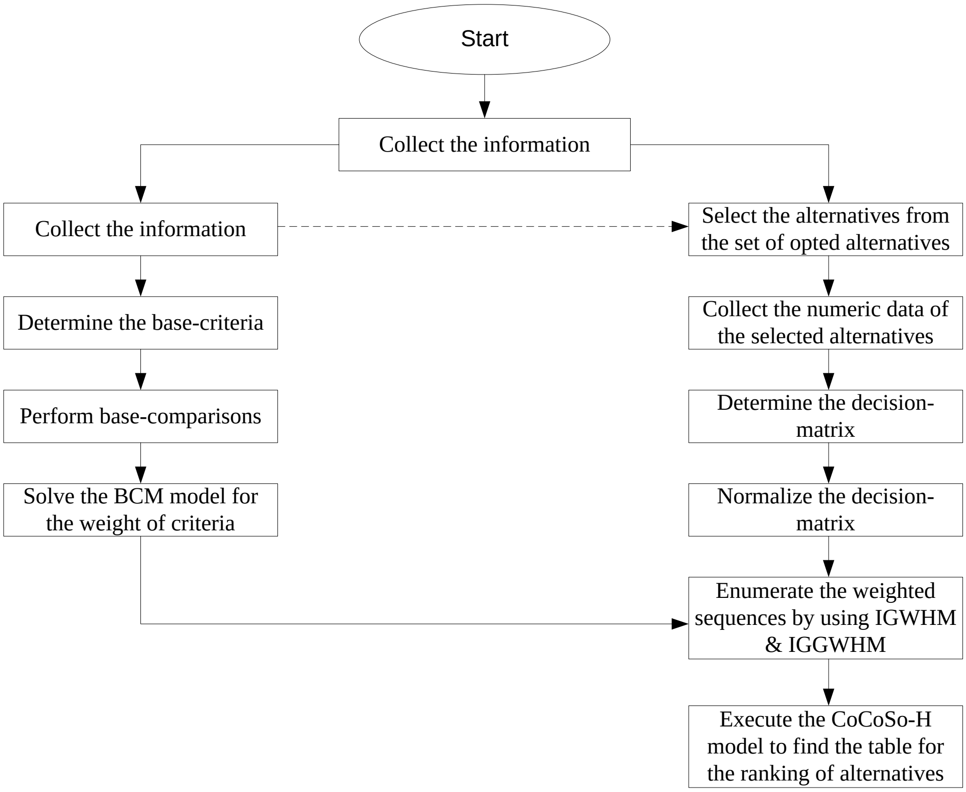

Figure 1. Flowchart of the F-CoCoSo-H-BCM model

3.3. A case study

Stock market plays a significant role in economic growth of a country. It is very challenging to choose an investor's stock because the stock market is full of uncertainties and very unpredictable. A profitable decision could be made by an investor based on some fundamental and technical analysis of stocks.

3.3.1. Structure of alternatives and criteria

Investors believe that there is no one right way to examine stocks because of the multidimensional uncertainties. Knowing the financial data and loss information of any company can go a long way in helping investors in the selection of stocks. There are many techniques to select the stocks for the investment like fundamental analysis and technical analysis. Fundamental approach sifts the key ratio of a market to shape its financial health and takes several factors into account like earnings ratios and risk, for future forecast. Several important fundamental factors are used by topmost investors and portfolio managers to evaluate and select stocks. With the help of extensive literature survey and based on the opinion of experts, we have opted 4 fundamental criteria. The criteria are as follows:

1. Revenue: Revenue reflects growth of the company. The increasing rate of revenue indicates the increasing demand of company's product in the market.

2. Return on Equity (ROE): Return on Equity measures the return or profit earned per share by equity holder. Company having high ROE consider good for investment.

3. Debt Equity Ratio (DER): Debt Equity Ratio of a company is calculated by dividing the debt of the company to the share holders' fund. It reflects the ability of a company to outstand of all the debts if the market downturn eventually. This ratio is used for selecting a safe investment.

4. Price to Earnings ratio (P/E): Price to Earnings ratio depicts the price per share corresponding to the earning per share. It tells the stocks of the company are overvalued or not.

So, the two criteria revenues (C_1) and ROE (C_2) are considered as beneficial criteria. DER (C_3) and P/E (C_4) are taken as non-beneficial or cost criteria.

15 stocks from NSE have been selected as alternatives to investigate them over four above mentioned criteria. The 11 years (from Jan 2010 to Dec 2020) historical data of the stocks is collected from http://www.investello.com and http://www.ratestar.in which serves as a collection of evidence to support or disprove the notion that the stock concerned is going to perform well in the future. Quarterly data has been used for this study. The single numeric value shown in Table 3 is taken as the evaluation value of each alternative to each criterion.

1. Tata Consultancy Services Limited (TCS) (st_1)
2. HDFC Bank Limited (HDFCBANK) (st_2)
3. TITAN Company Limited (TITAN) (st_3)
4. Oil & Natural Gas Corporation Limited (ONGC) (st_4)
5. Hindustan Uniliver Limited (HINDUNILVR) (st_5)
6. Divi's Laboratories Limited (DIVISLAB) (st_6)
7. Reliance Industries Limited (RELIANCE) (st_7)
8. Pidilite Industries Limited (PIDILITIND) (st_8)
9. JSW Steel Limited (JSWSTEEL) (st_9)
10. Aurobindo Pharma Limited (AUROPHARMA) (st_{10})
11. Bajaj Finance Limited (BAJFINANCE) (st_{11})
12. Dr. Reddy's Laboratories Limited (DRREDDY) (st_{12})
13. Kotak Mahindra Bank Limited (KOTAKBANK) (st_{13})
14. Asian Paints Limited (ASIANPAINT) (st_{14})
15. Jubilant Foodworks Limited (JUBLFOOD) (st_{15})

Table 3. EMA (Exponential moving average) of actual numerical values of each criterion

Stocks/Criteria	$C_1(Revenue)$	$C_2(ROE)$	$C_3(DER)$	$C_4(P/E)$
st_1	141259	35.43	0.0002	5.28
st_2	118167.2	15.78	0.3572	26.45
st_3	18139.04	22.64	0.2457	77.33
st_4	390249	9.567	0.481	9.729
st_5	37694.8	78.64	0.0079	70.78
st_6	4783.9	19.31	0.0085	40.61
st_7	511663.6	10.25	0.7372	22.35
st_8	66991.0	24.87	0.039	64.41
st_9	707483.7	14.70	1.60	14.52
st_{10}	19118.55	19.32	0.453	15.85
st_{11}	18341.78	17.55	4.52	43.23
st_{12}	16556.73	13.05	0.264	30.28
st_{13}	42500.38	12.64	0.8391	34.39

Stocks/Criteria	$C_1(Revenue)$	$C_2(ROE)$	$C_3(DER)$	$C_4(P/E)$
st_{14}	18434.92	17.55	0.0569	64.99
st_{15}	3346.76	22.09	0.0004	83.68

4. Results and discussions

In this part, the application of our proposed methodology to rank the performance of different stocks by investigating them under some selected criteria in the financial trading is discussed.

4.1. Enumeration of relative weights of criteria

In the first step, relative optimal weights of the criteria are calculated that directly affect the ranking of the alternatives in further work.

Step-1: At first, decision maker selects C_2:ROE as a base-criteria from a set of criteria {C_1:Revenue, C_2:ROE, C_3:DER, C_4:P/E}.

Step-2: The fuzzy base-comparisons are performed based on linguistic terms shown in Table 1.

Step-3: The fuzzy base-comparison vector based on TrFNs revealed by the decision maker for pairwise comparisons of the fuzzy base-criterion with other criteria as follows. $\tilde{T}_B = \{(4,5,6,7), (1,1,1,1), (6,7,8,9), (2,3,4,5)\}$;

Step-4: A non-linear constrained optimization problem is created based on the Equation 6 as follows.

$\min \tilde{\xi}^*$

Such that

$$\begin{cases} \left|\frac{(a_2,b_2,c_2,d_2)}{(a_1,b_1,c_1,d_1)} - (4,5,6,7)\right| \leq (k^*,k^*,k^*,k^*) \\ \left|\frac{(a_2,b_2,c_2,d_2)}{(a_3,b_3,c_3,d_3)} - (6,7,8,9)\right| \leq (k^*,k^*,k^*,k^*) \\ \left|\frac{(a_2,b_2,c_2,d_2)}{(a_4,b_4,c_4,d_4)} - (2,3,4,5)\right| \leq (k^*,k^*,k^*,k^*) \\ \frac{1}{18}\begin{pmatrix} 2a_1 + 7b_1 + 7c_1 + 2d_1 + 2a_2 + 7b_2 + 7c_2 + 2d_2 + 2a_3 \\ +7b_3 + 7c_3 + 2d_3 + 2a_4 + 7b_4 + 7c_4 + 2d_4 \end{pmatrix} = 1 \\ a_1 \leq b_1 \leq c_1 \leq d_1 \\ a_2 \leq b_2 \leq c_2 \leq d_2 \\ a_3 \leq b_3 \leq c_3 \leq d_3 \\ a_4 \leq b_4 \leq c_4 \leq d_4 \\ a_1, a_2, a_3, a_4 > 0 \\ k^* \geq 0 \end{cases} \tag{16}$$

by solving the Equation16, the optimal fuzzy weights are derived, which are:

$\breve{w}_1^* = (0.095,0.106,0.121,0.144)$; $\breve{w}_2^* = (0.576,0.608,0.638,0.665)$;

$\breve{w}_3^* = (0.073,0.079,0.086,0.096)$; $\breve{w}_4^* = (0.133,0.159,0.202,0.288)$;

by applying Equation 2, the optimal fuzzy weights of criteria are transformed into crisp values as:

$$\breve{w}_1 = 0.11,\ \breve{w}_2 = 0.62,\ \breve{w}_3 = 0.08,\ \breve{w}_4 = 0.19.$$

The relative weights will be used in the fuzzy CoCoSo-H model.

4.2. Selection of the extremely abiding stock through fuzzy CoCoSo-H

The second phase involves ranking the selected stocks based on the specified decision-criteria.

Step-1: The stocks on each criterion are rated by the decision maker using the actual numerical values of each criterion in Table 3 and fuzzy linguistic variables in Table 2, based on which initial decision matrix (Table 4) is derived.

Step-2: Using Equation 8 and 9, the normalized fuzzy decisional matrix (Table 5) is obtained.

Step-3: The weighted sequences SH_i and PH_i (Table 6) have been obtained using elements of the normalized decisional matrix and the relative weights calculated by F-BCM for the values of the parameters $p = q = 1$. We have validated $p = q = 1$ because they not only make calculation easier, but also fully capture the correlations among the specified decision criteria. Fuzzy weighted sequences are transformed into crisp sequences using Equation 2.

Table 4. Initial decision matrix of stocks.

Stocks	Decision-criteria			
	C_1	C_2	C_3	C_4
st_1	(6,7,8,9)	(5,6,7,8)	(7,8,9,9)	(6,7,8,9)
st_2	(6,7,8,9)	(1,2,3,4)	(4,5,6,7)	(6,7,8,9)
st_3	(1,2,3,4)	(3,4,5,6)	(5,6,7,8)	(1,2,3,4)
st_4	(7,8,9,9)	(1,1,2,3)	(4,5,6,7)	(8,9,9,9)
st_5	(2,3,4,5)	(8,9,9,9)	(7,8,9,9)	(1,2,3,4)
st_6	(1,1,2,3)	(2,3,4,5)	(7,8,9,9)	(4,5,6,7)
st_7	(8,9,9,9,)	(1,1,2,3)	(2,3,4,5)	(6,7,8,9)
st_8	(5,6,7,8)	(3,4,5,6)	(6,7,8,9)	(2,3,4,5)
st_9	(5,6,7,8)	(1,2,3,4)	(2,3,4,5)	(7,8,9,9)
st_{10}	(1,2,3,4)	(2,3,4,5)	(4,5,6,7)	(7,8,9,9)
st_{11}	(1,2,3,4)	(2,3,4,5)	(1,1,2,3)	(4,5,6,7)
st_{12}	(1,2,3,4)	(1,2,3,4)	(5,6,7,8)	(5,6,7,8)
st_{13}	(3,4,5,6)	(1,2,3,4)	(2,3,4,5)	(5,6,7,8)
st_{14}	(1,2,3,4)	(2,3,4,5)	(6,7,8,9)	(2,3,4,5)
st_{15}	(1,1,2,3)	(3,4,5,6)	(8,9,9,9)	(1,1,2,3)

Example 1: Fuzzy SH_i of st_1 for i=1, j=1 is derived by using Equation 10 is as follows:

=((0.11*0.67)*(0.11*0.67)+(0.11*0.67)*(0.62*0.56)+…….+(0.62*0.56)*(0.62*0.56)+(0.62*0.56)*(0.08*0.78)+…………….+(0.19*0.67)*(0.19*0.67))^0.5/(0.11*0.11+0.11*0.62+0.11*0.08+0.11*0.19+0.62*0.62+0.62*0.08+0.62*0.19+0.08*0.08+0.08*0.19+0.19*0.19)^0.5 = 0.58

Example 2: Fuzzy PH_i of st_1 for i=1, j=1is calculated by using Equation 11 is as follows:

=0.5*((0.67+0.67)^((2*4*0.11)/(20*(0.11+0.62+0.08+0.19))))*((0.11+0.62)^((2*4*0.62)/(20*(0.11+0.62+0.08+0.19))))*((0.11+0.08)^((2*4*0.08)/(20*(0.11+0.62+0.08+0.19))))*……………*(0.19*0.19)^((2*1*0.19)/(20*(0.19))) = 0.66

Example 3: Crisp value SH_1 of st_1 is calculated by make use of Equation 2 is as follows:

= (2*0.58+7*0.69+7*0.80+2*0.91)/18

=0.75

Step- 4: By using Equations 12,13 and 14 the relative significance of the stocks is procured. The coefficient λ is assumed to be 1/2 under the third pooling strategy. This value of λ is the middle most value between 0 and 1 and decision makers generally consider this value.

Step-5: Table 7 presents the final ranking of stocks based on Equation 15.

Table 5. Normalized decision matrix of stocks.

Stocks	Decision-criteria			
	C_1	C_1	C_3	C_4
st_1	(0.67,0.78,0.89,1.00)	(0.56,0.67,0.78,0.89)	(0.78,0.89,1.00,1.00)	(0.67,0.78,0.89,1.00)
st_2	(0.67,0.78,0.89,1.00)	(0.11,0.22,0.33,0.44)	(0.44,0.56,0.67,0.78)	(0.67,0.78,0.89,1.00)
st_3	(0.11,0.22,0.33,0.44)	(0.33,0.44,0.56,0.67)	(0.56,0.67,0.78,0.89)	(0.11,0.22,0.33,0.44)
st_4	(0.78,0.89,1.00,1.00)	(0.11,0.11,0.22,0.33)	(0.44,0.56,0.67,0.78)	(0.89,1.00,1.00,1.00)
st_5	(0.44,0.56,0.67,0.78)	(0.78,0.89,1.00,1.00)	(0.78,0.89,1.00,1.00)	(0.22,0.33,0.44,0.56)
st_6	(0.11,0.11,0.22,0.33)	(0.22,0.33,0.44,0.56)	(0.78,0.89,1.00,1.00)	(0.44,0.56,0.67,0.78)
st_7	(0.89,1.00,1.00,1.00)	(0.11,0.22,0.33,0.44)	(0.22,0.33,0.44,0.56)	(0.67,0.78,0.89,1.00)
st_8	(0.56,0.67,0.78,0.89)	(0.33,0.44,0.56,0.67)	(0.67,0.78,0.89,1.00)	(0.22,0.33,0.44,0.56)
st_9	(0.56,0.67,0.78,0.89)	(0.11,0.22,0.33,0.44)	(0.22,0.33,0.44,0.56)	(0.78,0.89,1.00,1.00)
st_{10}	(0.11,0.22,0.33,0.44)	(0.22,0.33,0.44,0.56)	(0.44,0.56,0.67,0.78)	(0.78,0.89,1.00,1.00)
st_{11}	(0.11,0.22,0.33,0.44)	(0.22,0.33,0.44,0.56)	(0.11,0.11,0.22,0.33)	(0.44,0.56,0.67,0.78)
st_{12}	(0.11,0.22,0.33,0.44)	(0.11,0.22,0.33,0.44)	(0.56,0.67,0.78,0.89)	(0.56,0.67,0.78,0.89)
st_{13}	(0.33,0.44,0.56,0.67)	(0.11,0.22,0.33,0.44)	(0.22,0.33,0.44,0.56)	(0.56,0.67,0.78,0.89)
st_{14}	(0.11,0.22,0.33,0.44)	(0.22,0.33,0.44,0.56)	(0.67,0.78,0.89,1.00)	(0.22,0.33,0.44,0.56)
st_{15}	(0.11,0.11,0.22,0.33)	(0.33,0.44,0.56,0.67)	(0.89,1.00,1.00,1.00)	(0.11,0.11,0.22,0.33)

Table 6. The weighted fuzzy sequences and their corresponding crisp values of the CoCoSo-H model

Stocks PH_i	Fuzzy SH_i	Fuzzy PH_i	Crisp SH_i	Crisp
st_1	(0.58,0.69,0.80,0.91)	(0.66,0.84,0.92,0.99)	0.75	0.87
st_2	(0.29,0.39,0.50,0.61)	(0.37,0.53,0.69,0.85)	0.45	0.61
st_3	(0.30,0.41,0.52,0.63)	(0.22,0.35,0.49,0.64)	0.46	0.42
st_4	(0.34,0.38,0.47,0.54)	(0.44,0.51,0.66,0.78)	0.43	0.59
st_5	(0.67,0.78,0.89,0.92)	(0.54,0.69,0.85,0.94)	0.83	0.76
st_6	(0.29,0.39,0.50,0.60)	(0.31,0.41,0.57,0.71)	0.44	0.50
st_7	(0.30,0.40,0.50,0.60)	(0.35,0.50,0.64,0.78)	0.45	0.57
st_8	(0.37,0.48,0.59,0.69)	(0.38,0.52,0.67,0.83)	0.52	0.60
st_9	(0.29,0.39,0.50,0.58)	(0.32,0.48,0.63,0.75)	0.44	0.52
st_{10}	(0.32,0.43,0.54,0.63)	(0.33,0.48,0.63,0.76)	0.48	0.55
st_{11}	(0.24,0.35,0.46,0.57)	(0.18,0.27,0.41,0.55)	0.40	0.35
st_{12}	(0.22,0.33,0.44,0.55)	(0.23,0.38,0.54,0.69)	0.38	0.46
st_{13}	(0.22,0.33,0.43,0.54)	(0.23,0.37,0.52,0.67)	0.38	0.45
st_{14}	(0.25,0.36,0.47,0.57)	(0.23,0.37,0.51,0.66)	0.41	0.44
st_{15}	(0.32,0.40,0.51,0.61)	(0.27,0.32,0.46,0.58)	0.46	0.40

Table 7. Relative significance and the final ranking of stocks.

Stocks	K_{iHa}	K_{iHb}	K_{iHc}	K_{iH}	Rank
st_1	0.105	4.45	0.952	2.601	1
st_2	0.069	2.92	0.623	1.707	4
st_3	0.057	2.42	0.521	1.420	10
st_4	0.066	2.83	0.603	1.65	6
st_5	0.103	4.36	0.939	2.341	2
st_6	0.061	2.58	0.553	1.513	9
st_7	0.066	2.82	0.602	1.53	7
st_8	0.073	3.10	0.663	1.813	3
st_9	0.064	2.74	0.585	1.603	8
st_{10}	0.067	2.86	0.613	1.675	5
st_{11}	0.049	2.06	0.443	1.207	15
st_{12}	0.055	2.33	0.497	1.361	13
st_{13}	0.054	2.28	0.448	1.334	14
st_{14}	0.055	2.34	0.502	1.371	12
st_{15}	0.056	2.34	0.505	1.375	11

On the basis of the obtained values, it is concluded that the final ranking of the stocks using the F-CoCoSo-H-BCM model is $st_1 > st_5 > st_8 > st_2 > st_{10} > st_4 > st_7 > st_9 > st_6 > st_3 > st_{15} > st_{14} > st_{12} > st_{13} > st_{11}$

4.3. Sensitivity analysis

The sensitivity analysis helps to examine the effects of changing criterion weightings on the ranking performance of alternatives. In this case study, the criterion c2 is determined as the most influential criterion because it has the highest weight value. When evaluating the effects of modification of criterion weighting on the preference rating of stocks, 10 scenarios have been developed. Table 8 represents the changes in weighting of criteria for different scenarios. While changing the weights, it has been kept in mind that the sum of the weights should be 1. Almost similar ranking result has been obtained for minor changes (<=5%) in criteria weighting. Whereas more changes (>5%) in the value of criteria weights change the ranking of alternatives. Figure 2 shows the ranking obtained by the original weighting and the variation in ranking upon change in the criterion weighting. The outcomes shows that the proposed model is sensitive to changes in the weighting of the criterion. It is also important to emphasize that st1, which represent the best solution, do not change the ranking in either scenario, meaning this is insensitive to changes in the significance of the criterion.

In the same order, the Spearman's Correlation Coefficient (SCC) between the prime ranking and variation in ranking based on change in weights has been calculated. Table 8 shows that the SCC [0.884,0.996], which is quite high.

Table 8. Changes in weightings of criteria for different scenarios

Changes in weights of criteria		Different Scenarios	SCC between prime ranking and different scenarios
Increase in w1, w3, w4	Decrease in w2		
1 %	1 %	Scenario 1	0.996
2 %	2 %	Scenario 2	0.985
3 %	3 %	Scenario 3	0.985
4 %	4 %	Scenario 4	0.985
5 %	5 %	Scenario 5	0.985
10 %	7 %	Scenario 6	0.975
15 %	10 %	Scenario 7	0.967
20 %	12 %	Scenario 8	0.964
30 %	15 %	Scenario 9	0.939
50 %	30 %	Scenario 10	0.884

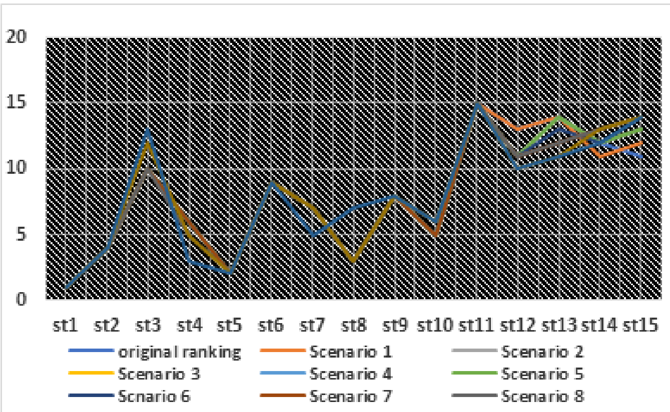

Figure 2. Effects of changing criterion weightings on the ranking performance of stocks

4.4. Portfolio analysis

In this section, a percentage of the total investment is obtained for each stock. The two main objectives of investors are- maximize the return and minimize the risk. An optimization method produces a more appropriate portfolio with respect to the risk and return of the portfolio.

Sortino ratio of the portfolio is considered as the objective function to optimize the total return under controllable risk. The sortino ratio is calculated by the following formula:

$$S.R.of\ portfolio = \frac{Portfolio\ return - risk\ free\ retun}{Portfolio\ downside\ deviation} = \frac{\sum_i x_i r_i - r_f}{\sum_i x_i d_i} \quad (17)$$

where, r_i = return of the i^{th} ranked asset of the portfolio
x_i = weight of the the i^{th} ranked asset of the portfolio
r_f = risk free rate
d_i = downside deviation of the i^{th} ranked asset of the portfolio

Sortino ratio penalize only negative volatility or downside deviation from the mean return as the upside deviation are beneficial for the investor. High value of sortino ratio is considered as a good investment. Hence, the optimization function for assigning the weights to the stocks is formulated as follows:

$$\max \frac{\sum_i x_i r_i - r_f}{\sum_i x_i d_i} \quad (18)$$

such that $\sum_i x_i = 1, \forall x_i > 0, \forall x_i < m$

4.3.1. Optimization using PSO

Particle swarm optimization technique is used to solve the aforementioned optimization problem. PSO is a meta-heuristic optimization technique inspired by the behavior flock of birds, proposed by Kennedy and Eberhart (1995). Suppose, a swarm consist of *m* particle in *n* dimensional search space and an optimization problem considering *N* candidate solutions such as $\{X_1, X_2 \dots X_N\}$. At *t* iteration, the position and the velocity of the i^{th} particle is denoted as $x_i(t) = (x_{i1}, x_{i2} \dots x_{in})$ and $v_i(t) = (v_{i1}, v_{i2} \dots v_{in})$, respectively.

The best position P_{best} visited by the i^{th} particle is denoted as $p_i(t) = (p_{i1}, p_{i2} \dots p_{in})$ and the particle that attained the best position in the previous iteration is denoted by $g_{best}(t) = (g_1, g_2 \dots g_n)$. At next iteration the new position $x_i(t+1)$ and velocity $v_i(t+1)$ of the i^{th} particle is calculated by the following equation:

$$v_{ij}(t+1) = \omega * v_{ij}(t) + c_1 * r_1 * \left(p_{ij}(t) - x_{ij}(t)\right) + c_2 * r_2 * \left(g_j(t) - x_{ij}(t)\right) \quad (19)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (20)$$

where ω is the inertia weight and c_1, c_2 are cognitive and social learning parameters, r_1 and r_2 are random numbers such that $r_1, r_2 \in (0,1)$. The P_{best} and g_{best} values are evaluated until the given number of iterations.

We used three years data 2018-2020 to construct three portfolios P_1, P_2 and P_3 consisting top 5, 7 and 10 ranked stocks respectively. The return and the downside deviation of each stock are calculated to implement the optimization problem using PSO. Here we consider 5 particles, $\omega = 0.5$, $c_1 = 1$, $c_2 = 2$ and optimize up to 50 iterations. Table 9 shows the weights of the stocks obtained by PSO.

Table 9. Weights of the stocks obtained by PSO

Stocks	st_1	st_5	st_8	st_2	st_{10}	st_4	st_7	st_9	st_6	st_3
For P_1	0.26	0.22	0.20	0.17	0.15					
For P_2	0.22	0.18	0.15	0.13	0.11	0.10	0.09			
For P_3	0.20	0.17	0.15	0.14	0.10	0.08	0.06	0.04	0.03	0.01

Sortino ratio greater than one is considered as a good investment. From the Table 10, we observe for two portfolios P_2 and P_3 , sortino ratio is greater than one. The investor prefers the portfolio with high sortino ratio as it depicts more return per unit for the downside risk. Hence, the portfolio P_2 attain maximum sortino ratio among all portfolios. It also gained maximum return of 16.72%. The downside risk is

high for P_1 with sortino ratio less than one. From the analysis, we can conclude that although all three portfolios gave similar return but P_2 and P_3 are profitable and save portfolios with less downside risk. We have also compared the results obtained

by our proposed model with earlier study in Table 11. This verifies the robustness and rootedness of the proposed ranking system in multi-dimensional decision analysis systems.

Table 10. Performance of different portfolio optimized by PSO

Portfolio	Portfolio return	Portfolio Downside deviation	Sortino ratio
P_1	0.1654	0.20	0.82
P_2	0.1672	0.14	1.194
P_3	0.1636	0.15	1.091

Table 11. Comparison of proposed model with earlier study

	Model Thakur et al. (2018)	Proposed model
Year	2016	2021
Ex. Return	0.1301	0.1672

5. Conclusions

In this paper, a new integrated F-CoCoSo-H-BCM strategy is proposed to solve the decision-making problem through some specific modifications to the main structure. The purpose of this study is to demonstrate that the HM aggregation operator can be used for fusion of criterion functions in the decision matrix of the CoCoSo model as it has ability to detect correlations in specified decision criteria. The improved CoCoSo-H model removes the impact of awkward data in stock selection. Furthermore, features of IGWHM and IGGWHM make decisions much more flexible. F-BCM forges the CoCoSo-H model more powerful by defining the optimal values of relative weights. Since it is an integrated model therefore there are some limitations for finding the criteria's weight and ranking of alternatives.

A hybrid approach (F-CoCoSo-H-BCM) based on a fusion of three strategies (the CoCoSo model, the heronian mean and the BCM) is proposed, which has major novelties and contributions as follows:

- This study is a presentation of the novel fuzzy CoCoSo-H-BCM model that serves the purpose of evaluating stocks in fuzzy environment.
- However, despite the uncertainty in the decision-making process and the lack of quantitative information, the presented methodology makes it possible to evaluate a set of alternatives.
- The F-CoCoSo-H-BCM approach enables the flexible decision-making with less computation. It also represents a new reference for researchers in the field of stock portfolio selection.
- The outcomes (expected return of 0.1672 or 16.72%) of the portfolios validate the rationality, stability, effectiveness and rootedness of the presented methodology.

The effectiveness and flexibility of the proposed F-CoCoSo-H-BCM are properties that make it recommended for use in management, supplier selection and engineering applications. In future, the pythagorean, type-2, neutrosophic, intuitionistic and hesitant concepts which are the generalizations of fuzzy set can be used in decision making process under uncertainty with the CoCoSo-H model.

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