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OBJECTIVE METHODS FOR DETERMINING CRITERIA WEIGHT COEFFICIENTS: A MODIFICATION OF THE CRITIC METHOD

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Abstract: *Determining criteria weight coefficients is a crucial step in multi-criteria decision making models. Therefore, this problem is given great attention in literature. This paper presents a new approach in modifying the CRiteria Importance Through Intercreteria Correlation (CRITIC) method, which falls under objective methods for determining criteria weight coefficients. Modifying the CRITIC method (CRITIC-M) entails changing the element normalization process of the initial decision matrix and changing data aggregation from the normalized decision matrix. By introducing a new normalization process, we achieve smaller deviations between normalized elements, which in turn causes lower values of standard deviation. Thus, the relationships between data in the initial decision matrix are presented in a more objective way. By introducing a new process of aggregation of weight coefficient values in the CRITIC-M method, a more comprehensive understanding of data in the initial decision matrix is made possible, leading to more objective values of weight coefficients. The presented CRITIC-M method has been tested in two examples, followed by a discussion of results via comparison to the classic CRITIC method.*

Key words: *CRITIC, criteria weights, multi-criteria decision making.*

1. Introduction

Determining criteria weight is one of the key problems of multi-criteria analysis models. Methodologies for determining criteria weight have been the topic of intensive research and scientific discussions for many years. Generally, most

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approaches to determining weight criteria can be divided into subjective and objective. Subjective approaches are based on determining criteria weight using information from decision makers or experts included in the decision process. Subjective approaches reflect the subjective opinion and intuition of decision makers which means that decision makers influence the decision making process. Contrary to subjective approaches, objective approaches are based on determining criteria weight using data that is present in the initial decision matrix. Objective approaches disregard the opinion of decision makers.

With the subjective approach, the decision maker or expert gives their opinion on the significance of criteria for a given process in accordance with their preferences. There are multiple ways of determining criteria weights using a subjective approach and they differ in the number of participants in the process of determining weights, applied methods and the way of forming final criteria weights. Subjective models used for aggregating partial values include: Trade-off method (Keeney & Raiffa, 1976); Swing method (Von Winterfeldt & Edwards, 1986); SMART method (the Simple Multi-Attribute Rating Technique) (Edwards & Barron, 1994); the new version of SMART method: SMARTER (SMART Exploiting Ranks) developed by Edwards and Barron (1994). SMARTER uses the centroid method for determining criteria weight coefficients.

Apart from the listed subjective approaches for determining criteria weights, there are also approaches based exclusively on pairwise comparisons. These approaches are called pairwise comparison methods. The pairwise comparison method was developed by Thurstone (1927) and it requires that comparisons be made by one or a team of experts. The pairwise comparison method is used for presenting relative significance of *m* alternatives in situations where it is not possible or meaningful to grade alternatives based on criteria. In pairwise methods, one or a team of experts compare an alternative to other alternatives from a set, in relation to a considered criterion. One of the best known methods for determining criteria weights using pairwise comparison is the Analytical Hierarchy Processes (AHP) method (Saaty, 1980). The AHP method is based on mutual comparison of criteria significance using Saaty's nine level scale. Apart from AHP, other pairwise comparison methods include: Decision-making Trial and Evaluation Laboratory (DEMATEL) method (Gabus & Fontela, 1972), Step-Wise Weight Assessment Ratio Analysis (SWARA) method (Keršuliene et al., 2010); Best Worst Method (BWM) (Rezaei, 2015); Full Consistency Method (FUCOM) (Pamučar et al., 2018); Level Based Weight Assessment (LBWA) (Žižović and Pamučar, 2019); Non-Decreasing Series at Criteria Significance Levels (NDSL) (Žižović et al., 2020) Resistance to Change Method (Roberts & Goodwin, 2002) which contains elements of the Swing and pairwise comparison methods.

Contrary to subjective methods, objective approaches eliminate, in a way, the decision maker, i.e. criteria weights are determined based on criteria values of alternatives. The emphasis is on the analysis of the decision matrix, i.e. values of alternatives are considered in relation to a set of criteria, followed by reaching data about values of criteria weights. The decision matrix allows cross referencing alternatives and criteria based on qualitative and quantitative values of each alternative in relation to each criteria. The best known models include: Entropy method (Shannon & Weaver, 1947), CRITIC method (CRiteria Importance Through Intercriteria Correlation), (Diakoulaki et al., 1995), FANMA method, named for its authors (Fan, 1996; Ma et al.,1999) and Data Envelopment Analysis (DEA) (Charnes et al., 1978).

The entropy method entails determining objective criteria weights based on Shannon's concept of entropic grading of data in the decision matrix (Shannon & Weaver, 1947). The method focuses on measuring lack of definition of data in the decision matrix. The Entropy method generates the set of weight coefficients based on mutual contrast of individual criteria value alternatives for each criterion and then for all criteria. Determining criteria weights using the FANMA method is based on using the principle of distance from the ideal point and the so-called early weight normalization (Srdjevic et al., 2003). Objective determination of criteria weights using the DEA method (Charnes et al, 1978) is based on solving linear optimisation models for alternatives and measuring efficiency of each alternative in relation to defined criteria. Criteria are categorized as input and output criteria. Then, a number of linear models equal to the number of options is solved. DEA objectively ranks options which is the end goal of a multi-criteria analysis, and features groups of criteria weight values for all options as a step to reaching the end goal.

The CRITIC method is part of the best known and most widely used objective methods. The CRITIC method is a correlation method, which uses standard deviation of ranked criteria values of options per column, as well as correlation coefficients of all paired columns to determine criteria contrasts. This paper identifies certain limitations when applying the classic CRITIC method and suggests a modification of the CRITIC method (CRITIC-M) that entails: 1) changing the normalization process of the initial matrix elements and 2) changing the function for aggregating data that represents values of weight coefficients. The presented modifications to the CRITIC method are aimed at reaching more objective values of weight coefficients.

The remainder of the paper is organized as follows. In the next section (Section 2) is presented the mathematical basis of the classic CRITIC method while sections 3 shows the motivation for developing the CRITIC-M method and the steps of the developed methodology. In the fourth section of the paper, we present the application of the CRITIC-M method on two examples and compare the results with the classic CRITIC method. Final observations and the direction of future research are presented in section 5.

2. The CRITIC method

The CRITIC method (CRiteria Importance Through Intercriteria Correlation), (Diakoulaki et al., 1995) is a correlation method. Standard deviations of ranked criteria values of options in columns, as well as correlation coefficients of all paired columns are used to determine criteria contrasts.

Step 1: Starting from an initial decision matrix, $X=\left[\xi_{ij}\right]_{m\times n}$, we normalize the element of the initial decision matrix and form the normalized matrix $X=\left[\xi_{ij}\right]_{m\times n}$.

$$X=\begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix}\begin{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{bmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1n} \\ \xi_{21} & \xi_{22} & \cdots & \xi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \xi_{m1} & \xi_{m2} & \cdots & \xi_{mn} \end{bmatrix}_{m\times n} \end{matrix} \tag{1}$$

The normalization of matrix elements $X=\left[\xi_{ij}\right]_{m\times n}$ is done by applying (2) and (3):

a) for maximizing criteria:

$$\xi_{ij} = \frac{\xi_{ij} - \xi_j^{\min}}{\xi_j^{\max} - \xi_j^{\min}}, \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,m; \tag{2}$$

b) for minimizing criteria:

$$\xi_{ij} = \frac{\xi_j^{\max} - \xi_{ij}}{\xi_j^{\max} - \xi_j^{\min}}, \quad i = 1,2,\ldots,n; \quad j = 1,2,\ldots,m; \tag{3}$$

where $\xi_j^{\max} = \max_j \left\{ \xi_{1j}, \xi_{2j}, \ldots, \xi_{mj} \right\}; \quad \xi_j^{\min} = \min_j \left\{ \xi_{1j}, \xi_{2j}, \ldots, \xi_{mj} \right\}$.

Upon normalizing criteria of the initial decision matrix, all elements ξ_{ij} are reduced to interval values [0, 1], so it can be said that all criteria have the same metrics.

Step 2: For criterion C_j $\left(j = 1,2,\ldots,n \right)$ we define the standard deviation σ_j, that represents the measure of deviation of values of alternatives for the given criterion of average value. Standard deviation of a given criterion is the measure considered in the further process of defining criteria weight coefficients.

Step 3: From the normalized matrix $X = \left[\xi_{ij} \right]_{m \times n}$ we separate the vector $\xi_j = \left(\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj} \right)$ that contains the values of alternatives A_i $\left(i = 1,2,..,m \right)$ for the given criterion C_j $\left(j = 1,2,\ldots,n \right)$. After forming the vector $\xi_j = \left(\xi_{1j}, \xi_{2j}, \ldots, \xi_{mj} \right)$, we construct the matrix $L = \left[l_{jk} \right]_{n \times n}$, that contains coefficients of linear correlation of vectors ξ_j and ξ_k. The bigger the discrepancy between criteria values of options for criteria j and k, the lower the value of coefficient l_{jk}. In that sense, the expression (4) represents the measure of conflict of criterion j in relation to other criteria in the given decision matrix.

$$\varphi_j = \sum_{k=1}^{n} (1 - l_{jk}) \tag{4}$$

The quantity of data W_j contained within criterion j is determined by combining previously listed measures σ_j and l_{jk} as follows:

$$W_j = \sigma_j \cdot \varphi_j = \sigma_j \sum_{k=1}^{n} (1 - l_{kj}) \tag{5}$$

Based on the previous analysis we can conclude da a higher value W_j means a larger quantity of data received from a given criterion, which in turn increases the relative significance of the given criterion for the given decision process.

Step 4: Objective weights of criteria are reached by normalizing measures W_j:

$$w_j = \frac{W_j}{\sum_{k=1}^{m} W_k} \tag{6}$$

Diakoulaki et al. (1995) and Deng et al. (2000) recommend determining criteria weights based on values of standard vector deviation, expression (7):

$$w_j = \frac{\sigma_j}{\sum_{k=1}^{m} \sigma_k} \tag{7}$$

where σ_j stands for standard deviation defined in Step 2..

3. Modification of CRITIC method: CRITIC-M method

The modification of the CRITIC method presented in this section of the paper is based on two assumptions: 1) Modification of normalizing data in the initial decision matrix and 2) Modification of expressions for determining final values of criteria weights.

1) Motivation for modifying the normalization of data in the initial decision matrix. In the original CRITIC method we apply linear normalization that entails that each column of a normalized matrix contains at least one element with values 0 and 1. An exception would only be a column in which all values are the same (which rarely happens), in which case this criterion has no influence on the final decision. Distribution of normalized values in the interval [0, 1] increases root-mean-square deviations, which in turn significantly influences values of criteria weight coefficients. If the standard deviation is close to zero for a certain criterion, then all elements regarding that criterion are centred around the average value of the element as per this criterion. In this situation, all values regarding this criterion are approximately equal so this criterion does not influence choice.

In the modified CRITIC method, normalization of the elements in the initial decision matrix entails dividing all the elements of the initial decision matrix with the maximum value in that column, expression (8).

$$\xi_{ij} = \frac{\xi_{ij}}{\xi_j^{\max}}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m; \tag{8}$$

where $\xi_j^{\max} = \max_j \left\{ \xi_{1j}, \xi_{2j}, \ldots, \xi_{mj} \right\}$.

By applying expression (8) we normalize maximized criteria in the initial decision matrix. Normalization of the minimized criteria is done in two steps. In the first step, values are normalized as with maximized criteria, i.e. by applying expression (8). In this way, we arrive at values $\overset{*}{\xi}_{ij}$. In the second phase, we normalize values by applying expression (9).

$$\xi_{ij} = -\overset{*}{\xi}_{ij} + \xi_j^{*\max} + \xi_j^{*\min}; \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m; \tag{9}$$

where $\xi_j^{*\max} = \max_j \left\{ \overset{*}{\xi}_{1j}, \xi_{2j}^{*}, \ldots, \xi_{mj}^{*} \right\}$; $\xi_j^{*\min} = \min_j \left\{ \overset{*}{\xi}_{1j}, \xi_{2j}^{*}, \ldots, \xi_{mj}^{*} \right\}$.

This normalization process decreases the root-mean-square deviation and resulting values of criteria weight coefficients better reflect the relationship between data in the initial decision matrix.

2) Motivation for modifying the expression for determining final criteria weight values. If the standard deviation is close to zero for a certain criterion, then all the elements regarding that criterion are centred around the average value of elements for this criterion. Therefore, all the values for this criterion are approximately equal and this criterion does not influence choice. Keeping this in mind, we adjust the expression for determining objective criteria weight values, expression (10)

$$w_j = \frac{\frac{\bar{\xi}_j}{1-\bar{\xi}_j}\cdot\sigma_j}{\sum_{j=1}^{n}\left(\frac{\bar{\xi}_j}{1-\bar{\xi}_j}\cdot\sigma_j\right)} \tag{10}$$

where $\bar{\xi}_j$ stands for the arithmetic average of elements of the normalized decision matrix as per criterion *j*, i.e. $\bar{\xi}_j = \frac{1}{m}\sum_{i=1}^{n}\xi_{ij}$. Expression (10) represents an extension of expression (7) by introducing average values which favores criteria with average values closer to the ideal value, i.e. closer to one. This means that regarding this criterion, many alternatives have maximum values. In this way, we introduce a certain amount of subjectivity in the objective methodology of the CRITIC method.

The following section presents the steps of the modified CRITIC (CRITIC-M) method:

Step 1: Starting with the initial decision matrix, $X=\left[\xi_{ij}\right]_{m\times n}$, we normalize the elements of the initial decision matrix and form a normalized matrix $X=\left[\xi_{ij}\right]_{m\times n}$.

Normalization of the elements of the matrix $X=\left[\xi_{ij}\right]_{m\times n}$ is done by applying expressions (8) and (9). Maximized criteria (higher values is better) are normalized by applying expression (8), while minimized criteria (lower values are better) are normalized by applying expression (9).

Step 2: Calculation of standard deviation of elements of the normalized matrix $X=\left[\xi_{ij}\right]_{m\times n}$. As with the classic CRITIC method, for each criterion C_j $\left(j=1,2,\ldots,n\right)$ we define standard deviation σ_j.

Step 3: Constructing the matrix of linear correlations $L=\left[l_{jk}\right]_{n\times n}$. For each criterion C_j from the normalized matrix $X=\left[\xi_{ij}\right]_{m\times n}$ we define the vector $\xi_j=\left(\xi_{1j},\ \xi_{2j},\ldots,\ \xi_{mj}\right)$ and calculate linear vector correlations ξ_j and ξ_k. Summing linear correlations per criteria results in measure of criteria conflict:

$$\varphi_j = \sum_{k=1}^{n}(1-l_{jk}) \tag{11}$$

Quantity of data W_j in the criterion j is determined by applying expression (12):

$$W_j = \sigma_j\sum_{k=1}^{n}(1-l_{kj}) \tag{12}$$

Step 4: Determining weight coefficients of criteria. Objective weights of criteria are reached by applying expression (13)

$$w_j = \frac{\frac{\bar{\xi}_j}{1-\bar{\xi}_j}\cdot W_j}{\sum_{j=1}^{n}\left(\frac{\bar{\xi}_j}{1-\bar{\xi}_j}\cdot W_j\right)} \tag{13}$$

Weights of criteria can be determined based on values of standard vector deviation, expression (14):

$$w_j = \frac{\frac{\bar{\xi}_j}{1-\bar{\xi}_j}\cdot\sigma_j}{\sum_{j=1}^{n}\left(\frac{\bar{\xi}_j}{1-\bar{\xi}_j}\cdot\sigma_j\right)} \tag{14}$$

where σ_j stands for standard deviation.

4. Determining criteria weights using the CRITIC-M method

Example 1: The following section demonstrates the application of the CRITIC-M method on an example that considers the evaluation of five alternatives A_i $(i=1,2,...,5)$ in relation to four criteria C_j $(j=1,2,...,4)$. All criteria in the initial decision matrix are maximized (*max*). The initial decision matrix ($X=\left[\xi_{ij}\right]_{m\times n}, i=1,2,...,m,\ j=1,2,...,n$) is presented using expression (15).

$$X = \begin{matrix} & \begin{matrix} C1 & C2 & C3 & C4 \end{matrix} \\ \begin{matrix} A1 \\ A2 \\ A3 \\ A4 \\ A5 \end{matrix} & \begin{bmatrix} 8 & 4 & 10 & 2 \\ 7 & 6 & 4 & 6 \\ 5 & 5 & 6 & 7 \\ 6 & 6 & 7 & 8 \\ 5 & 7 & 6 & 6 \end{bmatrix} \end{matrix} \tag{15}$$

In the following section we present the application of the CRITIC-M method in steps defined in the previous section of the paper:

Step 1: Normalization of the initial decision matrix (15). Since all criteria are maximized, we used expression (8) for normalizing elements. The normalized matrix is presented using expression (16).

$$X = \begin{matrix} & \begin{matrix} C1 & C2 & C3 & C4 \end{matrix} \\ \begin{matrix} A1 \\ A2 \\ A3 \\ A4 \\ A5 \end{matrix} & \begin{bmatrix} 1.000 & 0.571 & 1.000 & 0.250 \\ 0.875 & 0.857 & 0.400 & 0.750 \\ 0.625 & 0.714 & 0.600 & 0.875 \\ 0.750 & 0.857 & 0.700 & 1.000 \\ 0.625 & 1.000 & 0.600 & 0.750 \end{bmatrix} \\ & \begin{matrix} \max & \max & \max & \max \end{matrix} \end{matrix} \tag{16}$$

Normalization of elements A1-C2 in matrix (16) was done in the following way:

$$\xi_{12} = \frac{\xi_{12}}{\xi_2^{\max}} = \frac{4}{7} = 0.571;\ \xi_2^{\max} = \max_{C_2}\{4,6,5,6,7\} = 7\ .$$

Normalization of the remaining elements of matrix (16) was done in a similar way.

Step 2: Calculation of standard deviation of elements of normalized matrix (16). We arrive at standard deviation for criteria $\sigma_j = (0.1630,\ 0.1629,\ 0.2191,\ 0.2850)$.

Step 3: Matrix of linear correlation $L=\left[l_{jk}\right]_{4\times 4}$ is presented using expression (17).

$$L = \begin{matrix} & \begin{matrix} C1 & C2 & C3 & C4 \end{matrix} \\ \begin{matrix} C1 \\ C2 \\ C3 \\ C4 \end{matrix} & \begin{bmatrix} 1.000 & -0.605 & -0.473 & 0.740 \\ -0.605 & 1.000 & 0.681 & -0.635 \\ -0.473 & 0.681 & 1.000 & -0.671 \\ 0.740 & -0.635 & -0.671 & 1.000 \end{bmatrix} \end{matrix} \tag{17}$$

By applying expression (11) and matrix (17), we arrive at the measure of criteria conflict:

$\varphi_j = (3.873,\ 3.651,\ 3.878,\ 3.776)$

Element φ_1 for criterion C_1 is reached in the following way:

$\varphi_1 = (1-1)+(1+0.605)+(1+0.473)+(1-0.740) = 3.873$.

Remaining values φ_j we reach in a similar way.

By applying expression (12) we define quantity of data W_j:

$W_j = (0.6312,\ 0.5947,\ 0.8497,\ 1.0763)$

Quantity of data W_j for criterion C_1 is reached in the following way:

$W_1 = \sigma_1 \cdot \varphi_1 = 0.1630 \cdot 3.873 = 0.6312$.

Remaining values W_j are calculated in a similar way.

Step 4: Determining objective values of criteria weights. By applying expression (13) we arrive at criteria weight coefficients $w_j = (0.2405,\ 0.2632,\ 0.1825,\ 0.3139)$.

The value of criteria weight coefficient w_1 is reached in the following way:

$$w_1 = \frac{\frac{0.775}{1-0.775}\cdot 0.6312}{\left(\frac{0.775}{1-0.775}\cdot 0.631\right)+\left(\frac{0.800}{1-0.800}\cdot 0.594\right)+\left(\frac{0.660}{1-0.660}\cdot 0.849\right)+\left(\frac{0.725}{1-0.725}\cdot 1.076\right)} = 0.2405$$

In a similar way we arrive at the remaining criteria weight values.

Criteria weights can also be calculated by applying expression (14), i,e, based on the standard deviation σ_j. By applying expression (14) we arrive at weight coefficients:

$w_j = (0.2349,\ 0.2726,\ 0.1780,\ 0.3145)$

By applying expression (14), we arrive at the value of the weight coefficient of criterion w_1 in the following way:

$$w_1 = \frac{\frac{0.775}{1-0.775}\cdot 0.1630}{\left(\frac{0.775}{1-0.775}\cdot 0.1630\right)+\left(\frac{0.800}{1-0.800}\cdot 0.1629\right)+\left(\frac{0.660}{1-0.660}\cdot 0.2191\right)+\left(\frac{0.725}{1-0.725}\cdot 0.2850\right)} = 0.2349$$

The values of weight coefficients of the remaining criteria we reach in a similar way.

Example 2: In the following section, we present the application of CRITIC-M method on an example that considers the evaluation of six alternatives $A_i\ (i=1,2,\ldots,6)$ in relation to three criteria $C_j\ (j=1,2,3)$. Criteria C1 and C3 are maximized (*max*), while criterion C2 is minimized (*min*). The initial decision matrix ($X = \left[\xi_{ij}\right]_{6\times 3}$, $i=1,2,\ldots,6$, $j=1,2,3$) is presented using the expression (18).

$$X = \begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \end{array}\begin{array}{c} \begin{array}{ccc} C1 & C2 & C3 \end{array} \\ \begin{bmatrix} 15 & 525 & 7 \\ 30 & 400 & 5 \\ 50 & 210 & 8 \\ 30 & 350 & 5 \\ 30 & 400 & 1 \\ 20 & 350 & 3 \end{bmatrix} \end{array} \tag{18}$$

Application of CRITIC-M method on *Example* 2 is presented in the following section:

Step 1: Normalization of elements of matrix (18) is done by applying expressions (8) and (9). The normalized matrix is presented using the expression (19).

$$X = \begin{array}{c} \\ A1 \\ A2 \\ A3 \\ A4 \\ A5 \\ A6 \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} C1 & C2 & C3 \end{array} \\ \begin{bmatrix} 0.300 & 0.400 & 0.875 \\ 0.600 & 0.638 & 0.625 \\ 1.000 & 1.000 & 1.000 \\ 0.600 & 0.733 & 0.625 \\ 0.600 & 0.638 & 0.125 \\ 0.400 & 0.733 & 0.375 \end{bmatrix} \\ \begin{array}{ccc} \max & \min & \max \end{array} \end{array} \tag{19}$$

Normalization of elements A1-C1 in matrix (19) is done by applying the expression (8):

$$\xi_{11} = \frac{\xi_{11}}{\xi_1^{\max}} = \frac{15}{50} = 0.300 \; ; \; \xi_1^{\max} = \max_{C_1}\{15, 30, 50330, 30, 20\} = 50 \; .$$

Normalization of elements A1-C2 in matrix (19) is done by applying expression (9):

$$\xi_{12} = -\overset{*}{\xi}_{12} + \xi_2^{*\max} + \xi_2^{*\min} = -1.00 + 1.00 + 0.400 = 0.400 \; ,$$

where

$$\xi_2^{*\max} = \max_{C_2}\{1.000,\ 0.762,\ 0.400,\ 0.667,\ 0.762,\ 0.667\} = 1.00 \; ;$$

$$\xi_2^{*\min} = \min_{C_2}\{1.000,\ 0.762,\ 0.400,\ 0.667,\ 0.762,\ 0.667\} = 0.400 \; .$$

Normalization of the remaining elements of matrix (19) was done in a similar way.

Step 2: From the normalized matrix (19) we get standard deviations for criteria $C_j \; (j = 1,2,3)$: $\sigma_j = (0.240,\ 0.195,\ 0.320)$.

Step 3: Matrix of linear correlations $L = \left[l_{jk} \right]_{4\times 4}$ is presented using expression (20).

$$L = \begin{array}{c} \\ C1 \\ C2 \\ C3 \end{array} \begin{array}{c} \begin{array}{ccc} C1 & C2 & C3 \end{array} \\ \begin{bmatrix} 1.000 & 0.866 & 0.320 \\ 0.866 & 1.000 & 0.189 \\ 0.320 & 0.189 & 1.000 \end{bmatrix} \end{array} \tag{20}$$

By applying expression (11) and matrix (20), we get the measure of criteria conflict $\varphi_j = (0.814,\ 0.945,\ 1.491)$, while by applying expression (12) we define the quantity of data $W_j = (0.196,\ 0.184,\ 0.478)$.

Step 4: Determining objective values of criteria weight coefficients.

By applying expressions (13) and (14) respectively, we reach criteria weight coefficients: $w_j = (0.2670,\ 0.3447,\ 0.3883)$ and $w_j = (0.1937,\ 0.2903,\ 0.5160)$.

Table 1 presents criteria weight coefficients reached using the classic CRITIC method and the CRITIC-M method in examples 1 and 2.

Table 1. Criteria weight coefficients by applying CRITIC and CRITIC-M

Criteria	CRITIC	CRITIC-M, expression (13)	CRITIC-M, expression (14)
		Example 1	
C1	0.2221	0.2405	0.2349
C2	0.3994	0.2632	0.2726
C3	0.1979	0.1825	0.1780
C4	0.1805	0.3139	0.3145
		Example 2	
C1	0.2468	0.1937	0.2670
C2	0.2708	0.2903	0.3447
C3	0.4824	0.5160	0.3883

Table 1 presents two groups of data reached using the CRITIC-M method. The first group of data was reached using expression (13), while the second group of data was reached using expression (14). Based on data from Table 1 we note a very small difference between application of CRITIC-M, expressions (13) and (14). We note that determining conflict between criteria through coefficients of linear correlation, expression (13), does not identify significant differences that influence final values of criteria weights. However, the calculation of linear correlation matrix elements and the introduction of that data to the calculation of criteria weights significantly complicates the calculation of criteria weight coefficients. Therefore, we recommend the application of standard deviation (expression (14)) for calculating criteria weights, because it presents quite well the relationships between criteria in the initial decision matrix.

By comparing weight coefficients reached by using CRITIC and CRITIC-M methods we note that there are significant differences between resulting values. Differences in the weights are due to 1) different way of data normalization (CRITIC - linear normalization and CRITIC-M - percentual normalization) and 2) Application of different aggregation functions used for final values of criteria weight coefficients. Applying linear normalization in the CRITIC method results in higher values of standard deviation because normalization distributes all values in the interval [0, 1]. On the other hand, by applying percentual normalization in the CRITIC-M method, all normalized values are distributed in the interval $\left[\frac{\xi_j^{\min}}{\xi_j^{\max}},1\right]$. This shifts the distribution of all values towards the ideal value, i.e. towards one. As a consequence, standard deviation values are lower. Both examples in this paper show that criteria weight coefficients centre around average values. Also, we can point out that the CRITIC-M method contributes to a better objectivity of results. This can be noted in the second example and criteria C1 and C2. By applying the classic CRITIC method, there are very small differences between weight coefficients of criteria C1 and C2. On the other hand, by applying the CRITIC-M method, the differences between these criteria are clearly marked.

Further, in the CRITIC-M method, the function for aggregating values of weight coefficients has been changed by introducing average values. The reason for introducing average values and presenting their influence on criteria weights is favoring criteria whose average values are closer to the ideal value. By introducing this type of subjectivity to the CRITIC-M method, we eliminate one of the bad characteristics of the classic CRITIC method: assigning low values of criteria weight coefficients to criteria that, for most alternatives, have values close to the ideal value.

5. Conclusion

Weight coefficients are a calibration tool for decision models and the quality of their definition directly influences the quality of the decision. The reason for studying this problem lies in the fact that each of the subjective and objective methods for determining criteria weights has its advantages and flaws. This paper considers certain limitations of the CRITIC method and puts forward a modification with its new CRITIC-M algorithm.

The modification of the CRITIC method presented in this paper is based on a new approach to normalization of values in the initial decision matrix and on a new approach to aggregation of data from the initial decision matrix. The new normalization process in the CRITIC method makes it possible to reach lower standard deviation values for normalized values, which contributes to more objective representation of relationships between data in the initial decision matrix. Apart from modifying the CRITIC method using a new normalization process, we also present a new approach for aggregating values of weight coefficients. Aggregation of weight coefficient values in the CRITIC-M entails average values of normalized elements. Introduction of average values aims to favor criteria per which alternatives have values close to ideal values. Although this approach introduces a certain degree of subjectivity to this CRITIC methodology, authors maintain that this approach enables a more comprehensive understanding of data in the initial decision matrix and a more objective set of weight coefficient values.

It is clear that values reached by using objective and subjective methods can lead to completely different results, i.e. to completely different weight coefficient values. Keeping this in mind, objective methods for determining criteria weights can be used to correct criteria weights determined using subjective methods or based on subjective preferences of decision makers. Therefore, the presented CRITIC-M methodology can be a useful tool for correcting criteria weights. Further, future research can be directed towards defining absolute, ideal and anti-ideal values in the initial decision matrix. This would eliminate rank reversal problems in the case of adding new alternatives to the initial decision matrix and reduce its indirect influence on significant changes to criteria weights. Also, future research should also be directed towards application of uncertainty theories in the CRITIC-m method, such as fuzzy theory. This is supported by the significant position of fuzzy theory in the field of multi-criteria decision making, and as far as the authors are aware, there has, so far, been no presentation of expanded CRITIC methods in fuzzy environments.

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